Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 8, No. 4, (October 2014), pp. 561–570
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

A view on intuitionistic equiuniform action via intuitionistic $\mathcal{B}$-open symmetric member

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Received 28 03 2013; Revised 28 02 2014; Accepted 4 04 2014

Abstract. In this paper, the concepts of intuitionistic $\mathcal{B}$-open symmetric member, intuitionistic uniformly $\mathcal{B}$-continuous functions and intuitionistic bi-uniformly $\mathcal{B}$-continuous functions are introduced. The concepts of quasi intuitionistic $\mathcal{B}$-open symmetric functions and intuitionistic equiuniform actions are introduced. Some interesting properties are discussed.

2010 AMS Classification: 54A40,54E55

Keywords: Intuitionistic uniform topology, Intuitionistic t-open symmetric member, Intuitionistic $\mathcal{B}$-open symmetric member, Intuitionistic uniformly $\mathcal{B}$-continuous functions, Intuitionistic bi-uniformly $\mathcal{B}$-continuous functions, Quasi intuitionistic $\mathcal{B}$-open symmetric member and intuitionistic equiuniform action.

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1. Introduction

The concept of intuitionistic sets in topological spaces was introduced by Çoker in [2]. He studied topology on intuitionistic sets in [3]. In 1937, Andre Weil [8] formulated the concept of uniform space which is a generalization of a metric space. J. Tong [7] introduced the concept of $\mathcal{B}$-set in topological space. In this paper, the concepts of intuitionistic $\mathcal{B}$-open symmetric member, intuitionistic uniformly $\mathcal{B}$-continuous functions and intuitionistic bi-uniformly $\mathcal{B}$-continuous functions are introduced. The concepts of quasi intuitionistic $\mathcal{B}$-open symmetric functions and intuitionistic equiuniform actions are introduced. Some interesting properties are discussed.

2. Preliminaries

Definition 2.1 ([2])**.** Let X be a non empty set. An *intuitionistic set* (IS for short) A is an object having the form $A = \langle x, A^1, A^2\rangle$, where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \varnothing$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A. Every crisp set A on a nonempty set X is obviously an intuitionistic set having the form $\langle x, A, A^c\rangle$.

Definition 2.2 ([3])**.** Let X be a non empty set and let the intuitionistic sets A and B be in the form $A = \langle x, A^1, A^2\rangle$, $B = \langle x, B^1, B^2\rangle$, respectively. Furthermore, let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = \langle x, {A_i}^1, {A_i}^2\rangle$. Then

(i) $A \subseteq B$ if and only if $A^1 \subseteq B^1$ and $A^2 \supseteq B^2$.
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(iii) $\overline{A} = \langle x, A^2, A^1\rangle$.
(iv) $\cup A_i = \langle x, \cup {A_i}^1, \cap {A_i}^2\rangle$.
(v) $\cap A_i = \langle x, \cap {A_i}^1, \cup {A_i}^2\rangle$.
(vi) $\varnothing_\sim = \langle x, \varnothing, X\rangle$; $X_\sim = \langle x, X, \varnothing\rangle$.

Definition 2.3 ([3])**.** An intuitionistic topology (IT for short) on a nonempty set X is a family T of intuitionistic sets in X satisfying the following axioms:

(i) $\varnothing_\sim, X_\sim \in T$.
(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$.
(iii) $\cup G_i \in T$ for any arbitrary family $\{G_i : i \in J\} \subseteq T$.

In this case the pair (X, T) is called an intuitionistic topological space (ITS for short) and any intuitionistic set in T is called an intuitionistic open set(IOS for short) in X. The complement $\overline{A}$ of an intuitionistic open set A is called an intuitionistic closed set (ICS for short) in X.

Definition 2.4 ([3])**.** Let (X, T) be an intuitionistic topological space and $A = \langle x, A^1, A^2\rangle$ be an intuitionistic set in X. Then the closure and interior of A are defined by

$$Icl(A) = \cap\{K : K \text{ is an intuitionistic closed set in } X \text{ and } A \subseteq K\}.$$
$$Iint(A) = \cup\{G : G \text{ is an intuitionistic open set in } X \text{ and } G \subseteq A\}.$$

Definition 2.5 ([4])**.** Let X and Y be two nonempty sets and $f : X \to Y$ a function, $B = \langle y, B^1, B^2\rangle$ is an intuitionistic set in Y and $A = \langle x, A^1, A^2\rangle$ is an intuitionistic set in X. Then the preimage of B under f, denoted by $f^{-1}(B)$, is the intuitionistic set in X defined by $f^{-1}(B) = \langle x, f^{-1}(B^1), f^{-1}(B^2)\rangle$, and the image of A under f, denoted by $f(A)$, is the intuitionistic set in Y defined by $f(A) = \langle y, f(A^1), \underline{f}(A^2)\rangle$ where $\underline{f}(A^2) = Y - (f(X - A^2))$.

Definition 2.6 ([6])**.** A *uniform space* X with uniformity ξ is a set X with a nonempty collection ξ of subsets containing the diagonal Δ_x in $X \times X$ satisfying the following properties:

(i) If $E, F \in \xi$, then $E \cap F \in \xi$.
(ii) If $F \subset E$ and $E \in \xi$ then $F \in \xi$.
(iii) If $E \in \xi$ then $E^t = \{(x, y) : (y, x) \in E\} \in \xi$.

(iv) For any $E \in \xi$ there is some $F \in \xi$ such that $F^2 \subset E$.

Theorem 2.7 ([3]). *For any intuitionistic set A in (X, T), the following properties hold:*

(i) $cl(\overline{A}) = \overline{int(A)}$,
(ii) $int(\overline{A}) = \overline{cl(A)}$.

Definition 2.8 ([4]). Let A and B be two intuitionistic sets on X and Y, respectively. Then the product intuitionistic set (PIS for short) of A and B on $X \times Y$ is defined by $U \times V = \langle (X, Y), A^1 \times B^1, ((A^2)^c \times (B^2)^c)^c \rangle$, where $A = \langle X, A^1, A^2 \rangle$ and $B = \langle Y, B^1, B^2 \rangle$.

Definition 2.9 ([4]). Given the nonempty set X, we define the *diagonal* Δ_x as the following intuitionistic set in $X \times X$:

$$\Delta_x = \langle (x_1, x_2), \{(x_1, x_2) : x_1 = x_2\}, \{(x_1, x_2) : x_1 \neq x_2\} \rangle.$$

Definition 2.10 ([7]). Let (X, T) be a topological space. A subset S in X is said to be a *t-set* if $intcl(s) = int(S)$.

Definition 2.11 ([7]). Let (X, T) be a topological space. A subset S in X is said to be a *$\mathcal{B}$-set* if there is a $U \in T$ and a t-set A in X such that $S = U \cap A$.

Definition 2.12 ([7]). Let (X, T) and (Y, S) be any two topological space. Let $f : X \to Y$ be a mapping. If for each open set V in Y, $f^{-1}(V)$ is a $\mathcal{B}$-set in (X, T), then f is said to be *$\mathcal{B}$-continuous*.

Definition 2.13 ([1]). A mapping f of a uniform space X into a uniform space X' is said to be *uniformly continuous* if, for each entourage V' of X', there is an entourage V of X such that the relation $(x, y) \in V$ implies $(f(x), f(y)) \in V'$.

Definition 2.14 ([5]). A binary relation $\geq$ in a set D is said to *direct* D if and only if D is nonempty and the following three conditions are satisfied:

(i) If $a \in D$, then $a \geq a$.
(ii) If a, b, c are members of D such that $a \geq b$ and $b \geq c$, then $a \geq c$.
(iii) If a and b are members of D, then there exists a member $c \in D$ such that $c \geq a$ and $c \geq b$.

By a *directed set*, a set D furnished with a binary relation $\geq$ which directs D. In particular, the set N of all natural numbers together with the usual relation $\geq$ is a directed set. Let D be a given directed set and consider an arbitrary subset E of D. If, for every $d \in D$, there exists an $e \in E$ such that $e \geq d$, then E is said to be a *cofinal* subset of D.

Definition 2.15 ([1]). A *topological group* is a set G which carries a group structure and a topology and satisfy the following two axioms:

(i) The mapping $(x, y) \to xy$ of $G \times G$ into G is continuous.
(ii) The mapping $x \to x^{-1}$ of G into G (the symmetry of the group G) is continuous.

A group structure and a topology on a set G are said to be compatible if they satisfy (i) and (ii).

Definition 2.16 ([1]). The *right uniformity* on a topological group G is the uniformity for which a fundamental system of entourages is obtained by making correspond to each neighbourhood V of the identity element e, the set V_d of pairs (x, y) such that $yx^{-1} \in V$.

3. Intuitionistiv $\mathcal{B}$-open symmetric functions

Definition 3.1. An intuitionistic uniform topology (IUS for short) on a non-empty set X is a collection ξ of subsets containing the intuitionistic diagonal Δ_x in $X \times X$ which satisfies the following axioms

(i) $\varnothing_\sim$, $X_\sim \in \xi$.
(ii) $E_1 \cap E_2 \in \xi$ for any $E_1, E_2 \in \xi$.
(iii) $\cup E_i \in \xi$ for any arbitrary family $\{E_i : i \in J\} \subseteq \xi$.
(iv) If $E_1 \subset E_2$ and $E_1 \in \xi$ then $E_2 \in \xi$.
(v) If $E_1 \in \xi$ then $E_1^t = \{(x, y) : (y, x) \in E_1\} \in \xi$.
(vi) For any $E_1 \in \xi$ there is some $E_2 \in \xi$ such that $E_2^2 \subset E_1$.

In this case the pair (X, ξ) is called an intuitionistic uniform topological space ($IUTS$ for short) and any intuitionistic symmetric member in ξ is called an intuitionistic open symmetric member ($IOSM$ for short) in X. The complement $\overline{A}$ of an intuitionistic open symmetric member A is called an intuitionistic closed symmetric member ($ICSM$ for short) in X.

Notation 3.1. Let $(X \times X, \xi)$ be an intuitionistic uniform topological space and it is simply denoted by $(\mathbb{X}, \xi)$.

Notation 3.2. Let $X \times X$ be a non empty set.

(i) $\varnothing_\sim = \langle x, \varnothing, \mathbb{X}\rangle$
(ii) $\mathbb{X}_\sim = \langle x, \mathbb{X}, \varnothing\rangle$

Definition 3.2. Let $(\mathbb{X}, \xi)$ be an intuitionistic uniform topological space and $A = \langle x, A^1, A^2\rangle$ be an intuitionistic symmetric member in $\mathbb{X}$.Then the intuitionistic uniform closure ($IUcl$ for short) of A are defined by
$IUcl(A) = \cap\{K : K$ is an intuitionistic closed symmetric member in $\mathbb{X}$ and $A \subseteq K\}$.

Definition 3.3. Let $(\mathbb{X}, \xi)$ be an intuitionistic uniform topological space and $A = \langle x, A^1, A^2\rangle$ be an intuitionistic symmetric member in $\mathbb{X}$. Then the intuitionistic uniform interior ($IUint$ for short) of A are defined by
$IUint(A) = \cup\{G : G$ is an intuitionistic open symmetric member in $\mathbb{X}$ and $G \subseteq A\}$.

Definition 3.4. Let $(\mathbb{X}, \xi)$ be an intuitionistic uniform topological space and $S = \langle x, S^1, S^2\rangle$ be an intuitionistic symmetric member in $\mathbb{X}$ is said to be an intuitionistic t-open symmetric member if $IUint(IUcl(S)) = IUint(S)$.The complement of an intuitionistic t-open symmetric member S is called an intuitionistic t-closed symmetric member in $\mathbb{X}$.

Definition 3.5. Let $(\mathbb{X}, \xi)$ be an intuitionistic uniform topological space and $S = \langle x, S^1, S^2\rangle$ be an intuitionistic symmetric member in $\mathbb{X}$ is said to an intuitionistic $\mathcal{B}$-open symmetric member if there is a $U \in \xi$ and an intuitionistic t-open symmetric member A in $\mathbb{X}$ such that $S = U \cap A$. The complement of an intuitionistic $\mathcal{B}$-open symmetric member S is called an intuitionistic $\mathcal{B}$-closed symmetric member in $\mathbb{X}$.

Definition 3.6. Let $(\mathbb{X}, \xi_1)$ and $(\mathbb{Y}, \xi_2)$ be any two intuitionistic uniform topological spaces. A function $f : (\mathbb{X}, \xi_1) \to (\mathbb{Y}, \xi_2)$ is said to be intuitionistic uniformly $\mathcal{B}$-continuous if $f^{-1}(V)$ is an intuitionistic $\mathcal{B}$-open symmetric member in $(\mathbb{X}, \xi_1)$ for every intuitionistic open symmetric member V in $(\mathbb{Y}, \xi_2)$.

Definition 3.7. Let $(\mathbb{X}, \xi_1)$ and $(\mathbb{Y}, \xi_2)$ be any two intuitionistic uniform topological spaces.A surjection $f : (\mathbb{X}, \xi_1) \to (\mathbb{Y}, \xi_2)$ between intuitionistic uniform topological spaces are called intuitionistic bi-uniformly $\mathcal{B}$-continuous if the image of every intuitionistic $\mathcal{B}$-open symmetric member is intuitionistic $\mathcal{B}$-open symmetric and the inverse image of any intuitionistic $\mathcal{B}$-open symmetric is an intuitionistic $\mathcal{B}$-open symmetric member.

That is, f is an intuitionistic uniformly $\mathcal{B}$-continuous and if E is any intuitionistic $\mathcal{B}$-open symmetric member of $\mathbb{X}$ then $f(E)$ is an intuitionistic $\mathcal{B}$-open symmetric member of $\mathbb{Y}$. If f is not surjective then it is called intuitionistic bi-uniformly $\mathcal{B}$-continuous if it is intuitionistic bi-uniformly $\mathcal{B}$-continuous onto its image with the subspace intuitionistic uniformity.

Proposition 3.8. *Let $(\mathbb{X}, \xi_1)$, $(\mathbb{Y}, \xi_2)$ and $(\mathbb{Z}, \xi_3)$ be any three intuitionistic uniform topological spaces. A function $f : (\mathbb{X}, \xi_1) \to (\mathbb{Y}, \xi_2)$ is an intuitionistic bi-uniformly $\mathcal{B}$-continuous function and $g : (\mathbb{Y}, \xi_2) \to (\mathbb{Z}, \xi_3)$ is an intuitionistic bi-uniformly $\mathcal{B}$-continuous function. Then $g\ o\ f : (\mathbb{X}, \xi_1) \to (\mathbb{Z}, \xi_3)$ is an intuitionistic bi-uniformly $\mathcal{B}$-continuous function.*

Proof. Let $E = \langle z, E^1, E^2 \rangle$ be an intuitionistic $\mathcal{B}$-open symmetric member in $(\mathbb{Z}, \xi_3)$. Since g is an intuitionistic bi-uniformly $\mathcal{B}$-continuous function, $g^{-1}(E)$ is an intuitionistic $\mathcal{B}$-open symmetric member in $(\mathbb{Y}, \xi_2)$. Since f is an intuitionistic bi-uniformly $\mathcal{B}$-continuous function then $f^{-1}(g^{-1}(E))$ is an intuitionistic $\mathcal{B}$-open symmetric member in $(\mathbb{X}, \xi_1)$. Hence $g\ o\ f$ is an intuitionistic bi-uniformly $\mathcal{B}$-continuous function. □

An intuitionistic bi-uniformly continuous bijection between intuitionistic uniform topological spaces will be called an intuitionistic uniform homeomorphism. The inverse limit of this system is the subset $\mathbb{X} = \underleftarrow{lim}\mathbb{X}_\alpha$ of the intuitionistic product uniform topological space $\Pi_{\alpha\in\Lambda}\mathbb{X}_\alpha$ consisting of all (x_α) such that when $\alpha \le \beta$, $x_\alpha = \phi_{\alpha\beta}(x_\beta)$. The restriction to $\mathbb{X}$ of the natural projection of $\Pi_{\alpha\in\Lambda}\mathbb{X}_\alpha$ onto $\mathbb{X}_\alpha$ will be denoted by ϕ_α, and will also be referred to as a projection; these functions are intuitionistic uniformly $\mathcal{B}$-continuous. The intuitionistic uniformity on the inverse limit is the subspace intuitionistic uniformity induced by the intuitionistic product uniformity on $\Pi_{\alpha\in\Lambda}\mathbb{X}_\alpha$; more importantly for our purposes, a intuitionistic base for this intuitionistic uniformity consists of all an intuitionistic symmetric members of the form $\phi_\alpha^{-1}(E)$ where $E = \langle E_1, E_2 \rangle$ is an intuitionistic $\mathcal{B}$-open symmetric member in $\mathbb{X}_\alpha$ and $\alpha \in \Lambda$. Given an inverse system $\{\mathbb{X}_\alpha, \phi_{\alpha\beta}\}_{\alpha\in\Lambda}$ of intuitionistic uniform topological spaces, the inverse limit with respect to any cofinal subset of Λ is naturally intuitionistic uniformly homeomorphic to the inverse limit with respect to Λ.

Definition 3.9. Let $\{\mathbb{X}_\alpha, \phi_{\alpha\beta}\}_{\alpha\in\Lambda}$ be an inverse system of intuitionistic uniform topological spaces indexed over a directed set Λ. The function $\phi_{\alpha\beta} : \mathbb{X}_\beta \to \mathbb{X}_\alpha$, $\alpha \le \beta$ (called the intuitionistic bonding maps) are intuitionistic uniformly $\mathcal{B}$-continuous and satisfy, for $\alpha \le \beta \le \gamma$, $\phi_{\alpha\beta} \circ \phi_{\beta\gamma} = \phi_{\alpha\gamma}$.

Proposition 3.10. *Let $\{\mathbb{X}_\alpha, \phi_{\alpha\beta}\}_{\alpha\in\Lambda}$ be an inverse system of intuitionistic uniform topological spaces having intuitionistic bi-uniformly $\mathcal{B}$-continuous bonding maps. If each of the projections ϕ_α is surjective then each ϕ_α is intuitionistic bi-uniformly $\mathcal{B}$-continuous.*

Proof. Let each $\phi_{\alpha\beta}$ is surjective. Let $F = \langle x, F_1, F_2\rangle = \langle x, \phi_\alpha^{-1}(E_1), \phi_\alpha^{-1}(E_2)\rangle = \phi_\alpha^{-1}(E)$, where E is an intuitionistic $\mathcal{B}$-open symmetric member in $\mathbb{X}_\alpha$, be an intuitionistic base element of the intuitionistic uniformity on $\mathbb{X} = \underleftarrow{lim}\mathbb{X}_\alpha$. If $\beta \geq \alpha$ then since ϕ_β is surjective.

$$\begin{aligned}\phi_\beta(F) &= \langle x, \phi_\beta(F_1), \phi_\beta(F_2)\rangle\\ &= \langle x, \phi_\beta(\phi_\beta^{-1}(\phi_{\alpha\beta}^{-1}(E_1))), \phi_\beta(\phi_\beta^{-1}(\phi_{\alpha\beta}^{-1}(E_2)))\rangle,\\ &= \langle x, \phi_{\alpha\beta}^{-1}(E_1), \phi_{\alpha\beta}^{-1}(E_2)\rangle,\\ &= \phi_{\alpha\beta}^{-1}(E)\end{aligned}$$

Thus $\phi_\beta(F) = \phi_{\alpha\beta}^{-1}(E)$, which is an intuitionistic $\mathcal{B}$-open symmetric member. If $\beta \leq \alpha$ then we have

$$\begin{aligned}\phi_\beta(F) &= \langle x, \phi_\beta(F_1), \phi_\beta(F_2)\rangle\\ &= \langle x, \phi_{\beta\alpha}(\phi_\alpha(F_1)), \phi_{\beta\alpha}(\phi_\alpha(F_2))\rangle\\ &= \langle x, \phi_{\beta\alpha}(\phi_\alpha(\phi_\alpha^{-1}(E_1))), \phi_{\beta\alpha}(\phi_\alpha(\phi_\alpha^{-1}(E_2)))\rangle,\\ &= \langle x, \phi_{\beta\alpha}(E_1), \phi_{\beta\alpha}(E_2)\rangle,\\ &= \phi_{\beta\alpha}(E).\end{aligned}$$

Thus $\phi_\beta(F) = \phi_{\beta\alpha}(E)$. □

4. Quasi intuitionistic $\mathcal{B}$-open symmetric functions

Definition 4.1. Let $(\mathbb{X}, \xi_1)$ and $(\mathbb{Y}, \xi_2)$ be any two intuitionistic uniform topological spaces. A function $f : (\mathbb{X}, \xi_1) \to (\mathbb{Y}, \xi_2)$ is said to be quasi intuitionistic $\mathcal{B}$-open symmetric if the image of every intuitionistic $\mathcal{B}$-open symmetric member in $(\mathbb{X}, \xi_1)$ is an intuitionistic open symmetric member in $(\mathbb{Y}, \xi_2)$.

Definition 4.2. Let $(\mathbb{X}, \xi)$ be an intuitionistic uniform topological space and $A = \langle x, A_1, A_2\rangle$ be an intuitionistic symmetric member in $\mathbb{X}$. Then the intuitionistic uniform $\mathcal{B}$-interior ($IU\mathcal{B}int$ for short) of A are defined by
$IU\mathcal{B}int(A) = \cup\{G : G$ is an intuitionistic $\mathcal{B}$-open symmetric member in $\mathbb{X}$ and $G \subseteq A\}$.

Definition 4.3. Let $(\mathbb{X}, \xi)$ be an intuitionistic uniform topological space and $A = \langle x, A_1, A_2\rangle$ be an intuitionistic symmetric member in $\mathbb{X}$. Then the intuitionistic uniform $\mathcal{B}$-closure ($IU\mathcal{B}cl$ for short) of A are defined by
$IU\mathcal{B}cl(A) = \cap\{K : K$ is an intuitionistic $\mathcal{B}$-closed symmetric member in $\mathbb{X}$ and $A \subseteq K\}$.

Proposition 4.4. *Let $(\mathbb{X}, \xi_1)$ and $(\mathbb{Y}, \xi_2)$ be any two intuitionistic uniform topological spaces. A function $f : (\mathbb{X}, \xi_1) \to (\mathbb{Y}, \xi_2)$ is said to be quasi intuitionistic $\mathcal{B}$-open symmetric iff for every intuitionistic set $A = \langle x, A^1, A^2\rangle$ of $(\mathbb{X}, \xi_1)$, $f(IU\mathcal{B}int(A)) \subset Iint(f(A))$.*

Proof. Let f be a quasi intuitionistic $\mathcal{B}$-open symmetric function. Now, we have $IU\mathcal{B}int(A) \subseteq A$ and $IU\mathcal{B}int(A)$ is an intuitionistic $\mathcal{B}$-open symmetric member. Hence we obtain that $f(IU\mathcal{B}int(A)) \subseteq f(A)$. Since f is quasi intuitionistic $\mathcal{B}$-open symmetric member then $f(IU\mathcal{B}int(A))$ is intuitionistic open symmetric member. $Iint(f(IU\mathcal{B}int(A))) \subseteq Iint(f(A))$. That is, $f(IU\mathcal{B}int(A))) \subseteq Iint(f(A))$.

Conversely, assume that A is an intuitionistic $\mathcal{B}$-open symmetric member in $(\mathbb{X}, \xi_1)$. Then $f(A) = f(IU\mathcal{B}int(A)) \subseteq Iint(f(A))$. This implies that $f(A) \subseteq Iint(f(A))$ but $Iint(f(A)) \subseteq f(A)$.

Consequently $f(A) = Iint(f(A))$ and hence f is quasi intuitionistic $\mathcal{B}$-open symmetric member. □

Proposition 4.5. *Let $(\mathbb{X}, \xi_1)$ and $(\mathbb{Y}, \xi_2)$ be any two intuitionistic uniform topological spaces. If a function $f\colon (\mathbb{X}, \xi_1) \to (\mathbb{Y}, \xi_2)$ is quasi intuitionistic $\mathcal{B}$-open symmetric, then $IU\mathcal{B}int(f^{-1}(G)) \subseteq f^{-1}(Iint(G))$ for every intuitionistic set $G = \langle y, G^1, G^2\rangle$ of $(\mathbb{Y}, \xi_2)$.*

Proof. Let f be a quasi intuitionistic $\mathcal{B}$-open symmetric function. Let G be any arbitrary intuitionistic set of $(\mathbb{Y}, \xi_2)$. Then, $IU\mathcal{B}int(f^{-1}(G))$ is an intuitionistic $\mathcal{B}$-open symmetric member in $(\mathbb{X}, \xi_1)$ and f is quasi intuitionistic $\mathcal{B}$-open symmetric function. Then by Proposition (4.4),

$$f(IU\mathcal{B}int(f^{-1}(G))) \subseteq Iint(f(IU\mathcal{B}int(f^{-1}(G)))) \subseteq int(G).$$

Thus $IU\mathcal{B}int(f^{-1}(G)) \subset f^{-1}(int(G))$. □

5. Intuitionistic equiuniform action

Let G be a group of bijection of an intuitionistic symmetric member $\mathbb{X}$. We will denote the evaluation map by $\alpha : G \times \mathbb{X} \to \mathbb{X}$, where $\alpha(g, x) = g(x)$. As usual the group G is said to act freely if only the identity map in G has a fixed point. The action is transitive if $Gx = \mathbb{X}$ for some, and hence all, $x \in \mathbb{X}$. The orbit space $\mathbb{X}/G$ is defined to be the set of all orbits $Gx = \{g(x) : g \in G\}$ and the quotient map is $\pi : \mathbb{X} \to \mathbb{X}/G$, where $\pi(x) = Gx$. For any $x \in \mathbb{X}$ let $\phi_x : G \to Gx$ be defined by $\phi_x(g) = g(x)$.

Definition 5.1. Let G be a group of bijection of an intuitionistic uniform topological space $(\mathbb{X}, \xi)$. We will call G an intuitionistic equiuniform or say G acts intuitionistic equiuniformly, if for each intuitionistic $\mathcal{B}$-open symmetric member $E = \langle x, E^1, E^2\rangle$ there exists an intuitionistic $\mathcal{B}$-open symmetric member $F = \langle x, F^1, F^2\rangle$ such that for all $g \in G, g(F) \subseteq E$.

Proposition 5.2. *Let G be a group acting intuitionistic equiuniformly on an intuitionistic uniform topological space $(\mathbb{X}, \xi)$.For any intuitionistic $\mathcal{B}$-open symmetric member $E = \langle x, E^1, E^2\rangle$ there exists an intuitionistic $\mathcal{B}$-open symmetric member $F = \langle x, F^1, F^2\rangle$ such that if $(Gx, Gy) \in \pi(F)$ then for some $g \in G$,$(g(x), y) \in E$.*

Proof. Let G be a group acting intuitionistic equiuniformly on an intuitionistic uniform structure space $(\mathbb{X}, \xi)$. Let F be an intuitionistic $\mathcal{B}$-open symmetric member such that for all $g \in G$, $g(F) \subset E$ and suppose that $(Gx, Gy) \in \pi(F)$. This means that for some $g_1, g_2 \in G$, We have $(g_1(x), g_2(y))$. Thus $(g_1(x), g_2(y)) \in F$. But then

$(g_2^{-1}g_1(x), y) \in E$. Therefore $(g(x), y) \in E$, where $g = g_2^{-1} \circ g_1 \in G$ and the proof is finished. □

Notation 5.1. Let IH_X denote the group of intuitionistic uniform homeomorphisms of an intuitionistic uniform topological space $(\mathbb{X}, \xi)$ with composition as the operation.

Notation 5.2. For any intuitionistic uniformity E on $(\mathbb{X}, \xi)$.We define

$H(E) = \{(g, h) \in IH_X \times IH_X : (g(x), h(x)) \in E$ for all $x \in \mathbb{X}\}$ and
$U(E) = \{g \in IH_X : (x, g(x)) \in E$ for all $x \in \mathbb{X}\}$.

Proposition 5.3. *If H is a subgroup of G acting by intuitionistic left translation then for any intuitionistic open symmetric member $V = \langle x, V^1, V^2\rangle$ containing the identity, $U(E(V)) = V \cap H$.*

Proof. Let V be an intuitionistic open symmetric member, and let H be a subgroup of G acting by intuitionistic left translation. Some $h \in H$ lies in $U(E(V))$ if and only if $(g, hg) \in E(V)$ for all $g \in G$ that is, iff $gg^{-1}h^{-1} \in V$ for all $g \in G$. But the latter is equivalent to $h^{-1} \in V$, which is equivalent to $h \in V$. Hence $U(E(V)) = V \cap H$. □

Definition 5.4. An intuitionistic uniform topological group is a symmetric member G which carries a group structure and an intuitionistic uniform topology and satisfies the following two axioms:

(i) The mapping $(x, y) \to xy$ of $G \times G$ into G is an intuitionistic uniformly continuous.
(ii) The mapping $x \to x^{-1}$ of G into G (the symmetry of the group G) is an intuitionistic uniformly continuous.

A group structure and an intuitionistic uniform structure on a symmetric member G are said to be compatible if they satisfy (i) and (ii).

Definition 5.5. An intuitionistic uniform topology compatible with a group structure on G consists in giving an intuitionistic filter base $\mathfrak{B}$ satisfying the following axioms

(i) Given any $U \in \mathfrak{B}$,there exists $V \in \mathfrak{B}$ such that $V \circ V \subset U$.
(ii) Given any $U \in \mathfrak{B}$,there exists $V \in \mathfrak{B}$ such that $V^{-1} \subset U$.
(iii) Given any $a \in G$ and any $U \in \mathfrak{B}$, there exist $V \in \mathfrak{B}$ such that $V \subset a \circ U \circ a^{-1}$.

Proposition 5.6. *The collection of all intuitionistic symmetric members $U(E)$, where $E = \langle x, E^1, E^2\rangle$ is an intuitionistic $\mathcal{B}$-open symmetric in an intuitionistic uniform topological space $(\mathbb{X}, \xi)$ is a neighbourhood intuitionistic filter base at e that makes IH_X into an intuitionistic uniform topological group. The intuitionistic symmetric members $H(E)$ are the intuitionistic $\mathcal{B}$-open symmetric member of the intuitionistic right uniformity determined by this intuitionistic uniform topology. Moreover, if G is an intuitionistic uniform topological group with the intuitionistic right uniformity and H is a subgroup acting on G by intuitionistic left translation then this intuitionistic uniform topology coincides with subgroup intuitionistic uniform structure on H.*

Proof. Let E be an any intuitionistic $\mathcal{B}$-open symmetric member. Let $F = \langle x, E^1, E^2\rangle$ be intuitionistic $\mathcal{B}$-open symmetric such that $F^2 = \langle x, F^{2^1}, F^{2^2}\rangle$, thus $F^2 \subset E$. Let $g, h \in U(F)$. Then for all x,$(x, h(x)) \in F$. So$(h(x), g(h(x))) \in F$ for all x and therefore $(x, h(x)) \circ (h(x), g(h(x))) \in F \circ F$. $(x, g(h(x)) \in F^2 \subset E$ for all x. It follows that $U(F)^2$(this is product of $U(F)$ with itself with respect to the composition operation) is contained in $U(E)$ the first condition is proved. Now $U(E)^{-1} = \{g^{-1} : (g(x), x) \in E\}$. But if $g^{-1} \in U(E)^{-1}$ then $(x, g^{-1}(x)) = (g(g^{-1}(x), g^{-1}(x)) \in E$ for all x. Therefore $g^{-1} \in U(E)$. That is, $U(E)^{-1} \subset U(E)$. Hence the second condition.

For the third axiom let $E = \langle x, E^1, E^2\rangle$ be an intuitionistic $\mathcal{B}$-open symmetric member and let $g \in IH_X$. Since g is an intuitionistic uniform homeomorphism, $g(E)$ is intuitionistic $\mathcal{B}$-open symmetric. Suppose $k \in U(g(E))$ and $h = g^{-1} \circ k \circ g$. Since $(k(x), x) \in g(E)$ for all x. Therefore $(g^{-1}(k(x)), g^{-1}(x)) \in E$ for all x. In particular $(h(x), x) = (g^{-1}(k(g(x))), x) = (g^{-1}(k(g(x))), g^{-1}(g(x)))$. Thus $(h(x), x) = (g^{-1}(k(g(x))), x) = (g^{-1}(k(g(x))), g^{-1}g(x))) \in E$. Therefore $k = g \circ h \circ g^{-1} \in g(U(E))g^{-1}$. In otherwords, $U(g(E)) \subset g(U(E))g^{-1}$. Hence the third condition. To show the second statement simply note that $(g(x), h(x)) \in E$ for all x iff $(x, h(g^{-1}(x))) \in E$ for all x and therefore $(g, h) \in H(E)$ iff $g \circ h^{-1} \in U(E)$. To prove the last statement, Let V be an intuitionistic open symmetric member about e. Then since V is an intuitionistic symmetric member if H acts as intuitionistic left translations,

$$\begin{aligned} H \cap U(E(V)) &= \{g \in H : (x, gx) \in E(V) forallx \in G\} \\ &= \{g \in H : xx^{-1}g^{-1} \in V forallx \in G\} = H \cap V. \end{aligned}$$

□

Acknowledgements. The authors are thankful to the Editors-in-Chief for their valuable comments. The authors are also thankful to the Referees for their valuable suggestions in rewriting the paper in the present form. The first author refer's to thank the UGC for financial support.

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