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Fuzzy parameterized soft fuzzy matrix in a flood warning system

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Abstract. In this paper, Fuzzy Parameterized Soft Fuzzy (FPSF) sets are introduced and some of their basic properties are studied. Further, the notion of FPSF matrix is introduced, by using FPSF aggregation operator, which can be applied for decision making problems. Finally, by using FPSF matrix, a FPSF decision making algorithm is developed and implemented through flood alarm model.

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Keywords: Soft sets, Fuzzy Soft sets, Fuzzy Parameterized Soft sets, Fuzzy Parameterized Fuzzy Soft sets, Soft Fuzzy sets, FPSF-sets, FPSF-Matrix, FPSF-aggregation operator, FPSF-decision making algorithm.

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1. Introduction

A flood warning system is a non-structural measure for flood mitigation. Several parameters are responsible for flood related disasters. This work illustrates Fuzzy Parameterized Soft Fuzzy (FPSF) set analysis that has the capability to simulate the unknown relations between a set of meteorological and hydrological parameters.

In this paper, it has been demonstrated that FPSF model has its potential usage in flood prediction. The FPSF model presented for flood warning system has furnished very promising results. This model is applied for five selected stations of Uttarakhand, India. The five meteorological parameters for each station were analyzed by imposing weights on each parameter. A critical discussion on the results of proposed model has been conducted. It provides a new way that helps disaster management to cope with fatal and rapid changes in highly sensitive parameters.

The concept of soft set was originally proposed by Molodtsov [20], as a new mathematical tool for dealing with uncertainties. Maji et al. [13, 14] studied the theory of soft sets and applied it for solving some decision making problems. Further, in

[15, 16], they also introduced the concept of fuzzy soft set, which was a combination of fuzzy set and soft set and studied some of its properties. Majumdar and Samanta [17, 18] introduced generalization of fuzzy soft set, using the notion of parameterization of fuzzy sets. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [2, 3, 4, 6, 10, 12, 19, 23]. Aktas and Cagman [1] introduced the notion of soft groups and derived their basic properties. Cagman et al. [8, 9]proposed fuzzy soft set properties and successfully applied it, in various decision making problems. Recently Naim Cagman [7] defined and introduced fuzzy parameterized fuzzy soft (FPFS) sets and their operations. For further study see [11, 21, 22, 24, 25, 26, 27].

In this paper, the notion of FPSF sets are introduced by using soft fuzzy sets and Fuzzy Parameterized Fuzzy Soft sets. In a FPSF set, a degree is attached with the parameterization of fuzzy sets while defining a soft fuzzy set. This definition is more realistic as it involves uncertainty in the selection of fuzzy set corresponding to each value of the parameter. Further, the concept of FPSF matrix is developed by defining FPSF aggregation operators on FPSF sets. Moreover, an FPSF decision making algorithm has been constructed and successfully implemented in the Flood Alarm Model.

2. Preliminaries

In this section we give few definitions regarding soft sets, fuzzy soft set, fuzzy parameterized soft set, fuzzy parameterized fuzzy soft set and soft fuzzy set.

Definition 2.1 ([20])**.** Let U be an initial universe set and let E be a set of parameters. A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

In other words, a soft set over U is a parametrized family of subsets of the universe U. Every set $F(\epsilon)$, $\epsilon \in E$, from this family may be considered as the set of ϵ - elements of the soft set (F, E) or as the set of ϵ-approximate elements of the soft set.

Definition 2.2 ([15])**.** Let $\mathscr{P}(U)$ denotes the set of all fuzzy sets of U. Let $A_i \subset E$. A pair (F_i, A_i) is called a fuzzy-soft-set over U, where F_i is a mapping given by $F_i : A_i \to \mathscr{P}(U)$.

Note that the sets of fuzzy soft sets over U will be denoted by $\mathcal{FS}(U)$.

Definition 2.3 ([7])**.** Let U be an initial universe, $P(U)$ be the power set of U, E be the set of parameters and X be a fuzzy set over E with the membership function $\mu_X : E \to [0, 1]$. Then, an fuzzy parameterized soft set F_X over U is a set defined by a function f_X representing a mapping

$$f_X : E \to P(U) \text{ such that } f_X(x) = \phi \text{ if } \mu_X(x) = 0.$$

Here, f_X is called approximate function of the fuzzy parameterized soft set F_X, and the value $f_X(x)$ is a set called x-element of the fuzzy parameterized soft sets for all $x \in E$. Thus, an fuzzy parameterized soft set F_X over U can be represented by the set of ordered pairs

$$F_X = \{(\mu_X(x)/x, f_X(x)) : x \in E, f_X(x) \in P(U), \mu_X(x) \in [0, 1]\}$$

Note that the sets of all fuzzy parameterized soft sets over U will be denoted by $\mathcal{FPS}(U)$.

Definition 2.4 ([7]). Let U be an initial universe, E be the set of parameters, and X be a fuzzy set over E with membership function $\mu_X : E \to [0,1]$ and $\gamma_X(x)$ be a fuzzy set over U for all $x \in E$. Then, an fuzzy parameterized fuzzy soft set Γ_X over U is a set defined by a function $\gamma_X(x)$ representing a mapping

$$\gamma_X : E \to F(U) \text{ such that } \gamma_X(x) = \phi \text{ if } \mu_X(x) = 0.$$

Here, γ_X is called fuzzy approximate function of the fuzzy parameterized fuzzy soft set Γ_X, and the value $\gamma_X(x)$ is a fuzzy set called x-element of the fuzzy parameterized fuzzy soft set for all $x \in E$. Thus, an fuzzy parameterized fuzzy soft set Γ_X over U can be represented by the set of ordered pairs

$$\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(U), \mu_X(x) \in [0,1]\}$$

Note that from now on the sets of all fuzzy parameterized fuzzy soft sets over U will be denoted by $\mathcal{FPFS}(U)$.

Definition 2.5 ([5]). Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Let $A \subset E$. A pair (F, A) is called a soft fuzzy set over U, where F is a mapping given by $F : A \to P(U)$ and

$$F(x) = \{y \in U : R(x,y) \geq \alpha, x \in A, y \in U, \alpha \in [0,1]\}.$$

Here R denotes fuzzy relationship between E and U.

3. Fuzzy Parameterized Soft Fuzzy sets

In this section, we define Fuzzy Parameterized Soft Fuzzy sets and their related operations.

Definition 3.1. Let U be an initial universe, E be the set of parameters, $A \subset E$. Let X be a fuzzy set over A with membership function $\mu_X : A \to I = [0,1]$ and let $\gamma_{\alpha X} : A \to I^U_\alpha$, where I^U_α is the collection of all soft fuzzy subsets of U with $\alpha \in (0,1)$. Let $\Gamma_{\alpha X} : A \to I \times I^U_\alpha$ defined as follows:

$$\Gamma_{\alpha X}(e) = (\mu_X(e), \gamma_{\alpha X}(e))$$

Then the pair $(\Gamma_{\alpha X}, A)$ is called Fuzzy Parameterized Soft Fuzzy set over the soft universe (U, E).

Note that from now on the sets of all fuzzy parameterized soft fuzzy sets over U will be denoted by $\mathcal{FPSF}(U)$.

Example 3.2. Let $(\Gamma_{\alpha X}, E)$ describe the efficiency of the cars with respect to the given parameters for finding the best cars of an academic year. Let the set of cars under consideration be U = $\{c_1, c_2, c_3, c_4\}$. Let $E = \{e_1 = costly, e_2 = beautiful, e_3 = fuel\ efficient, e_4 = moderntechnology, e_5 = luxurious\}$ be the set of parameters framed to choose the best car and $A = \{e_1, e_2, e_3\} \subset E$. If $\mu_X(A) = \{0.5/e_1, 0.7/e_2, 1/e_3\}$ and $\gamma_X(e_1) = \{0.9/c_1, 0.3/c_4\}, \gamma_X(e_2) = \{0.8/c_1, 0.4/c_2, 1/c_4\}$, $\gamma_X(e_3) = \{1/c_3\}$, then the pair

$$(\Gamma_{\alpha X}, E) = \{(0.5/e_1, \{0.9/c_1\}), (0.7/e_2, \{0.8/c_1, 1/c_4\}), (1/e_3, \{1/c_3\})\}$$

is a FPSF set corresponding to $\alpha = 0.5$.

Remark 3.3. If $\alpha = 0$, then FPSF-sets reduced to FPFS-sets.

Definition 3.4. Let $\Gamma_{\alpha X}, \Gamma_{\alpha Y} \in \mathcal{FPSF}(U)$. Then $\Gamma_{\alpha X}$ is an FPSF subset of $\Gamma_{\alpha Y}$, denoted by $\Gamma_{\alpha X} \tilde{\subseteq} \Gamma_{\alpha Y}$, if $\mu_X(x) \leq \mu_Y(x)$ and $\gamma_{\alpha X}(x) \subseteq \gamma_{\alpha Y}(x)$ for all $x \in E$.

Example 3.5. Consider the FPSF $\Gamma_{\alpha X}$ over (U, E) given in example 3.2. Let $\Gamma_{\alpha Y}$ be another FPSF over (U, E) defined as follows:
Let $B = \{e_2, e_3\} \subset A$ and Y be a fuzzy set over B. If $\mu_Y(B) = \{0.5/e_2, 0.8/e_3\}$ and $\gamma_Y(e_2) = \{0.6/c_1, 0.3/c_2, 1/c_4\}, \gamma_Y(e_3) = \{0.7/c_3\}$, then the pair

$$(\Gamma_{\alpha Y}, B) = \{(0.5/e_2, \{0.6/c_1, 1/c_4\}), (0.8/e_3, \{0.7/c_3\})\}$$

is a FPSF set. Thus $\Gamma_{\alpha Y}$ is an FPSF subset of $\Gamma_{\alpha X}$, corresponding to $\alpha = 0.5$.

Definition 3.6. Let $\Gamma_{\alpha X}, \Gamma_{\alpha Y} \in \mathcal{FPSF}(U)$. Then $\Gamma_{\alpha X}$ and $\Gamma_{\alpha Y}$ are equal, denoted by $\Gamma_{\alpha X} = \Gamma_{\alpha Y}$, if and only if $\mu_X(x) = \mu_Y(x)$ and $\gamma_{\alpha X}(x) = \gamma_{\alpha Y}(x)$ for all $x \in E$.

Definition 3.7. Let $\Gamma_{\alpha X} \in \mathcal{FPSF}(U)$. Then the complement of $\Gamma_{\alpha X}$, denoted by $\Gamma^c_{\alpha X}$, is defined by $\mu_{X^c}(x) = 1 - \mu_X(x)$ and $\gamma_{\alpha X^c}(x) = \gamma^c_{\alpha X}(x)$ for all $x \in E$, where $\gamma^c_{\alpha X}(x)$ is complement of the set $\gamma_{\alpha X}(x)$.

Definition 3.8. Let $\Gamma_{\alpha X}, \Gamma_{\alpha Y} \in \mathcal{FPSF}(U)$. Then the union of $\Gamma_{\alpha X}$ and $\Gamma_{\alpha Y}$, denoted by $\Gamma_{\alpha X} \tilde{\cup} \Gamma_{\alpha Y}$, is defined by $\mu_{X \cup Y}(x) = max\{\mu_X(x), \mu_Y(x)\}$ and $\gamma_{\alpha(X \cup Y)}(x) = \gamma_{\alpha X}(x) \cup \gamma_{\alpha Y}(x)$ for all $x \in E$.

Definition 3.9. Let $\Gamma_{\alpha X}, \Gamma_{\alpha Y} \in \mathcal{FPSF}(U)$. Then the intersection of $\Gamma_{\alpha X}$ and $\Gamma_{\alpha Y}$, denoted by $\Gamma_{\alpha X} \tilde{\cap} \Gamma_{\alpha Y}$, is defined by $\mu_{X \cap Y}(x) = min\{\mu_X(x), \mu_Y(x)\}$ and $\gamma_{\alpha(X \cap Y)}(x) = \gamma_{\alpha X}(x) \cap \gamma_{\alpha Y}(x)$ for all $x \in E$.

Example 3.10. If $\Gamma_{0.4X} = \{(0.4/e_1, \{0.5/c_2, 0.6/c_3, 1/c_4\}), (0.8/e_2, \{0.6/c_1, 0.7/c_2, 0.9/c_4\}), (0.7/e_3, \{0.8/c_1, 1/c_2, 1/c_3\})\}$ and $\Gamma_{0.4Y} = \{(0.5/e_1, \{0.6/c_1, 0.8/c_2, 0.5/c_3\}), (0.6/e_2, \{1/c_1, 1/c_2, 0.8/c_4\}), (0.8/e_3, \{0.7/c_2, 1/c_3, 0.9/c_4\})\}$. Then
$\Gamma^c_{0.4X} = \{(0.6/e_1, \{1/c_1, 0.5/c_2, 0.4/c_3\}), (0.2/e_2, \{0.4/c_1, 0.3/c_2, 1/c_3, 0.1/c_4\}), (0.3/e_3, \{0.2/c_1, 1/c_4\})\}$
$\Gamma_{0.4X} \tilde{\cup} \Gamma_{0.4Y} = \{(0.5/e_1, \{0.6/c_1, 0.8/c_2, 0.6/c_3, 1/c_4\}), (0.8/e_2, \{1/c_1, 1/c_2, 0.9/c_4\}), (0.8/e_3, \{0.8/c_1, 1/c_2, 1/c_3, 0.9/c_4\})\}$
$\Gamma_{0.4X} \tilde{\cap} \Gamma_{0.4Y} = \{(0.4/e_1, \{0.5/c_2, 0.5/c_3\}), (0.6/e_2, \{0.6/c_1, 0.7/c_2, 0.8/c_4\}), (0.7/e_3, \{0.7/c_2, 1/c_3\})\}$

Proposition 3.11. *Let* $\Gamma_{\alpha X}, \Gamma_{\alpha Y} \in \mathcal{FPSF}(U)$,

(i) $\Gamma_{\alpha X} \tilde{\subseteq} \Gamma_{\alpha X}$
(ii) $\Gamma_{\alpha X} \tilde{\subseteq} \Gamma_{\alpha Y}$ *and* $\Gamma_{\alpha Y} \tilde{\subseteq} \Gamma_{\alpha Z} \Rightarrow \Gamma_{\alpha X} \tilde{\subseteq} \Gamma_{\alpha Z}$
(iii) $\Gamma_{\alpha X} \tilde{\subseteq} \Gamma_{\alpha Y}$ *and* $\Gamma_{\alpha Y} \tilde{\subseteq} \Gamma_{\alpha X} \Rightarrow \Gamma_{\alpha X} = \Gamma_{\alpha Y}$
(iv) $(\Gamma^c_{\alpha X})^c = \Gamma_{\alpha X}$

Proof: The Proofs are straightforward and follow from Definition 3.4, 3.6 and 3.7.

Proposition 3.12. *Let* $\Gamma_{\alpha X}, \Gamma_{\alpha Y} \in \mathcal{FPSF}(U)$,

(i) $\Gamma_{\alpha X} \tilde{\cup} \Gamma_{\alpha X} = \Gamma_{\alpha X}$
(ii) $\Gamma_{\alpha X} \tilde{\cup} \Gamma_{\alpha Y} = \Gamma_{\alpha Y} \tilde{\cup} \Gamma_{\alpha X}$
(iii) $(\Gamma_{\alpha X} \tilde{\cup} \Gamma_{\alpha Y}) \tilde{\cup} \Gamma_{\alpha Z} = \Gamma_{\alpha X} \tilde{\cup} (\Gamma_{\alpha Y} \tilde{\cup} \Gamma_{\alpha Z})$

Proof: The Proofs are straightforward and follow from Definition 3.8.

Proposition 3.13. *Let* $\Gamma_{\alpha X}, \Gamma_{\alpha Y} \in \mathcal{FPSF}(U)$*,*

(i) $\Gamma_{\alpha X} \widetilde{\cap} \Gamma_{\alpha X} = \Gamma_{\alpha X}$
(ii) $\Gamma_{\alpha X} \widetilde{\cap} \Gamma_{\alpha Y} = \Gamma_{\alpha Y} \widetilde{\cap} \Gamma_{\alpha X}$
(iii) $(\Gamma_{\alpha X} \widetilde{\cap} \Gamma_{\alpha Y}) \widetilde{\cap} \Gamma_{\alpha Z} = \Gamma_{\alpha X} \widetilde{\cap} (\Gamma_{\alpha Y} \widetilde{\cap} \Gamma_{\alpha Z})$

Proof: The Proofs are straightforward and follow from Definition 3.9.

Proposition 3.14. *Let* $\Gamma_{\alpha X}, \Gamma_{\alpha Y} \in \mathcal{FPSF}(U)$*.Then,De Morgan's laws are valid.*

(i) $(\Gamma_{\alpha X} \widetilde{\cup} \Gamma_{\alpha Y})^c = \Gamma^c_{\alpha X} \widetilde{\cap} \Gamma^c_{\alpha Y}$
(ii) $(\Gamma_{\alpha X} \widetilde{\cap} \Gamma_{\alpha Y})^c = \Gamma^c_{\alpha X} \widetilde{\cup} \Gamma^c_{\alpha Y}$

Proof of (i): For all $x \in E$,

$$\begin{aligned}
\mu_{(X\cup Y)^c}(x) &= 1-\mu_{(X\cup Y)}(x) \\
&= 1-\max\{\mu_X(x),\mu_Y(x)\} \\
&= \min\{1-\mu_X(x), 1-\mu_Y(x)\} \\
&= \min\{\mu_{X^c}(x),\mu_{Y^c}(x)\} \\
&= \mu_{X^c\cap Y^c}(x) \\
\textit{and}\ \ \gamma_{\alpha(X\cup Y)^c}(x) &= \gamma^c_{\alpha(X\cup Y)}(x) \\
&= (\gamma_{\alpha X}(x)\cup\gamma_{\alpha Y}(x))^c \\
&= (\gamma_{\alpha X}(x))^c\cap(\gamma_{\alpha Y}(x))^c \\
&= \gamma^c_{\alpha X}(x)\cap\gamma^c_{\alpha Y}(x) \\
&= \gamma_{(\alpha X)^c}(x)\cap\gamma_{(\alpha Y)^c}(x) \\
&= \gamma_{\alpha(X^c\cap Y^c)}(x).
\end{aligned}$$

Similarly, we can prove (ii).

Proposition 3.15. *Let* $\Gamma_{\alpha X}, \Gamma_{\alpha Y}, \Gamma_{\alpha Z} \in \mathcal{FPSF}(U)$*,*

(i) $\Gamma_{\alpha X} \widetilde{\cup} (\Gamma_{\alpha Y} \widetilde{\cap} \Gamma_{\alpha Z}) = (\Gamma_{\alpha X} \widetilde{\cup} \Gamma_{\alpha Y}) \widetilde{\cap} (\Gamma_{\alpha X} \widetilde{\cup} \Gamma_{\alpha Z})$
(ii) $\Gamma_{\alpha X} \widetilde{\cap} (\Gamma_{\alpha Y} \widetilde{\cup} \Gamma_{\alpha Z}) = (\Gamma_{\alpha X} \widetilde{\cap} \Gamma_{\alpha Y}) \widetilde{\cup} (\Gamma_{\alpha X} \widetilde{\cap} \Gamma_{\alpha Z})$

Proof of (i): For all $x \in E$,

$$\begin{aligned}
\mu_{X\cup(Y\cap Z)}(x) &= \max\{\mu_X(x),\mu_{Y\cap Z}(x)\} \\
&= \max\{\mu_X(x),\min\{\mu_Y(x),\mu_Z(x)\} \\
&= \min\{\max\{\mu_X(x),\mu_Y(x)\},\max\{\mu_X(x),\mu_Z(x)\}\} \\
&= \min\{\mu_{(X\cup Y)}(x),\mu_{(X\cup Z)}(x)\} \\
&= \mu_{(X\cup Y)\cap(X\cup Z)}(x) \\
\textit{and}\ \ \gamma_{\alpha(X\cup(Y\cap Z))}(x) &= \gamma_{\alpha X}(x)\cup\gamma_{\alpha(Y\cap Z)}(x) \\
&= \gamma_{\alpha X}(x)\cup(\gamma_{\alpha Y}(x)\cap\gamma_{\alpha Z}(x)) \\
&= (\gamma_{\alpha X}(x)\cup\gamma_{\alpha Y}(x))\cap(\gamma_{\alpha X}(x)\cup\gamma_{\alpha Z}(x)) \\
&= \gamma_{\alpha(X\cup Y)}(x)\cap\gamma_{\alpha(X\cup Z)}(x) \\
&= \gamma_{\alpha((X\cup Y)\cap(X\cup Z))}(x).
\end{aligned}$$

Similarly, we can prove (ii).

4. FPSF Aggregation Operator

In this section, first we define Fuzzy Parameterized Soft Fuzzy Matrix and its cardinal matrix. Next, we define FPSF aggregation operator that produce an aggregate fuzzy set from an FPSF set and its fuzzy parameter set. An FPSF-aggregation operator on the fuzzy sets is an operation by which several approximate functions of an FPSF-set are combined to produce a single fuzzy set which is the aggregate fuzzy set of the FPSF-set. Once an aggregate fuzzy set has been arrived at, it may be necessary to choose the best single crisp alternative from this set.

Definition 4.1. Let $\Gamma_{\alpha A} \in \mathcal{FPSF}(U)$, assume that $U = \{u_1, u_2, u_3, \ldots, u_m\}$, $E = \{x_1, x_2, x_3, \ldots, x_n\}$ and $A \subseteq E$, then the $\Gamma_{\alpha A}$ can be presented by the following table,

$\Gamma_{\alpha A}$	x_1	x_2	$\cdots$	x_n
u_1	$\mu_{\gamma_{\alpha A(x_1)}(u_1)}$	$\mu_{\gamma_{\alpha A(x_2)}(u_1)}$	$\cdots$	$\mu_{\gamma_{\alpha A(x_n)}(u_1)}$
u_2	$\mu_{\gamma_{\alpha A(x_1)}(u_2)}$	$\mu_{\gamma_{\alpha A(x_2)}(u_2)}$	$\cdots$	$\mu_{\gamma_{\alpha A(x_n)}(u_2)}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
u_m	$\mu_{\gamma_{\alpha A(x_1)}(u_m)}$	$\mu_{\gamma_{\alpha A(x_2)}(u_m)}$	$\cdots$	$\mu_{\gamma_{\alpha A(x_n)}(u_m)}$

Where $\mu_{\gamma_{\alpha A}(x)}$ is the membership function of $\gamma_{\alpha A}$.
If $a_{ij} = \mu_{\gamma_{\alpha A(x_j)}(u_i)}$ for $i = 1, 2, 3, \cdots, m$ and $j = 1, 2, 3, \cdots, n$, then the FPSF set $\Gamma_{\alpha A}$ is uniquely characterized by a matrix

$$[a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

is called an $m \times n$ Fuzzy Parameterized Soft Fuzzy (FPSF) matrix of the FPSF-set $\Gamma_{\alpha A}$over U, denoted by $M_{\Gamma_{\alpha A}}$.

Definition 4.2. Let $\Gamma_{\alpha A} \in \mathcal{FPSF}(U)$, then the cardinal set of $\Gamma_{\alpha A}$, denoted $c\Gamma_{\alpha A}$ and defined by

$$c\Gamma_{\alpha A} = \{\mu_{c\Gamma_{\alpha A}}(x)/x : x \in E\},$$

is fuzzy set over E. The membership function $\mu_{c\Gamma_{\alpha A}}$ of $c\Gamma_{\alpha A}$ is defined by $\mu_{c\Gamma_{\alpha A}} : E \to [0, 1]$ and $\mu_{c\Gamma_{\alpha A}}(x) = \frac{|\gamma_{\alpha A}(x)|}{|U|}$,where $|U|$ is the cardinality of universe U, and $|\gamma_{\alpha A}(x)|$ is the scalar cardinality of fuzzy set $\gamma_{\alpha A}(x)$, denoted by $_{\mathcal{C}}\mathcal{FPSF}(U)$.

Definition 4.3. Let $\Gamma_{\alpha A} \in \mathcal{FPSF}(U)$ and $c\Gamma_{\alpha A} \in {}_{\mathcal{C}}\mathcal{FPSF}(U)$, assume that $E = \{x_1, x_2, \ldots, x_n\}$ and $A \subseteq E$, then $c\Gamma_{\alpha A}$ can be presented by the following table

E	x_1	x_2	$\cdots$	x_n
$\mu_{c\Gamma_{\alpha A}}$	$\mu_{c\Gamma_{\alpha A}}(x_1)$	$\mu_{c\Gamma_{\alpha A}}(x_2)$	$\cdots$	$\mu_{c\Gamma_{\alpha A}}(x_n)$

If $a_{1j} = \mu_{c\Gamma_{\alpha A}}(x_j)$ for $j = 1, 2, \ldots, n$, then the cardinal set $c\Gamma_{\alpha A}$ is uniquely characterized by a matrix

$$[a_{1j}]_{1\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix}$$

which is called the cardinal matrix of the cardinal set $c\Gamma_{\alpha A}$ over E.

Example 4.4. The matrix representation of the example 3.2 is FPSF Matrix

$$M_{\Gamma_{\alpha A}} = \begin{bmatrix} 0.9 & 0.8 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

and its cardinal matrix

$$c\Gamma_{\alpha A} = \begin{bmatrix} 0.5 \\ 0.7 \\ 1 \end{bmatrix},$$

where α= 0.5.

Definition 4.5. Let $\Gamma_{\alpha A} \in \mathcal{FPSF}(U)$ and $c\Gamma_{\alpha A} \in c\mathcal{FPSF}(U)$, then FPSF-aggregation operator, denoted by $FPSF_{agg}$, is defined by $FPSF_{agg} : cFPSF(U) \times FPSF(U) \to F(U)$, $FPSF_{agg}(c\Gamma_{\alpha A}, \Gamma_{\alpha A}) = \Gamma^*_{\alpha A}$. Where $\Gamma^*_{\alpha A} = \{\mu_{\Gamma^*_{\alpha A}}(u)/u : u \in U\}$ is a fuzzy set over U. $\Gamma^*_{\alpha A}$ called the aggregate fuzzy set of the FPSP-set $\Gamma_{\alpha A}$. The membership function $\mu_{\Gamma^*_{\alpha A}}$ of $\Gamma^*_{\alpha A}$ is defined as follows:

$$\mu_{\Gamma^*_{\alpha A}} : U \to [0,1] \text{ and } \mu_{\Gamma^*_{\alpha A}}(u) = \frac{1}{|E|} \sum_{x\in E} \mu_{c\Gamma_{\alpha A}}(x) \mu_{\gamma_{\alpha A}(x)}(u),$$

where $|E|$ is the cardinality of E.

Definition 4.6. Let $\Gamma_{\alpha A} \in \mathcal{FPSF}(U)$ and $\Gamma^*_{\alpha A}$ be its aggregate fuzzy set. Assume that $U = \{u_1, u_2, \ldots, u_m\}$, then the $\Gamma^*_{\alpha A}$ can be presented by the following table

$\Gamma_{\alpha A}$	$\mu_{\Gamma^*_{\alpha A}}$
u_1	$\mu_{\Gamma^*_{\alpha A}}(u_1)$
u_2	$\mu_{\Gamma^*_{\alpha A}}(u_2)$
$\vdots$	$\vdots$
u_m	$\mu_{\Gamma^*_{\alpha A}}(u_m)$

If $a_{i1} = \mu_{\Gamma^*_{\alpha A}}(u_i)$ for $i = 1, 2, 3, \ldots, m$, then $\Gamma^*_{\alpha A}$ is uniquely characterized by the matrix,

$$[a_{i1}]_{m\times 1} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$$

which is called the aggregate matrix of $\Gamma^*_{\alpha A}$ over U.

Theorem 4.7. *Let $\Gamma_{\alpha A} \in \mathcal{FPSF}(U)$ and $A \subseteq E$. If $M_{\Gamma_{\alpha A}}, M_{c\Gamma_{\alpha A}}$ and $M_{\Gamma^*_{\alpha A}}$ are representation matrices of $\Gamma_{\alpha A}, c\Gamma_{\alpha A}$ and $\Gamma^*_{\alpha A}$ respectively, then $|E| \times M_{\Gamma^*_{\alpha A}} = M_{\Gamma_{\alpha A}} \times M^T_{c\Gamma_{\alpha A}}$ where $M^T_{c\Gamma_{\alpha A}}$ is the transposition of $M_{c\Gamma_{\alpha A}}$ and $|E|$ is the cardinality of E.*

Proof: It is sufficient to consider $[a_{i1}]_{m\times 1} = [a_{ij}]_{m\times n} \times [a_{1j}]^T_{1\times n}$.

Remark 4.8. Theorem 4.7 is applicable to computing the aggregate fuzzy set of an FPSF set.

The approximate functions of an FPSF set are fuzzy. The $FPSF_{agg}$ on the fuzzy sets is an operation by which several approximate functions of an FPSF set are combined to produce a single fuzzy set that is the aggregate fuzzy set of the FPSF set. Once an aggregate fuzzy set has been arrived at, it may be necessary to choose the best single crisp alternative from this set. Therefore, we can construct an FPSF decision making algorithm.

5. FPSF Set in Flood Alarm Model

In this section, first we construct an FPSF decision making algorithm.

Step 1: Construct an FPFS set Γ_A over U.
Step 2: Fix the value of α and construct an FPSF set $\Gamma_{\alpha A}$ over U.
Step 3: Find the cardinal set $c\Gamma_{\alpha A}$ of $\Gamma_{\alpha A}$.
Step 4: Find the aggregate fuzzy set $\Gamma^*_{\alpha A}$ of $\Gamma_{\alpha A}$.

Step 5: Find the best alternative from this set that has the largest membership grade by $max\{\mu_{\Gamma^*_{\alpha A}}(u)\}$.

We implemented the above algorithm through Flood Alarm Model. Reliable flood prediction cannot be done by subjecting available data to conventional methods of analysis. We therefore turn to FPSF sets and develop a simple but effective model (algorithm) which has been designed in such a way as to produce reliable output in the prediction of flood possibility. The inputs are basic parameters (real-valued variables) related to flood occurrence and fuzzy membership degrees and weights were assigned to each parameter. The model processes the FPSF set constructed from collected data and identifies the most flood prone location (the location which shows maximum discrimination factor).

The study intends to predict the possibility and severity of floods in five selected stations in Uttarakhand, India. We define FPSF set properties to establish an algorithm for a reliable prediction. The five selected location Uttarkashi, Badrinath, Kedarnath,Joshimath, Rudra prayag denoted by L_1, L_2, L_3, L_4 and L_5. The parameter set $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ respectively denotes wind speed , wind direction, relative humidity, surface pressure, river contribution, topography, and rainfall amount.

After a serious analysis of each locations with respect to each parameter, we will consider the set $A \subseteq E$ where $A = \{e_2, e_3, e_4, e_5, e_7\}$ and assign a weight for respective parameters. Let $\mu : A \to [0, 1]$ be a fuzzy subset of A, defined by $\{0.8/e_2, 0.6/e_3, 0.6/e_4, 1/e_5, 0.9/e_7\}$. Finally applies the following steps:

Step 1: An FPFS-set Γ_A over U.

$$\begin{aligned}\Gamma_A = \quad &\{(e_2, \{0.7/L_1, 0.6/L_2, 0.6/L_3, 0.9/L_4, 0.5/L_5\}),\\ &(e_3, \{0.8/L_1, 0.8/L_2, 1/L_3, 0.7/L_4, 0.9/L_5\}),\\ &(e_4, \{0.9/L_1, 0.9/L_2, 1/L_3, 0.8/L_4, 0.9/L_5\}),\\ &(e_5, \{1/L_1, 0.5/L_2, 1/L_3, 0.5/L_4, 0.5/L_5\}),\\ &(e_7, \{0.5/L_1, 1/L_2, 1/L_3, 0.5/L_4, 0.5/L_5\})\end{aligned}$$

Step 2: Fix the value of $\alpha = 0.6$, and construct an FPSF set $\Gamma_{\alpha A}$ over U.

$$\begin{aligned}\Gamma_{\alpha A} = \quad &\{(e_2, \{0.7/L_1, 0.6/L_2, 0.6/L_3, 0.9/L_4\}),\\ &(e_3, \{0.8/L_1, 0.8/L_2, 1/L_3, 0.7/L_4, 0.9/L_5\}),\\ &(e_4, \{0.9/L_1, 0.9/L_2, 1/L_3, 0.8/L_4, 0.9/L_5\}),\\ &(e_5, \{1/L_1, 1/L_3\}), (e_7, \{1/L_2, 1/L_3\})\end{aligned}$$

Step 3: The cardinal is computed, $c\Gamma_{\alpha A} = \{0.8/e_2, 0.6/e_3, 0.6/e_4, 1/e_5, 0.9/e_7\}$

Step 4: The aggregate fuzzy set is found by using theorem 4.7,

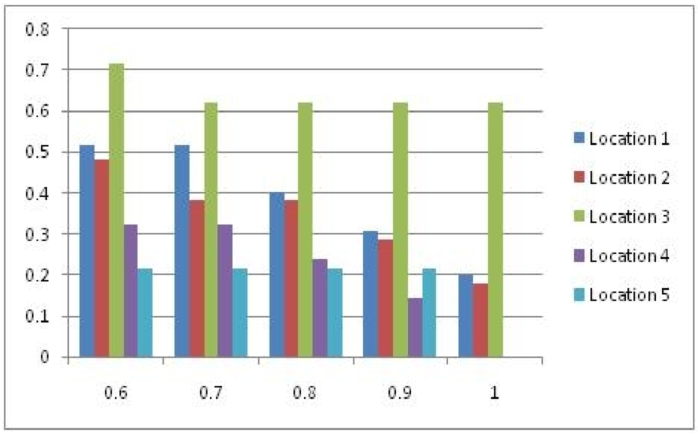
$$M_{\Gamma^*_{\alpha A}} = \frac{1}{5} \begin{bmatrix} 0.7 & 0.8 & 0.9 & 0.6 & 1 & 0 \\ 0.6 & 0.8 & 0.9 & 0 & 0 & 1 \\ 0.6 & 1 & 1 & 0 & 1 & 1 \\ 0.9 & 0.7 & 0.8 & 0 & 0 & 0 \\ 0 & 0.9 & 0.9 & 0.5 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.8 \\ 0.6 \\ 0.6 \\ 1 \\ 0.9 \end{bmatrix} = \begin{bmatrix} 0.5160 \\ 0.4800 \\ 0.7160 \\ 0.3240 \\ 0.2160 \end{bmatrix}$$

Step 5: Finally, we get $L_3 > L_1 > L_2 > L_4 > L_5$.

Thus the corresponding L_3 is the optimal solution. Therefore the highest possibility of flood occurrence is in Kedarnath, followed by Uttarkashi, Badrinath, Joshimath and Rudra prayag.

6. Analysis of behaviors of α value

Based on data collected from the individuals, a decision chart for the study of the behavior of α values has been developed.



From the above decision chart, the chosen values of α is 0.6, 0.7, 0.8, 0.9 and 1. The decision is in favour of the values of α. In the existing models, the decision obtained will always be unique, whereas, ideally in FPSF matrix method, the value of α reflects the decision. Since, the decision keeps on changing with the corresponding value of α. This procedure excludes unnecessary parameters and searches for suitable parameters of the decision making problems. Thus this procedure performs more efficiently.

7. Conclusion and Future Work

In this paper, we introduced the concept of Fuzzy Parameterized Soft Fuzzy Sets and their operations. Further, we studied some of its properties and FPSF decision making algorithm is presented. Moreover, the algorithm excludes unnecessary parameters and searches for suitable parameters. Finally, we provided a flood alarm model and demonstrated that this method can be successfully used for decision making problem containing uncertainty. These results of comparison show that the approaches presented in this paper are preferable to reflect the practical feasibility. This technique can be applied in many fields such as pattern recognition, image processing and fuzzy reasoning.

The concepts of FPSF Matrix explored in this paper can further be utilized in different areas of Decision Making which involves databases consisting of real world data. They can also be utilized to retrieve related information pertaining to queries which are fuzzy parameterized soft fuzzy in nature and also similar to the queries presented in the real world scenario.

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