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Similarity measures of interval-valued fuzzy soft sets and their application in decision making problems

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Abstract. In this paper we introduce three types of similarity measures of interval-valued fuzzy soft sets. Three types of similarity measures are done based on matching function, distance and set theoretic approach. An application of similarity measure between two interval-valued fuzzy soft sets in a decision making problem is illustrated.

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1. Introduction

Similarity measure between two fuzzy sets (interval-valued fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets) have been defined by many authors([4],[8],[10],[18],[20],[21],[22]). There are several techniques for defining similarity measure in such cases. Some of them are based on distances and some others are based on matching function.There are techniques based on set-theoretic approach also. Some properties are common to these measures and some are not, which influence the choice of the measure to be used in several applications.One of the significant differences between similarity measure based on matching function S and similarity measure $S^{'}$ based on distance is that if $A \cap B = \phi$ then S(A,B)=0 but $S^{'}$(A,B) may not be equal to zero, where A and B are two fuzzy sets. But it is easier to calculate the intermediate distance between two fuzzy sets or soft sets. Therefore, distance-based measures are also popular. Let U = $\{x_1, x_2, x_3, \ldots\ldots , x_n\}$ be the universe and A, B be two intuitionistic fuzzy sets (IFS) over U with their membership function μ_A,μ_B and non membership function ν_A,ν_B respectively. Then the distances between A and B defined by Szmidt & Kacprzyk[19]. Again in several problems it is often needed to compare two sets. The sets may be fuzzy, may be vague etc. We are often interested to know whether two patterns or images identical or approximately identical or at least to what degree they are identical. Several researchers like Chen([4],[5],[6]), Hu and Li[7] etc. have studied the problem of similarity measure between fuzzy sets and vague sets. Recently P. Majumdar and S. K. Samanta ([11],[12],[13]) have studied the similarity measure of soft sets , fuzzy soft sets and intuitionistic fuzzy soft sets. In [17], W.K.Min also introduced similarity in soft set theory. Cagman and Deli studied similarity measure of intuitionistic fuzzy soft sets [16]in [3] .

We have extended these concepts of similarity measure in interval-valued fuzzy soft sets. The aim of this paper is to introduce three types of similarity measures of interval-valued fuzzy soft sets. Three types of similarity measures are done based on matching function, distance and set theoretic approach. An application of similarity measure between two interval-valued fuzzy soft sets in a decision making problem is illustrated.

2. Preliminaries

In this section we briefly review some basic definitions related to interval-valued fuzzy soft sets which will be used in the rest of the paper.

Definition 2.1 ([24])**.** Let X be a non empty collection of objects denoted by x. Then a fuzzy set (FS for short) α in X is a set of ordered pairs having the form α=$\{(x, \mu_\alpha(x))\}$, where the function $\mu_\alpha : X \to [0, 1]$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in α .The interval $M = [0, 1]$ is called membership space.

Definition 2.2 ([25])**.** Let $D[0, 1]$ be the set of closed subintervals of the interval $[0, 1]$. An interval-valued fuzzy set in X, $X \neq \phi$ and Card(X) = n, is an expression A given by A=$\{$(x,M$_A$(x)):x$\in X\}$, where M$_A$:$\longrightarrow$D[0,1].

Definition 2.3 ([1])**.** Let X be a non empty set. Then an intuitionistic fuzzy set (IFS for short) A is a set having the form A=$\{$(x, μ_A(x), ν_A(x)) : x$\in$X$\}$, where the functions μ_A: X$\longrightarrow$[0,1] and ν_A: X$\longrightarrow$[0,1] represents the degree of membership and the degree of non-membership respectively of each element x$\in$X and 0$\leq\mu_A$(x) + ν_A(x) $\leq$1 for each x$\in$X .

Definition 2.4 ([2])**.** An interval valued intuitionistic fuzzy set A over a universe set U is defined as the object of the form A=$\{$(x,μ_A(x),ν_A(x)) : x$\in U\}$, where μ_A: U$\longrightarrow$D[0,1] and ν_A: U$\longrightarrow$D[0,1] are functions such that the condition, supμ_A(x)+supν_A(x)$\leq$1, $\forall$x$\in$U is satisfied, where D[0,1]is the set of all closed subintervals of [0,1].

Definition 2.5 ([9, 14])**.** Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and A$\subseteq$E . Then the pair (F, A) is called a soft set over U, where F is a mapping given by F:A$\longrightarrow$P(U).

Definition 2.6 ([15])**.** Let U be an initial universe and E be a set of parameters. Let I^U be the set of all fuzzy subsets of U and A$\subseteq$E . Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A$\longrightarrow$I^U .

Definition 2.7 ([23])**.** Let U be an initial universe and E be a set of parameters, a pair (F, E) is called an interval valued- fuzzy soft set over $F(U)$, where F is a mapping given by F:E⟶F(U) and $F(U)$ is the set of all interval-valued fuzzy sets of U .

An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of U, thus, its universe is the set of all interval-valued fuzzy sets of U, i.e.$F(U)$. An interval-valued fuzzy soft set is also a special case of a soft set because it is still a mapping from parameters to $F(U)$,∀ e∈E, $F(U)$ is referred as the interval fuzzy value set of parameters e , it is actually an interval-valued fuzzy set of U where x∈U and e∈E , it can be written as: F(e)={(x,$\mu_{F(e)}$(x)): x∈U},where $F(U)$ is the interval-valued fuzzy membership degree that object x holds on parameter.

3. Similarity Measure of interval-valued fuzzy soft sets

In this section we introduce similarity measure of interval-valued fuzzy soft sets based on matching function , distance and set theoretic approach.

Similarity measure of interval-valued fuzzy soft sets based on matching function:

Definition 3.1. Let U={x$_1$,x$_2$,x$_3$,$\cdots$,x$_n$} be the universe and E={e$_1$,e$_2$,e$_3$,$\cdots$,e$_m$} be the set of parameters. Let (F, E) be an interval valued- fuzzy soft set over $F(U)$, where F is a mapping given by F:E⟶F(U) and $F(U)$ is the set of all interval-valued fuzzy sets of U.

An interval-valued fuzzy soft set (IVFSS) can be represented in a tabular form as follows:

	e_1	e_2	e_3	......	e_m
x_1	$[a_{11}, b_{11}]$	$[a_{12}, b_{12}]$	$[a_{13}, b_{13}]$	......	$[a_{1m}, b_{1m}]$
x_2	$[a_{21}, b_{21}]$	$[a_{22}, b_{22}]$	$[a_{23}, b_{23}]$	......	$[a_{2m}, b_{2m}]$
.	......	......	......	......	..
.	......	......	......	......	..
x_n	$[a_{n1}, b_{n1}]$	$[a_{n2}, b_{n2}]$	$[a_{n3}, b_{n3}]$	......	$[a_{nm}, b_{nm}]$

Where $[a_{ij}, b_{ij}] \subseteq [0, 1]$ for all i = 1,2,3,.......,n and j = 1,2,3,........,m .

Now we represent this IVFSS as a matrix as follows:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & ..... & c_{1m} \\ c_{21} & c_{22} & c_{23} & ..... & c_{2m} \\ ..... & ...... & ...... & ..... & .. \\ ..... & ...... & ...... & ..... & .. \\ c_{n1} & c_{n2} & c_{n3} & ..... & c_{nm} \end{bmatrix}$$

Where c$_{ij}$= b$_{ij}$- a$_{ij}$ ∀ i = 1,2,3,.......,n and j = 1,2,3,......,m .

We denote $(c_{11}, c_{21}, c_{31}, ..., c_{n1})$ as $\overrightarrow{F(e_1)}$ etc.

Definition 3.2. Let (F, E) and (G, E) be two interval valued- fuzzy soft sets(IVFSSs) over $F(U)$ and $G(U)$ respectively, where F is a mapping given by F:E⟶F(U) and G is a mapping given by G:E⟶G(U) and $F(U)$, $G(U)$ are the sets of all interval-valued fuzzy sets of U. Then we define similarity measure between the IVFSSs (F, E) and (G, E) denoted by $S(F, G)$ as

$$S(F,G) = \frac{\sum_{i=1}^{m}(\overrightarrow{F(e_i)} \bullet \overrightarrow{G(e_i)})}{\sum_{i=1}^{m}((\overrightarrow{F(e_i)})^2 \vee (\overrightarrow{G(e_i)})^2)} \quad \text{.............................}(\mathbf{1})$$

where $\overrightarrow{F(e_i)} = (c_{1i}, c_{2i}, c_{3i}, ..., c_{ni})$ etc., i=1,2,3,.......,m .

Theorem 3.3. *If S(F,G) be the similarity measure between two IVFSSs (F,E) and (G,E) then*
(i) S(F,G) = S(G,F)
(ii) 0≤S(F,G)≤1
(iii) S(F,G) = 1 if and only if (F,E) = (G,E).

Proof. Obvious from the definition 3.2. □

Example 3.4. Let U={x_1,x_2,x_3} be the universe and E={e_1,e_2,e_3} be the set of parameters. We consider two IVFSSs (F, E) and (G, E) such that their tabular forms are as follows.

Tabular form of (F, E):

	e_1	e_2	e_3
x_1	$[0.7, 0.9]$	$[0.6, 0.7]$	$[0.5, 0.8]$
x_2	$[0.6, 0.8]$	$[0.2, 0.5]$	$[0.6, 0.9]$
x_3	$[0.5, 0.6]$	$[0.0, 0.7]$	$[0.2, 1.0]$

Tabular form of (G, E):

	e_1	e_2	e_3
x_1	$[0.2, 0.8]$	$[0.4, 0.9]$	$[0.3, 0.6]$
x_2	$[0.4, 0.7]$	$[0.4, 0.5]$	$[0.8, 0.9]$
x_3	$[0.0, 1.0]$	$[0.2, 0.5]$	$[0.8, 1.0]$

Therefore the corresponding matrices F and G of (F,E) and (G,E) respectively are given by

$$F = \begin{pmatrix} 0.2 & 0.1 & 0.3 \\ 0.2 & 0.3 & 0.3 \\ 0.1 & 0.7 & 0.8 \end{pmatrix} \quad and \quad G = \begin{pmatrix} 0.6 & 0.5 & 0.3 \\ 0.3 & 0.1 & 0.1 \\ 1.0 & 0.3 & 0.2 \end{pmatrix}$$

Therefore by equation (1) similarity measure between (F, E) and (G, E) is given by,

$$\begin{aligned} S(F,G) &= \frac{\sum_{i=1}^{3}(\overrightarrow{F(e_i)} \bullet \overrightarrow{G(e_i)})}{\sum_{i=1}^{3}((\overrightarrow{F(e_i)})^2 \vee (\overrightarrow{G(e_i)})^2)} \\ &= \frac{(0.12+0.06+0.1)+(0.05+0.03+0.21)+(0.09+0.03+0.16)}{(1.45+0.59+0.82)} \\ &= \frac{0.85}{2.86} \cong 0.2972. \end{aligned}$$

Example 3.5. Let $U = \{x_1, x_2, x_3, x_4\}$ be the universe and $E = \{e_1, e_2, e_3\}$ be the set of parameters. We consider two IVFSSs (F, E) and (G, E) such that their tabular forms are as follows.

Tabular form of (F, E):

	e_1	e_2	e_3
x_1	$[0.2, 0.9]$	$[0.0, 1.0]$	$[0.2, 0.8]$
x_2	$[0.4, 0.8]$	$[0.3, 0.9]$	$[0.3, 1.0]$
x_3	$[0.4, 1.0]$	$[0.3, 0.7]$	$[0.0, 0.7]$
x_4	$[0.1, 0.9]$	$[0.5, 1.0]$	$[0.3, 0.8]$

Tabular form of (G, E):

	e_1	e_2	e_3
x_1	$[0.1, 0.9]$	$[0.4, 1.0]$	$[0.0, 0.8]$
x_2	$[0.2, 0.7]$	$[0.1, 0.9]$	$[0.3, 1.0]$
x_3	$[0.0, 0.8]$	$[0.4, 0.9]$	$[0.2, 0.7]$
x_4	$[0.2, 1.0]$	$[0.0, 1.0]$	$[0.3, 1.0]$

Therefore the corresponding matrices F and G of (F,E) and (G,E) respectively are given by

$$F = \begin{pmatrix} 0.7 & 1.0 & 0.6 \\ 0.4 & 0.6 & 0.7 \\ 0.6 & 0.4 & 0.7 \\ 0.8 & 0.5 & 0.5 \end{pmatrix} \quad and \quad G = \begin{pmatrix} 0.8 & 0.6 & 0.8 \\ 0.5 & 0.8 & 0.7 \\ 0.8 & 0.5 & 0.5 \\ 0.8 & 1.0 & 0.7 \end{pmatrix}$$

Therefore by equation (1) similarity measure between (F, E) and (G, E) is given by,

$$S(F,G) = \frac{\sum_{i=1}^{3}(\overrightarrow{F(e_i)} \bullet \overrightarrow{G(e_i)})}{\sum_{i=1}^{3}((\overrightarrow{F(e_i)})^2 \vee (\overrightarrow{G(e_i)})^2)}$$
$$= \frac{1.88 + 1.78 + 1.67}{2.17 + 2.25 + 1.87}$$
$$= \frac{5.33}{6.29} \cong 0.847$$

Similarity Measure of interval-valued fuzzy soft sets based on distance:

Definition 3.6. Let U={x_1,x_2,x_3,$\cdots$,x_n} be the universe and E={e_1,e_2,e_3, $\cdots$, e_m} be the set of parameters. Let (F_1, E) and (G_1, E) be two interval valued-fuzzy soft sets(IVFSSs) over $F_1(U)$ and $G_1(U)$ respectively, where F_1 is a mapping given by F_1:E$\longrightarrow F_1$(U) and G_1 is a mapping given by G_1:E$\longrightarrow G_1$(U) and $F_1(U)$, $G_1(U)$ are the set of all interval-valued fuzzy sets of U. Then we define the following distances between (F_1, E) and (G_1, E).

a. Hamming distance:

$$d_H(F_1,G_1) = \frac{1}{2m}\Bigg[\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\Big|M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\Big| + \Big|M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\Big| + \Big|W_{F_1}(e_i)(x_j) - W_{G_1}(e_i)(x_j)\Big|\Big)\Bigg]$$

b. Normalized Hamming distance:

$$d_{NH}(F_1,G_1) = \frac{1}{2mn}\Bigg[\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\Big|M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\Big| + \Big|M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\Big| + \Big|W_{F_1}(e_i)(x_j) - W_{G_1}(e_i)(x_j)\Big|\Big)\Bigg]$$

c. Euclidean distance:

$$d_E(F_1,G_1) = \left(\frac{1}{2m}\Bigg[\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\Big(M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\Big)^2 + \Big(M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\Big)^2 + \Big(W_{F_1}(e_i)(x_j) - W_{G_1}(e_i)(x_j)\Big)^2\Big)\Bigg]\right)^{\frac{1}{2}}$$

d. Normalized Euclidean distance :

$$d_{NE}(F_1, G_1) = \left(\frac{1}{2mn} \left[\sum_{i=1}^{m} \sum_{j=1}^{n} \left(\left(M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j) \right)^2 + \left(M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j) \right)^2 + \left(W_{F_1}(e_i)(x_j) - W_{G_1}(e_i)(x_j) \right)^2 \right) \right] \right)^{\frac{1}{2}}$$

Where M_L , M_U, W denote respectively the lower limit, upper limit and amplitude (upper limit − lower limit) of the corresponding interval.

Definition 3.7. Let (F_1, E) and (G_1, E) be two interval valued- fuzzy soft sets (IVFSSs) over the universe U and the set of parameters E. Then the similarity measure between (F_1, E) and (G_1, E) denoted by $S(F_1, G_1)$ is defined as

$$S(F_1, G_1) = \frac{1}{1 + d(F_1, G_1)} \quad \text{............................(2)}$$

Where $d(F_1, G_1)$ denotes the distance between (F_1, E) and (G_1, E).
Clearly $S(F_1, G_1)$ satisfies all the properties stated in **Theorem** 3.3.

Example 3.8. Let U={x$_1$,x$_2$,x$_3$} be the universe and E={e$_1$,e$_2$,e$_3$} be the set of parameters. We consider two IVFSSs (F_1, E) and (G_1, E) such that their tabular forms are as follows.

Tabular form of (F_1, E):

	e_1	e_2	e_3
x_1	$[0.7, 0.9]$	$[0.6, 0.7]$	$[0.5, 0.8]$
x_2	$[0.6, 0.8]$	$[0.2, 0.5]$	$[0.6, 0.9]$
x_3	$[0.5, 0.6]$	$[0.0, 0.7]$	$[0.2, 1.0]$

Tabular form of (G_1, E):

	e_1	e_2	e_3
x_1	$[0.2, 0.8]$	$[0.4, 0.9]$	$[0.3, 0.6]$
x_2	$[0.4, 0.7]$	$[0.4, 0.5]$	$[0.8, 0.9]$
x_3	$[0.0, 1.0]$	$[0.2, 0.5]$	$[0.8, 1.0]$

Now by definition 3.6 the Hamming distance between (F_1, E) and (G_1, E) is given by $-$

$$d_H(F_1, G_1) = \frac{1}{2.3}\Bigg[\sum_{i=1}^{3}\sum_{j=1}^{3}\Big(\Big|M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\Big| +$$

$$\Big|M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\Big| + \Big|W_{F_1}(e_i)(x_j) - W_{G_1}(e_i)(x_j)\Big|\Big)\Bigg]$$

$$= \frac{1}{6}[(0.5 + 0.1 + 0.4) + (0.2 + 0.1 + 0.1) + (0.5 + 0.4 + 0.9)+$$
$$(0.2 + 0.2 + 0.4) + (0.2 + 0 + 0.2) + (0.2 + 0.2 + 0.4)+$$
$$(0.2 + 0.2 + 0) + (0.2 + 0 + 0.2) + (0.6 + 0 + 0.6)]$$
$$= \frac{7.2}{6} = 1.2$$

Therefore by equation (2) the similarity measure between (F_1, E) and (G_1, E) based on Hamming distance is given by
$S(F_1, G_1) = \frac{1}{1+d_H(F_1,G_1)} = \frac{1}{1+1.2} = \frac{1}{2.2} \cong 0.45.$

Example 3.9. Let U=$\{x_1,x_2,x_3,x_4\}$ be the universe and E=$\{e_1,e_2,e_3\}$ be the set of parameters. We consider two IVFSSs (F_1, E) and (G_1, E) such that their tabular forms are as follows.

Tabular form of (F_1, E):

	e_1	e_2	e_3
x_1	$[0.2, 0.9]$	$[0.0, 1.0]$	$[0.2, 0.8]$
x_2	$[0.4, 0.8]$	$[0.3, 0.9]$	$[0.3, 1.0]$
x_3	$[0.4, 1.0]$	$[0.3, 0.7]$	$[0.0, 0.7]$
x_4	$[0.1, 0.9]$	$[0.5, 1.0]$	$[0.3, 0.8]$

Tabular form of (G_1, E):

	e_1	e_2	e_3
x_1	$[0.1, 0.9]$	$[0.4, 1.0]$	$[0.0, 0.8]$
x_2	$[0.2, 0.7]$	$[0.1, 0.9]$	$[0.3, 1.0]$
x_3	$[0.0, 0.8]$	$[0.4, 0.9]$	$[0.2, 0.7]$
x_4	$[0.2, 1.0]$	$[0.0, 1.0]$	$[0.3, 1.0]$

Now by definition 3.6 the Hamming distance between (F_1, E) and (G_1, E) is given by

$$d_H(F_1, G_1) = \frac{1}{2.3}\Big[\sum_{i=1}^{3}\sum_{j=1}^{4}\Big(\Big|M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\Big| +$$

$$\Big|M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\Big| + \Big|W_{F_1}(e_i)(x_j) - W_{G_1}(e_i)(x_j)\Big|\Big)\Big]$$

$$= \frac{1}{6}[(0.1 + 0 + 0.1) + (0.4 + 0 + 0.4) + (0.2 + 0 + 0.2) +$$

$$(0.2 + 0.1 + 0.1) + (0.2 + 0 + 0.2) + (0 + 0 + 0) + (0.4 + 0.2 + 0.2) +$$

$$(0.1 + 0.2 + 0.1) + (0.2 + 0 + 0.2) + (0.1 + 0.1 + 0) +$$

$$(0.5 + 0 + 0.5) + (0 + 0.2 + 0.2)$$

$$= \frac{5.4}{6} = 0.9$$

Therefore by equation (2) the similarity measure between (F_1, E) and (G_1, E) based on Hamming distance is given by

$S(F_1, G_1) = \frac{1}{1+d_H(F_1,G_1)} = \frac{1}{1+0.9} = \frac{1}{1.9} \cong 0.52.$

Similarity Measure of interval-valued fuzzy soft sets based on set theoretic approach:

Definition 3.10. Let U=$\{x_1,x_2,x_3,\cdots,x_n\}$ be the universe and E=$\{e_1,e_2,e_3,\cdots,e_m\}$ be the set of parameters. Let (F_1, E) and (G_1, E) be two interval valued-fuzzy soft sets(IVFSSs) over the universe U and the set of parameters E. Then the similarity measure between (F_1, E) and (G_1, E) denoted by $S(F_1, G_1)$ is defined as

$$S(F_1, G_1) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\Big|M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\Big| \wedge \Big|M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\Big|\Big)}{\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\Big|M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\Big| \vee \Big|M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\Big|\Big)} \quad ...(3)$$

Example 3.11. Let U=$\{x_1,x_2,x_3\}$ be the universe and E=$\{e_1,e_2,e_3\}$ be the set of parameters. We consider two IVFSSs (F_1, E) and (G_1, E) such that their tabular forms are as follows.

Tabular form of (F_1, E):

	e_1	e_2	e_3
x_1	$[0.7, 0.9]$	$[0.6, 0.7]$	$[0.5, 0.8]$
x_2	$[0.6, 0.8]$	$[0.2, 0.5]$	$[0.6, 0.9]$
x_3	$[0.5, 0.6]$	$[0.0, 0.7]$	$[0.2, 1.0]$

Tabular form of (G_1, E):

	e_1	e_2	e_3
x_1	$[0.2, 0.8]$	$[0.4, 0.9]$	$[0.3, 0.6]$
x_2	$[0.4, 0.7]$	$[0.4, 0.5]$	$[0.8, 0.9]$
x_3	$[0.0, 1.0]$	$[0.2, 0.5]$	$[0.8, 1.0]$

Now by definition 3.10 the similarity measure between (F_1, E) and (G_1, E) is given by

$$S(F_1, G_1)$$
$$= \frac{\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\left|M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\right| \wedge \left|M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\right|\right)}{\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\left|M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\right| \vee \left|M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\right|\right)}$$
$$= \frac{(0.5\wedge0.1)+(0.2\wedge0.2)+(0.2\wedge0.2)+(0.2\wedge0.1)+(0.2\wedge0)+(0.2\wedge0)+(0.5\wedge0.4)+(0.2\wedge0.2)+(0.6\wedge0)}{(0.5\vee0.1)+(0.2\vee0.2)+(0.2\vee0.2)+(0.2\vee0.1)+(0.2\vee0)+(0.2\vee0)+(0.5\vee0.4)+(0.2\vee0.2)+(0.6\vee0)}$$
$$= \frac{0.1+0.2+0.2+0.1+0+0+0.4+0.2+0}{0.5+0.2+0.2+0.2+0.2+0.2+0.5+0.2+0.6}$$
$$= \frac{1.2}{2.8} \cong 0.42$$

Example 3.12. Let U={x_1,x_2,x_3,x_4} be the universe and E={e_1,e_2,e_3} be the set of parameters. We consider two IVFSSs (F_1, E) and (G_1, E) such that their tabular forms are as follows.

Tabular form of (F_1, E):

	e_1	e_2	e_3
x_1	$[0.2, 0.9]$	$[0.0, 1.0]$	$[0.2, 0.8]$
x_2	$[0.4, 0.8]$	$[0.3, 0.9]$	$[0.3, 1.0]$
x_3	$[0.4, 1.0]$	$[0.3, 0.7]$	$[0.0, 0.7]$
x_4	$[0.1, 0.9]$	$[0.5, 1.0]$	$[0.3, 0.8]$

Tabular form of (G_1, E):

	e_1	e_2	e_3
x_1	$[0.1, 0.9]$	$[0.4, 1.0]$	$[0.0, 0.8]$
x_2	$[0.2, 0.7]$	$[0.1, 0.9]$	$[0.3, 1.0]$
x_3	$[0.0, 0.8]$	$[0.4, 0.9]$	$[0.2, 0.7]$
x_4	$[0.2, 1.0]$	$[0.0, 1.0]$	$[0.3, 1.0]$

Now by definition 3.10 the similarity measure between (F_1, E) and (G_1, E) is given by

$$S(F_1, G_1)$$
$$= \frac{\sum_{i=1}^{3}\sum_{j=1}^{4}\Big(\Big|M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\Big| \wedge \Big|M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\Big|\Big)}{\sum_{i=1}^{3}\sum_{j=1}^{4}\Big(\Big|M_{F_1L}(e_i)(x_j) - M_{G_1L}(e_i)(x_j)\Big| \vee \Big|M_{F_1U}(e_i)(x_j) - M_{G_1U}(e_i)(x_j)\Big|\Big)}$$
$$= \frac{(0.1 \wedge 0) + (0.4 \wedge 0) + (0.2 \wedge 0) + (0.2 \wedge 0.1) + (0.2 \wedge 0) + (0 \wedge 0) +}{(0 \vee 0) + (0.1 \vee 0) + (0.4 \vee 0) + (0.2 \vee 0) + (0.2 \vee 0.1) + (0.2 \vee 0) +}$$
$$\frac{(0.4 \wedge 0.2) + (0.1 \wedge 0.2) + (0.2 \wedge 0) + (0.1 \wedge 0.1) + (0.5 \wedge 0) + (0 \wedge 0.2)}{(0.4 \vee 0.2) + (0.1 \vee 0.2) + (0.2 \vee 0) + (0.1 \vee 0.1) + (0.5 \vee 0) + (0 \vee 0.2)}$$
$$= \frac{(0 + 0 + 0 + 0.1 + 0 + 0 + 0.2 + 0.1 + 0 + 0.1 + 0 + 0)}{(0.1 + 0.4 + 0.2 + 0.2 + 0.2 + 0 + 0.4 + 0.2 + 0.2 + 0.1 + 0.5 + 0.2)}$$
$$= \frac{0.5}{2.7} \cong 0.185$$

Definition 3.13. Let (F, A) and (G, B) be two IVFSSs over U. Then (F, A) and (G, B) are said be $\alpha-$similar , denoted by $(F, A) \overset{\alpha}{=} (G, B)$ if and only if $S((F, A), (G, B)) > \alpha$ for $\alpha \in (0, 1)$. We call the two IVFSSs significantly similar if $S((F, A), (G, B)) > \frac{1}{2}$.

Example 3.14. Let us consider the example **3.4** . In this example the similarity measure between the IVFSSs (F, A) and (G, B) , where $A = B = E = \{e_1, e_2, e_3\}$ is $S(F, G) = 0.2972 < \frac{1}{2}$. Therefore (F, A) and (G, B) are not significantly similar. But if we consider the example **3.5** then $S(F, G) = 0.847 > \frac{1}{2}$. Therefore (F, A) and (G, B) are significantly similar, where $A = B = E = \{e_1, e_2, e_3\}$.

4. Decision making method

In this section we construct a decision making method based on similarity measure of two interval valued fuzzy soft sets (IVFSSs). The algorithm of this method can be given as follows:

Step 1. Construct a IVFSS (F,A) over the universe U based on an expert.
Step 2. Construct a IVFSS (G,A) over the universe U based on a responsible person for the problem.
Step 3. Calculate the distances of (F,A) and (G,A).
Step 4. Calculate similarity measure of (F,A) and (G,A).
Step 5. Estimate result by using the similarity.

Now we are giving an example for the decision making method.The similarity measure of two IVFSSs based on Hamming distance can be applied to detect whether a ill person is suffering from a certain disease or not. In this problem we will try to estimate the possibility that an ill person having certain symptoms is suffering from typhoid . For this we first construct a IVFSS for illness and IVFSS for ill person . Then we find the similarity measure of these two IVFSSs. If they are significantly similar then we conclude that the person is possibly suffering from typhoid.

Example 4.1. Assume that the universal set U contains only two elements x_1 (typhoid) and x_2 (not typhoid) i.e. $U = \{x_1, x_2\}$. Here the set of parameters E, is a set of certain visible symptoms. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, where e_1 = bone pain, e_2 = headache, e_3 = loss of appetite, e_4 = weight loss, e_5 = wounds, e_6 = chest pain.

Step 1: Construct a IVFSS (F, A) over U for typhoid as given below, which can be prepared with the help of a medical person.

(F, A)	e_1	e_2	e_3	e_4	e_5	e_6
x_1	[0.2,0.7]	[0.3,0.8]	[0.7,1.0]	[0.4,0.8]	[0.5,0.7]	[0.2,0.6]
x_2	[0.1,0.3]	[0.2,0.5]	[0.4,0.6]	[0.3,0.4]	[0.5,0.6]	[0.3,0.5]

Step 2: Construct a IVFSS (G, B) over U based on data of ill person as given below.

(G, B)	e_1	e_2	e_3	e_4	e_5	e_6
x_1	[0.8,1.0]	[0.0,0.2]	[0.1,0.3]	[0.0,0.2]	[0.1,0.2]	[0.7,1.0]
x_2	[0.8,0.9]	[0.7,1.0]	[0.0,0.1]	[0.9,1.0]	[0.9,1.0]	[0.4,1.0]

Where $A = B = E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.

Step 3: Calculate Hamming distance of (F, A) and (G, B):

Now by definition **3.6** the Hamming distance between (F, A) and (G, B) is given by

$$
\begin{aligned}
d_H(F,G) = \frac{1}{2.6}\Bigg[\sum_{i=1}^{6}\sum_{j=1}^{2}\Big(\Big|M_{FL}(e_i)(x_j) - M_{GL}(e_i)(x_j)\Big| + \\
\Big|M_{FU}(e_i)(x_j) - M_{GU}(e_i)(x_j)\Big| + \Big|W_F(e_i)(x_j) - W_G(e_i)(x_j)\Big|\Big)\Bigg] \\
= \frac{1.2 + 1.2 + 1.4 + 1.2 + 1.0 + 1.0 + 1.4 + 1.0 + 1.0 + 1.2 + 0.8 + 1.0}{12} \\
= \frac{13.4}{12} \cong 1.117.
\end{aligned}
$$

Step 4: Calculate similarity measure of (F, A) and (G, B):

By equation (**2**) the similarity measure between (F, A) and (G, B) based on Hamming distance is given by
$S(F,G) = \frac{1}{1+d_H(F,G)} = \frac{1}{1+1.117} = \frac{1}{2.117} \cong 0.47 < \frac{1}{2}$.
Step 5: Here the two IVFSSs i.e. two sets of symptoms (F, A) and (G, B) are not significantly similar, therefore we conclude that the person is not possibly suffering from typhoid.

Example 4.2. Let us consider example **4.1** with different person.

Step 1: Construct a IVFSS (F, A) over U for typhoid as given below, which can be prepared with the help of a medical person.

Step 2: Construct a IVFSS (J, C) over U based on data of ill person as given below.

(J, C)	e_1	e_2	e_3	e_4	e_5	e_6
x_1	[0.1,0.6]	[0.1,0.4]	[0.3,0.9]	[0.3,0.8]	[0.3,0.7]	[0.3,0.7]
x_2	[0.2,0.5]	[0.3,0.8]	[0.4,0.7]	[0.2,0.5]	[0.4,0.8]	[0.4,0.6]

Where $A = C = E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.

Step 3: Calculate Hamming distance of (F, A) and (J, C):

Now by definition **3.6** the Hamming distance between (F, A) and (J, C) is given by

$$d_H(F, J) = \frac{1}{2.6}\Bigg[\sum_{i=1}^{6}\sum_{j=1}^{2}\Big(\Big|M_{FL}(e_i)(x_j) - M_{JL}(e_i)(x_j)\Big| + \Big|M_{FU}(e_i)(x_j) - M_{JU}(e_i)(x_j)\Big| + \Big|W_F(e_i)(x_j) - W_J(e_i)(x_j)\Big|\Big)\Bigg]$$

$$= \frac{0.2 + 0.8 + 0.8 + 0.2 + 0.4 + 0.2 + 0.4 + 0.6 + 0.2 + 0.4 + 0.6 + 0.2}{12}$$

$$= \frac{5}{12} \cong 0.42.$$

Step 4: Calculate similarity measure of (F, A) and (J, C):

By equation (**2**) the similarity measure between (F, A) and (J, C) based on Hamming distance is given by

$S(F, J) = \frac{1}{1+d_H(F,J)} = \frac{1}{1+0.42} = \frac{1}{1.42} \cong 0.704 > \frac{1}{2}$.

Step 5: Here the two IVFSSs i.e. two sets of symptoms (F, A) and (J, C) are significantly similar, therefore we conclude that the person is possibly suffering from typhoid.

5. Conclusion

In this paper we have defined three types of similarity measure between two IVFSSs and proposed similarity measures of two IVFSSs. Then we construct a decision making method based on similarity measures. Finally we give two simple examples to show the possibilities of diagnosis of diseases. In these examples if we use the other distances, we can obtain similar results. Thus we can use the method to solve the problem that contain uncertainty such as problem in social, economic system, pattern recognition, medical diagnosis, game theory coding theory and so on. In future we will develop the theory of similarity measure of interval valued intuitionistic fuzzy soft sets.

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