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Multi fuzzy subrings and ideals

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Abstract. In this paper the notions of multi fuzzy subring, multi fuzzy ideal were introduced and some elementary properties of the same were studied.

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1. Introduction

Zadeh generalized the notion of subset of a set to that of fuzzy subset of a set in his pioneering paper Zadeh [6] in the year 1965. According to Zadeh, a fuzzy subset of a set X is any function $A : X \to [0,1]$, where $[0,1]$ is the closed interval 0, 1 of real numbers. For any element x in X, the degree of belonging of x to A is the number Ax in $[0,1]$. If $Ax = 1$, then x belongs to A *wholly* and if $Ax = 0$ then x does *not* belong to A and if Ax is some number between 0 and 1 then x belongs to A to the degree of belonging Ax.

Recently, generalizing the notion of fuzzy subset of a set, the notion of multi fuzzy subset of a set is introduced by Sabu Sebastian and T. V. Ramakrishnan [1], according to which a multi fuzzy subset A of a set X is a family of fuzzy subsets $(A_j)_{j\in J}$, where $A_j : X \to L_j$ is an L_j-fuzzy subset of X for each $j \in J$.

If A is any multi fuzzy subset of a crisp set X, for any element x in X, the degree of belonging of x to A, Ax is the element $(A_jx)_{j\in J}$ in $\Pi_{j\in J}L_j$.

After introducing, multi fuzzy subsets of a crisp set, they have also introduced and studied some elementary properties of multi fuzzy subgroups in Sabu Sebastian and T. V. Ramakrishnan [2].

Now the aim of this paper is to introduce and study some elementary properties of multi fuzzy subrings, multi fuzzy (left, right) ideals.

Through out the paper X, Y, Z are crisp sets; I, J, K are index sets; $(L_j)_{j\in J}$ and $(M_i)_{i\in I}$ are families of complete lattices and P^Q is the set of all functions from Q to P. The Cartesian products $\Pi_{j\in J}L_j$ and $\Pi_{i\in I}M_i$ are products of complete lattices and are themselves complete lattices with point wise ordering. The Cartesian products $\Pi_{j\in J}L_j^Y$ and $\Pi_{i\in I}M_i^X$ are also complete lattices.

2. Preliminaries

Definition 2.1 ([4]). Let $'$: $M \to M$ and $'$: $L \to L$ be order reversing involutions. A mapping $h : M \to L$ is called an order homomorphism, if it satisfies the conditions $h(0) = 0$, $h(\vee a_i) = \vee h(a_i)$ and $h^{-1}(b') = (h^{-1}(b))'$.

$h^{-1} : L \to M$ is defined by $\forall b \in L$, $h^{-1}(b) = \vee\{a \in M : h(a) \leq b\}$. Wang[7] proved the following properties of order homomorphism. For every $a \in M$ and $p \in L$; $a \leq h^{-1}(h(a))$, $h(h^{-1}(p)) \leq p$, $h^{-1}(1_L) = 1_M$, $h^{-1}(0_L) = 0_M$ and $a \leq h^{-1}(p)$ iff $h(a) \leq p$ iff $h^{-1}(p') \leq a'$. Both h and h^{-1} are order preserving and arbitrary join preserving maps. Moreover $h^{-1}(\wedge a_i) = \wedge h^{-1}(a_i)$.

Proposition 2.2 ([5]). *Let $f : L_1 \to L_2$ be a mapping. If f is injective, then $f^{-1}(f(a)) = a, \forall a \in L_1$ and if f is surjective, then $f(f^{-1}(b)) = b, \forall b \in L_2$.*

Definition 2.3 ([1]). Let $A = (A_i)_{i\in I}$ and $B = (B_i)_{i\in I}$ be a pair of multi fuzzy subsets of X with A_i, $B_i : X \to L_i$ for each $i \in I$. Then
(a) $A \subseteq B$ iff $A_ix \leq B_ix$ for each $i \in I$ and for each $x \in X$
(b) $A = B$ iff $A_ix = B_ix$ for each $i \in I$ and for each $x \in X$
(c) $(A \cup B)(x) = (A_ix \vee B_ix)_{i\in I}$ for each $x \in X$
(d) $(A \cap B)(x) = (A_ix \wedge B_ix)_{i\in I}$ for each $x \in X$.

Proposition 2.4 ([2]). *Let $A, B, C \in \Pi M_i^X$ be any multi fuzzy sets in X then:*
(a) $A \cup A = A$, $A \cap A = A$;
(b) $A \subseteq A \cup B$, $B \subseteq A \cup B, A \cap B \subseteq A$ and $A \cap B \subseteq B$;
(c) $A \subseteq B$ iff $A \cup B = B$ iff $A \cap B = A$.

Proposition 2.5 ([3]). *Let $A \in \Pi M_i^X$ and for any $\alpha \in \Pi M_i$, the set $A_\alpha = \{x \in X : A(x) \geq \alpha, \alpha \in \Pi M_i\}$ be the α-level of A. $A, B \in \Pi M_i^X$, then for every $\alpha, \beta \in \Pi M_i$:*
(a) $\alpha \leq \beta$ implies $A_\beta \subseteq A_\alpha$
(b) $A \subseteq B$ iff $A_\alpha \subseteq B_\alpha$
(c) $A = B$ iff $A_\alpha = B_\alpha$.

Definition 2.6 ([3]). Let $f : X \to Y$ and $h : \Pi M_i \to \Pi L_j$ be functions. The multi-fuzzy extension of f and the inverse of the extension are $f : \Pi M_i^X \to \Pi L_j^Y$ and $f^{-1} : \Pi L_j^Y \to \Pi M_i^X$ defined by $f(A)(y) = \bigvee_{y=f(x)} h(A(x))$, $A \in \Pi M_i^X, y \in Y$ and $f^{-1}(B)(x) = h^{-1}(B(f(x)))$, $B \in \Pi L_j^Y, x \in X$

where h^{-1} is the upper adjoint [7] of h. The function $h : \Pi M_i \to \Pi L_j$ is called the **Bridge function** of the multi-fuzzy extension of f.

Theorem 2.7 ([3]). *If an order homomorphism $h : \Pi M_i \to \Pi L_j$ is the bridge function for the multi-fuzzy extension of a crisp function $f : X \to Y$, then for any $k \in K$, $A_k \in \Pi M_i^X, B_k \in \Pi L_j^Y$:*
(a) $f(0_X) = 0_Y$ (b) $A_1 \subseteq A_2$ implies $f(A_1) \subseteq f(A_2)$
(c) $f(\cup A_k) = \cup f(A_k)$ (d) $f(\cap A_k) \subseteq \cap f(A_k)$
(e) $f(A_\alpha) \subseteq f(A)_{h(\alpha)}$ (f) $f^{-1}(1_Y) = 1_X$ and $f^{-1}(0_Y) = 0_X$
(g) $B_1 \subseteq B_2$ implies $f^{-1}(B_1) \subseteq f^{-1}(B_2)$ (h) $f^{-1}(\cup B_k) = \cup f^{-1}(B_k)$

(i) $f^{-1}(\cap B_k) = \cap f^{-1}(B_k)$ *(j)* $(f^{-1}(B))' = f^{-1}(B')$
(k) $A \subseteq f^{-1}(f(A))$ *(l)* $f(f^{-1}(B)) \subseteq B$.

Theorem 2.8 ([3]). *(i) If $f : X \to Y$ and $h : \Pi M_i \to \Pi L_j$ are injective (that is $f^{-1}(f(x)) = x, \forall x \in X$ and $h^{-1}(h(m)) = m \in \Pi M_i$), then $f^{-1}(f(A)) = A$, for every $A \in \Pi M_i^X$. Moreover the multi-fuzzy extension $f : \Pi M_i^X \to \Pi L_j^Y$ is injective.*

(ii) If $f : X \to Y$ and $h : \Pi M_i \to \Pi L_j$ are surjective, then $f(f^{-1}(B)) = B$, for every $B \in \Pi L_j^Y$.

3. Multi fuzzy subrings

In this section first we introduce the notion of multi fuzzy sub ring then show that any intersection of multi fuzzy sub rings is a multi fuzzy sub ring, image and inverse image of a multi fuzzy sub ring is a multi fuzzy sub ring.

Definition 3.1. A multi fuzzy sub set A of a ring R is called a multi fuzzy sub ring of R iff
(1) $A(x-y) \geq A(x) \wedge A(y)$
(2) $A(xy) \geq A(x) \wedge A(y)$.

Examples: (1) Let R be a ring and A be an L-fuzzy sub ring of R. Then by the definition of L-fuzzy sub ring, A satisfies $A(x-y) \geq A(x) \wedge A(y)$ and $A(xy) \geq A(x) \wedge A(y)$, for all x, y in R. Clearly, letting $I = \{1\}$ and letting $A = \{A_1\}$, A is a multi fuzzy sub ring of R.

(2) Let R be a ring and L_1 and L_2 be a pair of complete lattices. Let α_1 in L_1 and α_2 in L_2 be a pair of arbitrary but fixed elements. Let $I = \{1, 2\}$ and let A_i be the constant map from R to L_i assuming the value α_i, for $i = 1, 2$. Then one can easily see that $A = \{A_1, A_2\}$ is a multi fuzzy sub ring of R.

Theorem 3.2. *If $\{A_i/i \in I\}$ is a family of multi fuzzy sub rings of a ring R, then $\cap_{i\in I} A_i$ is a multi fuzzy sub ring of R.*

Proof. Let $A = \cap A_i$ and let $x, y \in R$.

(a) $A(x-y) = (\wedge A_i)(x-y) = \wedge_{i\in I} A_i(x-y) \geq \wedge_{i\in I}(A_i x \wedge A_i y) = (\wedge_{i\in I} A_i x) \wedge (\wedge_{i\in I} A_i y) = (\cap_{i\in I} A_i)(x) \wedge (\cap_{i\in I} A_i)(y) = Ax \wedge Ay$.

(b) $A(xy) = (\wedge A_i)(xy) = \wedge_{i\in I} A_i(xy) \geq \wedge_{i\in I}(A_i x \wedge A_i y) = (\wedge_{i\in I} A_i)x \wedge (\wedge_{i\in I} A_i)y = (\wedge_{i\in I} A_i)(x) \wedge (\wedge_{i\in I} A_i)(y) = Ax \wedge Ay$.

From (a) and (b) it follows that, $\cap_{i\in I} A_i$ is a multi fuzzy sub ring of R. □

Note: Union of two multi fuzzy sub rings need not be a multi fuzzy sub ring.
Example: Since every crisp sub ring S can be regarded as a $[0, 1]$-fuzzy sub ring via its characteristic map χ_S and every $[0, 1]$-fuzzy sub ring μ can be regarded as a multi fuzzy sub ring via the singleton family $\{\mu\}$, the sub rings $2Z, 3Z$ of the ring of integers Z can be regarded as a multi fuzz sub rings of Z. It is easy to see that their (multi fuzzy) union is not a (multi fuzzy) sub ring of Z.

Remark: When a multi fuzzy sub group of a group G is constant on a kernel of a group homomorphism $f : G \to H$, for all $y \in H$ there exists $u \in f^{-1}y$ such that $Au = \vee A f^{-1} y$ whenever $f^{-1}y \neq \phi$. First observe that for all $u, v \in f^{-1}y$, $uv^{-1} \in$

ker(f) which contains the identity of G. So $Auv^{-1} = Ae$. Now $Ae = \vee AG$ because for all $g \in G$, $Ae = Agg^{-1} \geq Ag \wedge Ag = Ag$, implying the assertion.

So $Au = Auv^{-1}v \geq Ae \wedge Av = Av$ and vice versa, implying $Au = Av$. Obviously, $Au = \vee Af^{-1}y$ for any $u \in f^{-1}y$.

Theorem 3.3. *Let R and S be a pair of rings, f be a ring homomorphism from R to S and a finite meet preserving order homomorphism $h : \Pi M_i \to \Pi L_j$ be the bridge function for the multi fuzzy extension of f such that each L_j is complete infinite meet distributive lattice.Then if A is a multi fuzzy sub ring of R, then $f(A)$ is a multi fuzzy sub ring of S.*

Proof. Let $c, d \in S$. If either $f^{-1}c$ or $f^{-1}d$ is empty, then $fAc \wedge fAd = 0$ and trivially we have $f(A)(x-y) \geq f(A)x \wedge f(A)y$ and $f(A)(xy) \geq f(A)x \wedge f(A)y$. Assume that neither $f^{-1}c$ nor $f^{-1}d$ is empty. Then there exist $u \in f^{-1}c$ and $v \in f^{-1}d$ such that $fu = c$ and $fv = d$.

Since each L_j is a complete infinite meet distributive lattice we shall get that $\Pi_{j \in J} L_j$ is a complete infinite meet distributive lattice and hence for any $\alpha \in \Pi_{j \in J} L_j$ and $(\beta_k)_{k \in K} \subseteq \Pi_{j \in J} L_j$, we have $\alpha \wedge (\vee_{k \in K} \beta_k) = \vee_{k \in K}(\alpha \wedge \beta_k)$.

(a) $c - d = fu - fv = f(u - v)$ and $u - v \in f^{-1}(c - d)$.

Further, $fA(c - d) = \vee hAf^{-1}(c - d) \geq hA(u - v) \geq h(Au \wedge Av) = hAu \wedge hAv$ for all $u \in f^{-1}c$ and $v \in f^{-1}d$.

Therefore $fA(c - d) \geq \vee_{v \in f^{-1}d}(hAu \wedge hAv) = hAu \wedge \vee_{v \in f^{-1}d} hAv = hAu \wedge \vee hAf^{-1}d = hAu \wedge fAd$ for all $u \in f^{-1}c$, since $\Pi_{j \in J} L_j$ is complete infinite meet distributive lattice.

Again $fA(c-d) \geq \vee_{u \in f^{-1}c}(hAu \wedge fAd) = (\vee_{u \in f^{-1}c} hAu) \wedge fAd = \vee hAf^{-1}c \wedge fAd = fAc \wedge fAd$, again since $\Pi_{j \in J} L_j$ is a complete infinite meet distributive lattice.

(b) $c.d = fu.fv = f(u.v)$ and $u.v \in f^{-1}(c.d)$.

Further, $fA(c.d) = \vee hAf^{-1}(c.d) \geq hA(uv) \geq h(Au \wedge Av) = hAu \wedge hAv$ for all $u \in f^{-1}c$ and $v \in f^{-1}d$.

Therefore $fA(cd) \geq \vee_{v \in f^{-1}d}(hAu \wedge hAv) = hAu \wedge \vee_{v \in f^{-1}d} hAv = hAu \wedge \vee hAf^{-1}d = hAu \wedge fAd$ for all $u \in f^{-1}c$, since $\Pi_{j \in J} L_j$ is complete infinite meet distributive lattice.

From (a) and (b) it follows that $f(A)$ is a multi fuzzy sub ring of S. □

Theorem 3.4. *Let R and S be rings $f : R \to S$ be a ring homomorphism from R to S and order homomorphism $h : \Pi M_i \to \Pi L_j$ be a the bridge function for the multi fuzzy extension of f. Then for any multi-fuzzy sub ring B of S, $f^{-1}(B)$ is a multi-fuzzy sub ring of R.*

Proof. By the discussion after definition 1.1, we have h^{-1} is meet preserving. Let $x, y \in R$.

(a) $f^{-1}(B)(x - y) = h^{-1}B(f(x - y)) = h^{-1}B(f(x) - f(y)) \geq h^{-1}(B(f(x)) \wedge B(f(y))) = h^{-1}(B(f(x))) \wedge h^{-1}(B(f(y))) = f^{-1}(B)(x) \wedge f^{-1}(B)(y)$.

(b) $f^{-1}(B)(xy) = h^{-1}B(f(xy)) = h^{-1}B(f(x)f(y)) \geq h^{-1}(B(f(x)) \wedge B(f(y))) = h^{-1}(B(f(x))) \wedge h^{-1}(B(f(y))) = f^{-1}(B)(x) \wedge f^{-1}(B)(y)$.

From (a) and (b) it follows that $f^{-1}(B)$ is a multi fuzzy sub ring of R. □

Theorem 3.5. *Let f be an injective ring homomorphism from ring R to S, an injective meet preserving order homomorphism $h : \Pi M_i \to L_j$ be the bridge function* *for the multi fuzzy extension of f and $\{A_{i/i\in I}\}$ be a family of multi fuzzy sub rings of R. Then the multi fuzzy set $\cup A_i$ is a multi fuzzy sub ring of R iff $\cup f(A_i)$ is a multi fuzzy sub ring of S.*

Proof. Assume that $\cup A_i$ is a multi fuzzy sub ring of R. Theorem 2.7(c) and 3.3 together imply $\cup f(A_i) = f(\cup A_i)$ is a multi fuzzy sub ring of S. Conversely assume that $\cup f(A_i)$ is a multi fuzzy sub ring of S.

Theorem 2.8 and 2.7(h) and 3.4 together imply

$$\cup A_i = \cup f^{-1}(f(A_i)) = f^{-1}(\cup f(A_i))$$

is a multi fuzzy sub ring of R. □

4. Multi fuzzy ideals

In this section we introduce the notion of multi fuzzy (left,right) ideal of a ring R. Further we show that any intersection of multi fuzzy (left,right) ideals is a multi fuzzy (left,right) ideal, level set criteria for multi fuzzy sub rings, multi fuzzy sub ring and multi fuzzy (left,right) ideals and (inverse) image of a multi fuzzy (left,right) ideal is a multi fuzzy (left,right) ideal.

Definition 4.1. A multi fuzzy sub ring A of a ring R is called a multi fuzzy left ideal iff $A(rx) \geq A(x)\ \forall r, x \in R$.

Definition 4.2. A multi fuzzy sub ring A of a ring R is called a multi fuzzy right ideal iff $A(xr) \geq A(x)\ \forall r, x \in R$.

Definition 4.3. A multi fuzzy sub ring A of a ring R is called a multi fuzzy ideal iff $A(rx) \wedge A(xr) \geq A(x)\ \forall r, x \in R$.

Examples: (1) Let R be a ring and A be an L-fuzzy left ideal of R. Then by the definition of L-fuzzy (left,right) ideal, A satisfies $A(x - y) \geq A(x) \wedge A(y)$ and $A(xy) \geq A(y)$, for all x, y in R. Clearly, letting $I = \{1\}$ and letting $A = \{A_1\}$, A is a multi fuzzy left ideal of R.

(2) Let R be a ring and L_1 and L_2 be a pair of complete lattices. Let α_1 in L_1 and α_2 in L_2 be a pair of arbitrary but fixed elements. Let $I = \{1, 2\}$ and let A_i be the constant map from R to L_i assuming the value α_i, for $i = 1, 2$. Then one can easily see that $A = \{A_1, A_2\}$ is a multi fuzzy (left,right) ideal of R.

Theorem 4.4. *If $\{A_i/i \in I\}$ is a family of multi fuzzy left ideals of a ring R, then $\cap A_i$ is a multi fuzzy left ideal of R.*

Proof. Let $A = \cap_{i\in I} A_i$. Since meet of any family of multi fuzzy sub rings is a multi fuzzy sub ring.

So, it is enough to show that $A(rx) \geq A(x) \forall r, x \in R$, to show A is a multi fuzzy left ideal.

$A(rx) = (\wedge_{i\in I} A_i)(rx) = \wedge_{i\in I}(A_i(rx)) \geq \wedge_{i\in I}(A_i(x)) = (\wedge_{i\in I} A_i)(x) = A(x)$. □

Theorem 4.5. *If $\{A_i/i \in I\}$ is a family of multi fuzzy right ideals of a ring R, then $\cap A_i$ is a multi fuzzy right ideal of R.*

Proof. Let $A = \cap_{i\in I} A_i$. Since meet of any family of multi fuzzy sub rings is a multi fuzzy sub ring.

so, it is enough to show that $A(xr) \geq A(x) \forall r, x \in R$, to show A is a multi fuzzy right ideal.

$A(xr) = (\wedge_{i\in I} A_i)(xr) = \wedge_{i\in I}(A_i(xr)) \geq \wedge_{i\in I}(A_i(x)) = (\wedge_{i\in I} A_i)(x) = A(x)$. □

Theorem 4.6. *If $\{A_i / i \in I\}$ is a family of multi fuzzy ideals of a ring R, then $\cap A_i$ is a multi fuzzy ideal of R.*

Proof. It follows from above two theorems. □

Lemma 4.7. *Let A be a multi fuzzy sub set of a ring R. Then*

(1) A is a multi fuzzy sub ring R iff each non empty level sub set A_α of A is a sub ring of R.

(2) A is a multi fuzzy left ideal R iff each non empty level sub set A_α of A is a left ideal of R.

(3) A is a multi fuzzy right ideal R iff each non empty level sub set A_α of A is a right ideal of R.

(4) A is a multi fuzzy ideal iff each non empty level sub set A_α of A is a ideal of R.

Proof. (1)($\Rightarrow$): Assume that A is a multi fuzzy sub ring of R. Let $\alpha \in R$ be arbitrary but fixed. $x, y \in A_\alpha$ implies $\alpha \leq A(x)$, $\alpha \leq A(y)$. Then (a) $\alpha \leq A(x) \wedge A(y) \leq A(x-y)$ implying $x - y \in A_\alpha$.

(b) $\alpha \leq A(x) \wedge A(y) \leq A(xy)$ implying $xy \in A_\alpha$.

Therefore A_α is a sub ring of R.

($\Leftarrow$): Assume that each non empty level sub set A_α of A is a sub ring of R.

To show that A is multi fuzzy sub ring R, we need to show that (a) $A(x-y) \geq Ax \wedge Ay$ and (b) $A(xy) \geq Ax \wedge Ay$.

(a) Let $\alpha = Ax \wedge Ay$. Then $x, y \in A_\alpha$ because $Ax, Ay \geq Ax \wedge Ay = \alpha$.

Since A_α is a non empty sub ring of R, $x - y \in A_\alpha$ implies $A(x-y) \geq \alpha = Ax \wedge Ay$.

(b) Let $\alpha = Ax \wedge Ay$. Then $x, y \in A_\alpha$ because $Ax, Ay \geq Ax \wedge Ay = \alpha$.

Since A_α is a non empty sub ring of R, $xy \in A_\alpha$ implies $A(xy) \geq \alpha = Ax \wedge Ay$.

(2)($\Rightarrow$): Suppose A is a multi fuzzy left ideal.

To show that A_α is left ideal of R for each $\alpha \in \Pi_{j\in J} L_j$, whenever A_α is non empty.

$x \in A_\alpha$ implies $\alpha \leq A(x)$. Since A is a multi fuzzy left ideal, $A(rx) \geq A(x)$. Therefore $\alpha \leq A(x) \leq A(rx)$ that implies $rx \in A_\alpha$.

($\Leftarrow$): Suppose each non empty level sub set A_α of A is left ideal of R.

To show that A is a multi fuzzy left ideal of R, we need to show that $A(rx) \geq Ax$ for all $x, r \in R$.

Let $r, x \in R$ and $\alpha = Ax$. Then $x \in A_\alpha$. Since A_α is a left ideal of R, $rx \in A_\alpha$. Therefore $A(rx) \geq \alpha = Ax$.

(3) Proof of this is similar to the one of (2) above.

(4) Proof of this follows by (2) and (3) above. □

Theorem 4.8. *Let R and S be rings $f : R \to S$ be a ring homomorphism from R to S and order homomorphism $h : \Pi M_i \to \Pi L_j$ be the bridge function for the multi fuzzy extension off f.Then the following are true for any multi fuzzy subset B of S:*

(1) B is a multi fuzzy left ideal of S implies $f^{-1}B$ is a multi fuzzy left ideal of R

(2) B is a multi fuzzy right ideal of S implies $f^{-1}B$ is a multi fuzzy right ideal of R

(3) B is a multi fuzzy ideal of S implies $f^{-1}B$ is a multi fuzzy ideal of R.

Proof. (1) In view of Theorem 2.4 above, it is enough to show that $(f^{-1}B)(rx) \geq f^{-1}(Bx)$.

But $(f^{-1}B)rx = h^{-1}(B(f(rx))) = h^{-1}(B(frfx)) \geq h^{-1}(B(fx)) = (f^{-1}B)x$.

(2) Proof of this is similar to the one of (1) above.

(3) Proof of this follows from (2) and (3). □

Theorem 4.9. *Let R, S be a pair of rings and let f be a ring homomorphism from R onto S. Let $h : \Pi_{i \in I} M_i \to \Pi_{j \in J} L_j$ be a bridge function which is meet preserving order homomorphism for the multi fuzzy extensions of f such that each L_j is a complete infinite meet distributive lattice. Then*

(a) If A is a multi fuzzy left ideal of R, then $f(A)$ is a multi fuzzy left ideal of S

(b) If A is a multi fuzzy right ideal of R, then $f(A)$ is a multi fuzzy right ideal of S

(c) If A is a multi fuzzy ideal of R, then $f(A)$ is a multi fuzzy ideal of S.

Proof. (a) In view of Theorem 2.3, it is enough to show that for all s, y in S, $fA(sy) \geq fAy$, which by definition of fA, amounts to showing $\vee hAf^{-1}(sy) \geq \vee hAf^{-1}y$.

If $f^{-1}y$ is empty then the inequality is trivially true.

Let $x \in f^{-1}y$ be arbitrary but fixed. Then $fx = y$. Since f is on to then is an $r \in R$ such that $fr = s$. So $sy = frfx = frx$ and $rx \in f^{-1}sy$.

Consequently, $(fA)(sy) = \vee hAf^{-1}(sy) \geq hArx \geq hAx$ for all $x \in f^{-1}y$ and so $(fA)(sy) \geq \vee hAf^{-1}y = (fA)(y)$.

(b) Proof of this is similar to that of (a) above.

(c) Proof of this follows from (a) and (b) above. □

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