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$\mathcal{C}$-set and a decomposition of bicontinuous difunctions

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Abstract. In this paper, the counterparts of $\mathcal{C}$-sets and and co-$\mathcal{C}$-sets for ditopological texture spaces are introduced. The $\mathcal{C}$-bicontinuity is defined and given a decomposition of bicontinuous difunction.

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1. Introduction

In general topology, decomposition of continuous function has been remarkable area. Tong [15] introduced the notion of $\mathcal{A}$-continuity and decomposed continuity into α-continuity and $\mathcal{A}$-continuity. In [9], some characterizations were given depending on these concepts for bicontinuous difunctions. On the other hand, in [11] (and [14]), the notion of $\mathcal{C}$-set for topological spaces (fuzzy topological spaces) and $\mathcal{C}$-continuity were introduced and given another decomposition of continuity. In this study we generalize these notions to ditopological texture spaces and give a decomposition of bicontinuous difunction.

Textures and ditopological texture spaces were first introduced by L. M. Brown as a point-based setting for the study fuzzy topology. This section concludes with some basic definitions from the theory, and the reader is referred to [1, 2, 3, 4, 5, 6] for more background material.

Definition 1.1 ([2])**.** Let S be a set. A texturing is a point-separating, complete, completely distributive lattice with respect to inclusion, which contains S and $\emptyset$, and for which arbitrary meets coincide with intersections, and finite joins with unions. If $\mathcal{S}$ is a texturing of S, then the pair $(S, \mathcal{S})$ is called a *texture space* or shortly, *texture*.

For a texture $(S,\mathcal{S})$, most properties are conveniently defined in terms of the *p-sets* $P_s=\bigcap\{A\in\mathcal{S}\mid s\in A\}$ and, as a dually, the *q-sets*, $Q_s=\bigvee\{A\in\mathcal{S}\mid s\notin A\}$. We note in particular that $A^\flat=\{s\in S\mid A\not\subseteq Q_s\}$ is called the *core* of $A\in\mathcal{S}$.

Examples 1.2. (1) If X is a set and $\mathcal{P}(X)$ the powerset of X, then $(X,\mathcal{P}(X))$ is the *discrete texture* on X. For $x\in X$, $P_x=\{x\}$ and $Q_x=X\setminus\{x\}$.

(2) The texture $(L,\mathcal{L})$ is defined by $L=(0,1]$, $\mathcal{L}=\{(0,r]\mid r\in\mathbb{I}\}$. For $r\in L$, $P_r=(0,r]=Q_r$.

(3) Setting $\mathbb{I}=[0,1]$, $\mathcal{I}=\{[0,r),[0,r]\mid r\in\mathbb{I}\}$ gives the *unit interval texture* $(\mathbb{I},\mathcal{I})$. For $r\in\mathbb{I}$, $P_r=[0,r]$ and $Q_r=[0,r)$.

Definition 1.3 ([1])**.** A *ditopology* on a texture $(S,\mathcal{S})$ is a pair (τ,κ) of subsets of $\mathcal{S}$, where the set of *open sets* τ and the set of closed sets κ satisfies

$$\begin{array}{ll} S,\varnothing\in\tau, & S,\varnothing\in\kappa,\\ G_1,G_2\in\tau \implies G_1\cap G_2\in\tau, & K_1,K_2\in\kappa\implies K_1\cup K_2\in\kappa,\\ G_i\in\tau,\ i\in I\implies \bigvee_i G_i\in\tau. & K_i\in\kappa,\ i\in I\implies \bigcap K_i\in\kappa. \end{array}$$

Hence a ditopology is essentially a "topology" for which there is no *a priori* relation between the open and closed sets. For $A\in\mathcal{S}$ we define the *closure* and the *interior* of A under (τ,κ) by the equalities

$$cl\,(A)=\bigcap\{K\in\kappa\mid A\subseteq K\},\quad int\,(A)=\bigvee\{G\in\tau\mid G\subseteq A\}.$$

We denote by $O(S,\mathcal{S},\tau,\kappa)$, or when there can be no confusion by $O(S)$, the set of open sets in $\mathcal{S}$. Likewise, $C(S,\mathcal{S},\tau,\kappa)$, $C(S)$ will denote the set of closed sets.

If a ditopology (τ,κ) on $(S,\mathcal{S})$ satisfies the following conditions it is called *clint* ditopology [9],

(1) $\forall A\in\tau$, $A\cap cl\,(B)\subseteq cl\,(A\cap B)$, $B\in\mathcal{S}$,
(2) $\forall A\in\kappa$, $int\,(A\cup B)\subseteq A\cup int\,(B)$, $B\in\mathcal{S}$.

Definition 1.4 ([2])**.** A mapping $\sigma:\mathcal{S}\to\mathcal{S}$ satisfying $\sigma(\sigma(A))=A$, $\forall A\in\mathcal{S}$ and $A\subseteq B\implies\sigma(B)\subseteq\sigma(A),\forall A,B\in\mathcal{S}$ is called a complementation on $(S,\mathcal{S})$ and $(S,\mathcal{S},\sigma)$ is said to be a *complemented texture*.

If (τ,κ) is a ditopology on a complemented texture $(S,\mathcal{S},\sigma)$ we say (τ,κ) is complemented if $\kappa=\sigma(\tau)$. In this case we have $\sigma(cl\,(A))= int(\sigma(A))$ and $\sigma(int(A))=cl(\sigma(A))$.

Let $(S,\mathcal{S}),(T,\mathcal{T})$ be textures. In the following definition we consider the product texture $\mathcal{P}(S)\otimes\mathcal{T}$. To avoid confusion $\overline{P}_{(s,t)},\overline{Q}_{(s,t)}$ are used to denote the p-sets and q-sets for the product texture $(S\times T,\mathcal{P}(S)\otimes\mathcal{T})$.

Definition 1.5 ([4])**.** Let $(S,\mathcal{S})$, $(T,\mathcal{T})$ be textures. Then

(1) $r\in\mathcal{P}(S)\otimes\mathcal{T}$ is called a *relation from* $(S,\mathcal{S})$ *to* $(T,\mathcal{T})$ if it satisfies
  R1 $r\not\subseteq\overline{Q}_{(s,t)},P_{s'}\not\subseteq Q_s\implies r\not\subseteq\overline{Q}_{(s',t)}$.
  R2 $r\not\subseteq\overline{Q}_{(s,t)}\implies\exists s'\in S$ such that $P_s\not\subseteq Q_{s'}$ and $r\not\subseteq\overline{Q}_{(s',t)}$.
(2) $R\in\mathcal{P}(S)\otimes\mathcal{T}$ is called a *corelation from* $(S,\mathcal{S})$ *to* $(T,\mathcal{T})$ if it satisfies
  CR1 $\overline{P}_{(s,t)}\not\subseteq R,P_s\not\subseteq Q_{s'}\implies\overline{P}_{(s',t)}\not\subseteq R$.
  CR2 $\overline{P}_{(s,t)}\not\subseteq R\implies\exists s'\in S$ such that $P_{s'}\not\subseteq Q_s$ and $\overline{P}_{(s',t)}\not\subseteq R$.

(3) A pair (r,R), where r is a relation and R a corelation from $(S,\mathcal{S})$ to $(T,\mathcal{T})$, is called a *direlation from* $(S,\mathcal{S})$ *to* $(T,\mathcal{T})$.

Definition 1.6 ([4]). Let (f,F) be a direlation from $(S,\mathcal{S})$ to $(T,\mathcal{T})$. Then (f,F) is called a *difunction from* $(S,\mathcal{S})$ *to* $(T,\mathcal{T})$ if it satisfies the following two conditions.

DF1 For $s,s' \in S$, $P_s \not\subseteq Q_{s'} \implies \exists\, t \in T$ with $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s',t)} \not\subseteq F$.

DF2 For $t,t' \in T$ and $s \in S$, $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s,t')} \not\subseteq F \implies P_{t'} \not\subseteq Q_t$.

Definition 1.7 ([5]). Let $(f,F):(S,\mathcal{S}) \to (T,\mathcal{T})$ be a difunction.

(1) For $A \in \mathcal{S}$, the *image* $f^{\to}(A)$ and the *co-image* $F^{\to}(A)$ are defined by
$f^{\to}(A) = \bigcap\{Q_t \mid \forall\, s,\ f \not\subseteq \overline{Q}_{(s,t)} \implies A \subseteq Q_s\}$,
$F^{\to}(A) = \bigvee\{P_t \mid \forall\, s,\ \overline{P}_{(s,t)} \not\subseteq F \implies P_s \subseteq A\}$.

(2) For $B \in \mathcal{T}$, the *inverse image* $f^{\leftarrow}(B)$ and the *inverse co-image* $F^{\leftarrow}(B)$ are defined by
$f^{\leftarrow}(B) = \bigvee\{P_s \mid \forall\, t,\ f \not\subseteq \overline{Q}_{(s,t)} \implies P_t \subseteq B\}$,
$F^{\leftarrow}(B) = \bigcap\{Q_s \mid \forall\, t,\ \overline{P}_{(s,t)} \not\subseteq F \implies B \subseteq Q_t\}$.

For a difunction, the inverse image and the inverse co-image are equal; and the image and co-image can't be equal.

Definition 1.8 ([4]). Let (f,F) be a difunction between the complemented textures $(S,\mathcal{S},\sigma)$ and $(T,\mathcal{T},\theta)$. The complement $(f,F)' = (F',f')$ of the difunction (f,F) is a difunction, where $f' = \bigcap\{\overline{Q}_{(s,t)} \mid \exists\, u,v$ with $f \not\subseteq \overline{Q}_{(u,v)}$, $\sigma(Q_s) \not\subseteq Q_u$ and $P_v \not\subseteq \theta(P_t)\}$ and $F' = \bigvee\{\overline{P}_{(s,t)} \mid \exists\, u,v$ with $\overline{P}_{(u,v)} \not\subseteq F$, $P_u \not\subseteq \sigma(P_s)$ and $\theta(Q_t) \not\subseteq Q_v\}$.

If $(f,F) = (f,F)'$ then the difunction (f,F) is called complemented.

Definition 1.9 ([5]). A difunction $(f,F):(S,\mathcal{S},\tau_S,\kappa_S) \to (T,\mathcal{T},\tau_T,\kappa_T)$ is called *continuous* if $B \in \tau_T \implies F^{\leftarrow}B \in \tau_S$, *cocontinuous* if $B \in \kappa_T \implies f^{\leftarrow}B \in \kappa_S$ and *bicontinuous* if it is both.

Finally, we also recall the some classes of ditopological texture spaces.

Definition 1.10. Let $(S,\mathcal{S},\tau,\kappa)$ be a ditopological texture space. $A \in \mathcal{S}$ is called pre-open [12] (respectively, semi-open [8], β-open [10], α-open [7], regular open [13]) if $A \subseteq int\,cl(A)$ (respectively, $A \subseteq cl\,int\,(A)$, $A \subseteq cl\,int\,cl(A)$, $A \subseteq int\,cl\,int\,(A)$, $A = int\,cl\,(A)$). A is called $\mathcal{A}$-set [9], if $A = G \cap K$, where G is open and K is regular closed. A is called locally closed [9], if $A = G \cap K$, where G is open and K is closed.

We denote by $PO(S,\mathcal{S},\tau,\kappa)$, more simply by $PO(S)$ the set of pre-open sets in $\mathcal{S}$. Likewise, it is denoted by $SO(S)$ (respectively, $\beta O(S)$, $O_\alpha(S)$, $RO(S)$, $\mathcal{A}O(S)$, $LC(S)$) the set of semi-open (respectively, β-open, α-open, regular open, $\mathcal{A}$-set, locally closed).

On the other hand, $A \in \mathcal{S}$ is called pre-closed (respectively, semi-closed, β-closed, α-closed, regular closed) if $cl\,int(A) \subseteq A$ (respectively, $int\,cl(A) \subseteq A$, $int\,cl\,int\,(A) \subseteq A$, $cl\,int\,cl(A) \subseteq A$, $A = cl\,int(A)$). A is called co-$\mathcal{A}$-set, if $A = K \cup G$, where K is closed and G is regular open. A is called locally-coclosed, if $A = K \cup G$, where K is closed and G is open.

We denote by $PC(S,\mathcal{S},\tau,\kappa)$, more simply by $PC(S)$ the set of pre-closed sets in $\mathcal{S}$. Likewise, it is denoted by $SC(S)$ (respectively, $\beta C(S)$, $C_\alpha(S)$, $RC(S)$, $\mathcal{A}C(S)$,

$LCC(S)$) the set of semi-closed (respectively, β-closed, α-closed, regular closed, co-$\mathcal{A}$-set, locally-coclosed).

Corresponding to the above concepts, we have variations of continuity.

Definition 1.11. A difunction $(f, F) : (S, \mathcal{S}, \tau_S, \kappa_S) \to (T, \mathcal{T}, \tau_T, \kappa_T)$ is called pre-continuous [13] (resp. semi-continuous [8], β-continuous [10], α-continuous [7],$\mathcal{A}$-continuous [9], locally-continuous [9]) if the inverse image under (f, F) of open set in τ_T is an pre-open (semi-open, β-open, α-open, $\mathcal{A}$-set, locally-closed).

On the other hand, (f, F) is called pre-cocontinuous (resp. semi-cocontinuous, β-cocontinuous, α-cocontinuous, $\mathcal{A}$-cocontinuous, locally-cocontinuous) if the inverse co-image under (f, F) of closed set in κ_T is an pre-closed (semi-closed, β-closed, α-closed, co-$\mathcal{A}$-set, locally-coclosed).

2. $\mathcal{C}$-sets in ditopological texture spaces

We begin by recalling [11] that a subset A of a topological space X is called $\mathcal{C}$-set if $A = O \cap C$, where O is open and C is pre-closed set. This leads to the following analogous concepts in a ditopological texture space. As expected, there is also the dual notion.

Definition 2.1. Let $(S, \mathcal{S}, \tau, \kappa)$ be ditopological texture space and $A \in \mathcal{S}$.

(1) A is called $\mathcal{C}$-set, if $A = G \cap K$, where $G \in O(S)$ and $K \in PC(S)$.
(2) A is called co-$\mathcal{C}$-set, if $A = K \cup G$, where where $K \in C(S)$ and $G \in PO(S)$.

We denote by $\mathcal{C}O(S, \mathcal{S}, \tau, \kappa)$, or when there can be no confusion by $\mathcal{C}O(S)$, the set of $\mathcal{C}$-sets in $\mathcal{S}$. Likewise, $\mathcal{C}C(S, \mathcal{S}, \tau, \kappa)$, or $\mathcal{C}C(S)$ will denote the set of co-$\mathcal{C}$-sets.

Remark 2.2. For any ditopological space $(S, \mathcal{S}, \tau, \kappa)$:

(1) $O(S), C(S) \subseteq \mathcal{C}O(S)$ and $O(S), C(S) \subseteq \mathcal{C}C(S)$.
Because, we may write $G = G \cap S$ and $K = \varnothing \cup K$ for some $G \in \tau$, $K \in \kappa$.
(2) Likewise, $PC(S) \subseteq \mathcal{C}O(S)$ and $PO(S) \subseteq \mathcal{C}C(S)$.
(3) Since every open(closed) set is pre-open(pre-closed) set, it is obvious that $\mathcal{A}O(S) \subseteq LC(S) \subseteq \mathcal{C}O(S)$ and $\mathcal{A}C(S) \subseteq LCC(S) \subseteq \mathcal{C}C(S)$, by [9, Proposition 3.4].

Definition 2.3. Let $(S, \mathcal{S}, \tau, \kappa)$ be a ditopological texture space. For $A \in \mathcal{S}$, we define:

(1) The pre-closure $pcl\,(A)$ of A under (τ, κ) by the equality $pcl(A) = \bigcap\{B \mid B \in PC(S) \text{ and } A \subseteq B\}$.
(2) The pre-interior $pint\,(A)$ of A under (τ, κ) by the equality $pint(A) = \bigvee\{B \mid B \in PO(S) \text{ and } B \subseteq A\}$.

By [10, Lemma 2.3], since the family of pre-closed (pre-open) sets is closed under arbitrary intersections(joins) we have, $pint(A) \in PO(S)$, $pcl(A) \in PC(S)$, while $A \in PO(S) \iff A = pint(A)$ and $A \in PC(S) \iff A = pcl(A)$.

Obviously, $pint(A)$ is the greatest pre-open set which is contained in A and $pcl(A)$ is pre-closed set which contains A and we have, $A \subseteq pcl(A) \subseteq clA$ and $intA \subseteq pint(A) \subseteq A$.

Proposition 2.4. *Let $(S, \mathcal{S}, \tau, \kappa)$ be a clint ditopological texture space and $A \in \mathcal{S}$.*

(1) $pcl\,(A) = A \cup cl\,int\,(A)$,
(2) $pint\,(A) = A \cap int\,cl\,(A)$

Proof. We prove (1), leaving the dual proof of (2) to the interested reader. Since $pcl\,(A)$ is pre-closed, we have $cl\,int\,(A) \subseteq cl\,int\,(pcl\,(A)) \subseteq pcl\,(A)$ and hence $A \cup cl\,int\,(A) \subseteq pcl\,(A)$.

On the other hand, $cl\,int\,(A \cup cl\,int\,(A)) \subseteq cl\,(int\,(A) \cup cl\,int\,(A)) = cl\,int\,(A) \subseteq A \cup cl\,int\,(A)$ by the definition of *clint* ditopology. Hence $A \cup cl\,int\,(A)$ is pre-closed and $pcl\,(A) \subseteq A \cup cl\,int\,(A)$. □

Lemma 2.5. *Let $(S, \mathcal{S}, \tau, \kappa)$ be a ditopological space. Then:*
(1) $A \in \mathcal{C}O(S)$ if and only if $A = G \cap pcl(A)$ for some $G \in O(S)$.
(2) $A \in \mathcal{C}C(S)$ if and only if $A = K \cap pint(A)$ for some $K \in C(S)$.

Proof. We prove (1), leaving the dual proof of (2) to the interested reader. Suppose that $A = G \cap pcl(A)$ for some $G \in O(S)$. Since $pcl(A)$ is pre-closed, $A \in \mathcal{C}O(S)$.

Conversely, let $A \in \mathcal{C}O(S)$. Then $A = G \cap K$, where $G \in O(S)$ and $K \in PC(S)$. Since $A \subseteq G \cap K \subseteq K$ and K is pre-closed, we have $pcl\,(A) \subseteq pcl\,(K) = K$. Thus, $G \cap pcl\,(A) \subseteq G \cap K = A$. Since $A \subseteq G$ and $A \subseteq pcl(A)$, we have $A \subseteq G \cap pcl\,(A)$, which shows that $A = G \cap pcl\,(A)$, with $G \in O(S)$ as required. □

Lemma 2.6. *Let $(S, \mathcal{S}, \tau, \kappa)$ be a ditopological space and $A \in \mathcal{S}$. Then:*
(1) $A = G \cap cl\,int\,(A)$ for some $G \in O(S)$ if and only if $A \in \mathcal{C}O(S) \cap SO(S)$.
(2) $A = K \cup int\,cl\,(A)$ for some $K \in C(S)$ if and only if $A \in \mathcal{C}C(S) \cup SC(S)$.

Proof. We prove only (1), leaving the dual proof of (2) to the interested reader. Suppose that $A = G \cap cl\,int\,(A)$ for some $G \in O(S)$. Then $A \subseteq cl\,int\,(A)$, which show that $A \in SO(S)$. Since $cl\,int\,(A)$ is closed and so pre-closed, we have $A \in \mathcal{C}O(S)$.

Conversely, let $A \in \mathcal{C}O(S) \cap SO(S)$. Then there exists $G \in O(S)$ such that $A = G \cap pcl\,(A)$, by Lemma 2.5. Because of $A \in SO(S)$, $A \subseteq cl\,int\,(A)$ and so $pcl\,(A) \subseteq cl\,int\,(A)$. Since $(A \cup cl\,int(A)) \subseteq pcl\,(A)$, we have $(G \cap cl\,int(A)) \subseteq (G \cap pcl\,(A)) = A \subseteq (G \cap cl\,int(A))$ and $A = G \cap cl\,int\,(A)$ with $G \in O(S)$, as required. □

Theorem 2.7. *Let $(S, \mathcal{S}, \tau, \kappa)$ be a clint ditopological texture space.*
(1) $\mathcal{A}O(S) = \mathcal{C}O(S) \cap SO(S)$.
(2) $\mathcal{A}C(S) = \mathcal{C}C(S) \cap SC(S)$.

Proof. We prove (1), leaving the dual proof of (2) to the interested reader. From Remark 2.2. and [8, Proposition 3.4], it is clear that $\mathcal{A}O(S) \subseteq \mathcal{C}O(S) \cap SO(S)$.

Conversely, suppose that $A \in \mathcal{C}O(S) \cap SO(S)$. Then, by Lemma 2.6, $A = G \cap cl\,int\,(A)$, where G is open. Since $int\,(G)$ is open, $cl\,int\,(A)$ is regular closed, by [9, Lemma 3.13(1)]. Thus, $A \in \mathcal{A}O(S)$, which is required. □

Generally there is no relation between the $\mathcal{C}$-sets and co-$\mathcal{C}$-closed sets, but for a complemented ditopological texture space we have the following result.

Proposition 2.8. *Let $(S, \mathcal{S}, \sigma, \tau, \kappa)$ be a complemented ditopological texture space.*

$$A \in \mathcal{S} \quad \textit{is} \quad \mathcal{C}-\textit{set if and only if} \quad \sigma(A) \textit{ is} \quad co-\mathcal{C}-\textit{set}$$

Proof. Let $A \in \mathcal{S}$ and $A = G \cap K$, where $G \in O(S)$, $K \in PC(S)$. Since $\sigma(\tau) = \kappa$ and $\sigma(A) = \sigma(G \cap K) = \sigma(G) \cup \sigma(K)$, the proof is trivial. □

Example 2.9. (1) If $(X, \mathcal{T})$ is a topological space then $(X, \mathcal{P}(X), \pi_X, \mathcal{T}, \mathcal{T}^c)$ is a complemented ditopological texture space. Here $\pi_X(Y) = X \setminus Y$ for $Y \subseteq X$ is the usual complementation on $(X, \mathcal{P}(X))$ and $\mathcal{T}^c = \{\pi_X(G) \mid G \in \mathcal{T}\}$. Clearly the $\mathcal{C}$-sets, co-$\mathcal{C}$-sets in $(X, \mathcal{T})$ correspond precisely to the $\mathcal{C}$-sets, co-$\mathcal{C}$-sets respectively, in $(X, \mathcal{P}(X), \pi_X, \mathcal{T}, \mathcal{T}^c)$.

(2) For the unit interval texture $(\mathbb{I}, \mathcal{I})$ of Examples 1.2.(3), let ι be the complementation $\iota([0, r)) = [0, 1-r]$, $\iota([0, r]) = [0, 1-r)$, and $(\tau_{\mathbb{I}}, \kappa_{\mathbb{I}})$ the standard complemented ditopology given by

$$\tau_{\mathbb{I}} = \{[0, r) \mid r \in \mathbb{I}\} \cup \{\mathbb{I}\}, \quad \kappa_{\mathbb{I}} = \{[0, r] \mid r \in \mathbb{I}\} \cup \{\varnothing\}.$$

Since $[0, r]$, $r \in \mathbb{I}$ is pre-closed, $[0, r) = [0, r) \cap [0, r]$ is $\mathcal{C}$-set. Likewise, $[0, r)$ is pre-open, $[0, r] = [0, r) \cup [0, r]$ is co-$\mathcal{C}$-set. Thus, $\mathcal{C}\,O(\mathbb{I}) = \mathcal{I} = \mathcal{C}\,C(\mathbb{I})$.

(3)The $\mathcal{C}$-set (co-$\mathcal{C}$-set) is different from semi-open(semi-closed) or locally-closed (locally-coclosed) set. Now, we consider $X = \{a, b, c\}$ and the ditopology (τ, κ) on the discrete texture $(X, \mathcal{P}(X))$, where $\tau = \{\varnothing, \{b\}, \{a, c\} X\}$, $\kappa = \{\varnothing, \{a, c\}, \{b\}, X\}$. Then $\{a\}$ is pre-closed. Thus it is a $\mathcal{C}$-set, but is neither semi-open nor locally-closed. Besides, $\{b, c\}$ is semi-open,but it is not $\mathcal{C}$-set.

Now, we discuss $\mathcal{C}$-bicontinuous difunction and its relation other classes of difunctions. We recall that a function between topological spaces is called $\mathcal{C}$-continuous [11] if the inverse image of each open set is $\mathcal{C}$-set. This leads to the following concepts for a difunction between ditopological texture spaces.

Definition 2.10. Let $(S_j, \mathcal{S}_j, \tau_j, \kappa_j)$, $j = 1, 2$, be ditopological texture spaces and $(f, F) : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ a difunction.

(1) It is called $\mathcal{C}$-continuous, if $F^{\leftarrow}(G) \in \mathcal{C}O(S_1)$, for every $G \in O(S_2)$.
(2) It is called $\mathcal{C}$-cocontinuous, if $f^{\leftarrow}(K) \in \mathcal{C}C(S_1)$, for every $K \in C(S_2)$.
(3) It is called $\mathcal{C}$-bicontinuous, if it is $\mathcal{C}$-continuous and $\mathcal{C}$-cocontinuous.

The following proposition is obtained immediate from Theorem 2.7.

Proposition 2.11. *Let (f, F) be a difunction between clint ditopological texture spaces.*

(1) (f, F) is $\mathcal{A}$-continuous if and only if it is $\mathcal{C}$-continuous and semi-continuous.
(2) (f, F) is $\mathcal{A}$-cocontinuous if and only if it is $\mathcal{C}$-cocontinuous and semi-cocontinuous.

In [9], it is proved that a difunction is bicontinuous if and only if it is both α-bicontinuous and semi-bicontinuous. Since α-bicontinuity implies semi-bicontinuity, with Theorem 2.7, we obtain the following decomposition of bicontinuity.

Corollary 2.12. *Let (f, F) be a difunction between clint ditopological texture spaces.*

(1) (f, F) is continuous if and only if it is $\mathcal{C}$-continuous and α-continuous.
(2) (f, F) is cocontinuous if and only if it is $\mathcal{C}$-cocontinuous and α-cocontinuous.

Proposition 2.13. *Let* $(S_j, \mathcal{S}_j, \sigma_j, \tau_j, \kappa_j)$, $j = 1, 2$, *complemented ditopology and* $(f, F) : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ *be complemented difunction. Then* (f, F) *is* $\mathcal{C}$*-continuous if and only if* (f, F) *is* $\mathcal{C}$*-cocontinuous.*

Proof. Since (f, F) is complemented, $(F', f') = (f, F)$. From [5, Lemma 2.20], $\sigma_1((f')^{\leftarrow}(B)) = f^{\leftarrow}(\sigma_2(B))$ and $\sigma_1((F')^{\leftarrow}(B)) = F^{\leftarrow}(\sigma_2(B))$ for all $B \in \mathcal{S}_2$. Hence the proof is clear from these equalities. □

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