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Pairwise ordered soft $\tilde{L}$-fuzzy C-basically disconnected ideal $\mathcal{V}$ space

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Abstract. In this paper, a new structure called soft $\tilde{L}$-fuzzy $\mathcal{V}$-structure is introduced and studied. Uryshon Lemma and Tietze Extension Theorem in a pairwise ordered soft $\tilde{L}$-fuzzy C-basically disconnected $\mathcal{V}$-space via ideals is established.

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1. Introduction

The fundamental definitions and theories of scientific studies, specially in mathematical ones, with respect to the ordinary sets are considered as a particular case of the corresponding fuzzy notions. Therefore, it is natural to extend the concept of point set topology to fuzzy sets which is characterized by a membership function in the sense of L. A. Zadeh [17], resulting in the theory of fuzzy topology due to C. L. Chang [2]. Fuzzy sets have applications in many fields such as information [10] and control [11]. D. Molodtsov [8] proposed the soft set which is free from parameterization inadequacy syndrome of fuzzy set theory. The notion of soft fuzzy set was introduced and studied by Ismail U. Triyaki [12]. The concept of soft fuzzy C-open set was introduced by T. Yogalakshmi, E. Roja, M. K. Uma [14].

Kuratowski [7], Vaidyanathaswamy [13] and several other authors introduced the notion of ideal theory in general topology. Debasis Sarkar [9] introduced the notions of fuzzy ideal and fuzzy local function in fuzzy set theory. T. Yogalakshmi, E. Roja, M. K. Uma [16] introduced the concepts of soft fuzzy ideal and soft fuzzy local function theory. Bruce Hutton [4] constructed an interesting L-fuzzy topological space, called L-fuzzy unit interval. Thomas Kubiak [6] introduced and studied about the properties of L-sets in fuzzy sense. The concept of soft $\tilde{L}$-fuzzy set was introduced by T. Yogalakshmi, E. Roja, M. K. Uma [15].

In this paper, a new structure, called soft $\tilde{L}$-fuzzy $\mathcal{V}$-structure is introduced by using the notion of the Fell topology [3] of the hyperspaces which is bit better than the Vietoris topology [5]. Urysohn Lemma and Tietz Extension Theorem in a pairwise ordered soft $\tilde{L}$-fuzzy C-basically disconnected ideal $\mathcal{V}$-space is introduced and studied.

2. Preliminaries

Definition 2.1 ([6])**.** Let X be a set and L be a complete lattice. An L-fuzzy set on X is a map from X into L. That is, if λ is a L-fuzzy subset of X then $\lambda \in L^X$, where L^X denotes the collection of all maps from X into L.

Definition 2.2 ([1])**.** A fuzzy topological space (X, τ) is said to be a fuzzy basically disconnected space if the closure of a fuzzy open and fuzzy F_σ set is fuzzy open.

Definition 2.3 ([14])**.** Let X be a nonempty set and I=[0,1] be the unit interval. Let μ be a fuzzy subset of X such that $\mu : X \to [0,1]$ and M be any crisp subset of X. Then, the pair (μ, M) is called as a *soft fuzzy set* in X. The family of all soft fuzzy subsets of X,will be denoted by $SF(X)$.

3. On soft $\tilde{L}$-fuzzy $\mathcal{V}$-structure

Throughout this paper $\tilde{L} = \tilde{L} <\sqsubseteq, \sqcup, \sqcap,' >$ is an infinitely distributive lattice with an order-reversing involution. Such a lattice being complete has a least element $(0_X, \psi_\phi)$ and greatest element $(1_X, \psi_X)$.

Definition 3.1 ([15])**.** Let X be a non-empty set and $N \subseteq X$. Let L be any lattice. Associated to each soft fuzzy set (λ, N), a soft $\tilde{L}$-fuzzy set (λ, ψ_N) is defined as a function from X to $L \times L$ such that $(\lambda, \psi_N)(x) = (\mathfrak{l}_x, \psi_N(x))$ where $\mathfrak{l}_x = \wedge\{\alpha \in L : \lambda(x) \leq \alpha\}$ and

$$\psi_N(x) = \begin{cases} 1, & \text{if} x \in N = X \\ \mathfrak{l}_x, & \text{if} x \in N \subset X \\ 0, & \quad \text{otherwise} \end{cases}$$

The family of all soft $\tilde{L}$-fuzzy sets is denoted by $\tilde{L}^X$.

Example 3.2. Let $X = \{a, b, c\}$ be a non-empty set and $L = \{0, 1/4, 2/4, 3/4, 1\}$ be any lattice. Let (λ, N) be any soft fuzzy set where $\lambda : X \to [0,1]$ such that $\lambda(a) = 0.2; \lambda(b) = 0.3; \lambda(c) = 0.5;$ $N = \{a, c\}$. Then (λ, ψ_N) is a soft $\tilde{L}$-fuzzy set where $\mathfrak{l}_a = 1/4; \mathfrak{l}_b = 2/4; \mathfrak{l}_c = 2/4; \psi_N(a) = 1/4; \psi_N(b) = 0; \psi_N(c) = 2/4$.

Definition 3.3. If f is a function from X to Y and $(\lambda, \psi_N) \in \tilde{L}^X$, then the *image* of (λ, ψ_N), $f(\lambda, \psi_N)$ is the soft $\tilde{L}$-fuzzy set in Y defined by

$$f((\lambda, \psi_N))(y) = sup_{x \in f^{-1}(y)}\{(\lambda, \psi_N)(x)\}.$$

Definition 3.4. If f is a function from X to Y and $(\mu, \psi_M) \in \tilde{L}^Y$, then the *inverse image* of (μ, ψ_M), $f^{-1}(\mu, \psi_M)$ is the soft $\tilde{L}$-fuzzy set in X defined by

$$f^{-1}((\mu, \psi_M)) = (\mu, \psi_M) \circ f.$$

Example 3.5. Let $X = \{a, b, c\}$ and $Y = \{p, q\}$ be any two non-empty sets. Let $L = \{0, 1/4, 2/4, 3/4, 1\}$ be any lattice. Define a function $f : X \to Y$ as $f(a) = f(b) = p; f(c) = q$. If (λ, ψ_N) is a soft $\tilde{L}$-fuzzy set in X such that $(\lambda, \psi_N)(a) = (1/4, 1/4); (\lambda, \psi_N)(b) = (2/4, 0); (\lambda, \psi_N)(c) = (2/4, 2/4)$, then $f((\lambda, \psi_N))$ is a soft $\tilde{L}$-fuzzy set in Y such that $f((\lambda, \psi_N))\ (p) = (2/4, 1/4); f((\lambda, \psi_N))(q) = (2/4, 2/4)$. If (μ, ψ_M) is a soft $\tilde{L}$-fuzzy set in Y such that $(\mu, \psi_M)(p) = (2/4, 1/4); (\mu, \psi_M)(q) = (2/4, 2/4)$, then $f^{-1}((\mu, \psi_M))$ is a soft $\tilde{L}$-fuzzy set in X such that $f^{-1}((\mu, \psi_M))(a) = (2/4, 1/4); f^{-1}((\mu, \psi_M))(b) = (2/4, 1/4); f^{-1}((\mu, \psi_M))(c) = (2/4, 2/4)$.

Definition 3.6. A soft $\tilde{L}$-fuzzy topology on a non-empty set X is a collection, τ of soft $\tilde{L}$-fuzzy sets in X satisfying the following axioms:

(1) $(0_X, \psi_\phi), (1_X, \psi_X) \in \tau$.
(2) For any family of soft $\tilde{L}$-fuzzy sets $(\lambda_j, \psi_{N_j}) \in \tau, j \in J$, $\Rightarrow \sqcup_{j\in J}(\lambda_j, \psi_{N_j}) \in \tau$.
(3) For any finite number of soft $\tilde{L}$-fuzzy sets $(\lambda_j, \psi_{N_j}) \in \tau$, j=1,2,3,. . . n , $\Rightarrow \sqcap_{j=1}^{n}(\lambda_j, \psi_{N_j}) \in \tau$.

Then the pair $(X, \tau)$is called as a *soft $\tilde{L}$-fuzzy topological space.* (in short,$S\tilde{L}FTS$). Any soft $\tilde{L}$-fuzzy set in τ is said to be a *soft $\tilde{L}$-fuzzy open set* (in short, $S\tilde{L}FOS$) in X. The complement of S$\tilde{L}$FOS in a S$\tilde{L}$FTS (X, τ) is called as a *soft $\tilde{L}$-fuzzy closed set*, denoted $S\tilde{L}FCS$ in X.

Example 3.7. Let $X = \{a, b, c\}$ be a non-empty set and $L = \{0, 1/5, 2/5, 3/5, 4/5, 1\}$ be any lattice. Define a soft $\tilde{L}$-fuzzy topology $\tau = \{(0_X, \psi_\phi), (1_X, \psi_X), (\lambda_i, \psi_{N_i})\}$ for $i = 1, 2, 3, 4$ such that
$(\lambda_1, \psi_{N_1})(a) = (1/5, 1/5)$, $(\lambda_1, \psi_{N_1})(b) = (2/5, 0)$,
$(\lambda_1, \psi_{N_1})(c) = (0, 0)$; $(\lambda_2, \psi_{N_2})(a) = (1/5, 0)$,
$(\lambda_2, \psi_{N_2})(b) = (0, 0)$, $(\lambda_2, \psi_{N_2})(c) = (3/5, 3/5)$; $(\lambda_3, \psi_{N_3})(a) = (1/5, 1/5)$,
$(\lambda_3, \psi_{N_3})(b) = (2/5, 0)$, $(\lambda_3, \psi_{N_3})(c) = (3/5, 3/5)$;
$(\lambda_4, \psi_{N_4})(a) = (1/5, 0)$, $(\lambda_4, \psi_{N_4})(b) = (\lambda_4, \psi_{N_4})(c) = (0, 0)$.
Then the pair (X, τ) is a soft $\tilde{L}$-fuzzy topological space.

Definition 3.8 ([15])**.** Let (X, τ) be a soft $\tilde{L}$-fuzzy topological space. Let $(\lambda, \psi_N) \in \tilde{L}^X$. Then, the soft $\tilde{L}$-fuzzy real line $\tilde{L}^{\mathbb{R}}$ i. e. $\mathbb{R}(L \times L)$is the set of all monotone decreasing element $[(\lambda, \psi_N)] \in \tilde{L}^{\mathbb{R}}$ satisfying

$$\sqcup\{(\lambda, \psi_N)(t) : t \in \mathbb{R}\} = (1, 1)$$
$$\sqcap\{(\lambda, \psi_N)(t) : t \in \mathbb{R}\} = (0, 0)$$

after the identification of (λ, ψ_N), $(\mu, \psi_M) \in \tilde{L}^{\mathbb{R}}$ iff

$$(\lambda, \psi_N)(t-) = (\mu, \psi_M)(t-)$$
$$(\lambda, \psi_N)(t+) = (\mu, \psi_M)(t+)$$

for all $t \in \mathbb{R}$, where,

$$(\lambda, \psi_N)(t-) = \sqcap_{s<t}(\lambda, \psi_N)(s) = lt_{s\to t-}(\lambda, \psi_N)(s).$$
$$(\lambda, \psi_N)(t+) = \sqcup_{s>t}(\lambda, \psi_N)(s) = lt_{s\to t+}(\lambda, \psi_N)(s).$$

Definition 3.9. A *partial order on* $\mathbb{R}(L \times L)$ is defined by $[(\lambda, \psi_N)] \sqsubseteq [(\mu, \psi_M)]$ iff $(\lambda, \psi_N)(t-) \sqsubseteq (\mu, \psi_M)(t-)$ and $(\lambda, \psi_N)(t+) \sqsubseteq (\mu, \psi_M)(t+)$, for all $t \in \mathbb{R}$.

Definition 3.10. Let (X, τ) be a soft $\tilde{L}$-fuzzy topological space. Let $(\lambda, \psi_N) \in \tilde{L}^X$. The *natural soft $\tilde{L}$-fuzzy topology* on $\mathbb{R}(L\times L)$ is generated from the sub-basis $\{L_t, R_t : t \in \mathbb{R}\}$, where, $L_t, R_t : \mathbb{R} \to L\times L$ and $L_t(\lambda, \psi_N) = (\lambda, \psi_N)(t-)'=(1,1)-(\lambda, \psi_N)(t-)$ and $R_t(\lambda, \psi_N) = (\lambda, \psi_N)(t+)$, for all $(\lambda, \psi_N) \in \tilde{L}^{\mathbb{R}}$. This topology is called as the *usual topology* for $\mathbb{R}(L \times L)$. $\mathcal{L} = \{L_t : t \in \mathbb{R}\} \cup \{(0_X, \psi_\phi), (1_X, \psi_X)\}$ and $\mathcal{R} = \{R_t : t \in \mathbb{R}\} \cap \{(0_X, \psi_\phi), (1_X, \psi_X)\}$ are called the left and right hand I-topologies respectively.

Definition 3.11 ([15])**.** Let (X, τ) be a soft $\tilde{L}$-fuzzy topological space. The *soft $\tilde{L}$-fuzzy unit interval* $I(L \times L)$ is a subset of $\mathbb{R}(L \times L)$ such that $[(\lambda, \psi_N)] \in I(L \times L)$ i. e. $[(\lambda, \psi_N)] \in \tilde{L}^I$, if

$$(\lambda, \psi_N)(t) = (1,1)\text{for} t < 0, t \in \mathbb{R}$$
$$(\lambda, \psi_N)(t) = (0,0)\text{for} t > 1, t \in \mathbb{R}$$

It is equipped with the soft $\tilde{L}$-fuzzy subspace topology.

Definition 3.12. Let (X, τ) be a soft $\tilde{L}$-fuzzy topological space. A soft $\tilde{L}$-fuzzy set (λ, ψ_N) is said to be a *soft $\tilde{L}$-fuzzy compact set* iff each soft $\tilde{L}$-fuzzy open cover of (λ, ψ_N) has a finite subcover.

Definition 3.13. A soft $\tilde{L}$-fuzzy topological space (X, τ) is said to be a *soft $\tilde{L}$-fuzzy locally compact space* iff for every soft $\tilde{L}$-fuzzy point (x_p, ψ_x) in (X, τ), there exists a soft $\tilde{L}$-fuzzy open set $(\lambda, \psi_N) \in \tau$ such that
(i) $(x_p, \psi_x) \in (\lambda, \psi_N)$ and
(ii) (λ, ψ_N) is soft $\tilde{L}$-fuzzy compact.

Example 3.14. Let $X = \{a, b, c\}$ be a non-empty set and $L = \{0, 1/5, 2/5, 3/5, 4/5, 1\}$ be any lattice. Define a soft $\tilde{L}$-fuzzy topology $\tau = \{(0_X, \psi_\phi), (1_X, \psi_X), (\lambda_i, \psi_{N_i})\}$ for $i = 1, 2, ...6$ such that
$(\lambda_1, \psi_{N_1})(a) = (1, 1)$, $(\lambda_1, \psi_{N_1})(b) = (0, 0)$,
$(\lambda_1, \psi_{N_1})(c) = (0, 0)$; $(\lambda_2, \psi_{N_2})(a) = (0, 0)$,
$(\lambda_2, \psi_{N_2})(b) = (1, 1)$, $(\lambda_2, \psi_{N_2})(c) = (0, 0)$;
$(\lambda_3, \psi_{N_3})(a) = (0, 0)$, $(\lambda_3, \psi_{N_3})(b) = (0, 0)$,
$(\lambda_3, \psi_{N_3})(c) = (1, 1)$; $(\lambda_4, \psi_{N_4})(a) = (1, 1)$,
$(\lambda_4, \psi_{N_4})(b) = (0, 0)$, $(\lambda_4, \psi_{N_4})(c) = (1, 1)$;
$(\lambda_5, \psi_{N_5})\ (a) = (0, 0)$, $(\lambda_5, \psi_{N_5})(b) = (1, 1)$,
$(\lambda_5, \psi_{N_5})(c) = (1, 1)$; $(\lambda_6, \psi_{N_6})(a) = (1, 1)$,
$(\lambda_6, \psi_{N_6})(b) = (1, 1)$, $(\lambda_6, \psi_{N_6})(c) = (0, 0)$.
Then the pair (X, τ) is a soft $\tilde{L}$-fuzzy locally compact space.

Definition 3.15. Let (X, τ) be a soft $\tilde{L}$-fuzzy topological space and a soft $\tilde{L}$-fuzzy locally compact space. Let $C_F(X)$ be the hyperspace of all soft $\tilde{L}$-fuzzy sets, which are both soft $\tilde{L}$-fuzzy closed and soft $\tilde{L}$-fuzzy compact sets in (X, τ). Let

$$(\lambda, \psi_N)^+ = \{(\gamma, \psi_K) \in C_F(X) : (\lambda, \psi_N) \sqcap (\gamma, \psi_K) \neq (0_X, \psi_\phi)\}$$
$$(\lambda, \psi_N)^- = \{(\gamma, \psi_K) \in C_F(X) : (\lambda, \psi_N) \sqcap (\gamma, \psi_K) = (0_X, \psi_\phi)\}$$

Soft $\tilde{L}$-fuzzy $\mathcal{V}$-structure on $C_F(X)$ is the collection $\mathcal{V}$ which is generated by the sub-base consisting of soft $\tilde{L}$-fuzzy sets of the form $(\lambda, \psi_N)^+$ and $(\lambda, \psi_N)^-$, where (λ, ψ_N) is both soft $\tilde{L}$-fuzzy open set and soft $\tilde{L}$-fuzzy compact set in (X, τ). Then the pair $(X, \mathcal{V})$ is said to be a *soft $\tilde{L}$-fuzzy $\mathcal{V}$-space*. The member of soft $\tilde{L}$-fuzzy $\mathcal{V}$-structure is said to be *soft $\tilde{L}$-fuzzy $\mathcal{V}$-open set*. It is denoted by $S\tilde{L}F\mathcal{V}OS$. The complement of soft $\tilde{L}$-fuzzy $\mathcal{V}$-open set is said to be a *soft $\tilde{L}$-fuzzy $\mathcal{V}$-closed set*. It is denoted by $S\tilde{L}F\mathcal{V}CS$.

Example 3.16. Let $X = \{a, b, c\}$ be a non-empty set and

$$L = \{0, 1/5, 2/5, 3/5, 4/5, 1\}$$

be any lattice. Define a soft $\tilde{L}$-fuzzy topology $\tau = \{(0_X, \psi_\phi), (1_X, \psi_X), (\lambda_i, \psi_{N_i})\}$ for $i = 1, 2, ...6$ such that
$(\lambda_1, \psi_{N_1})(a) = (1, 1), (\lambda_1, \psi_{N_1})(b) = (0, 0),$
$(\lambda_1, \psi_{N_1})(c) = (0, 0)$; $(\lambda_2, \psi_{N_2})(a) = (0, 0),$
$(\lambda_2, \psi_{N_2})(b) = (1, 1), (\lambda_2, \psi_{N_2})(c) = (0, 0)$;
$(\lambda_3, \psi_{N_3})(a) = (0, 0), (\lambda_3, \psi_{N_3})(b) = (0, 0),$
$(\lambda_3, \psi_{N_3})(c) = (1, 1)$; $(\lambda_4, \psi_{N_4})(a) = (1, 1),$
$(\lambda_4, \psi_{N_4})(b) = (0, 0), (\lambda_4, \psi_{N_4})(c) = (1, 1)$;
$(\lambda_5, \psi_{N_5})\ (a) = (0, 0), (\lambda_5, \psi_{N_5})(b) = (1, 1),$
$(\lambda_5, \psi_{N_5})(c) = (1, 1)$; $(\lambda_6, \psi_{N_6})(a) = (1, 1),$
$(\lambda_6, \psi_{N_6})(b) = (1, 1), (\lambda_6, \psi_{N_6})(c) = (0, 0).$
Then the pair (X, τ) is a soft $\tilde{L}$-fuzzy locally compact space. Let

$$\mathfrak{b} = \{(0_X, \psi_\phi), (1_X, \psi_X), (\lambda_4, \psi_{N_4}), (\lambda_5, \psi_{N_5}), (\lambda_6, \psi_{N_6})\}$$

be a subbase. Now, soft $\tilde{L}$-fuzzy $\mathcal{V}$-structure $\mathcal{V}$ is the collection which generated by $\mathfrak{b}$. Then the pair $(X, \mathcal{V})$ is the soft $\tilde{L}$-fuzzy $\mathcal{V}$-space.

Definition 3.17. Let $(X, \mathcal{V})$ be a soft $\tilde{L}$-fuzzy $\mathcal{V}$-space. Let $(\lambda, \psi_N) \in S\tilde{L}FS$ in X. Then, the *soft $\tilde{L}$-fuzzy $\mathcal{V}$-interior* and the *soft $\tilde{L}$-fuzzy $\mathcal{V}$-closure of* (λ, ψ_N) are defined as

$$S\tilde{L}F\mathcal{V}\text{-}int(\lambda, \psi_N) = \sqcup\{(\mu, \psi_M) : (\mu, \psi_M) \text{ is a soft } \tilde{L}\text{-fuzzy } \mathcal{V}\text{-open set and } (\lambda, \psi_N) \sqsupseteq (\mu, \psi_M)\}$$

$$S\tilde{L}F\mathcal{V}\text{-}cl(\lambda, \psi_N) = \sqcap\{(\mu, \psi_M) : (\mu, \psi_M) \text{ is a soft } \tilde{L}\text{-fuzzy } \mathcal{V}\text{-closed set and } (\lambda, \psi_N) \sqsubseteq (\mu, \psi_M)\}$$

4. On soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space

Definition 4.1 ([9]). A*soft $\tilde{L}$-fuzzy ideal* $\mathcal{I}$ on X is a non-empty collection of soft $\tilde{L}$-fuzzy sets which satisfies the following axiom:
(i) If $(\mu, \psi_M) \in \mathcal{I}$ and $(\mu, \psi_M) \sqsupseteq (\lambda, \psi_N)$ then $(\lambda, \psi_N) \in \mathcal{I}$. (heredity)
(ii) If $(\mu, \psi_M), (\lambda, \psi_N) \in \mathcal{I}$, then, $(\mu, \psi_M) \sqcup (\lambda, \psi_N) \in \mathcal{I}$. (finite additivity)

Example 4.2. Let $X = \{a, b, c\}$ be a non-empty set and

$$L = \{0, 1/10, 2/10, 3/10, 4/10, 5/10,\ 6/10, 7/10,\ 8/10, 9/10, 1\}$$

be any lattice. Then $\mathcal{I} = \{(\lambda, \psi_N) :\ \text{for all } x \in X, 0 \leq (\lambda, \psi_N)(x) \leq 6/10\}$ is a soft $\tilde{L}$-fuzzy ideal.

Definition 4.3. A *soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space*, denoted by $(X, \mathcal{V}, \mathcal{I})$ means a soft $\tilde{L}$-fuzzy $\mathcal{V}$-space with a soft $\tilde{L}$-fuzzy ideal, $\mathcal{I}$ and soft $\tilde{L}$-fuzzy $\mathcal{V}$-structure, $\mathcal{V}$.

Definition 4.4 ([16])**.** Given a soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space, $(X, \mathcal{V}, \mathcal{I})$ and if $\tilde{L}^X$ is the set of all soft $\tilde{L}$-fuzzy sets, $(\lambda, \psi_N) : X \to L \times L$, then the soft $\tilde{L}$-fuzzy set operator $(.)^* : \tilde{L}^X \to \tilde{L}^X$ called the *soft $\tilde{L}$-fuzzy local function of* (λ, ψ_N) with respect to $\mathcal{V}$ and $\mathcal{I}$, is defined as follows:

$$\begin{aligned}(\lambda, \psi_N)^*(\mathcal{V}, \mathcal{I}) = \sqcap\{(\gamma, \psi_K) \in \tilde{L}^X :\ &\text{if } (\mu, \psi_M) \in \mathcal{V},\ \text{then there exists}\\ &(\lambda, \psi_N) \sqcap (\mu, \psi_M) \notin \mathcal{I} \text{ such that } (\lambda, \psi_N) \sqcap (\mu, \psi_M) \sqsupseteq (\gamma, \psi_K)\\ &\text{with } (\gamma, \psi_K) \text{is a} S\tilde{L}F\mathcal{V} \text{closed set}\}\end{aligned}$$

Definition 4.5. A *soft $\tilde{L}$-fuzzy closure operator*, $Cl^*(.)$ for a soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space $(\mathcal{V}, \mathcal{I})$ is defined by

$$S\tilde{L}F\mathcal{V}cl^*(\lambda, \psi_N) = (\lambda, \psi_N) \sqcup (\lambda, \psi_N)^*$$

Definition 4.6. Let $(X, \mathcal{V}, \mathcal{I})$ be a soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let (λ, ψ_N) be a soft $\tilde{L}$-fuzzy set. Then, (λ, ψ_N) is said to be a *soft $\tilde{L}$-fuzzy $\mathcal{I}$ $\mathcal{V}$-open set* if $(\lambda, \psi_N) \sqsubseteq int(\lambda, \psi_N)^*$.

Definition 4.7. Let $(X, \mathcal{V}, \mathcal{I})$ be a soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let (λ, ψ_N) be a soft $\tilde{L}$-fuzzy set. Then, (λ, ψ_N) is said to be a *soft $\tilde{L}$-fuzzy α^*-$\mathcal{I}$ $\mathcal{V}$-open set* if $int(\lambda, \psi_N) = int(cl^*(int(\lambda, \psi_N)))$.

5. Ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space :

Definition 5.1. An ordered $\mathcal{I}$-set on which there is given a soft $\tilde{L}$-fuzzy $\mathcal{V}$-structure is called as an *ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space.* (for short. ordered S$\tilde{L}$F$\mathcal{I}\mathcal{V}$S)

Definition 5.2. A soft $\tilde{L}$-fuzzy set (λ, ψ_N) in a partially ordered set $(X, \mathcal{V}, \sqsubseteq)$ is said to be an

(1) *Increasing soft $\tilde{L}$-fuzzy set* (for short. $\uparrow$ S$\tilde{L}$FS) if $x \leq y \Rightarrow (\lambda, \psi_N)(x) \sqsubseteq (\mu, \psi_M)(y)$
(2) *Decreasing soft $\tilde{L}$-fuzzy set* (for short. $\downarrow$ S$\tilde{L}$FS) if $x \leq y \Rightarrow (\lambda, \psi_N)(x) \sqsupseteq (\mu, \psi_M)(y)$

Definition 5.3. Let $(X, \mathcal{V}, \mathcal{I}, \sqsubseteq)$ be an ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let $(\lambda, \psi_N) \in \uparrow$ (resp. $\downarrow$) S$\tilde{L}$FS in X. Then,(λ, ψ_N) is said to be an *increasing (resp. decreasing) soft $\tilde{L}$-fuzzy $\mathcal{V}$-closure* of* (λ, ψ_N) if $I^{*\mathcal{V}}(\lambda, \psi_N) = (\lambda, \psi_N) \sqcup (\lambda, \psi_N)^*$ (resp. $D^{*\mathcal{V}}(\lambda, \psi_N) = (\lambda, \psi_N) \sqcup (\lambda, \psi_N)^*$).

Definition 5.4. Let $(X, \mathcal{V}, \mathcal{I}, \sqsubseteq)$ be an ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let (λ, ψ_N) be any soft $\tilde{L}$-fuzzy set. Then, (λ, ψ_N) is said to be an *increasing (resp. decreasing) soft $\tilde{L}$-fuzzy $\mathcal{I}\mathcal{V}$-open set* if $(\lambda, \psi_N) \sqsubseteq I_0^{\mathcal{V}}(\lambda, \psi_N)^*$ (resp. $(\lambda, \psi_N) \sqsubseteq D_0^{\mathcal{V}}(\lambda, \psi_N)^*$). It is denoted by $\uparrow S\tilde{L}F\mathcal{I}\mathcal{V}OS$ (resp. $\downarrow S\tilde{L}F\mathcal{I}\mathcal{V}OS$). The complement of $\uparrow S\tilde{L}F\mathcal{I}\mathcal{V}OS$ (resp. $\downarrow S\tilde{L}F\mathcal{I}\mathcal{V}OS$) is *decreasing (resp. increasing) soft $\tilde{L}$-fuzzy $\mathcal{I}\mathcal{V}$-closed set* . It is denoted by $\downarrow S\tilde{L}F\mathcal{I}\mathcal{V}CS$ (resp. $\uparrow S\tilde{L}F\mathcal{I}\mathcal{V}CS$).

Definition 5.5. Let $(X, \mathcal{V}, \mathcal{I}, \sqsubseteq)$ be an ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let (λ, ψ_N) be any soft $\tilde{L}$-fuzzy set. Then, (λ, ψ_N) is said to be an *increasing (resp.*

decreasing) soft $\tilde{L}$-fuzzy $\alpha^*\mathcal{IV}$-open set if $I_0^{\mathcal{V}}(\lambda,\psi_N) = I_0^{\mathcal{V}}(I^{*\mathcal{V}}(I_0^{\mathcal{V}}(\lambda,\psi_N)))$ (resp. $D_0^{\mathcal{V}}(\lambda,\psi_N) = D_0^{\mathcal{V}}(D^{*\mathcal{V}}(D_0^{\mathcal{V}}(\lambda,\psi_N)))$). It is denoted by $\uparrow$ (resp. $\downarrow$)$S\tilde{L}F\alpha^*\mathcal{IV}OS$. The complement of $\uparrow$ (resp. $\downarrow$)$S\tilde{L}F\alpha^*\mathcal{IV}OS$ is *decreasing (resp. increasing) soft $\tilde{L}$-fuzzy $\alpha^*\mathcal{IV}$-closed set* . It is denoted by $\downarrow$ (resp. $\uparrow$)$S\tilde{L}F\alpha^*\mathcal{IV}CS$.

Definition 5.6. Let $(X,\mathcal{V},\mathcal{I},\sqsubseteq)$ be an ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let (λ,ψ_N) be any soft $\tilde{L}$-fuzzy set in $(X,\mathcal{V},\mathcal{I},\sqsubseteq)$. Then, (λ,ψ_N) is said to be an *increasing (resp. decreasing) soft $\tilde{L}$-fuzzy $c\mathcal{IV}$-open set* if

$$(\lambda,\psi_N) = (\mu,\psi_M) \sqcap (\gamma,\psi_K)$$

where, (μ,ψ_M) is an $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$F$\mathcal{IV}$-open set and (γ,ψ_K) is an $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$F$\alpha^*\mathcal{IV}$-open set. It is denoted by $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$Fc$\mathcal{IV}$OS. The complement of an $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$Fc$\mathcal{IV}$OS is a *decreasing (resp. increasing) soft $\tilde{L}$-fuzzy $c\mathcal{IV}$-closed set.* It is denoted by $\downarrow$ (resp. $\uparrow$) S$\tilde{L}$Fc$\mathcal{IV}$CS.

Definition 5.7. Let $(X,\mathcal{V},\mathcal{I},\sqsubseteq)$ be an ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let (λ,ψ_N) be a soft $\tilde{L}$-fuzzy set in $(X,\mathcal{V},\mathcal{I},\sqsubseteq)$. Then, (λ,ψ_N) is said to be an *increasing (resp. decreasing) soft $\tilde{L}$-fuzzy G_δ $\mathcal{IV}$ set* if

$$(\lambda,\psi_N) = \sqcap_{i=1}^{\infty}(\lambda_i,\psi_{N_i})$$

where, each (λ_i,ψ_{N_i}) is an $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$F$\mathcal{IV}$-open set. The complement of an $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$F$G_\delta\mathcal{IV}$S is a *decreasing (resp. increasing) soft $\tilde{L}$-fuzzy $F_\sigma\mathcal{IV}$ set.* It is denoted by $\downarrow$ (resp. $\uparrow$) S$\tilde{L}$F$F_\sigma\mathcal{IV}$S.

Definition 5.8. Let $(X,\mathcal{V},\mathcal{I},\sqsubseteq)$ be an ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let (λ,ψ_N) be a soft $\tilde{L}$-fuzzy set in $(X,\mathcal{V},\mathcal{I},\sqsubseteq)$. Then, (λ,ψ_N) is said to be an *increasing (resp. decreasing) soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{IV}$ set* if it is both increasing (resp. decreasing) soft $\tilde{L}$-fuzzy c$\mathcal{IV}$-open set and increasing (resp. decreasing) soft $\tilde{L}$-fuzzy $F_\sigma\mathcal{IV}$ set. It is denoted by $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$F$cF_\sigma\mathcal{IV}$S. The complement of $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$F$cF_\sigma\mathcal{IV}$ set is called as a *decreasing (resp. increasing)soft $\tilde{L}$-fuzzy $cG_\delta\mathcal{IV}$ set.* It is denoted by $\downarrow$ (resp. $\uparrow$) S$\tilde{L}$F$cG_\delta\mathcal{IV}$S.

Definition 5.9. Let $(X,\mathcal{V},\mathcal{I},\sqsubseteq)$ be an ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let (λ,ψ_N) be a soft $\tilde{L}$-fuzzy set in $(X,\mathcal{V},\mathcal{I},\sqsubseteq)$. Then, (λ,ψ_N) is said to be an *increasing (resp. decreasing) soft $\tilde{L}$-fuzzy C-clopen$G_\delta F_\sigma\mathcal{IV}$ set* if it is both increasing (resp. decreasing) S$\tilde{L}$F$cF_\sigma\mathcal{IV}$ set and increasing (resp. decreasing) S$\tilde{L}$F$cG_\delta\mathcal{IV}$ set. It is denoted by $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$F$cG_\delta F_\sigma\mathcal{IV}$S.

Remark 5.10. Let $(X,\mathcal{V},\mathcal{I},\sqsubseteq)$ be an ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. If (λ,ψ_N) is $S\tilde{L}FcF_\sigma\mathcal{IV}$-set, then $D_0^{S\tilde{L}FcF_\sigma\mathcal{IV}}(\lambda,\psi_N) = D_0^{S\tilde{L}Fc\mathcal{IV}}(\lambda,\psi_N)$

Proof. Proof is obvious. □

Definition 5.11. An ordered soft $\tilde{L}$-fuzzy set which is both $\downarrow$ (resp. $\uparrow$) $S\tilde{L}Fc\mathcal{IV}$-open set and $\downarrow$ (resp. $\uparrow$) $S\tilde{L}Fc\mathcal{IV}$-closed set is called as a *$\downarrow$ (resp. $\uparrow$) $S\tilde{L}Fc\mathcal{IV}$-clopen set.*

Definition 5.12. An ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space $(X,\mathcal{V},\mathcal{I},\sqsubseteq)$ is said to have a *property* $\sharp$, if the union of any family of soft $\tilde{L}$-fuzzy C-$\mathcal{IV}$-open set is soft $\tilde{L}$-fuzzy C-$\mathcal{IV}$-open.

Definition 5.13. Let $(X, \mathcal{V}, \mathcal{I}, \sqsubseteq)$ be an ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let (λ, ψ_N) be a soft $\tilde{L}$-fuzzy set in $(X, \mathcal{V}, \mathcal{I}, \sqsubseteq)$. Then,

$$\begin{aligned} I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) &= \uparrow S\tilde{L}Fc\mathcal{IV}\text{-closure of } (\lambda, \psi_N) \\ &= \sqcap\{(\mu, \psi_M) : (\mu, \psi_M) \text{ is an } \uparrow S\tilde{L}Fc\mathcal{IV}\text{-closed set and } (\lambda, \psi_N) \sqsubseteq (\mu, \psi_M)\} \end{aligned}$$

$$\begin{aligned} D^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) &= \downarrow S\tilde{L}Fc\mathcal{IV}\text{-closure of } (\lambda, \psi_N) \\ &= \sqcap\{(\mu, \psi_M) : (\mu, \psi_M) \text{is a } \downarrow S\tilde{L}Fc\mathcal{IV}\text{-closed set and} (\lambda, \psi_N) \sqsubseteq (\mu, \psi_M)\} \end{aligned}$$

$$\begin{aligned} I_0^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) &= \uparrow S\tilde{L}Fc\mathcal{IV}\text{-interior of} (\lambda, \psi_N) \\ &= \sqcup\{(\mu, \psi_M) : (\mu, \psi_M) \text{is an } \uparrow S\tilde{L}Fc\mathcal{IV}\text{-open set and} (\lambda, \psi_N) \sqsupseteq (\mu, \psi_M)\} \end{aligned}$$

$$\begin{aligned} D_0^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) &= \downarrow S\tilde{L}Fc\mathcal{IV}\text{-interior of } (\lambda, \psi_N) \\ &= \sqcup\{(\mu, \psi_M) : (\mu, \psi_M) \text{is a } \downarrow S\tilde{L}Fc\mathcal{IV}\text{-open set and} (\lambda, \psi_N) \sqsupseteq (\mu, \psi_M)\} \end{aligned}$$

Clearly, $I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N)$ (resp. $D^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N)$) is the smallest increasing (resp. decreasing) soft $\tilde{L}$-fuzzy $c\mathcal{IV}$-closed set containing (λ, ψ_N) and $I_0^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N)$ (resp. $D_0^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N)$) is the largest increasing (resp. decreasing) soft $\tilde{L}$-fuzzy $c\mathcal{IV}$-open set contained in (λ, ψ_N).

Proposition 5.14. *For any soft $\tilde{L}$-fuzzy set, (λ, ψ_N) in $(X, \mathcal{V}, \mathcal{I}, \sqsubseteq)$, the following statements are hold.*
(i) $(1_X, \psi_X) - I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) = D_0^{S\tilde{L}Fc\mathcal{IV}}((1_X, \psi_X) - (\lambda, \psi_N))$.
(ii) $(1_X, \psi_X) - D^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) = I_0^{S\tilde{L}Fc\mathcal{IV}}((1_X, \psi_X) - (\lambda, \psi_N))$.
(iii) $(1_X, \psi_X) - I_0^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) = D^{S\tilde{L}Fc\mathcal{IV}}((1_X, \psi_X) - (\lambda, \psi_N))$.
(iv) $(1_X, \psi_X) - D_0^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) = I^{S\tilde{L}Fc\mathcal{IV}}((1_X, \psi_X) - (\lambda, \psi_N))$.

Proof. (i) Let $I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N)$ be an increasing $S\tilde{L}Fc\mathcal{IV}$-closed set containing (λ, ψ_N). Then, $(1_X, \psi_X) - I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N)$ is a decreasing $S\tilde{L}Fc\mathcal{IV}$-open set such that

$$(1_X, \psi_X) - I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) \sqsubseteq (1_X, \psi_X) - (\lambda, \psi_N).$$

Now, consider (μ, ψ_M) is another decreasing $S\tilde{L}Fc\mathcal{IV}$-open set such that $(\mu, \psi_M) \sqsubseteq (1_X, \psi_X) - (\lambda, \psi_N)$. Then, $(1_X, \psi_X) - (\mu, \psi_M)$ is an increasing $S\tilde{L}Fc\mathcal{IV}$-closed set such that $(1_X, \psi_X) - (\mu, \psi_M) \sqsupseteq (\lambda, \psi_N)$. It follows that, $I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) \sqsubseteq I^{S\tilde{L}Fc\mathcal{IV}}((1_X, \psi_X) - (\mu, \psi_M)) = (1_X, \psi_X) - (\mu, \psi_M)$. This implies that, $(\mu, \psi_M) \sqsubseteq (1_X, \psi_X) - I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N)$. Thus, $(1_X, \psi_X) - I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N)$ is the largest $\downarrow S\tilde{L}Fc\mathcal{IV}$-open set such that $(1_X, \psi_X) - I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) \sqsubseteq (1_X, \psi_X) - (\lambda, \psi_N)$. This implies that, $(1_X, \psi_X) - I^{S\tilde{L}Fc\mathcal{IV}}(\lambda, \psi_N) = D_0^{S\tilde{L}Fc\mathcal{IV}}((1_X, \psi_X) - (\lambda, \psi_N))$. Hence, (i) is proved. Similarly, (ii), (iii), (iv) can be proved. □

6. Pairwise ordered C-basically disconnected ideal $\mathcal{V}$-space

Definition 6.1. A *pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space* is a 5-tuples $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$, where X is a set , $\mathcal{V}_1, \mathcal{V}_2$ are any two soft $\tilde{L}$-fuzzy $\mathcal{V}$-structures on X, $\mathcal{I}$ is a soft $\tilde{L}$-fuzzy ideal and $\sqsubseteq$ is an ordered set.

Notation: An $\uparrow$ (resp. $\downarrow$) $S\tilde{L}Fc\mathcal{I}\mathcal{V}_1$-$cl(\lambda,\psi_N)$ (or) $\uparrow$ (resp. $\downarrow$) $S\tilde{L}Fc\mathcal{I}\mathcal{V}_2$-$cl(\lambda,\psi_N)$ is denoted by $I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\lambda,\psi_N)$ (resp. $D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\lambda,\psi_N)$). Similarly, for interior is denoted by $I_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\lambda,\psi_N)$ (resp. $D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\lambda,\psi_N)$).

Definition 6.2. Let $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$ be a pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let (λ,ψ_N) be any $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$F$cF_\sigma\mathcal{I}\mathcal{V}_1$ set or $\uparrow$ (resp. $\downarrow$) S$\tilde{L}$F$cF_\sigma\mathcal{I}\mathcal{V}_2$ set in $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$. If $I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\lambda,\psi_N)$ (resp. $D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\lambda,\psi_N)$) is $\uparrow$ (resp. $\downarrow$) $S\tilde{L}Fc\mathcal{I}\mathcal{V}_1$-open or $\mathcal{V}_2$-open set, then, $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$ is said to be *upper (resp. lower) $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}_1$ or $\mathcal{V}_2$-space.*

Definition 6.3. A pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$ is said to be *pairwise upper (resp. lower) $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}$-space* if it is both upper (resp. lower) $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}_1$-space and upper (resp. lower) $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}_2$-space.

Definition 6.4. A pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$ is said to be *pairwise ordered $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}$-space* if it is both pairwise upper $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}$-space and pairwise lower $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}$-space.

Proposition 6.5. *For a pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$, the following statements are equivalent:*

(a) $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$ *is pairwise upper soft $\tilde{L}$-fuzzy C-basically disconnected ideal $\mathcal{V}$-space.*

(b) *For each decreasing soft $\tilde{L}$-fuzzy $cG_\delta\mathcal{I}\mathcal{V}_1$ set or $\mathcal{V}_2$ set (λ,ψ_N), $D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N)$ is a decreasing soft $\tilde{L}$-fuzzy $c\mathcal{I}\mathcal{V}_2$ or $\mathcal{V}_1$-closed set.*

(c) *For each increasing soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set (λ,ψ_N), we have*

$$I_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N)) = I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N).$$

(d) *For each pair of increasing soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set (λ,ψ_N) and decreasing soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set (μ,ψ_M) with $D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((1_X,\psi_X)-(\lambda,\psi_N)) = (\mu,\psi_M)$, we have $(1_X,\psi_X) - I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N) = D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\mu,\psi_M)$.*

Proof. $(a)\Rightarrow(b)$: Let (λ,ψ_N) be a decreasing soft $\tilde{L}$-fuzzy $cG_\delta\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set. Now, $(\lambda,\psi_N)'$ is an increasing soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set. By (a), $I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((1_X,\psi_X)-(\lambda,\psi_N))$ is an increasing soft $\tilde{L}$-fuzzy $c\mathcal{I}\mathcal{V}_2$-open or $\mathcal{V}_1$-open set. Now,

$$I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((1_X,\psi_X)-(\lambda,\psi_N)) = (1_X,\psi_X) - D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N)$$

This implies that, $D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N)$ is a decreasing soft $\tilde{L}$-fuzzy $c\mathcal{I}\mathcal{V}_2$-closed or $\mathcal{V}_1$-closed set.

$(b)\Rightarrow(c)$: Let (λ,ψ_N) be an increasing soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set. Then, $(\lambda,\psi_N)'$ is a decreasing soft $\tilde{L}$-fuzzy $cG_\delta\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set. By (b), $D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((\lambda,\psi_N)')$

is a decreasing $S\tilde{L}Fc\mathcal{I}\mathcal{V}_2$-closed or $\mathcal{V}_1$-closed set. Now,

$$\begin{aligned}&(1_X,\psi_X)-I_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N))\\&=D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((1_X,\psi_X)-(\lambda,\psi_N)))\\&=D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((1_X,\psi_X)-(\lambda,\psi_N))\\&=(1_X,\psi_X)-I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N)\end{aligned}$$

Hence, $I_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N))=I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N)$

$(c)\Rightarrow(d)$: Let (λ,ψ_N) be an increasing soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set and (μ,ψ_M) be a decreasing soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set in $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$ with $D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((1_X,\psi_X)-(\lambda,\psi_N))=(\mu,\psi_M)$. By (c), we have

$$I_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N))=I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N).$$

Now,

$$\begin{aligned}&D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((1_X,\psi_X)-(\lambda,\psi_N)))\\&=(1_X,\psi_X)-I_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N))\\&=(1_X,\psi_X)-I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N)\\&=D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((1_X,\psi_X)-(\lambda,\psi_N))\\&=(\mu,\psi_M)\end{aligned}$$

This implies that, $D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\mu,\psi_M)=(\mu,\psi_M)$. Now,

$$\begin{aligned}(1_X,\psi_X)-I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N)&=D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((1_X,\psi_X)-(\lambda,\psi_N)\\&=(\mu,\psi_M)=D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\mu,\psi_M).\end{aligned}$$

$(d)\Rightarrow(a)$: Let (λ,ψ_N) be an increasing soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set in $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$. Consider a decreasing soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ set (μ,ψ_M) with $D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}((1_X,\psi_X)-(\lambda,\psi_N))=(\mu,\psi_M)$. By (d),

$$(1_X,\psi_X)-I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N)=D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\mu,\psi_M).$$

This implies that, $I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\lambda,\psi_N)=(1_X,\psi_X)-D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}(\mu,\psi_M)$. It follows that, $I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_2/\mathcal{V}_1}$ (λ,ψ_N) is an increasing $S\tilde{L}Fc\mathcal{I}\mathcal{V}_2$-open or $\mathcal{V}_1$-open set. Therefore, $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$ is pairwise upper $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}_1$-space and pairwise upper $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}_2$-space. Hence, $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq$ $)$ is pairwise upper $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}$-space. □

Proposition 6.6. *Let $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$ be a pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Then, $(X,\mathcal{V}_1,\mathcal{V}_2,\mathcal{I},\sqsubseteq)$ is pairwise upper $S\tilde{L}F$ C-basically disconnected ideal $\mathcal{V}$-space iff for each $\downarrow S\tilde{L}F$ $cF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set, (λ,ψ_N) and $\downarrow S\tilde{L}F$ $cG_\delta F_\sigma\mathcal{I}\mathcal{V}_2$ or $\mathcal{V}_1$-set, (μ,ψ_M) such that $(\lambda,\psi_N)\sqsubseteq(\mu,\psi_M)$, we have*

$$D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\lambda,\psi_N)\sqsubseteq D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\mu,\psi_M).$$

Proof. Using Proposition 6. 5 and Remark 5. 10, it is clear. ☐

Remark 6.7. Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ be a pairwise upper soft $\tilde{L}$-fuzzy C-basically disconnected ideal $\mathcal{V}$-space. Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ possess the property $\sharp$. Let

$$\{(\lambda_i, \psi_{N_i}), (\mu_j, \psi_{M_j}) : i, j \in \mathbb{N}\}$$

be a collection such that each (λ_i, ψ_{N_i})'s are $\downarrow S\tilde{L}FcF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-sets and (μ_j, ψ_{M_j})'s are $\downarrow S\tilde{L}FcG_\delta F_\sigma\mathcal{I}\mathcal{V}_2$ or $\mathcal{V}_1$-sets. Let (λ, ψ_N) and (μ, ψ_M) be the decreasing $S\tilde{L}FcF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set and $\downarrow S\tilde{L}FcG_\delta F_\sigma\mathcal{I}\mathcal{V}_2$ or $\mathcal{V}_1$-set respectively. If $(\lambda_i, \psi_{N_i}) \sqsubseteq (\lambda, \psi_N) \sqsubseteq (\mu_j, \psi_{M_j})$ and $(\lambda_i, \psi_{N_i}) \sqsubseteq (\mu, \psi_M) \sqsubseteq (\mu_j, \psi_{M_j})$, for all $i, j \in \mathbb{N}$, then there exists a $\downarrow$ soft $\tilde{L}$-fuzzy c$\mathcal{I}\mathcal{V}_1$ and $\mathcal{V}_2$-clopen set (γ, ψ_K) such that $D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}$ $(\lambda_i, \psi_{N_i}) \sqsubseteq (\gamma, \psi_K) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}$ (μ_j, ψ_{M_j}) for all $i, j \in \mathbb{N}$.

Proof. By Proposition 6. 6,

$$\begin{aligned} D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}\ (\lambda_i, \psi_{N_i}) &\sqsubseteq D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}\ (\lambda, \psi_N) \sqcap D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}\ (\mu, \psi_M) \\ &\sqsubseteq D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}\ (\mu_j, \psi_{M_j}), \end{aligned}$$

for all $i, j \in \mathbb{N}$. Since $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ is pairwise upper soft $\tilde{L}$-fuzzy C-basically disconnected ideal $\mathcal{V}$-space, it follows that, $(\gamma, \psi_K) = D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1\mathcal{V}_2}$ $(\lambda, \psi_N) \sqcap D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}$ (μ, ψ_M) is a $\downarrow$ soft $\tilde{L}$-fuzzy c$\mathcal{I}\mathcal{V}_1$ and $\mathcal{V}_2$-clopen set satisfying the required condition. ☐

Proposition 6.8. *Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ be a pairwise upper soft $\tilde{L}$-fuzzy C-basically disconnected ideal $\mathcal{V}$-space. Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ possess the property $\sharp$. Let*

$$\{(\lambda_q, \psi_{N_q})\}_{q\in\mathbb{Q}} \text{ and } \{(\mu_q, \psi_{M_q})\}_{q\in\mathbb{Q}}$$

be the monotone increasing collections of $\downarrow S\tilde{L}FcF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$ sets and $\downarrow$ soft $\tilde{L}$-fuzzy $cG_\delta F_\sigma\mathcal{I}\mathcal{V}_2$ or $\mathcal{V}_1$ sets of $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ respectively. ($\mathbb{Q}$ is the set of all rational numbers). If $(\lambda_{q_1}, \psi_{N_{q_1}}) \sqsubseteq (\mu_{q_2}, \psi_{M_{q_2}})$, whenever $q_1 < q_2$,$(q_1, q_2 \in \mathbb{Q})$, where $(\lambda_{q_1}, \psi_{N_{q_1}})$ is $\downarrow S\tilde{L}FcF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set and $(\mu_{q_2}, \psi_{M_{q_2}})$ is $\downarrow S\tilde{L}FcG_\delta F_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set, then, there exists a monotone increasing collection $\{(\gamma_q, \psi_{K_q})\}_{q\in\mathbb{Q}}$ of soft $\tilde{L}$-fuzzy C-$\mathcal{I}\mathcal{V}_1$ and $\mathcal{V}_2$-clopen sets of $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ such that $D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}$ $(\lambda_{q_1}, \psi_{N_{q_1}}) \sqsubseteq (\gamma_{q_2}, \psi_{K_{q_2}})$ and $(\gamma_{q_1}, \psi_{K_{q_1}}) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}$ $(\mu_{q_2}, \psi_{M_{q_2}})$ whenever $q_1 < q_2$.

Proof. Let us arrange into a sequence $\{q_n\}$ of rational numbers without repetitions. For every $n \geq 2$, define inductively a collection $\{(\gamma_{q_i}, \psi_{K_{q_i}}) : 1 \leq i < n\} \subseteq \tilde{L}^X$ such that

$$\left\{ \begin{array}{ll} D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\lambda_q, \psi_{N_q}) \sqsubseteq (\gamma_{q_i}, \psi_{K_{q_i}}), & \text{if } q < q_i \\ (\gamma_{q_i}, \psi_{K_{q_i}}) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\mu_q, \psi_{M_q}), & \text{if } q_i < q \end{array} \right\} \dashrightarrow (S_n)$$

for all $i < n$. By Proposition 6. 6, the countable collections

$$\{D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}\ (\lambda_q, \psi_{N_q})\}_{q\in\mathbb{Q}} \text{ and } \{D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}\ (\mu_q, \psi_{M_q})\}_{q\in\mathbb{Q}}$$

satisfying $D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}$ $(\lambda_{q_1}, \psi_{N_{q_1}}) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}$ $(\mu_{q_2}, \psi_{M_{q_2}})$, if $q_1 < q_2$. By Remark 6. 7, there exists a $\downarrow S\tilde{L}Fc\mathcal{I}\mathcal{V}_1$ and $\mathcal{V}_2$-open set, (δ, ψ_L) such that $D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}$ $(\lambda_{q_1}, \psi_{N_{q_1}}) \sqsubseteq (\delta, \psi_L) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}$ $(\mu_{q_2}, \psi_{M_{q_2}})$. By setting $(\gamma_{q_1}, \psi_{K_{q_1}}) = (\delta, \psi_L)$, we get (S_2). Assume that soft $\tilde{L}$-fuzzy sets $(\gamma_{q_i}, \psi_{K_{q_i}})$ (already defined), for $i < n$ and satisfy (S_n). Define

$$\Phi = \sqcup\{(\gamma_{q_i}, \psi_{K_{q_i}}) : i < n, q_i < q_n\} \sqcup (\lambda_{q_n}, \psi_{N_{q_n}})$$
$$\Omega = \sqcap\{(\gamma_{q_j}, \psi_{K_{q_j}}) : j < n, q_j > q_n\} \sqcap (\mu_{q_n}, \psi_{M_{q_n}})$$

Then, we have, $D^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\gamma_{q_i}, \psi_{K_{q_i}}) \sqsubseteq D^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\Phi) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\gamma_j, \psi_{K_j})$ and $D^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}\,(\gamma_{q_i}, \psi_{K_{q_i}}) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}\,(\Omega) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\gamma_j, \psi_{K_j})$ whenever $q_i < q_n < q_j$ $(i, j < n)$, as well as $(\lambda_q, \psi_{N_q}) \sqsubseteq D^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\Phi) \sqsubseteq (\mu_{q'}, \psi_{M_{q'}})$ and $(\lambda_q, \psi_{N_q}) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\Omega) \sqsubseteq (\mu_{q'}, \psi_{M_{q'}})$, whenever $q < q_n < q'$. This shows that the countable collections $\{(\gamma_{q_i}, \psi_{K_{q_i}}) : i < n, q_i < q_n\} \bigcup\{(\lambda_q, \psi_{N_q}) : q < q_n\}$ and $\{(\gamma_{q_j}, \psi_{K_{q_j}}) : j < n, q_j > q_n\} \bigcup\{(\mu_{q'}, \psi_{M_{q'}}) : q' > q_n\}$ together with Φ and Ω, fulfil all the conditions of the Remark 6. 7. Hence, there exists a decreasing soft $\tilde{L}$-fuzzy C-$\mathcal{IV}_1$ and $\mathcal{V}_2$-clopen set, (δ_n, ψ_{L_n}) such that $D^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\delta_n, \psi_{L_n}) \sqsubseteq (\mu_q, \psi_{M_q})$ if $q_n < q$, and $(\lambda_q, \psi_{N_q}) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\delta_n, \psi_{L_n})$ if $q < q_n$. Also,

$$D^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\gamma_{q_i}, \psi_{K_{q_i}}) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\delta_n, \psi_{L_n}), \text{ if } q_i < q_n$$

and $D^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\delta_n, \psi_{L_n}) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(\gamma_{q_j}, \psi_{K_{q_j}})$ if $q_n < q_j$, where $1 \leq i, j \leq n-1$. Now setting $(\gamma_{q_n}, \psi_{K_{q_n}}) = (\delta_n, \psi_{L_n})$, we obtain the soft $\tilde{L}$-fuzzy sets $(\gamma_{q_1}, \psi_{K_{q_1}})$, $(\gamma_{q_2}, \psi_{K_{q_2}}), \cdots, (\gamma_{q_n}, \psi_{K_{q_n}})$ that satisfy (S_{n+1}). Therefore, the collection $\{(\gamma_{q_i}, \psi_{K_{q_i}}) : i = 1, 2, 3, ...\}$ has the required property. This completes the proof. □

Definition 6.9. Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ be a pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. A mapping $f : X \to \mathbb{R}(L \times L)$ is called as the *$\mathcal{V}_i$-lower (resp. $\mathcal{V}_i$-upper) soft $\tilde{L}$-fuzzy C-$\mathcal{I}$-continuous function*, if $f^{-1}R_t$ (respy. $f^{-1}L_t$) is an increasing or decreasing soft $\tilde{L}$-fuzzy $cF_\sigma\mathcal{IV}_i$-set (soft $\tilde{L}$-fuzzy $cG_\delta F_\sigma\mathcal{IV}_i$-set), for each $t \in \mathbb{R}$, i = 1,2.

Proposition 6.10. *Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ be a pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let $(\lambda, \psi_N) \in \tilde{L}^X$. Let $f : X \to \mathbb{R}(L \times L)$ be such that*

$$f(x)(t) = \begin{cases} (1,1), & if\, t < 0 \\ (\lambda, \psi_N)(x), & if\, t \in [0,1] \\ (0,0), & if\, t > 1 \end{cases} \quad , \quad \textit{for all } x \in X$$

Then, f is $\mathcal{V}_i$-lower (resp. $\mathcal{V}_i$-upper) soft $\tilde{L}$-fuzzy C-$\mathcal{I}$-continuous function iff (λ, ψ_N) is an ↑ or ↓ soft $\tilde{L}$-fuzzy C-$\mathcal{IV}_i$-open (resp. $S\tilde{L}Fc\mathcal{IV}_i$-closed) set, for i = 1,2.

Proof. Proof is obvious. □

Definition 6.11. The *$\mathcal{V}$-characteristic function of* $(\lambda, \psi_N) \in \tilde{L}^X$ is the map $\chi_{(\lambda,\psi_N)}$ $: X \to \mathbb{R}(L \times L)$ defined by

$$\chi_{(\lambda,\psi_N)}(x) = \begin{cases} (0,0), & \text{if } t < 0 \\ (\lambda, \psi_N)(x), & \text{if } t \in [0,1] \\ (1,1), & \text{if } t > 1 \end{cases} \quad , \quad \text{for all } x \in X$$

Proposition 6.12. *Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ be a pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let $(\lambda, \psi_N) \in \tilde{L}^X$. Then, $\chi_{(\lambda,\psi_N)}$ is $\mathcal{V}_i$-lower (resp. $\mathcal{V}_i$-upper) soft $\tilde{L}$-fuzzy C-$\mathcal{I}$-continuous function iff (λ, N) is an $\uparrow$ or $\downarrow$ soft $\tilde{L}$-fuzzy C-$F_\sigma \mathcal{I}\mathcal{V}_i$ ($S\tilde{L}FcG_\delta F_\sigma \mathcal{I}\mathcal{V}_i$) set, for i = 1,2.*

Proof. It follows from the above Proposition 6. 10. □

Definition 6.13. Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ and $(Y, \mathcal{V}_3, \mathcal{V}_4, \mathcal{I}, \sqsubseteq)$ be a pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-spaces. A function $f : (X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq) \to (Y, \mathcal{V}_3, \mathcal{V}_4, \mathcal{I}, \sqsubseteq)$ is called an $\uparrow$ *(resp. $\downarrow$) $S\tilde{L}Fc\mathcal{I}\mathcal{V}_i$ continuous function* if $f^{-1}(\lambda, \psi_N)$ is an increasing (resp. decreasing) $S\tilde{L}Fc\mathcal{I}\mathcal{V}_i$-open set in $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$, for every $\uparrow$(resp. $\downarrow$) $S\tilde{L}F\mathcal{I}\mathcal{V}_3$-open set or $\uparrow$(resp. $\downarrow$) $S\tilde{L}F\mathcal{I}\mathcal{V}_4$-open set, (λ, ψ_N) in $(Y, \mathcal{V}_3, \mathcal{V}_4, \mathcal{I}, \sqsubseteq)$, for i = 1,2 . If f is both $\uparrow$ and $\downarrow$ S$\tilde{L}Fc\mathcal{I}\mathcal{V}_i$ continuous function, then it is called the *ordered $S\tilde{L}Fc\mathcal{I}\mathcal{V}_i$ continuous function*, for i = 1,2.

Proposition 6.14. *Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ be a pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Then, the following statements are equivalent.*

(a) *$(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ be a pairwise upper soft $\tilde{L}$-fuzzy C-basically disconnected space.*

(b) *(Insertion Theorem)Let $g, h : X \to \mathbb{R}(L \times L)$. Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ possesses the property $\sharp$. If g is $\mathcal{V}_1$ or $\mathcal{V}_2$-lower soft $\tilde{L}$-fuzzy C-$\mathcal{I}$-continuous and h is $\mathcal{V}_2$ or $\mathcal{V}_1$-upper soft $\tilde{L}$-fuzzy C-$\mathcal{I}$-continuous functions with $g \sqsubseteq h$, then there exists an increasing soft $\tilde{L}$-fuzzy C-$\mathcal{I}\mathcal{V}_1$ and $\mathcal{V}_2$-continuous function, $f : (X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq) \to \mathbb{R}(L \times L)$ such that $g \sqsubseteq f \sqsubseteq h$.*

(c) *(Urysohn Lemma)If $(\lambda, \psi_N)'$ is an increasing soft $\tilde{L}$-fuzzy C-$G_\delta F_\sigma \mathcal{I}\mathcal{V}_2$ or $\mathcal{V}_1$-set and (μ, ψ_M) is a decreasing soft $\tilde{L}$-fuzzy C-$F_\sigma \mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set such that $(\mu, \psi_M) \sqsubseteq (\lambda, \psi_N)$, then there exists a function, $f : (X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq) \to [0, 1](L \times L)$, which is both an increasing soft $\tilde{L}$-fuzzy C-$\mathcal{I}\mathcal{V}_1$-continuous function and an increasing soft $\tilde{L}$-fuzzy C-$\mathcal{I}\mathcal{V}_2$-continuous function such that $(\mu, \psi_M) \sqsubseteq (L_1)'f \sqsubseteq R_0 f \sqsubseteq (\lambda, \psi_N)$.*

Proof. $(a) \Rightarrow (b)$: Define $H_r = L_r h$ and $G_r = R'_r g$, $r \in \mathbb{Q}$. Then, we have two monotone increasing families respectively, $\downarrow S\tilde{L}FcF_\sigma \mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set and $\downarrow S\tilde{L}FcG_\delta F_\sigma \mathcal{I}\mathcal{V}_2$ or $\mathcal{V}_1$-set of $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$. Moreover, $H_r \sqsubseteq G_s$, if $r < s$. By Proposition 6. 8, there exists a monotone increasing family $\{F_r\}_{r\in\mathbb{Q}}$ of $\downarrow S\tilde{L}Fc\mathcal{I}\mathcal{V}_1$ and $\mathcal{V}_2$-clopen sets of $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ such that $D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(H_r) \sqsubseteq F_s$ and $F_r \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(G_s)$, whenever $r < s$. Let $U_t = \sqcap_{r<t} F'_r$, for all $t \in \mathbb{R}$, we define a monotone decreasing family $\{U_t : t \in \mathbb{R}\} \subseteq \tilde{L}^X$. Moreover, we have $I^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(U_t) \sqsubseteq I_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(U_s)$, whenever $s < t$. Now,

$$\begin{aligned}
\sqcup_{t\in\mathbb{R}} U_t &= \sqcup_{t\in\mathbb{R}} \sqcap_{r<t} F'_r \\
&\sqsupseteq \sqcup_{t\in\mathbb{R}} \sqcap_{r<t} (G_r)' \\
&= \sqcup_{t\in\mathbb{R}} \sqcap_{r<t} g^{-1} R_r \\
&= \sqcup_{t\in\mathbb{R}} g^{-1} R_t \\
&= g^{-1}(\sqcup_{t\in\mathbb{R}} R_t) \\
&= (1_X, \psi_X)
\end{aligned}$$

Similarly,

$$\sqcap_{t\in\mathbb{R}}U_t = (0_X, \psi_{1_\phi})$$

We now define a function $f : X \to \mathbb{R}(L \times L)$ possessing the required properties. Let $f(x)(t) = U_t(x)$, for all $x \in X, t \in \mathbb{R}$. By the above discussion, it follows that f is well defined. To prove f is $\uparrow S\tilde{L}Fc\mathcal{IV}_1$ and $\mathcal{V}_2$-continuous function, we observe that,

$$\sqcup_{s>t}U_s = \sqcup_{s>t}I_0^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(U_s)$$

and

$$\sqcap_{s<t}U_s = \sqcap_{s<t}I^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(U_s)$$

Then, $f^{-1}R_t = \sqcup_{s>t}U_s = \sqcup_{s>t}I_0^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(U_s)$ is $\uparrow S\tilde{L}Fc\mathcal{IV}_1$ and $\mathcal{V}_2$-open set and also, $f^{-1}(L_t') = \sqcap_{s<t}U_s = \sqcap_{s<t}I^{S\tilde{L}Fc\mathcal{IV}_1/\mathcal{V}_2}(U_s)$ is $\uparrow S\tilde{L}Fc\mathcal{IV}_1$ and $\mathcal{V}_2$-closed set. Therefore, f is an increasing soft $\tilde{L}$-fuzzy C-$\mathcal{IV}_1$ and $\mathcal{V}_2$-continuous function. To conclude the proof it remains to show that $g \sqsubseteq f \sqsubseteq h$. It is enough to show that, $g^{-1}(L_t') \sqsubseteq f^{-1}(L_t') \sqsubseteq h^{-1}(L_t')$ and $g^{-1}R_t \sqsubseteq f^{-1}R_t \sqsubseteq h^{-1}R_t$, for each $t \in \mathbb{R}$. Now, we have

$$\begin{aligned}
g^{-1}(L_t') &= \sqcap_{s<t}g^{-1}(L_s')\\
&= \sqcap_{s<t}\sqcap_{r<s}g^{-1}R_r\\
&= \sqcap_{s<t}\sqcap_{r<s}G_r'\\
&\sqsubseteq \sqcap_{s<t}\sqcap_{r<s}((1_X,\psi_X)-F_r)\\
&= \sqcap_{s<t}U_s\\
&= f^{-1}((1_X,\psi_X)-L_t)
\end{aligned}$$

Now,

$$\begin{aligned}
f^{-1}(L_t') &= \sqcap_{s<t}U_s\\
&= \sqcap_{s<t}\sqcap_{r<s}F_r'\\
&\sqsubseteq \sqcap_{s<t}\sqcap_{r<s}H_r'\\
&= \sqcap_{s<t}\sqcap_{r<s}h^{-1}L_r'\\
&= \sqcap_{s<t}h^{-1}(L_s')\\
&= h^{-1}(L_t')
\end{aligned}$$

Similarly, we obtain,

$$\begin{aligned}
g^{-1}R_t &= \sqcup_{s>t}g^{-1}R_s\\
&= \sqcup_{s>t}\sqcup_{r>s}g^{-1}R_r\\
&= \sqcup_{s>t}\sqcup_{r>s}((1_X,\psi_X)-G_r)\\
&\sqsubseteq \sqcup_{s>t}\sqcap_{r<s}((1_X,\psi_X)-F_r)\\
&= \sqcup_{s>t}U_s\\
&= f^{-1}R_t
\end{aligned}$$

Now,

$$\begin{aligned} f^{-1}R_t &= \sqcup_{s>t} U_s \\ &= \sqcup_{s>t} \sqcap_{r<s} ((1_X, \psi_X) - F_r) \\ &\sqsubseteq \sqcup_{s>t} \sqcup_{r>s} ((1_X, \psi_X) - H_r) \\ &= \sqcup_{s>t} \sqcup_{r>s} h^{-1}(L'_r) \\ &= \sqcup_{s>t} h^{-1} R_s \\ &= h^{-1} R_t \end{aligned}$$

Thus, (b) is proved.

$(b) \Rightarrow (c)$: Suppose that (λ, ψ_N) is $\downarrow S\tilde{L}FcG_\delta F_\sigma \mathcal{I}\mathcal{V}_2$ or $\mathcal{V}_1$-set and (μ, ψ_M) is $\downarrow S\tilde{L}FcF_\sigma \mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set such that $(\mu, \psi_M) \sqsubseteq (\lambda, \psi_N)$. Then, $\chi_{(\mu,\psi_M)} \sqsubseteq \chi_{(\lambda,\psi_N)}$, where $\chi_{(\mu,\psi_M)}$ and $\chi_{(\lambda,\psi_N)}$ are the $\mathcal{V}_1$ or $\mathcal{V}_2$-lower and $\mathcal{V}_2$ or $\mathcal{V}_1$-upper soft $\tilde{L}$-fuzzy C-$\mathcal{I}$-continuous functions respectively. Hence, by (b), there exists an $\uparrow$ soft $\tilde{L}$-fuzzy C-$\mathcal{I}\mathcal{V}_1$ and $\mathcal{V}_2$-continuous function, $f : X \to \mathbb{R}(L \times L)$ such that $\chi_{(\mu,\psi_M)} \sqsubseteq f \sqsubseteq \chi_{(\lambda,\psi_N)}$. Clearly, $f(x) \in \tilde{L}^{\mathbb{R}}$, for all $x \in \mathbb{R}$ and

$$\begin{aligned} (\mu, \psi_M) &= L'_1 \chi_{(\mu,\psi_M)} \\ &\sqsubseteq L'_1 f \\ &\sqsubseteq R_0 f \\ &\sqsubseteq R_0 \chi_{(\lambda,\psi_N)} \\ &= (\lambda, \psi_N) \end{aligned}$$

Therefore, $(\mu, \psi_M) \sqsubseteq L'_1 f \sqsubseteq R_0 f \sqsubseteq (\lambda, \psi_N)$.

$(c) \Rightarrow (a)$: Let (λ, ψ_N) be $\downarrow S\tilde{L}FcG_\delta F_\sigma \mathcal{I}\mathcal{V}_2$ or $\mathcal{V}_1$-set and (μ, ψ_M) be $\downarrow S\tilde{L}FcF_\sigma \mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set such that $(\mu, \psi_M) \sqsubseteq (\lambda, \psi_N)$. Then, there exists an $\uparrow S\tilde{L}Fc\mathcal{I}\mathcal{V}_1$ and $\mathcal{V}_2$-continuous function, $f : X \to [0,1](L \times L)$ such that $L'_1 f \sqsubseteq R_0 f$. In fact that, L'_1 is a soft $\tilde{L}$-fuzzy closed set and R_0 is a soft $\tilde{L}$-fuzzy open set. Since $(\mu, \psi_M) \sqsubseteq L'_1 f \sqsubseteq R_0 f \sqsubseteq (\lambda, \psi_N)$, it follows that, $D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\mu, \psi_M) \sqsubseteq D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(L'_1 f) = L'_1 f$. Similarly, $R_0 f = D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(R_0 f) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\lambda, \psi_N)$. This implies that, $D^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\mu, \psi_M) \sqsubseteq D_0^{S\tilde{L}Fc\mathcal{I}\mathcal{V}_1/\mathcal{V}_2}(\lambda, \psi_N)$. By Proposition 6. 6, $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ is a pairwise upper soft $\tilde{L}$-fuzzy C-basically disconnected ideal $\mathcal{V}$-space. □

Note: The Proposition 6. 5, Proposition 6. 6 and Prposition: 6. 8, Proposition 6. 14 and Remark 6. 7 can also be discussed for pairwise lower soft $\tilde{L}$-fuzzy C-basically disconnected ideal $\mathcal{V}$-space.

Definition 6.15. Let $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ be a pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space. Let A be any subset of X. Then, the pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-space $(A, \mathcal{V}_1|A,\ \mathcal{V}_2|A, \mathcal{I}|A, \sqsubseteq)$ is called a pairwise ordered soft $\tilde{L}$-fuzzy ideal $\mathcal{V}$-subspace of $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$. Where $\mathcal{V}_1|A = \{(\lambda, \psi_N)|A : (\lambda, \psi_N) \in \mathcal{V}_1\}$ and $\mathcal{V}_2|A = \{(\mu, \psi_M)|A : (\mu, \psi_M) \in \mathcal{V}_2\}$ are the soft $\tilde{L}$-fuzzy $\mathcal{V}$-structures on A.

Tietze Extension Theorem on pairwise ordered soft $\tilde{L}$-fuzzy C-basically disconnected ideal $\mathcal{V}$-space

Definition 6.16. Let X be any non-empty crisp set. Let A be any subset of X and χ_A^*: $X \to \{(1_X, \psi_X), (0_X, \psi_\phi)\}$. Then, the *characteristic* function of* A, χ_A^* is defined as

$$\chi_A^*(x) = \begin{cases} (1_X, \psi_X), & \text{if } x \in A \\ (0_X, \psi_\phi), & \text{if } x \notin A \end{cases} \quad , \text{ for all } x \in X$$

Proposition 6.17. *Let* $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ *be a pairwise ordered soft* $\tilde{L}$*-fuzzy C-basically disconnected ideal* $\mathcal{V}$*-space. Let* $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$ *possesses the property* $\sharp$*. Let* $A \subseteq X$ *such that* χ_A^* *is a soft* $\tilde{L}$*-fuzzy C-*$G_\delta F_\sigma \mathcal{I}\mathcal{V}_1$ *or* $\mathcal{V}_2$*-set and let* $f : (A, \mathcal{V}_1|A, \mathcal{V}_2|A, \mathcal{I}|A, \sqsubseteq) \to [0,1](L \times L)$ *be an* $\uparrow$ *soft* $\tilde{L}$*-fuzzy C-*$\mathcal{I}\mathcal{V}_1$ *and* $\mathcal{V}_2$*-continuous and isotone function. Then,* f *admits an extension* $\mathcal{F} : (X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq) \to [0,1](L \times L)$ *with all its properties preserved if* f *satisfies the following* θ *property.*
(θ) $[(\lambda, \psi_N)] \sqsubseteq [(\mu, \psi_M)] \Rightarrow f^{-1}\{\chi_{\{[(0_X, \psi_\phi)], [(\lambda, \psi_N)]\}}\} \sqsubseteq f^{-1}\{\chi_{\{[(\mu, \psi_M)], [(1_X, \psi_X)]\}}\}$ *where* $\eta \sqsubseteq \xi \Rightarrow D_{\mathcal{V}_2}(f(\eta)) \sqcap I_{\mathcal{V}_2}(f(\xi)) = (0_X, \psi_\phi)$ *and* $\{[(\lambda, \psi_N)], [(\mu, \psi_M)]\} = \{[(\mu, \psi_M)] \in I(L \times L) : [(\lambda, \psi_N)] \sqsubseteq [(\gamma, \psi_K)] \sqsubseteq [(\mu, \psi_M)]\}$.

Proof. Define two functions $g, h : X \to [0,1](L \times L)$ by

$$g(x) = \begin{cases} f(x), & \text{if } x \in A \\ [(\lambda_0, \psi_{N_0})], & \text{if } x \notin A \end{cases}$$

and

$$h(x) = \begin{cases} f(x), & \text{if } x \in A \\ [(\lambda_1, \psi_{N_1})], & \text{if } x \notin A \end{cases}$$

where, $[(\lambda_0, \psi_{N_0})]$ and $[(\lambda_1, \psi_{N_1})]$ are the equivalence classes determined by

$$(\lambda_0, \psi_{N_0}), (\lambda_1, \psi_{N_1}) : \mathbb{R} \to L \times L$$

such that

$$(\lambda_0, \psi_{N_0})(t) = \begin{cases} (1,1), & \text{if } t \leq 0 \\ (0,0), & \text{if } t > 0 \end{cases}$$

and

$$(\lambda_1, \psi_{N_1})(t) = \begin{cases} (1,1), & \text{if } t < 1 \\ (0,0), & \text{if } t \geq 1 \end{cases}$$

Now, we have to show that g and h are $\mathcal{V}_1$ or $\mathcal{V}_2$-lower and $\mathcal{V}_1$ or $\mathcal{V}_2$-upper $S\tilde{L}Fc\mathcal{I}$ continuous functions. Indeed, let $t \geq 1$. Then,

$$L_t h(x) = \begin{cases} L_t f(x), & \text{if } x \in A \\ (1_X, \psi_X), & \text{if } x \notin A \end{cases}$$

where, $L_t h$ being $\uparrow$ or $\downarrow$ $S\tilde{L}FcG_\delta F_\sigma \mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set in $(A, \mathcal{V}_1|A, \mathcal{V}_2|A, \mathcal{I}|A, \sqsubseteq)$ is of the form $(\mu_t, \psi_{M_t})|A$ where (μ_t, ψ_{M_t}) is $S\tilde{L}FcG_\delta F_\sigma \mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set so that

$$L_t h = \begin{cases} (\mu_t, \psi_{M_t}) \sqcap \chi_A^*, & \text{if } t < 1 \\ (0_X, \psi_\phi), & \text{if } t \geq 1 \end{cases} \quad , \text{ for all } t \in \mathbb{R}$$

is $\uparrow$ or $\downarrow$ $S\tilde{L}FcG_\delta F_\sigma \mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set in $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$. Thus, h is a $\mathcal{V}_1$ or $\mathcal{V}_2$-upper soft $\tilde{L}$-fuzzy C-$\mathcal{I}$-continuous function.

$$R_t g(x) = \begin{cases} R_t f(x), & \text{if } x \in A \\ (1_X, \psi_X), & \text{if } x \notin A \end{cases}$$

where, $R_t g$ being $\uparrow$ or $\downarrow$ $S\tilde{L}FcF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set in $(A, \mathcal{V}_1|A,\ \mathcal{V}_2|A, \mathcal{I}|A, \sqsubseteq)$ is of the form $(\lambda_t, \psi_{N_t})|A$ where (λ_t, ψ_{N_t}) is $S\tilde{L}FcF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set so that

$$R_t g = \begin{cases} (\lambda_t, \psi_{N_t}) \sqcap \chi_A^*, & \text{if } t > 0 \\ (1_X, \psi_X), & \text{if } t \leq 0 \end{cases}, \quad \text{for all } t \in \mathbb{R}$$

is $\uparrow$ or $\downarrow$ $S\tilde{L}FcF_\sigma\mathcal{I}\mathcal{V}_1$ or $\mathcal{V}_2$-set in $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$. Thus, g is a $\mathcal{V}_1$ or $\mathcal{V}_2$-lower soft $\tilde{L}$-fuzzy C-$\mathcal{I}$-continuous function. Clearly, $g \sqsubseteq h$. By Proposition 6. 6, there exists an $\uparrow$ soft $\tilde{L}$-fuzzy C-$\mathcal{I}\mathcal{V}_1$ and $\mathcal{V}_2$-continuous function, $\mathcal{F} : X \to [0,1](L \times L)$ such that $g(x) \sqsubseteq \mathcal{F}(x) \sqsubseteq h(x)$, for all $x \in X$. Hence, for all $x \in A$, we have $g(x) \sqsubseteq f(x) \sqsubseteq h(x)$ so that $\mathcal{F}$ is the required extension of f over $(X, \mathcal{V}_1, \mathcal{V}_2, \mathcal{I}, \sqsubseteq)$. Moreover, $\mathcal{F}$ is isotone as f satisfies the θ property. □

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