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Soft semi separation axioms and some types of soft functions

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Abstract. Shabir and Naz in [23] introduced the notion of soft topological spaces. They defined basic notions of soft topological spaces such as open soft sets, closed soft sets, soft subspaces, soft closure, soft nbd of a soft point, soft separation axioms, soft regular spaces, soft normal spaces and they established their several properties. Min in [17] investigated some properties of such soft separation axioms. In the present paper, we have continued to study the properties of soft topological spaces. We introduce new soft separation axioms based on the semi open soft sets which are more general than of the open soft sets. We show that the properties of soft semi T_i-spaces ($i = 1, 2$) are soft topological properties under the bijection and irresolute open soft mapping. Also, the property of being soft semi regular and soft semi normal are soft topological properties under bijection, irresolute soft and irresolute open soft functions. Further, we show that the properties of being soft semi T_i-spaces ($i = 1, 2, 3, 4$) are hereditary properties.

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Keywords: Soft set, Soft topological space, Soft interior, Soft closure, Open soft, Closed soft, Soft semi separation axioms, Soft semi T_i-spaces ($i = 1, 2, 3, 4$), Soft semi regular, Soft semi normal, Irresolute soft functions, Irresolute open soft functions.

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1. Introduction

The concept of soft sets was first introduced by Molodtsov [19] in 1999 as a general mathematical tool for dealing with uncertain objects. In [19, 18], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration,

probability, theory of measurement, and so on. After presentation of the operations of soft sets [15], the properties and applications of soft set theory have been studied increasingly [4, 11, 18, 21]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1, 3, 5, 7, 13, 14, 15, 16, 18, 20, 22, 25]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [8]. Recently, in 2011, Shabir and Naz [23] initiated the study of soft topological spaces. They defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Min in [17] investigate some properties of these soft separation axioms mentioned in [23]. Banu and Halis in [6] studied some properties of soft Hausdorff space.
The main purpose of this paper is to introduce the notion of soft semi separation axioms. In particular we study the properties of the soft semi regular spaces and soft semi normal spaces. We show that if x_E is semi closed soft set for all $x \in X$ in a soft topological space (X, τ, E), then (X, τ, E) is soft semi T_1-space. Also, we show that if a soft topological space (X, τ, E) is soft semi T_3-space, then $\forall\, x \in X$, x_E is semi closed soft set. This paper, not only can form the theoretical basis for further applications of topology on soft sets, but also lead to the development of information systems.

2. Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1 ([19])**.** Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A). The set of all these soft sets over X denoted by $SS(X)_A$.

Definition 2.2 ([15])**.** Let $F_A,\ G_B \in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if

- **(1):** $A \subseteq B$, and
- **(2):** $F(e) \subseteq G(e), \forall e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition 2.3 ([15])**.** Two soft subset F_A and G_B over a common universe set X are said to be soft equal if F_A is soft subset of G_B and G_B is soft subset of F_A.

Definition 2.4 ([4])**.** The complement of a soft set (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$, $F^c : A \to P(X)$ is a mapping given by $F^c(e) = X - F(e),\ \forall\, e \in A$ and F^c is called the soft complement function of F.
Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

Definition 2.5 ([23]). The difference of two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E) - (G, E)$ is the soft set (H, E) where for all $e \in E$, $H(e) = F(e) - G(e)$.

Definition 2.6 ([23]). Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$.

Definition 2.7 ([23]). The soft set (F, E) over X such that $F(e) = \{x\}\ \forall e \in E$ is called singleton soft point and denoted by x_E or (x, E).

Definition 2.8 ([15]). A soft set (F, A) over X is said to be a NULL soft set denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A,\ F(e) = \phi$.

Definition 2.9 ([15]). A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A,\ F(e) = X$. Clearly, we have $X_A^c = \phi_A$ and $\phi_A^c = X_A$.

Definition 2.10 ([15]). The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e),\ e \in A - B, \\ G(e),\ e \in B - A, \\ F(e) \cup G(e),\ e \in A \cap B \end{cases} .$$

Definition 2.11 ([15]). The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.
Note that, in order to efficiently discuss, we consider only soft sets (F, E) over a universe X in which all the parameter sets E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition 2.12 ([26]). Let I be an arbitrary indexed set and $L = \{(F, E)_i, i \in I\}$ be a subfamily of $SS(X)_E$.

- **(1):** The union of L is the soft set (H, E), where $H(e) = \bigcup_{i \in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcup}_{i \in I}(F, E)_i = (H, E)$.
- **(2):** The intersection of L is the soft set (M, E), where $M(e) = \bigcap_{i \in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcap}_{i \in I}(F, E)_i = (M, E)$.

Definition 2.13 ([23]). Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

- **(1):** $\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X,\ \forall e \in E$,
- **(2):** the union of any number of soft sets in τ belongs to τ,
- **(3):** the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.14 ([9]). Let (X, τ, E) be a soft topological space. A soft set (F, A) over X is said to be closed soft set in X, if its relative complement $(F, A)^c$ is open soft set.

Definition 2.15 ([9]). Let(X, τ, E) be a soft topological space. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X, \tau, E)$, or $OS(X)$ and the set of all closed soft sets by $CS(X, \tau, E)$, or $CS(X)$.

Definition 2.16 ([23]). Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft closure of (F, E), denoted by $cl(F, E)$ is the intersection of all closed soft super sets of (F, E) i.e
$cl(F, E) = \tilde{\cap}\{(H, E) : (H, E)\ \textit{is closed soft set and}\ (F, E)\tilde{\subseteq}(H, E)\})$.

Definition 2.17 ([26]). Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft interior of (G, E), denoted by $int(G, E)$ is the union of all open soft subsets of (G, E) i.e
$int(G, E) = \tilde{\cup}\{(H, E) : (H, E)\ \textit{is an open soft set and}\ (H, E)\tilde{\subseteq}(G, E)\})$.

Definition 2.18 ([26]). The soft set $(F, E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by x_e.

Definition 2.19 ([26]). The soft point x_e is said to be belonging to the soft set (G, A), denoted by $x_e\tilde{\in}(G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

Proposition 2.1 ([24]). *The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it.*

Definition 2.20 ([23]). Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$ and Y be a non null subset of X. Then the sub soft set of (F, E) over Y denoted by (F_Y, E), is defined as follows:

$$F_Y(e) = Y \cap F(e)\ \forall e \in E.$$

In other words $(F_Y, E) = \tilde{Y}\tilde{\cap}(F, E)$.

Definition 2.21 ([23]). Let (X, τ, E) be a soft topological space and Y be a non null subset of X. Then

$$\tau_Y = \{(F_Y, E) : (F, E) \in \tau\}$$

is called the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Theorem 2.1 ([23]). *Let (Y, τ_Y, E) be a soft subspace of a soft topological space (X, τ, E) and $(F, E) \in SS(X)_E$. Then*

- **(1):** *If (F, E) is open soft set in Y and $\tilde{Y} \in \tau$, then $(F, E) \in \tau$.*
- **(2):** *(F, E) is open soft set in Y if and only if $(F, E) = \tilde{Y}\tilde{\cap}(G, E)$ for some $(G, E) \in \tau$.*
- **(3):** *(F, E) is closed soft set in Y if and only if $(F, E) = \tilde{Y}\tilde{\cap}(H, E)$ for some (H, E) is τ-closed soft set.*

Definition 2.22 ([10]). Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. If $(F, E)\tilde{\subseteq}cl(int(F, E))$, then (F, E) is called semi-open soft set. We denote the set of all semi-open soft sets by $SOS(X, \tau, E)$, or $SOS(X)$ and the set of all semi-closed soft sets by $SCS(X, \tau, E)$, or $SCS(X)$.

Definition 2.23 ([2]). Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets on X and Y respectively, $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a mapping. Then;

(1): If $(F, A) \in SS(X)_A$. Then the image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is soft set in $SS(Y)_B$ such that
$$f_{pu}(F)(b) = \begin{cases} \cup_{a \in p^{-1}(b) \cap A} \ u(F(a)), & p^{-1}(b) \cap A \neq \phi, \\ \phi, & otherwise. \end{cases}$$
for all $b \in B$.

(2): If $(G, B) \in SS(Y)_B$. Then the inverse image of (G, B) under f_{pu}, written as $f^{-1}_{pu}(G, B) = (f^{-1}_{pu}(G), p^{-1}(B))$, is soft set in $SS(X)_A$ such that
$$f^{-1}_{pu}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B, \\ \phi, & otherwise. \end{cases}$$
for all $a \in A$.

The soft function f_{pu} is called surjective if p and u are surjective, also it is said to be injective if p and u are injective.

Definition 2.24 ([10, 12, 26]). Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, The function f_{pu} is said to be

(1): Continuous soft if $f^{-1}_{pu}(G, B) \in \tau_1 \ \forall \ (G, B) \in \tau_2$.
(2): Open soft if $f_{pu}(G, A) \in \tau_2 \forall \ (G, A) \in \tau_1$.
(3): Semi open soft if $f_{pu}(G, A) \in SOS(Y) \forall \ (G, A) \in \tau_1$.
(4): Semi continuous soft function if $f^{-1}_{pu}(G, B) \in SOS(X) \forall \ (G, B) \in \tau_2$.
(5): Irresolute soft if $f^{-1}_{pu}(G, B) \in SOS(X) \forall \ (G, B) \in SOS(Y) [f^{-1}_{pu}(F, B) \in SCS(X) \forall \ (F, B) \in SCS(Y)]$.
(6): Irresolute open soft (resp. irresolute closed soft) if $f_{pu}(G, A) \in SOS(Y)$ $\forall \ (G, A) \in SOS(X)$ (resp. $f_{pu}(F, A) \in SCS(Y) \ \forall \ (F, A) \in SCS(Y)$).

Theorem 2.2 ([2]). *Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. For the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$, the following statements hold,*

(a): $f^{-1}_{pu}((G, B)^c) = (f^{-1}_{pu}(G, B))^c \forall \ (G, B) \in SS(Y)_B$.
(b): $f_{pu}(f^{-1}_{pu}((G, B))) \tilde{\subseteq} (G, B) \forall \ (G, B) \in SS(Y)_B$. *If f_{pu} is surjective, then the equality holds.*
(c): $(F, A) \tilde{\subseteq} f^{-1}_{pu}(f_{pu}((F, A))) \forall \ (F, A) \in SS(X)_A$. *If f_{pu} is injective, then the equality holds.*
(d): $f_{pu}(\tilde{X}) \tilde{\subseteq} \tilde{Y}$. *If f_{pu} is surjective, then the equality holds.*
(e): $f^{-1}_{pu}(\tilde{Y}) = \tilde{X}$ *and* $f_{pu}(\tilde{\phi}_A) = \tilde{\phi}_B$.
(f): *If* $(F, A) \tilde{\subseteq} (G, A)$, *then* $f_{pu}(F, A) \tilde{\subseteq} f_{pu}(G, A)$.
(g): *If* $(F, B) \tilde{\subseteq} (G, B)$, *then* $f^{-1}_{pu}(F, B) \tilde{\subseteq} f^{-1}_{pu}(G, B) \ \forall \ (F, B), (G, B) \in SS(Y)_B$.
(h): $f^{-1}_{pu}[(F, B) \tilde{\cup} (G, B)] = f^{-1}_{pu}(F, B) \tilde{\cup} f^{-1}_{pu}(G, B)$ *and* $f^{-1}_{pu}[(F, B) \tilde{\cap} (G, B)] = f^{-1}_{pu}(F, B) \tilde{\cap} f^{-1}_{pu}(G, B) \ \forall \ (F, B), (G, B) \in SS(Y)_B$.
(I): $f_{pu}[(F, A) \tilde{\cup} (G, A)] = f_{pu}(F, A) \tilde{\cup} f_{pu}(G, A)$ *and*
$$f_{pu}[(F, A) \tilde{\cap} (G, A)] \tilde{\subseteq} f_{pu}(F, A) \tilde{\cap} f_{pu}(G, A)$$
$\forall (F, A), (G, A) \in SS(X)_A$. *If f_{pu} is injective, then the equality holds.*

3. Soft semi separation axioms

Definition 3.1. Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. Then (X, τ, E) is called a soft semi T_o-space if there exist semi open soft sets (F, E) and (G, E) such that either $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$.

Proposition 3.1. *Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. If there exist semi open soft sets (F, E) and (G, E) such that either $x \in (F, E)$ and $y \in (F, E)^c$ or $y \in (G, E)$ and $x \in (G, E)^c$. Then (X, τ, E) is soft semi T_o-space.*

Proof. Let $x, y \in X$ such that $x \neq y$. Let (F, E) and (G, E) be semi open soft sets such that either $x \in (F, E)$ and $y \in (F, E)^c$ or $y \in (G, E)$ and $x \in (G, E)^c$. If $x \in (F, E)$ and $y \in (F, E)^c$. Then $y \in (F(e))^c$ for all $e \in E$, This implies that, $y \notin F(e)$ for all $e \in E$. Therefore, $y \notin (F, E)$. Similarly, if $y \in (G, E)$ and $x \in (G, E)^c$, then $x \notin (G, E)$. Hence (X, τ, E) is soft semi T_o-space. □

Theorem 3.1. *A soft subspace (Y, τ_Y, E) of a soft semi T_o-space (X, τ, E) is soft semi T_o.*

Proof. Let $x, y \in Y$ such that $x \neq y$. Then $x, y \in X$ such that $x \neq y$. Hence there exist semi open soft sets (F, E) and (G, E) in X such that either $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$. Since $x \in Y$. Then $x \in \tilde{Y}$. Hence $x \in \tilde{Y} \tilde{\cap} (F, E) = (F_Y, E)$, (F, E) is semi open soft set. Consider $y \notin (F, E)$, This implies that, $y \notin F(e)$ for some $e \in E$. Therefore, $y \notin \tilde{Y} \tilde{\cap} (F, E) = (F_Y, E)$. Similarly, if $y \in (G, E)$ and $x \notin (G, E)$, then $y \in (G_Y, E)$ and $x \notin (G_Y, E)$. Thus, (Y, τ_Y, E) is soft semi T_o. □

Definition 3.2. Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. Then (X, τ, E) is called a soft semi T_1-space if there exist semi open soft sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$.

Proposition 3.2. *Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. If there exist semi open soft sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \in (F, E)^c$ and $y \in (G, E)$ and $x \in (G, E)^c$. Then (X, τ, E) is soft semi T_1-space.*

Proof. It is similar to the proof of Theorem 3.1. □

Theorem 3.2. *A soft subspace (Y, τ_Y, E) of a soft semi T_1-space (X, τ, E) is soft semi T_1.*

Proof. It is similar to the proof of Theorem 3.1. □

Theorem 3.3. *Let (X, τ, E) be a soft topological space. If x_E is semi closed soft set in τ for all $x \in X$, then (X, τ, E) is soft semi T_1-space.*

Proof. Suppose that $x \in X$ and x_E is semi closed soft set in τ. Then x_E^c is semi open soft set in τ. Let $x, y \in X$ such that $x \neq y$. For $x \in X$ and x_E^c is semi open soft set such that $x \notin x_E^c$ and $y \in x_E^c$. Similarly, y_E^c is semi open soft set in τ such that $y \notin y_E^c$ and $x \in y_E^c$. Thus, (X, τ, E) is soft semi T_1-space over X. □

Definition 3.3. Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. Then (X, τ, E) is called a soft semi Hausdorff space or a soft semi T_2-space if there exist semi open soft sets (F, E) and (G, E) such that $x \in (F, E)$, $y \in (G, E)$ and $(F, E)\tilde{\cap}(G, E) = \tilde{\phi}$.

Theorem 3.4. *For a soft topological space (X, τ, E) we have:*
soft semi T_2-space $\Rightarrow$ soft semi T_1-space $\Rightarrow$ soft semi T_o-space.

Proof. Straightforward. □

Remark 3.1. The converse of Theorem 3.4 is not true in general, as shown in the following examples.

Examples 3.1. **(1):** Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and

$$\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$$

where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined as follows:
$F_1(e_1) = X$, $F_1(e_2) = \{h_2\}$,
$F_2(e_1) = \{h_1\}$, $F_2(e_2) = X$,
$F_3(e_1) = \{h_1\}$, $F_3(e_2) = \{h_2\}$.
Then τ defines a soft topology on X. Also (X, τ, E) is soft semi T_1-space but it is not a soft semi T_2-space, for $h_1, h_2 \in X$ and $h_1 \neq h_2$, but there is no semi open soft sets (F, E) and (G, E) such that $h_1 \in (F, E)$, $h_2 \in (G, E)$ and $(F, E)\tilde{\cap}(G, E) = \tilde{\phi}$.

(2): Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E)\}$ where (F_1, E) is soft set over X defined as follows by $F_1(e_1) = X$, $F_1(e_2) = \{h_2\}$.
Then τ defines a soft topology on X. Also (X, τ, E) is soft semi T_o-space but not a soft semi T_1-space, since $h_1, h_2 \in X$, $h_1 \neq h_2$, but all the open soft sets which contain h_1 also contain h_2.

Theorem 3.5. *A soft subspace (Y, τ_Y, E) of a soft semi T_2-space (X, τ, E) is soft semi T_2.*

Proof. Let $x, y \in Y$ such that $x \neq y$. Then $x, y \in X$ such that $x \neq y$. Hence there exist semi open soft sets (F, E) and (G, E) in X such that $x \in (F, E)$, $y \in (G, E)$ and $(F, E)\tilde{\cap}(G, E) = \tilde{\phi}$. It follows that $x \in F(e)$, $y \in G(e)$ and $F(e) \cap G(e) = \phi$ for all $e \in E$. This implies that, $x \in Y \cap F(e)$, $y \in Y \cap G(e)$ and $F(e) \cap G(e) = \phi$ for all $e \in E$. Thus, $x \in \tilde{Y}\tilde{\cap}(F, E) = (F_Y, E)$, $y \in \tilde{Y}\tilde{\cap}(G, E) = (G_Y, E)$ and $(F_Y, E)\tilde{\cap}(G_Y, E) = \tilde{\phi}$, where $(F_Y, E), (G_Y, E)$ are semi open soft sets in Y. Therefore, (Y, τ_Y, E) is soft semi T_2-space. □

Definition 3.4. Let (X, τ, E) be a soft topological space, (G, E) be a semi closed soft set in X and $x \in X$ such that $x \notin (G, E)$. If there exist semi open soft sets (F_1, E) and (F_2, E) such that $x \in (F_1, E)$, $(G, E)\tilde{\subseteq}(F_2, E)$ and $(F_1, E)\tilde{\cap}(F_2, E) = \tilde{\phi}$, then (X, τ, E) is called a soft semi regular space. A soft semi regular T_1-space is called a soft semi T_3-space.

Proposition 3.3. *Let (X, τ, E) be a soft topological space, (G, E) be a semi closed soft set in X and $x \in X$ such that $x \notin (G, E)$. If (X, τ, E) is soft semi regular space, then there exists a semi open soft set (F, E) such that $x \in (F, E)$ and $(F, E)\tilde{\cap}(G, E) = \tilde{\phi}$.*

Proof. It is obvious from Definition 3.4. ☐

Proposition 3.4. *Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$ and $x \in X$. Then:*

(1): *$x \in (F, E)$ if and only if $x_E \tilde{\subseteq} (F, E)$.*
(2): *If $x_E \tilde{\cap} (F, E) = \tilde{\phi}$, then $x \notin (F, E)$.*

Proof. Obvious. ☐

Theorem 3.6. *Let (X, τ, E) be a soft topological space and $x \in X$. If (X, τ, E) is soft semi regular space, then:*

(1): *$x \notin (F, E)$ if and only if $x_E \tilde{\cap} (F, E) = \tilde{\phi}$ for every semi closed soft set (F, E).*
(2): *$x \notin (G, E)$ if and only if $x_E \tilde{\cap} (G, E) = \tilde{\phi}$ for every semi open soft set (G, E).*

Proof. **(1):** Let (F, E) be a semi closed soft set such that $x \notin (F, E)$. Since (X, τ, E) is soft semi regular space. Then by Proposition 3.3 there exists a semi open soft set (G, E) such that $x \in (G, E)$ and $(F, E) \tilde{\cap} (G, E) = \tilde{\phi}$. It follows that $x_E \tilde{\subseteq} (G, E)$ from Proposition 3.4 (1). Hence $x_E \tilde{\cap} (F, E) = \tilde{\phi}$. Conversely, if $x_E \tilde{\cap} (F, E) = \tilde{\phi}$, then $x \notin (F, E)$ from Proposition 3.4 (2).

(2): Let (G, E) be a semi open soft set such that $x \notin (G, E)$. If $x \notin G(e)$ for all $e \in E$, then we get the proof. If $x \notin G(e_1)$ and $x \in G(e_2)$ for some $e_1, e_2 \in E$, then $x \in G^c(e_1)$ and $x \notin G^c(e_2)$ for some $e_1, e_2 \in E$. This means that, $x_E \tilde{\cap} (G, E) \neq \tilde{\phi}$. Hence $(G, E)^c$ is semi closed soft set such that $x \notin (G, E)^c$. It follows by (1) $x_E \tilde{\cap} (G, E)^c = \tilde{\phi}$. This implies that, $x_E \tilde{\subseteq} (G, E)$ and so $x \in (G, E)$, which is contradiction with $x \notin G(e_1)$ for some $e_1 \in E$. Therefore, $x_E \tilde{\cap} (G, E) = \tilde{\phi}$. Conversely, if $x_E \tilde{\cap} (G, E) = \tilde{\phi}$, then it is obvious that $x \notin (G, E)$. This completes the proof.

☐

Corollary 3.1. *Let (X, τ, E) be a soft topological space and $x \in X$. If (X, τ, E) is soft semi regular space, then the following are equivalent:*

(1): *(X, τ, E) is soft semi T_1-space.*
(2: *$\forall\, x, y \in X$ such that $x \neq y$, there exist semi open soft sets (F, E) and (G, E) such that $x_E \tilde{\subseteq} (F, E)$ and $y_E \tilde{\cap} (F, E) = \tilde{\phi}$ and $y_E \tilde{\subseteq} (G, E)$ and $x_E \tilde{\cap} (G, E) = \tilde{\phi}$.*

Proof. It is obvious from Theorem 3.6. ☐

Theorem 3.7. *Let (X, τ, E) be a soft topological space and $x \in X$. Then the following are equivalent:*

(1): *(X, τ, E) is soft semi regular space.*
(2): *For every semi closed soft set (G, E) such that $x_E \tilde{\cap} (G, E) = \tilde{\phi}$, there exist semi open soft sets (F_1, E) and (F_2, E) such that $x_E \tilde{\subseteq} (F_1, E)$, $(G, E) \tilde{\subseteq} (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$.*

Proof. **(1) ⇒ (2) :** Let (G,E) be a semi closed soft set such that $x_E\tilde{\cap}(G,E) = \tilde{\phi}$. Then $x \notin (G,E)$ from Theorem 3.6 (1). It follows by (1), there exist semi open soft sets (F_1,E) and (F_2,E) such that $x \in (F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. This means that, $x_E\tilde{\subseteq}(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$.

(2) ⇒ (1) : Let (G,E) be a semi closed soft set such that $x \notin (G,E)$. Then $x_E\tilde{\cap}(G,E) = \tilde{\phi}$ from Theorem 3.6 (1). It follows by (2), there exist semi open soft sets (F_1,E) and (F_2,E) such that $x_E\tilde{\subseteq}(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Hence $x \in (F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Thus, (X,τ,E) is soft semi regular space. □

Theorem 3.8. *Let (X,τ,E) be a soft topological space. If (X,τ,E) is soft semi T_3-space, then $\forall\, x \in X$, x_E is semi closed soft set.*

Proof. We want to prove that x_E is semi closed soft set, which is sufficient to prove that x^c_E is semi open soft set for all $y \in \{x\}^c$. Since (X,τ,E) is soft semi T_3-space, then there exist semi open soft sets $(F,E)_y$ and (G,E) such that $y_E\tilde{\subseteq}(F,E)_y$ and $x_E\tilde{\cap}(F,E)_y = \tilde{\phi}$ and $x_E\tilde{\subseteq}(G,E)$ and $y_E\tilde{\cap}(G,E) = \tilde{\phi}$. It follows that $\bigcup_{y\in\{x\}^c}(F,E)_y \tilde{\subseteq} x^c_E$. Now we want to prove that $x^c_E\tilde{\subseteq}\bigcup_{y\in\{x\}^c}(F,E)_y$. Let $\bigcup_{y\in\{x\}^c}(F,E)_y = (H,E)$, where $H(e) = \bigcup_{y\in\{x\}^c} F(e)_y$ for all $e \in E$. Since $x^c_E(e) = \{x\}^c$ for all $e \in E$ from Definition 2.7. So, for all $y \in \{x\}^c$ and $e \in E$, $x^c_E(e) = \{x\}^c = \bigcup_{y\in\{x\}^c}\{y\} = \bigcup_{y\in\{x\}^c} y_E(e)\tilde{\subseteq}\bigcup_{y\in\{x\}^c} F(e)_y = H(e)$. Thus, $x^c_E\tilde{\subseteq}\bigcup_{y\in\{x\}^c}(F,E)_y$ from Definition 2.2 and so $x^c_E = \bigcup_{y\in\{x\}^c}(F,E)_y$. This means that, x^c_E is semi open soft set for all $y \in \{x\}^c$. Therefore, x_E is semi closed soft set. □

Theorem 3.9. *Every soft semi T_3-space is soft semi T_2-space.*

Proof. Let (X,τ,E) be a soft semi T_3-space and $x,y \in X$ such that $x \neq y$. By Theorem 3.8, y_E is semi closed soft set and $x \notin y_E$. It follows from the soft semi regularity, there exist semi open soft sets (F_1,E) and (F_2,E) such that $x \in (F_1,E)$, $y_E\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Thus, $x \in (F_1,E)$, $y \in y_E\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Therefore, (X,τ,E) is soft semi T_2-space. □

Theorem 3.10. *A soft subspace (Y,τ_Y,E) of a soft semi T_3-space (X,τ,E) is soft semi T_3.*

Proof. By Theorem 3.1 (Y,τ_Y,E) is soft semi T_1-space. Now we want to prove that (Y,τ_Y,E) is soft semi regular space. Let $y \in Y$ and (G,E) be a semi closed soft set in Y such that $y \notin (G,E)$. Then $(G,E) = (Y,E)\tilde{\cap}(F,E)$ for some semi closed soft set (F,E) in X from Theorem 2.1. Hence $y \notin (Y,E)\tilde{\cap}(F,E)$. But $y \in (Y,E)$, so $y \notin (F,E)$. Since (X,τ,E) is soft semi T_3-space, so there exist semi open soft sets (F_1,E) and (F_2,E) in X such that $y \in (F_1,E)$, $(F,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Take $(G_1,E) = (Y,E)\tilde{\cap}(F_1,E)$ and $(G_2,E) = (Y,E)\tilde{\cap}(F_2,E)$, then $(G_1,E),(G_2,E)$ are semi open soft sets in Y such that $y \in (G_1,E)$, $(G,E)\tilde{\subseteq}(Y,E)\tilde{\cap}(F_2,E) = (G_2,E)$ and $(G_1,E)\tilde{\cap}(G_2,E)\tilde{\subseteq}(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Thus, (Y,τ_Y,E) is soft semi T_3-space. □

Definition 3.5. Let (X, τ, E) be a soft topological space, $(F, E), (G, E)$ be semi closed soft sets in X such that $(F, E)\tilde{\cap}(G, E) = \tilde{\phi}$. If there exist semi open soft sets (F_1, E) and (F_2, E) such that $(F, E)\tilde{\subseteq}(F_1, E)$, $(G, E)\tilde{\subseteq}(F_2, E)$ and $(F_1, E)\tilde{\cap}(F_2, E) = \tilde{\phi}$, then (X, τ, E) is called a soft semi normal space. A soft semi normal T_1-space is called a soft semi T_4-space.

Theorem 3.11. *Let (X, τ, E) be a soft topological space and $x \in X$. Then the following are equivalent:*

- **(1):** *(X, τ, E) is soft semi normal space.*
- **(2):** *For every semi closed soft set (F, E) and semi open soft set (G, E) such that $(F, E)\tilde{\subseteq}(G, E)$, there exists a semi open soft set (F_1, E) such that (F, E) $\tilde{\subseteq}(F_1, E)$, $SScl(F_1, E)\tilde{\subseteq}(G, E)$, where $SScl(F_1, E)$ is the soft semi closure of (F_1, E) mentioned in [10].*

Proof. **(1) ⇒ (2) :** Let (F, E) be a semi closed soft set and (G, E) be a semi open soft set such that $(F, E)\tilde{\subseteq}(G, E)$. Then $(F, E), (G, E)^c$ are semi closed soft sets such that $(F, E)\tilde{\cap}(G, E)^c = \tilde{\phi}$. It follows by (1), there exist semi open soft sets (F_1, E) and (F_2, E) such that $(F, E)\tilde{\subseteq}(F_1, E)$, $(G, E)^c\tilde{\subseteq}(F_2, E)$ and $(F_1, E)\tilde{\cap}(F_2, E) = \tilde{\phi}$. Now $(F_1, E)\tilde{\subseteq}(F_2, E)^c$, so

$$SScl(F_1, E)\tilde{\subseteq}SScl(F_2, E)^c = (F_2, E)^c,$$

where (G, E) is semi open soft set. Also $(F_2, E)^c\tilde{\subseteq}(G, E)$. Hence

$$SScl(F_1, E)\tilde{\subseteq}(F_2, E)^c\tilde{\subseteq}(G, E).$$

Thus, $(F, E)\tilde{\subseteq}(F_1, E)$, $SScl(F_1, E)\tilde{\subseteq}(G, E)$.

(2) ⇒ (1) : Let $(G_1, E), (G_2, E)$ be semi closed soft sets such that $(G_1, E)\ \tilde{\cap}\ (G_2, E) = \tilde{\phi}$. Then $(G_1, E)\tilde{\subseteq}(G_2, E)^c$, then by hypothesis, there exists a semi open soft set (F_1, E) such that $G_1, E)\tilde{\subseteq}(F_1, E)$, $SScl(F_1, E)\tilde{\subseteq}(G_2, E)^c$. So, $(G_2, E)\tilde{\subseteq}[SScl(F_1, E)]^c$, $G_1, E)\tilde{\subseteq}(F_1, E)$ and $[SScl(F_1, E)]^c\tilde{\cap}(F_1, E) = \tilde{\phi}$, where (F_1, E) and $[SScl(F_1, E)]^c$ are semi open soft sets. Thus, (X, τ, E) is soft semi normal space.

□

Theorem 3.12. *A semi closed soft subspace (Y, τ_Y, E) of a soft semi normal space (X, τ, E) is soft semi normal.*

Proof. Let $(G_1, E), (G_2, E)$ be semi closed soft sets in Y such that $(G_1, E)\tilde{\cap}(G_2, E) = \tilde{\phi}$. Then $(G_1, E) = (Y, E)\tilde{\cap}(F_1, E)$ and $(G_2, E) = (Y, E)\tilde{\cap}(F_1, E)$ for some semi closed soft sets $(F_1, E), (F_2, E)$ in X from Theorem 2.1. Since Y is sa semi closed soft subset of X. Then $(G_1, E), (G_2, E)$ are semi closed soft sets in X such that $(G_1, E)\tilde{\cap}(G_2, E) = \tilde{\phi}$. Hence by soft semi normality there exist semi open soft sets (H_1, E) and (H_2, E) such that $(G_1, E)\ \tilde{\subseteq}\ (H_1, E)$, $(G_2, E)\ \tilde{\subseteq}\ (H_2, E)$ and $(H_1, E)\ \tilde{\cap}\ (H_2, E) = \tilde{\phi}$. Since (G_1, E), $(G_2, E)\ \tilde{\subseteq}\ (Y, E)$, then $(G_1, E)\ \tilde{\subseteq}\ (Y, E)\ \tilde{\cap}\ (H_1, E)$, $(G_2, E)\ \tilde{\subseteq}\ (Y, E)\ \tilde{\cap}\ (H_2, E)$ and $[(Y, E)\tilde{\cap}(H_1, E)]\tilde{\cap}[(Y, E)\tilde{\cap}(H_2, E)] = \tilde{\phi}$, where $(Y, E)\tilde{\cap}(H_1, E)$ and $(Y, E)\tilde{\cap}(H_2, E)$ are semi open soft sets in Y. Therefore, (Y, τ_Y, E) is soft semi normal space. □

Theorem 3.13. *Let (X, τ, E) be a soft topological space. If (X, τ, E) is soft semi normal space and x_E is semi closed soft set in τ for all $x \in X$, then (X, τ, E) is soft semi T_3-space.*

Proof. Since x_E is semi closed soft set for all $x \in X$, then (X, τ, E) is soft semi T_1-space from Theorem 3.3. Also (X, τ, E) is soft semi regular space from Theorem 3.7 and Definition 3.5. Hence (X, τ, E) is soft semi T_3-space. □

4. Some types of soft functions

Theorem 4.1. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be soft function which is bijective and irresolute open soft. If (X, τ_1, A) is soft semi T_o-space, then (Y, τ_2, B) is also a soft semi T_o-space.*

Proof. Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then $\exists x_1, x_2 \in X$ such that $u(x_1) = y_1$, $u(x_2) = y_2$ and $x_1 \neq x_2$. By hypothesis, there exist semi open soft sets (F, A) and (G, A) in X such that either $x_1 \in (F, A)$ and $x_2 \notin (F, A)$ or $x_2 \in (G, A)$ and $x_1 \notin (G, A)$. So, either $x_1 \in F_A(e)$ and $x_2 \notin F_A(e)$ or $x_2 \in G_A(e)$ and $x_1 \notin G_A(e)$ for all $e \in E$. This implies that, either $y_1 = u(x_1) \in u[F_A(e)]$ and $y_2 = u(x_2) \notin u[F_A(e)]$ or $y_2 = u(x_2) \in u[G_A(e)]$ and $y_1 = u(x_1) \notin u[G_A(e)]$ for all $e \in E$. Hence either $y_1 \in f_{pu}(F, A)$ and $y_2 \notin f_{pu}(F, A)$ or $y_2 \in f_{pu}(G, A)$ and $y_1 \notin f_{pu}(G, A)$. Since f_{pu} is irresolute open soft function, then $f_{pu}(F, A), f_{pu}(G, A)$ are semi open soft sets in Y. Hence (Y, τ_2, B) is also a soft semi T_o-space. □

Theorem 4.2. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be soft function which is bijective and irresolute open soft. If (X, τ_1, A) is soft semi T_1-space, then (Y, τ_2, B) is also a soft semi T_1-space.*

Proof. It is similar to the proof of Theorem 4.1. □

Theorem 4.3. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be soft function which is bijective and irresolute open soft. If (X, τ_1, A) is soft semi T_2-space, then (Y, τ_2, B) is also a soft semi T_2-space.*

Proof. $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then $\exists x_1, x_2 \in X$ such that $u(x_1) = y_1$, $u(x_2) = y_2$ and $x_1 \neq x_2$. By hypothesis, there exist semi open soft sets (F, A) and (G, A) in X such that $x_1 \in (F, A)$, $x_2 \in (G, A)$ and $(F, A)\tilde{\cap}(G, A) = \tilde{\phi}_A$. So, $x_1 \in F_A(e)$, $x_2 \in G_A(e)$ and $F_A(e)\tilde{\cap}G_A(e) = \phi$ for all $e \in E$. This implies that, $y_1 = u(x_1) \in u[F_A(e)]$, $y_2 = u(x_2) \in u[G_A(e)]$ for all $e \in E$. Hence $y_1 \in f_{pu}(F, A)$, $y_2 \in f_{pu}(G, A)$ and $f_{pu}(F, A)\tilde{\cap} f_{pu}(G, A) = f_{pu}[(F, A)\tilde{\cap}(G, A)] = f_{pu}[\tilde{\phi}_A] = \tilde{\phi}_B$ from Theorem 2.2. Since f_{pu} is irresolute open soft function, then $f_{pu}(F, A), f_{pu}(G, A)$ are semi open soft sets in Y. Thus, (Y, τ_2, B) is also a soft semi T_2-space. □

Theorem 4.4. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be soft function which is bijective, irresolute soft and irresolute open soft. If (X, τ_1, A) is soft semi regular space, then (Y, τ_2, B) is also a soft semi regular space.*

Proof. Let (G,B) be a semi closed soft set in Y and $y \in Y$ such that $y \notin (G,B)$. Since f_{pu} is surjective and irresolute soft , then $\exists x \in X$ such that $u(x) = y$ and $f_{pu}^{-1}(G,B)$ is semi closed soft set in X such that $x \notin f_{pu}^{-1}(G,B)$. By hypothesis, there exist semi open soft sets (F,A) and (H,A) in X such that $x \in (F,A)$, $f_{pu}^{-1}(G,B)\tilde{\subseteq}(H,A)$ and $(F,A)\tilde{\cap}(H,A) = \tilde{\phi}_A$. It follows that $x \in F_A(e)$ for all $e \in E$ and $(G,B) = f_{pu}[f_{pu}^{-1}(G,B)]\tilde{\subseteq} f_{pu}(H,A)$ from Theorem 2.2. So, $y = u(x_1) \in u[F_A(e)]$ for all $e \in E$ and $(G,B)\tilde{\subseteq} f_{pu}(H,A)$. Hence $y \in f_{pu}(F,A)$ and $(G,B)\tilde{\subseteq} f_{pu}(H,A)$ and $f_{pu}(F,A)\tilde{\cap} f_{pu}(H,A) = f_{pu}[(F,A)\tilde{\cap}(H,A)] = f_{pu}[\tilde{\phi}_A] = \tilde{\phi}_B$ from Theorem 2.2. Sincef_{pu} is irresolute open soft function. Then $f_{pu}(F,A), f_{pu}(H,A)$ are semi open soft sets in Y. Thus, (Y,τ_2,B) is also a soft semi regular space. □

Theorem 4.5. *Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu}: SS(X)_A \to SS(Y)_B$ be soft function which is bijective, irresolute soft and irresolute open soft. If (X,τ_1,A) is soft semi T_3-space, then (Y,τ_2,B) is also a soft semi T_3-space.*

Proof. Since (X,τ_1,A) is soft semi T_3-space, then (X,τ_1,A) is soft semi regular T_1-space. It follows that (Y,τ_2,B) is also a soft semi T_1-space from Theorem 4.2 and soft semi regular space from Theorem 4.4. Hence, (Y,τ_2,B) is also a soft semi T_3-space. □

Theorem 4.6. *Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu}: SS(X)_A \to SS(Y)_B$ be soft function which is bijective, irresolute soft and irresolute open soft. If (X,τ_1,A) is soft semi normal space, then (Y,τ_2,B) is also a soft semi normal space.*

Proof. Let $(F,B),(G,B)$ be semi closed soft sets in Y such that $(F,B)\tilde{\cap}(G,B) = \tilde{\phi}_B$. Since f_{pu} is irresolute soft, then $f_{pu}^{-1}(F,B)$ and $f_{pu}^{-1}(G,B)$ are semi closed soft set in X such that $f_{pu}^{-1}(F,B)\tilde{\cap} f_{pu}^{-1}(G,B) = f_{pu}^{-1}[(F,B)\tilde{\cap}(G,B)] = f_{pu}^{-1}[\tilde{\phi}_B] = \tilde{\phi}_A$ from Theorem 2.2. By hypothesis, there exist semi open soft sets (K,A) and (H,A) in X such that $f_{pu}^{-1}(F,B)\tilde{\subseteq}(K,A)$, $f_{pu}^{-1}(G,B)\tilde{\subseteq}(H,A)$ and $(F,A)\tilde{\cap}(H,A) = \tilde{\phi}_A$. It follows that $(F,B) = f_{pu}[f_{pu}^{-1}(F,B)]\tilde{\subseteq} f_{pu}(K,A)$,$(G,B) = f_{pu}[f_{pu}^{-1}(G,B)]\tilde{\subseteq} f_{pu}(H,A)$ from Theorem 2.2 and $f_{pu}(K,A)\tilde{\cap} f_{pu}(H,A) = f_{pu}[(K,A)\tilde{\cap}(H,A)] = f_{pu}[\tilde{\phi}_A] = \tilde{\phi}_B$ from Theorem 2.2. Since f_{pu} is irresolute open soft function. Then $f_{pu}(K,A), f_{pu}(H,A)$ are semi open soft sets in Y. Thus, (Y,τ_2,B) is also a soft semi normal space. □

Corollary 4.1. *Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu}: SS(X)_A \to SS(Y)_B$ be soft function which is bijective, irresolute soft and irresolute open soft. If (X,τ_1,A) is soft semi T_4-space, then (Y,τ_2,B) is also a soft semi T_4-space.*

Proof. It is obvious from Theorem 4.2 and Theorem 4.6. □

5. Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [19] and easily applied to many problems having uncertainties from social life. In this paper, we introduce the notion of soft semi separation axioms. In particular we study the properties of the soft semi regular spaces and soft semi normal spaces. We show that if x_E is semi closed soft set for all $x \in X$ in a soft topological space (X, τ, E), then (X, τ, E) is soft semi T_1-space. Also, we show that if a soft topological space (X, τ, E) is soft semi T_3-space, then $\forall\ x \in X$, x_E is semi closed soft set. Also, we show that the property of being semi T_i-spaces $(i = 1, 2)$ is soft topological property under a bijection and irresolute open soft mapping. Further, the properties of being soft semi regular and soft semi normal are soft topological properties under a bijection, irresolute soft and irresolute open soft functions. Finally, we show that the property of being semi T_i-spaces $(i = 1, 2, 3, 4)$ is a hereditary property. We hope that the results in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical li

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References

[1] B. Ahmad and A. Kharal, On fuzzy soft sets, Adv. Fuzzy Syst. 2009, Art. ID 586507, 6 pp.
[2] B. Ahmad and A. Kharal, Mappings of soft classes, to appear in New Math. Nat. Comput.
[3] H. Aktas and N. Cagman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2726–2735.
[4] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57 (2009) 1547–1553.
[5] S. Atmaca and I. Zorlutuna, On fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 5 (2013) 377–386.
[6] Banu Pazar Varol and Halis Aygun, On soft Hausdorff spaces, Ann. Fuzzy Math. Inform. 5 (2013) 15–24.
[7] N. Cagman, F. Citak and S. Enginoglu, Fuzzy parameterized fuzzy soft set theory and its applications, Turkish Journal of Fuzzy Systems 1(1) (2010) 21–35.
[8] N. Cagman and S. Enginoglu, Soft set theory and uni-int decision making, European J. Oper. Res. 207(2) (2010) 848–855.
[9] S. Hussain and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl. 62 (2011) 4058–4067.
[10] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity in soft topological spaces, Ann. Fuzzy Math. Inform. 7 (2014) 181–196.
[11] D. V. Kovkov, V. M. Kolbanov and D. A. Molodtsov, Soft sets theory-based optimization, Journal of Computer and Systems Sciences International 46(6) (2007) 872–880.
[12] J. Mahanta and P. K. Das, On soft topological space via semi open and semi closed soft sets, arXiv:1203.4133v1,2012.
[13] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9(3) (2001) 589–602.
[14] P. K. Maji, R. Biswas and A. R. Roy, Intuitionistic fuzzy soft sets, J. Fuzzy Math. 9(3) (2001) 677–691.
[15] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[16] P. Majumdar and S. K. Samanta, Generalised fuzzy soft sets, Comput. Math. Appl. 59 (2010) 1425–1432.
[17] W. K. Min, A note on soft topological spaces, Comput. Math. Appl. 62 (2011) 3524–3528.
[18] D. Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1(1) (2006) 8–39.

[19] D. A. Molodtsov, Soft set theory-firs tresults, Comput. Math. Appl. 37 (1999) 19–31.
[20] A. Mukherjee and S. B. Chakraborty, On intuitionistic fuzzy soft relations, Bull. Kerala Math. Assoc. 5(1) (2008) 35–42.
[21] D. Pei and D. Miao, From soft sets to information systems, in: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager, B. Zhang (Eds.), Proceedings of Granular Computing, in: IEEE, vol.2, 2005, pp. 617–621.
[22] S. Roy and T. K. Samanta, A note on fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 3(2) (2012) 305–311.
[23] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786–1799.
[24] Sujoy Das and S. K. Samanta, Soft metric, Ann. Fuzzy Math. Inform. 6 (2013) 77–94.
[25] Y. Zou and Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowledge-Based Systems 21 (2008) 941–945.
[26] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3 (2012) 171–185.

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