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Faintly compatible maps and existence of common fixed points in fuzzy metric space

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Abstract. Recently, Bisht and Shahzad [Faintly compatible mappings and common fixed points, Fixed point theory and applications,2013,2013:156] introduced the notion of faintly compatible maps and proved some new fixed point theorems under both contractive and noncontractive conditions which allowed the existence of a common fixed point or multiple fixed point or coincidence points. The aim of this paper is to demonstrate the applicability of faintly compatible maps in the existence of common fixed point in fuzzy metric space. Our results fuzzify, generalize and improve their result without containment and continuity requirement of involved maps. We also furnish example in support of our result.

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1. Introduction

Most of the mathematical tools used for formal modeling, reasoning and computing are crisp, deterministic and precise in character. But in real life, the problems in economics, engineering, environment, social science, medical science etc., do not always involve crisp data. Consequently to deal with uncertainty, we need techniques other than classical ones wherein some specific logic is required. Fuzzy set theory is one of the uncertainty approaches wherein topological structures are basic tools to develop mathematical models compatible to concrete real life situations. That is why; so many researchers are trying to fuzzify classical mathematical concepts. The fruitful and productive idea of fuzzy set was initiated by Zadeh [18] where the concept of uncertainty was introduced in the theory of sets, in a non probabilistic manner. The flexibility in fuzzy concepts allows the fuzzification of different mathematical structures in more than one ways. We consider here the definition of fuzzy metric space suggested by George and Veeramani [4] which is a modification of the definition given in [7], done for topological reasons. Fuzzy fixed point theory has been developed mostly on this fuzzy metric space. This is probably because the space has certain salient features necessary for a successful development of a metric fixed point theory, one of these being that the topology on this space is Hausdorff topology. Fuzzy fixed point theory has applications in applied sciences such as neural network theory, stability theory, mathematical programming, modeling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication, color image processing etc.

Weak commutativity of a pair of maps was introduced by Sessa [16] in fixed point considerations. There after number of generalizations of this notion have been obtained. Jungck [5] enlarged the class of non-commuting maps by compatible maps. Also the concept of compatible maps was further improved by Jungck and Rhoades [6] with the notion of weakly compatible maps which merely commute at coincidence points. For a brief development of weaker forms of commuting maps, one may refer to Singh and Tomar [17]. In fact weak compatibility is most widely used concept among all weaker forms of commuting maps. Al-Thagafi and Shahzad [2] weakened the notion of nontrivial weakly compatible maps by occasionally weakly compatible (owc) maps and Pant and Pant [14] further redefined it as conditionally commuting maps. Recently, Bisht and Shahzad [3] introduced the notion of faintly compatible maps as a improvement of conditionally compatible maps introduced by Pant and Bisht[13], which allowed the existence of a common fixed point or multiple fixed point or coincidence points under both contractive and noncontractive conditions.

The purpose of this paper is to demonstrate the effectiveness of the faintly compatible maps in the existence of common fixed point in fuzzy metric space .Our results fuzzify, generalize and improve the results of Bisht and Shahzad [3] without containment requirement of involved maps and replacing continuity of maps by a weaker condition, reciprocal continuity. We also furnish example in support of our result.

2. Preliminaries

In this section, we recall some definitions and useful results which are already in the literature. The concept of triangular norms (t-norms) is originally introduced by Menger [10] in study of statistical metric spaces.

Definition 2.1 ([15])**.** A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if * satisfies the following conditions:
(i) * is commutative and associative;
(ii) * is continuous;
(iii) $a * 1 = a$ for all $a \in [0,1]$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in$[0,1].

Example 2.2 ([15])**.** The following are classical examples of continuous t-norm:
(i) $a * b = min\{a, b\}$, minimum t-norm.
(ii) $a * b = ab$, product t-norm.
(iii) $a * b = max\{a + b - 1, 0\}$, Lukasiewict t-norm.

In order to introduce an Hausdorff topology on the fuzzy metric spaces, George and Veeramani [4] modified the notion of fuzzy metric space of Kramosil and Michalek [7] in a slight but appealing way as follows:

Definition 2.3. The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X, t, s > 0$;
$(GV-1)$ $M(x, y, t) > 0$;
$(GV-2)$ $M(x, y, t) = 1$ iff $x = y$;
$(GV-3)$ $M(x, y, t) = M(y, x, t)$;
$(GV-4)$ $M(x, y, t) * M(y, z, s) \leq M(x, z, t+s)$;
$(GV-5)$ $M(x, y, .) : [0, \infty) \to [0, 1]$ is continuous.

Example 2.4. (Induced fuzzy metric space) Let (X, d) be a metric space. Denote $a * b = a.b$ for all $a, b \in [0, 1]$ and let M_d be fuzzy set on $X^2 \times (0, \infty)$ defined as follows: $M_d(x, y, t) = \frac{ht^n}{ht^n + md(x,y)}$ for all $h, m, n \in R^+$. Then $(X, M_d, *)$ is a fuzzy metric space.

Remark 2.5. It should be noted that the above example holds even when continuous t-norm is replaced by $a * b = min\{a, b\}$ and hence function M defined in above example is a fuzzy metric with respect to any continuous t-norm. In the above example, by taking $h = m = n = 1$, we get $M_d(x, y, t) = \frac{t}{t+d(x,y)}$.

We call this fuzzy metric induced by a metric d as the standard fuzzy metric.

Definition 2.6 ([4])**.** A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be
(i) convergent to a point $x \in X$, if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in N$ such that
$M(x_n, x, t) > 1 - \epsilon$ for all $n \geq n_0$.
(ii) Cauchy sequence if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x_{n+p}, t) > 1 - \epsilon$ for all $n \geq n_0$.

Definition 2.7. A pair of self-maps (A, S) on a fuzzy metric space $(X, M, *)$ is said to be
(a) compatible [5], if $lim_{n\to\infty} M(ASx_n, SAx_n, t)$=1, for all $t > 0$ whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = x$, for some $x \in X$.
(b) non-compatible, if (A, S) is not compatible, i.e., if there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = x$, for some $x \in X$, and
$lim_{n\to\infty} M(ASx_n, SAx_n, t) \neq 1$ or non-existent for all $t > 0$.
(c) weakly compatible [6], if the pair commute on the set of their coincidence points, i.e., for $x \in X, Ax = Sx$ implies $ASx = SAx$.
(d) conditionally compatible [13], iff whenever the set of sequences $\{x_n\}$ satisfying $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n$, is non-empty, there exists a sequence $\{z_n\}$ in X such that $lim_{n\to\infty} Az_n = lim_{n\to\infty} Sz_n = t$, for some $t \in X$ and $lim_{n\to\infty} M(ASx_n, SAx_n, t)$ $= 1$ for all $t > 0$.
(e) reciprocally continuous[12], if $lim_{n\to\infty} ASx_n = Ax$, $lim_{n\to\infty} SAx_n = Sx$, whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = x$, for some $x \in X$.

(f) faintly compatible [3], iff (A, S) is conditionally compatible and A and S commute on a non-empty subset of the set of coincidence points, whenever the set of coincidence points is nonempty.
(g) satisfy the property $(E.A)$ [1], if there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = x$, for some $x \in X$.

It is interesting to mention here that faint compatibility, compatibility and non-compatibility are independent concepts. In fact faint compatibility, like most of the weaker forms of compatibility existing in literature [8-9,17], does not reduce to the class of compatibility in the presence of unique common fixed point (or unique coincidence point)(refer examples [3]) and is applicable for maps satisfying both contractive and non contractive condition.

Lemma 2.8 ([11]). *Let $(X, M, *)$ be a fuzzy metric space and for all $x, y \in X$, $t > 0$ and if there exists a constant $k \in (0, 1)$ such that $M(x, y, kt) \geq M(x, y, t)$ then $x = y$.*

3. Major section

Theorem 3.1. *Let a non-compatible, faintly compatible pair of self maps (A, S) of a fuzzy metric space $(X, M, *)$ satisfies:*
(3.1) $M(Ax, Ay, kt) \geq M(Sx, Sy, t)$, *for every $x, y \in X$ and some $0 < k < 1$.*
Then A and S have a unique common fixed point provided that the pair of self maps (A, S) is reciprocally continuous.

Proof. Let $\{x_n\}$ be a sequence in X such that $lim_{n\to\infty}\ Ax_n = lim_{n\to\infty}\ Sx_n = t$, for some $t \in X$. Since pair of self maps (A, S) is non-compatible, this gives $lim_{n\to\infty} M(ASx_n, SAx_n, t) \neq 1$ or non-existent. Since pair (A, S) is also faintly compatible, there exist a sequence $\{z_n\}$ in X satisfying $lim_{n\to\infty} Az_n = lim_{n\to\infty} Sz_n = u$, for some $u \in X$ such that $lim_{n\to\infty} M(ASz_n, SAz_n, t) = 1$. As pair (A, S) is also reciprocally continuous, we get $lim_{n\to\infty} ASz_n = Au$, $lim_{n\to\infty} SAz_n = Su$ and so $Au = Su$. Since pair (A, S) is faintly compatible, we get $ASu = SAu$ and so $AAu = ASu = SAu = SSu$. By (3.1),$M(Au, AAu, kt) \geq M(Su, SAu, t) = M(Au, AAu, t)$. By Lemma 2.8, $Au = AAu$. So $Au = AAu = SAu$ and Au is a common fixed point of A and S. For uniqueness, if $\alpha, \beta \in X$ such that $A\alpha = S\alpha = \alpha, A\beta = S\beta = \beta$, we get $M(\alpha, \beta, kt) = M(A\alpha, A\beta, kt) \geq M(S\alpha, S\beta, t) = M(\alpha, \beta, t)$ which gives, by Lemma 2.8, $\alpha = \beta$ and hence the common fixed point is unique. □

The following example illustrates the Theorem 3.1.

Example 3.2. Let $X = [0, 4]$ and M be the usual fuzzy metric on X. Let a pair of self maps (A, S) of X be defined as $Ax = 1$, if $x \leq 1$, $Ax = 2$, if $x > 1$, $Sx = 2 - x$, if $x \leq 1$, $Sx = 4$, if $x > 1$.
(i) Let $x_n = 1 - \frac{1}{n} \in X$. Now $Ax_n \to 1, Sx_n \to 1, ASx_n \to 2, SAx_n \to 1$ and so $M(ASx_n, SAx_n, t)$ does not converge to 1. Therefore, a pair of self maps (A, S) is non-compatible.
(ii) Let $z_n = 1$. Now $Az_n \to 1, Sz_n \to 1$ and $ASz_n \to 1, SAz_n \to 1$ and so $M(ASz_n, SAz_n, t) \to 1$. Therefore, a pair of self maps (A, S) is conditionally compatible. Also $A1 = S1$ and $AS1 = SA1$. Hence a pair of self maps (A, S) is faintly

compatible.
(iii) Condition (3.1) is satisfied with $k = \frac{1}{2}$.
(iv) Let $x_n \in X$ be such that $Ax_n \to z, Sx_n \to z \in X$. Then $z = 2$ if $x \geq 1$. Also $ASx_n \to Az$, $SAx_n \to Sz$. Therefore, a pair of self maps (A, S) is reciprocally continuous.
Hence, all the conditions of the Theorem 3.1 are satisfied and $x = 1 \in X$ is the unique common fixed point of A and S. Moreover both the self maps are discontinuous at common fixed point $x = 1$

It is well known that the strict contractive condition do not ensure the existence of common fixed points unless some strong conditions like completeness or closedness of space/subspace, continuity and containment requirement of involved maps are imposed. The next theorem illustrates the applicability of faintly compatible maps in finding the existence of common fixed point for maps satisfying the strict contractive condition in fuzzy metric space.

Theorem 3.3. *Let a non-compatible, faintly compatible pair of self maps (A, S) of a fuzzy metric space $(X, M, *)$ satisfies*
(3.2) $M(Ax, Ay, t) > M(Sx, Sy, t)$, *whenever* $Sx \neq Sy$, $x, y \in X$.
Then A and S have a unique common fixed point provided that the pair of self maps (A, S) is reciprocally continuous.

Proof. Let $\{x_n\}$ be a sequence in X such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = t$, for some $t \in X$. Since pair of self maps (A, S) is non-compatible $lim_{n\to\infty}M(ASx_n, SAx_n, t) \neq 1$ or non-existent. Since pair (A, S) is also faintly compatible there exist a sequence $\{z_n\}$ in X satisfying $lim_{n\to\infty}Az_n = lim_{n\to\infty}Sz_n = u$, for some $u \in X$ such that $lim_{n\to\infty}M(ASz_n, SAz_n, t) = 1$. As pair (A, S) is also reciprocally continuous, we get $lim_{n\to\infty}ASz_n = Au$, $lim_{n\to\infty}SAz_n = Su$ and so $Au = Su$. As pair (A, S) is faintly compatible, we get $ASu = SAu$ and so $AAu = ASu = SAu = SSu$. If $Au \neq AAu$, i.e., $Su \neq SAu$, we get by (3.2), $M(Au, AAu, t) > M(Su, SAu, t) = M(Au, AAu, t)$, a contradiction. So $Au = AAu = SAu$ and Au is a common fixed point of A and S. Uniqueness of common fixed point may be proved on the lines of Theorem 3.1 using (3.2). □

We now furnish an example to illustrate the Theorem 3.3.

Example 3.4. Let $X = [2, \infty)$ and M be the usual fuzzy metric on X. Let a pair of self maps (A, S) of X be defined as $Ax = 2$, if $x = 2$, $Ax = 6$, if $x > 2$, $Sx = 2$, if $x = 2$, $Sx = x + 4$, if $x > 2$.
(i) Let $x_n = 2 + \frac{1}{n} \in X$. Then $Ax_n \to 6, Sx_n \to 6$ and $ASx_n \to 6$, $SAx_n \to 10$. Therefore, $M(ASx_n, SAx_n, t)$ does not converge to 1 and so a pair of self maps $(A, S)$is non- compatible.
(ii) With $z_n = 2 \in X$, we get $Az_n \to 2, Sz_n \to 2$ and $ASz_n \to 2, SAz_n \to 2$ and so $M(ASz_n, SAz_n, t) \to 1$. Also A, S commute at the only point of coincidence $2 \in X$.Therefore , a pair of self maps (A, S) is faintly compatible.
(iii) Condition (3.2) is satisfied for all $x, y \in X$.
(iv) Let $x_n \in X$ be such that $Ax_n \to z, Sx_n \to z$ in X. Then $z = 2$ if $x = 2$. Also $ASx_n \to Az$,$SAx_n \to Sz$. Therefore, a pair of self maps (A, S) is reciprocally continuous.

Thus, all the conditions of the Theorem 3.3 are satisfied and we note that $2 \in X$ is the unique common fixed point of A and S. Moreover both the self maps are discontinuous at common fixed point $x = 2$.

The next theorem illustrates the applicability of faintly compatible maps in finding the existence of common fixed point for maps satisfying Lipschitz-type condition in fuzzy metric space.

Theorem 3.5. *Let a non-compatible, faintly compatible pair of self maps (A, S) of a fuzzy metric space $(X, M, *)$ satisfies*
(3.3) $M(Ax, AAy, t) \neq min\{M(Ax, SAy, t), M(SAx, AAy, t)\}$, *whenever the right side is $\neq 1, x, y \in X$.*
Then A and S have a unique common fixed point provided that the pair of self maps (A, S) is reciprocally continuous.

Proof. As a pair of self maps (A, S) is non-compatible, there exist a sequence $\{x_n\}$ in X such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = t$, for some $t \in X$ and $lim_{n\to\infty}M(ASx_n, SAx_n, t) \neq 1$ or non-existent. Also a pair of self maps (A, S) is faintly compatible, so there exist a sequence $\{z_n\}$ in X satisfying $lim_{n\to\infty}Az_n = lim_{n\to\infty}Sz_n = u$, for some $u \in X$, and $lim_{n\to\infty}M(ASz_n, SAz_n, t) = 1$. Since a pair (A, S) is reciprocally continuous, we get $lim_{n\to\infty}ASz_n = Au$, $lim_{n\to\infty}SAz_n = Su$. These give $Au = Su$. In view of (3.3), we get $ASu = SAu$. Hence we get, $AAu = ASu = SAu = SSu$. If $Au \neq AAu$, we get by (3.3), $M(Au, AAu, t) \neq \min\{M(Au, SAu, t), M(SAu, AAu, t)\} = M(Au, SAu, t) = M(Au, AAu, t)$, which is a contradiction and so $Au = AAu = SAu$. Therefore, Au is a common fixed point of A and S. If $\alpha, \beta \in X$ are common fixed points of A and S, we would get $M(\alpha, \beta, t) = M(A\alpha, AA\beta, t) \neq \min\{M(A\alpha, SA\beta, t), M(SA\alpha, AA\beta, t)\} = M(\alpha, \beta, t)$, which is a contradiction and so $\alpha = \beta$. Hence the common fixed point is unique. □

The following example illustrates Theorem 3.5.

Example 3.6. Let $X = [2, \infty)$ and M be the usual fuzzy metric on X. Let a pair of self maps (A, S) of X be defined as $Ax = 2$, if $2 \leq x \leq 5, Ax = 8$, if $x > 5$, $Sx = 2$, if $2 \leq x \leq 5$, $Sx = x + 3$, if $x > 5$.
(i) Let $x_n = 5 + \frac{1}{n} \in X$. Now, $Ax_n \to 8$, $Sx_n \to 8$ and $ASx_n \to 8, SAx_n \to 11$. Hence, $M(ASx_n, SAx_n, t)$ does not converge to 1 and so, a pair of self maps $(A, S)$is non-compatible.
(ii) With $z_n = 2+\frac{1}{n} \in X$, we see that $Az_n \to 2, Sz_n \to 2$ and $ASz_n \to 2, SAz_n \to 2$. Therefore, $M(ASz_n, SAz_n, t) \to 1$ and so a pair of self maps $(A, S)$is conditionally compatible. Also for $x \in X$, $Ax = Sx$ implies $ASx = SAx$ if $2 \leq x \leq 5$. Therefore a pair of self maps (A, S) is faintly compatible.
(iii) Condition (3.3) is satisfied for all $x \in X$.
(iv) Let $x_n \in X$ be such that $Ax_n \to z$,$Sx_n \to z$ in X. Then $z = 2$ in $2 \leq x_n \leq 5$ and $ASx_n \to Az$, $SAx_n \to Sz$. Therefore, (A, S) is reciprocally continuous. Therefore, all the conditions of the Theorem 3.5 are satisfied and we note that $2 \in X$ is the unique common fixed point of A and S. Moreover both self maps are discontinuous at common fixed point $x = 2$.

Substituting $x = y$ in(3.3), we get the following result, which is a generalization of a result due to Bisht and Shahzad [3, Theorem 2.4].

Corollary 3.7. *Let a non-compatible, faintly compatible pair of self maps (A, S) of a fuzzy metric space $(X, M, *)$ satisfies*
$(3.4) M(Ax, AAx, t) \neq min\{M(Ax, SAx, t), M(SAx, AAx, t)\}$*, whenever the right side is* $\neq 1, x \in X$.
Then A and S have common fixed point provided that the pair of self maps (A, S) is reciprocally continuous.

Proof. The proof is similar to the proof of the Theorem 3.5 without the uniqueness part. □

Remark 3.8. (i) Theorems $3.1, 3.3$ and 3.5 remain true if we replace the condition of non-compatiblity of a pair of self maps by the property $(E.A.)$.
(ii) Theorem 3.5 remains true if we replace the condition (3.3) by
$M(Sx, SSy, t) \neq \min\{M(Sx, ASy, t), M(ASx, SSy, t)\}$, whenever the right side is $\neq 1, x \in X$.
(iii) It is interesting to note that Examples $3.2, 3.4$ and 3.6 cannot be covered by all those common fixed point theorems which require containment and continuity of involved maps along with completeness (or closedness) of underlying space. Our results fuzzify, generalize, extend and improve multitude of common fixed point results existing in the literature (for instance Bisht and Shahzad [3] and references there in) and guarantee the existence of common fixed point for noncompatible discontinuous maps satisfying both contractive and noncontractive conditions in noncomplete fuzzy metric space without containment requirement of the involved maps.

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