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Soft fuzzy soft compatibility relation on soft fuzzy soft $\mathcal{C}^1$ atlases

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Abstract. In this paper the concept of soft fuzzy soft $\mathcal{C}^1$ diffeomorphism, soft fuzzy soft $\mathcal{C}^1$ atlas, soft fuzzy soft chart, the equivalence relation SFScompatibility between soft fuzzy soft $\mathcal{C}^1$ atlases and soft fuzzy soft $\mathcal{C}^1$ manifolds are established and discussed. In this connection soft fuzzy soft tangent vector and soft fuzzy soft tangent space are introduced and some of their properties are discussed.

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Keywords: Soft fuzzy soft $\mathcal{C}^1$ diffeomorphism, Soft fozzy soft $\mathcal{C}^1$ atlas, Soft fuzzy soft chart at x, SFS compatibility relation, Soft fuzzy soft $\mathcal{C}^1$ manifold, Soft fuzzy soft tangent vector, Soft fuzzy soft tangent space.

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1. Introduction

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper[8]. Fuzzy sets have applications in many fields such as information[4] and control[5]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Chang[1] introduced and developed the concept of fuzzy topological spaces. The concept of soft fuzzy topological space is introduced by Ismail U. Tiryaki[6]. Maji, Biswas and Roy[2] presented the definition of fuzzy soft sets and some applications of this notion to decision making problems. Soft fuzzy soft topological vector spaces and soft fuzzy soft differentiations are introduced in[7] and some of their basic properties which are needed for this paper are discussed in[7].

In this paper the soft fuzzy soft $\mathcal{C}^1$ atlas is defined on the basis of the definition of a classical atlas given by Serge lang[3]. Soft fuzzy soft $\mathcal{C}^1$ diffeomorphisms, soft fuzzy soft $\mathcal{C}^1$ atlas, soft fuzzy soft chart, the equivalence relation SFScompatibility between soft fuzzy soft $\mathcal{C}^1$ atlases, soft fuzzy soft $\mathcal{C}^1$ manifolds, soft fuzzy soft tangent vector and soft fuzzy soft tangent space are introduced and studied. Moreover in this paper we proved the soft fuzzy soft differentiable structure is SFScompatible with a soft fuzzy soft topology and that soft fuzzy soft $\mathcal{C}^1$ manifolds and soft fuzzy soft differentiable mappings between them retain some of the properties of classical manifolds.

2. Preliminaries

Definition 2.1 ([2]). Let U be an intial universe set and E be a set of parameters. Let $F(U)$ denotes the fuzzy power set of U. Let $A \subset E$. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to F(U)$.

Definition 2.2 ([7]). Let X be a nonempty set and E be the set of all parameters. Let I be the unit interval and $P(E)$ be the power set of E. A soft fuzzy soft set λ_E on X is a function from X to $I \times P(E)$ such that $\lambda_E(x) = (\lambda(x), F)$ where λ : $X \to I$, $F \in P(E)$. The family of all soft fuzzy soft sets over X is denoted by $SFS(X,\ E)$.

Definition 2.3 ([7]). Let A be a subset of X, then the soft fuzzy soft characteristic function $\chi_A : X \to \{(1, E), (0, \phi)\}$ is defined as

$$\chi_A(x) = \begin{cases} (1, E), & \text{if } x \in A; \\ \\ (0, \phi), & \text{otherwise.} \end{cases}$$

Definition 2.4 ([7]). Let X be a nonempty set and E be the set of all parameters. Let $\delta_E : X \to I \times P(E)$ be a soft fuzzy soft set and $x \in X$, $F \in P(E)$. Define

$$x_{\delta_E}(y) = \begin{cases} (\delta(x), F)(0 < \delta(x) \leq 1), & \text{if } x = y; \\ \\ (0, \phi), & \text{otherwise.} \end{cases}$$

Then the soft fuzzy soft set x_{δ_E} is called as the soft fuzzy soft point (in short, SFSP) of X with support x and value $(\delta(x), F)$.

Definition 2.5 ([7]). Let X be a nonempty set and E be the set of all parameters. If the soft fuzzy soft set λ_E is such that $\lambda_E(x) = (1, E)$, $\forall x \in X$, then λ_E is called the universal soft fuzzy soft set and it is denoted by $(1, E)^{\sim}$. If the soft fuzzy soft set λ_E is such that $\lambda_E(x) = (0, \phi)$, $\forall x \in X$, then λ_E is called the null soft fuzzy soft set and it is denoted by $(0, \phi)^{\sim}$.

Definition 2.6 ([7]). Let X be a nonempty set and E be the set of all parameters. If λ_E is a soft fuzzy soft set such that $\lambda_E(x) = (\lambda(x), A)$, then the complement of λ_E is denoted by λ_E^c where $\lambda_E^c(x) = (1, E)^{\sim}(x) - \lambda_E(x) = (1 - \lambda(x), E \setminus A)$, $\forall x \in X$.

Definition 2.7 ([7]). Let λ_E and μ_E be any two soft fuzzy soft sets such that $\lambda_E(x)$ $= (\lambda(x), A)$ and $\mu_E(x) = (\mu(x), B)$. Then

(i) $\lambda_E(x) \sqsubseteq \mu_E(x) \Leftrightarrow \lambda(x) \leq \mu(x)$, $\forall x \in X$, $A \subseteq B$.
(ii) $\lambda_E(x) \sqsupseteq \mu_E(x) \Leftrightarrow \lambda(x) \geq \mu(x)$, $\forall x \in X$, $A \supseteq B$.
(iii) $\lambda_E(x) \sqcap \mu_E(x) = (\min\{\lambda(x), \mu(x)\}, A \cap B)$, $\forall x \in X$.
(iv) $\lambda_E(x) \sqcup \mu_E(x) = (\max\{\lambda(x), \mu(x)\}, A \cup B)$, $\forall x \in X$.

Definition 2.8 ([7]). Let X be a non-empty set and E be the set of all parameters. Let λ_E and μ_E be any two soft fuzzy soft sets. Then

(i) $\lambda_E \Subset \mu_E \Leftrightarrow \lambda_E(x) \sqsubseteq \mu_E(x), \forall\, x \in X$.
(ii) $\lambda_E \Supset \mu_E \Leftrightarrow \lambda_E(x) \sqsupseteq \mu_E(x), \forall\, x \in X$.
(iii) $\lambda_E = \mu_E \Leftrightarrow \lambda_E(x) = \mu_E(x), \forall\, x \in X$.

Definition 2.9 ([7])**.** Let X be a non-empty set and E be the set of all parameters. Let λ_E and μ_E be any two soft fuzzy soft sets. Then

(i) $\beta_E = (\lambda_E \Cup \mu_E) \Rightarrow \beta_E(x) = \lambda_E(x) \sqcup \mu_E(x), \forall\, x \in X$.
(ii) $\gamma_E = (\lambda_E \Cap \mu_E) \Rightarrow \gamma_E(x) = \lambda_E(x) \sqcap \mu_E(x), \forall\, x \in X$.

Similarly arbitrary union and arbitrary intersection can be defined.

Definition 2.10 ([7])**.** Let $f : X \to Y$ be a function. If λ_E is a soft fuzzy soft set in Y, then its preimage under f is given by $f^{-1}(\lambda_E)(x) = \lambda_E \circ f(x) = \lambda_E(f(x))\ \forall x \in X$.

Definition 2.11 ([7])**.** Let $f : X \to Y$ be a function. If μ_E is a soft fuzzy soft set in X, then its image under f denoted by $f(\mu_E)$, is defined as

$$f(\mu_E)(y) = \begin{cases} \sqcup_{x \in f^{-1}(y)} \mu_E(x), & \text{if } f^{-1}(y) \neq \phi; \\ (0, \phi), & \text{otherwise.} \end{cases}$$

Definition 2.12 ([7])**.** Let X be a nonempty set. A fuzzy set $\mathfrak{K}_c$ in X is said to be a constant membership function, if $\mathfrak{K}_c(x) = c$ for all $x \in X$, where $0 \leq c \leq 1$.

Definition 2.13 ([7])**.** A soft fuzzy soft topology on a non empty set X is a family T of soft fuzzy soft sets in X satisfying the following axioms:

(i) For all constant membership function $\mathfrak{K}_c$ in X and for all $A \in P(E)$, $\mathfrak{K}_{c_A} \in T$ where $\mathfrak{K}_{c_A}(x) = (\mathfrak{K}_c(x), A)$ for all $x \in X$.
(ii) For any family of soft fuzzy soft sets $\lambda_{j_E} \in T$, $j \in J \Rightarrow \Cup_{j \in J} \lambda_{j_E} \in T$.
(iii) For any finite no of soft fuzzy soft sets $\lambda_{j_E} \in T$, $j = 1, 2, 3, 4....n \Rightarrow \Cap_{j \in J} \lambda_{j_E} \in T$.

Then the pair $(X,\ T)$ is called a soft fuzzy soft topological space (in short SFSTS). Any soft fuzzy soft set in T is said to be a soft fuzzy soft open set (in short SFSOS) in X. The complement of a SFSOS in a SFSTS $(X,\ T)$ is called as a soft fuzzy soft closed set denoted by SFSCS in X.

Definition 2.14 ([7])**.** Let $(X,\ \tau)$ be a topological space. Then X has the usual soft fuzzy soft topology T given by

$$T = \{\chi_M : \forall M \in \tau\} \cup \{\mathfrak{K}_{c_E} : \forall \text{ constant membership function } \mathfrak{K}_c \text{ in } X \text{ and for all } A \in P(E)\}$$

Then the pair $(X,\ T)$ is a soft fuzzy soft topological space.

Proposition 2.15 ([7])**.** *Let f be a function from a set X into Y. If λ_{1_E}, λ_{2_E} be any two soft fuzzy soft sets in X and μ_{1_E}, μ_{2_E} be any two soft fuzzy soft sets in Y. Then*

(i) $\lambda_{1_E} \Subset \lambda_{2_E} \Rightarrow f(\lambda_{1_E}) \Subset f(\lambda_{2_E})$.
(ii) $\mu_{1_E} \Subset \mu_{2_E} \Rightarrow f^{-1}(\mu_{1_E}) \Subset f^{-1}(\mu_{2_E})$.

Definition 2.16 ([7]). Let $(X,\ T)$ be a soft fuzzy soft topological space and E be the set of all parameters. A soft fuzzy soft set λ_E in X is called a soft fuzzy soft neighbourhood of a soft fuzzy soft point x_{δ_E} in X iff there exists a soft fuzzy soft open set $\mu_E \in T$ such that $x_{\delta_E} \Subset \mu_E \Subset \lambda_E$.

Definition 2.17 ([7]). Let $(X,\ T)$ be a soft fuzzy soft topological space and E be the set of all parameters. A fundamental system of soft fuzzy soft neighbourhoods of a soft fuzzy soft point x_{δ_E} is a set $\mathbb{B}(x_{\delta_E})$ of soft fuzzy soft neighbourhoods of x_{δ_E} such that for each soft fuzzy soft neighbourhood λ_E of x_{δ_E} there is a $\mu_E \in \mathbb{B}(x_{\delta_E})$ such that $\mu_E \Subset \lambda_E$.

Definition 2.18 ([7]). Let $(X,\ T)$, $(Y,\ S)$ be any two soft fuzzy soft topological spaces and E be the set of all parameters. If f is a function from $(X,\ T)$ into $(Y,\ S)$, then

(i) f is said to be soft fuzzy soft continuous iff for each soft fuzzy soft open set λ_E in $(Y,\ S)$, the inverse image $f^{-1}(\lambda_E)$ is soft fuzzy soft open in $(X,\ T)$.

(ii) f is said to be soft fuzzy soft open iff for each soft fuzzy soft open set λ_E in $(X,\ T)$, the image $f(\lambda_E)$ is soft fuzzy soft open in $(Y,\ S)$.

(iii) f is said to be soft fuzzy soft continuous at a point $x \in X$ iff for each soft fuzzy soft open set μ_E in $(Y,\ S)$ and each soft fuzzy soft point $y_{\delta_E} \Subset \mu_E$, where $y_{\delta_E}(y) = (\delta(y), A)$ and $y_{\delta_E} = (f(x))_{\delta_E}$, $(0, \phi) \sqsubset (\delta(y), A) \sqsubseteq (1, E)$, the inverse image $f^{-1}(\mu_E)$ is a soft fuzzy soft open set in $(X,\ T)$ such that $x_{\lambda_E} \Subset f^{-1}(\mu_E)$, where $x_{\lambda_E}(x) = (\lambda(x), F)$, $(0, \phi) \sqsubset (\lambda(x), F) \sqsubseteq (\delta(y), A)$.

Definition 2.19 ([7]). Let $(X,\ T)$, $(Y,\ S)$ be any two soft fuzzy soft topological spaces and E be the set of all parameters. A bijective function from $(X,\ T)$ into $(Y,\ S)$ is a soft fuzzy soft homeomorphism iff both f and f^{-1} are soft fuzzy soft continuous.

Proposition 2.20 ([7]). *If $(X,\ T)$, $(Y,\ S)$ be any two soft fuzzy soft topological spaces, E be the set of all parameters and f is a function from $(X,\ T)$ into $(Y,\ S)$, then the following assertions are equivalent.*

(i) *The function f is soft fuzzy soft continuous.*

(ii) *For each soft fuzzy soft set λ_E in X and each soft fuzzy soft neighbourhood δ_E of $f(\lambda_E)$, there is a soft fuzzy soft neighbourhood μ_E of λ_E such that $f(\mu_E) \Subset \delta_E$.*

Definition 2.21 ([7]). Given a family $\{(X_j,\ T_j)\}$, $j \in J$, of soft fuzzy soft topological spaces, E be the set of all parameters, their product $\prod_{j\in J}(X_j,\ T_j)$ to be the soft fuzzy soft topological space $(X,\ T)$, where $X = \prod_{j\in J} X_j$ is the usual set product and T is the coarsest soft fuzzy soft topology on X for which the projections p_j of X onto X_j are soft fuzzy soft continuous for each $j \in J$. The soft fuzzy soft topology T is called the soft fuzzy soft product topology on X, and $(X,\ T)$ is the soft fuzzy soft product topological space.

Remark 2.22 ([7]). Let $\{X_j\}$, $j = 1, 2, 3, \ldots, n$, be a finite family of sets and for each j let λ_{j_E} be a soft fuzzy soft set in X_j. Then the product $\lambda_E = \prod_{j=1}^{n} \lambda_{j_E}$ of the family $\{\lambda_{j_E}\}$ as the soft fuzzy soft set in $X = \prod_{j\in J}^{n} X_j$ that has the membership function given by

$\lambda_E(x_1, x_2, x_3, \ldots, x_n) = \sqcap\{\lambda_{1_E}(x_1), \lambda_{2_E}(x_2), \ldots, \lambda_{n_E}(x_n)\}, (x_1, x_2, \ldots, x_n) \in X.$
It follows that if X_j has soft fuzzy soft topology T_j, $j = 1, 2, \ldots, n$, the soft fuzzy soft product topology T on X has a base as the set of product soft fuzzy soft sets of the form $\prod_{j\in J} \mu_{j_E}$, where $\mu_{j_E} \in T_j$, $j = 1, 2, 3, \ldots, n$.

Definition 2.23 ([7]). A soft fuzzy soft topological space $(X,\ T)$ is called a soft fuzzy soft T_1 space iff every soft fuzzy soft point is a soft fuzzy soft closed set.

Proposition 2.24 ([7]). *A soft fuzzy soft topological space $(X,\ T)$ is a soft fuzzy soft T_1 space iff for each $x \in X$ and each $(\mu(x), P) \in I \times P(E)$ there exists $\mu_E \in T$ such that $\mu_E(x) = (1 - \mu(x), E/P)$, $\mu_E(y) = (1, E)$, for all $y \neq x$.*

Definition 2.25 ([7]). Let $\{\lambda_{j_E}\}$, $j = 1, 2, 3, \ldots, n$, be a finite family of soft fuzzy soft sets in a vector space V over K with E as the set of parameters. The sum $\lambda_E = \lambda_{1_E} + \lambda_{2_E} + \lambda_{3_E} + \ldots + \lambda_{n_E}$ of the family $\{\lambda_{j_E}\}$, $j = 1, 2, 3, \ldots, n$, is the soft fuzzy soft set in V whose membership function is given by

$$\lambda_E(x) = \sqcup_{x_1+\ldots+x_n=x}(\lambda_{1_E}(x_1) \sqcap \lambda_{2_E}(x_2) \ldots \sqcap \lambda_{n_E}(x_n)),\ x \in V$$

The scalar product $\alpha\lambda_E$, of $\alpha \in K$ and λ_E is a soft fuzzy soft set in V that has membership function $\alpha\lambda_E(x)$, $x \in V$, given by $\alpha\lambda_E(x) = \lambda_E(x/\alpha)$ for all $\alpha \neq 0$, $x \in V$. For $\alpha = 0$,

$$\begin{aligned}\alpha\lambda_E(x) &= (0, \phi),\ x \neq 0\\ &= \sqcup_{y\in V}\lambda_E(y),\ x = 0,\end{aligned}$$

Definition 2.26 ([7]). A soft fuzzy soft set λ_E in a vector space V over K with E as the set of parameters is said to be soft fuzzy soft balanced, if $\alpha\lambda_E \Subset \lambda_E$ for all $\alpha \in K$, $\mid \alpha \mid \leq 1$.

Proposition 2.27 ([7]). *Let $(V,\ T)$ be a soft fuzzy soft topological vector space over K with E as the set of parameters. For every soft fuzzy soft point 0_{δ_E} such that 0_{δ_E} $= (\delta(0), A)$, $(0, \phi) \sqsubset (\delta(0), A) \sqsubseteq (1, E)$, there exists a fundamental system of soft fuzzy soft neighbourhoods $\mathbb{B}(0_{\delta_E})$ in V for which the following results hold.*

(i) *For each $\lambda_E \in \mathbb{B}(0_{\delta_E})$ there is a $\mu_E \in \mathbb{B}(0_{\delta_E})$ with $\mu_E + \mu_E \Subset \lambda_E$.*
(ii) *For each $\lambda_E \in \mathbb{B}(0_{\delta_E})$ there is a $\mu_E \in \mathbb{B}(0_{\delta_E})$ for which $k\mu_E \Subset \lambda_E$ for all $k \in K$, $\mid k \mid \leq 1$.*
(iii) *Every $\lambda_E \in \mathbb{B}(0_{\delta_E})$ is soft fuzzy soft balanced.*

Definition 2.28 ([7]). A soft fuzzy soft topological vector space $(V,\ T)$ is a vector space V over K with E as the set of parameters equipped with a soft fuzzy soft topology T such that the two functions

(i) $\varphi : V \times V \to V$, $< x, y > \mapsto x + y$.
(ii) $\psi : K \times V \to V$, $< \alpha, x > \mapsto \alpha x$.

are soft fuzzy soft continuous where K has the usual soft fuzzy soft topology and $V \times V$, $K \times V$ are the soft fuzzy soft product topological spaces.

Let $(V_1,\ T_1)$ and $(V_2,\ T_2)$ be any two soft fuzzy soft topological vector spaces over K with E as the set of parameters and let σ be a function from $(V_1,\ T_1)$ into $(V_2,\ T_2)$. Let $\rho(t)$ denote any function of a real variable t such that $\lim_{t\to 0} \rho(t)/t = 0$.

Definition 2.29 ([7]). The function σ is said to be soft fuzzy soft tangent to 0 if given a soft fuzzy soft neighbourhood μ_E of 0_{δ_E} where $0_{\delta_E}(0) = (\delta(0), A)$, $(0, \phi) \sqsubset (\delta(0), A) \sqsubseteq (1, E)$, in V_2 there exists a soft fuzzy soft neighbourhood λ_E of 0_{γ_E} where $0_{\gamma_E}(0) = (\gamma(0), F)$, $(0, \phi) \sqsubset (\gamma(0), F) \sqsubseteq (\delta(0), A)$ in V_1 such that

$$\sigma(t\lambda_E) \Subset \rho(t)\mu_E$$

for some function $\rho(t)$.

Definition 2.30 ([7]). Let $(V_1,\ T_1)$ and $(V_2,\ T_2)$ be any two soft fuzzy soft topological vector spaces over K with E as the set of parameters, each of them is a soft fuzzy soft T_1 space. Let $f : V_1 \to V_2$ be a soft fuzzy soft continuous function. f is said to be soft fuzzy soft differentiable at a point $x \in V_1$ if there exists a linear soft fuzzy soft continuous function u of V_1 into V_2 such that

$$f(x + y) = f(x) + u(y) + \sigma(y),\ \ y \in V_1$$

where σ is soft fuzzy soft tangent to 0. The function u is called the soft fuzzy soft derivative of f at x. The soft fuzzy soft derivative is denoted by $f^{'}(x)$; it is an element of set of all linear soft fuzzy soft continuous function from V_1 into V_2.

Proposition 2.31 ([7]). *Let $(V_1,\ T_1)$, $(V_2,\ T_2)$, $(V_3,\ T_3)$ be any three soft fuzzy soft topological vector spaces over K with E as the set of parameters, f a soft fuzzy soft continuous function from V_1 into V_2, and g a soft fuzzy soft continuous function from V_2 into V_3. Let $x \in V_1$ and $y = f(x)$. If f is soft fuzzy soft differentiable at x and g is soft fuzzy soft differentiable at y, then the composition $h = g \circ f$ is soft fuzzy soft differentiable at x.*

Further the soft fuzzy soft derivative of $g \circ f$ at $x \in V$ is $g^{'}(f(x)) \circ f^{'}(x)$.

3. Soft fuzzy soft $\mathcal{C}^1$ diffeomorphism

Definition 3.1. Let X be a nonempty set and E be the set of all parameters. Let T be a soft fuzzy soft topology on a set X. A Subfamily $\mathscr{B}$ of T is called a base for T if each member λ_E of T can be expressed as $\Cup_{\lambda_{j_E} \in \mathscr{B}} \lambda_{j_E}$.

Proposition 3.2. *Let X be a nonempty set and E be the set of all parameters. A family $\mathscr{B}$ of soft fuzzy soft sets in X is a base for a soft fuzzy soft topology on X if it satisfies the following conditions.*

(i) $\sqcup_{\lambda_{j_E} \in \mathscr{B}} \{\lambda_{j_E}(x)\} = (1, E)$, $\forall\ x \in X$.
(ii) *If λ_{1_E}, $\lambda_{2_E} \in \mathscr{B}$ then $\lambda_{1_E} \Cap \lambda_{2_E} \in \mathscr{B}$.*
(iii) *For every constant membership function $\mathfrak{K}_c$ in X and every $\lambda_E \in \mathscr{B}$, $\mathfrak{K}_{c_E} \Cap \lambda_E \in \mathscr{B}$.*

Proof. Let T be the family of soft fuzzy soft sets then each λ_E expressed as $\Cup_{\lambda_{j_E} \in \mathscr{B}} \lambda_{j_E}$. From condition (i), $(1, E)^{\sim} \in T$, and it is obvious that if $\lambda_{j_E} \in T$, $j \in J$, then $\Cup_{j \in J} \lambda_{j_E} \in T$. Let $\{\mu_{j_E}\}$ and $\{\mu_{l_E}\}$ be subfamilies of $\mathscr{B}$ (j and l ranging in index sets J and L, respectively) and let $\gamma_E(x) = \sqcup_j \{\mu_{j_E}(x)\}$ and $\delta_E(x) = \sqcup_l \{\mu_{l_E}(x)\}$, $x \in X$. Then

$$\begin{aligned}\gamma_E(x) \sqcap \delta_E(x) &= (\sqcup_j \{\mu_{j_E}(x)\}) \sqcap (\sqcup_l \{\mu_{l_E}(x)\}) \\ &= \sqcup_{j,l} \{\mu_{j_E}(x) \sqcap \mu_{l_E}(x)\},\ x \in X.\end{aligned}$$

This shows that if γ_E, $\delta_E \in T$, then $\gamma_E \sqcap \delta_E \in T$. Finally, it is necessary to prove that $\mathfrak{K}_{c_E} \in T$,for all constant membership function in X. Condition (iii) implies that for each constant membership function $\mathfrak{K}_c$ in X , the soft fuzzy soft set with membership function

$$\sqcup_{\mu_E \in \mathscr{B}}\{\mathfrak{K}_{c_E}(x) \sqcap \mu_E(x)\},\ x \in X,$$

belongs to T. By condition (i), for each $x \in X$ and for each constant membership function $\mathfrak{K}_c$ in X, there exists a soft fuzzy soft set $\mu_E \in \mathscr{B}$ with membership $\mu_E(x) \sqsupseteq \mathfrak{K}_{c_E}(x)$. Then $\sqcup_{\mu_E \in \mathscr{B}}\{\mathfrak{K}_{c_E}(x) \sqcap \mu_E(x)\} = \mathfrak{K}_{c_E}(x)$, for all $x \in X$ and for all constant membership function $\mathfrak{K}_c$ in X, which shows that $\mathfrak{K}_{c_E} \in T$. Hence the proof. □

Proposition 3.3. *Let $(X,\ T)$, $(Y,\ S)$ and $(Z,\ R)$ be any three soft fuzzy soft topological spaces. Let f be a soft fuzzy soft continuous (resp. soft fuzzy soft open) function from (X,T) into (Y,S) and g be a soft fuzzy soft continuous (resp. soft fuzzy soft open) function from (Y,S) into (Z,R). Then the composition $g \circ f$ is a soft fuzzy soft continuous function (resp. soft fuzzy soft open) function from (X,T) into (Z,R).*

Proof. Let λ_E be a soft fuzzy soft open set in $(Z,\ R)$. Since g is soft fuzzy soft continuous, $g^{-1}(\lambda_E)$ is soft fuzzy soft open set in $(Y,\ S)$. Since f is soft fuzzy soft continuous, $f^{-1}(g^{-1}(\lambda_E))$ is soft fuzzy soft open set in $(X,\ T)$. That is $(g \circ f)^{-1}(\lambda_E)$ is soft fuzzy soft open set in $(X,\ T)$. Hence $g \circ f$ is a soft fuzzy soft continuous function.

Let μ_E be a soft fuzzy soft open set in $(X,\ T)$. Since f is soft fuzzy soft open, $f(\mu_E)$ is soft fuzzy soft open set in $(Y,\ S)$. Since g is soft fuzzy soft open, $g \circ f(\mu_E)$ is soft fuzzy soft open set in $(Z,\ R)$. Hence $g \circ f$ is a soft fuzzy soft open function. □

Proposition 3.4. *Let $\{(X_j,\ T_j)\}_{j \in J}$ be a family of soft fuzzy soft topological spaces, $(X,\ T)$ be the soft fuzzy soft product topological space and f is a function from a soft fuzzy soft topological space $(Y,\ S)$ into $(X,\ T)$. Then f is soft fuzzy soft continuous iff $p_j \circ f$ is soft fuzzy soft continuous for each $j \in J$, where p_j is the projection map of j^{th} coordinate.*

Proof. Proof is simple. □

Proposition 3.5. *Let $\{(X_j,\ T_j)\}_{j \in J}$, $\{(Y_j,\ S_j)\}_{j \in J}$ be two families of soft fuzzy soft topological spaces and $(X,\ T)$, $(Y,\ S)$ be the respective soft fuzzy soft product topological spaces. For each $j \in J$, let f_j be a function from $(X_j,\ T_j)$ into $(Y_j,\ S_j)$. Then the product mapping $f = \prod_{j \in J} f_j : (x_j) \to (f_j(x_j))$ of $(X,\ T)$ into $(Y,\ S)$ is soft fuzzy soft continuous if f_j is soft fuzzy soft continuous for each $j \in J$.*

Proof. The function f can be written as $x \mapsto (f_j(p_j(x)))$, where $x = (x_j)$ and is therefore soft fuzzy soft continuous by Proposition 3.4. Hence the proof. □

Proposition 3.6. *Let $\{(X_j,\ T_j)\}_{1 \le j \le n}$, $\{(Y_j,\ S_j)\}_{1 \le j \le n}$ be two finite families of soft fuzzy soft topological spaces and let $(X,\ T)$, $(Y,\ S)$ be the respective soft fuzzy soft product topological spaces. For each j, $1 \le j \le n$, let f_j be a function from $(X_j,\ T_j)$ into (Y_j, vS_j). Then the product function $f = \prod_{j=1}^{n} f_j : (x_j) \mapsto (f(x_j))$ of $(X,\ T)$ into $(Y,\ S)$ is soft fuzzy soft open if f_j is soft fuzzy soft open for each j, $1 \le j \le n$.*

Proof. Let λ_E be a soft fuzzy soft open set in $(X,\ T)$. Then there exists soft fuzzy soft open sets λ_{jm_E} in $(X_j,\ T_j)$, $m \in M$, $j = 1, 2, 3, \ldots, n$ such that

$$\lambda_E = \Cup_{m\in M} \prod_{j=1}^{n} \lambda_{jm_E}$$

The image $f(\lambda_E)$ has membership function for all $y \in Y$,

$$\begin{aligned}
f(\lambda_E)(y) &= f(\sqcup_{m\in M} \prod_{j=1}^{n} \lambda_{jm_E})(y) \\
&= \sqcup_{z\in f^{-1}(y)}(\sqcup_{m\in M} \prod_{j=1}^{n} \lambda_{jm_E}(z) \\
&= \sqcup_{m\in M}(\sqcup_{z\in f^{-1}(y)} \prod_{j=1}^{n} \lambda_{jm_E}(z_1, z_2, \ldots, z_n) \\
&= \sqcup_{m\in M}\{(\sqcup_{z_1\in f_1{}^{-1}(y_1)}\lambda_{1m_E}(z_1)) \sqcap \ldots \sqcap (\sqcup_{z_n\in f_n{}^{-1}(y_n)}\lambda_{nm_E}(z_n))\} \\
&= \sqcup_{m\in M}\{f_1(\lambda_{1m_E})(y_1) \sqcap \ldots \sqcap f_n(\lambda_{nm_E})(y_n)\} \\
&= \sqcup_{m\in M} \prod_{j=1}^{n} f_j(\lambda_{jm_E})(y)
\end{aligned}$$

Since each f_j is soft fuzzy soft open, $f(\lambda_E)$ is soft fuzzy soft open set in $(Y,\ S)$. Hence f is soft fuzzy soft open function. □

Proposition 3.7. *Let $(V,\ T) = (V_1,\ T_1) \times (V_2,\ T_2)$ be the soft fuzzy soft product topological vector spaces of two soft fuzzy soft topological vector spaces $(V_1,\ T_1)$, $(V_2,\ T_2)$. Each soft fuzzy soft neighbourhood λ_E of 0_{δ_E} where $0_{\delta_E}(0) = (\delta(0), A)$, $(0, \phi) \sqsubset (\delta(0), A) \sqsubseteq (1, E)$ in V_2 such that $\lambda_E \Supset \mu_{1_E} \times \mu_{2_E}$ where μ_{1_E} and μ_{2_E} are soft fuzzy soft neighbourhoods of 0_{ν_E} where $0_{\nu_E}(0) = (\nu(0), F)$ in V_1, V_2, respectively, for every $(\nu(0), F)$, $(0, \phi) \sqsubset (\nu(0), F) \sqsubset (\delta(0), A)$.*

Proof. Since λ_E is a soft fuzzy soft neighbourhood of 0_{δ_E}, there exists a soft fuzzy soft open set μ'_E in V such that $0_{\delta_E} \Subset \mu'_E \Subset \lambda_E$. The set μ'_E can be expressed as $\Cup_{\substack{i\in I\\ j\in J}} \gamma_{i_E} \times \nu_{j_E}$ where γ_{i_E} and ν_{j_E} are soft fuzzy soft open sets in V_1, V_2 respectively. Hence,

$$\mu'_E(0) = \sqcup_{\substack{i\in I\\ j\in J}}[\gamma_{i_E}(0) \sqcap \nu_{j_E}(0)] \sqsupseteq (\delta(0), A).$$

Therefore for each $(\nu(0), F)$, $(0, \phi) \sqsubset (\nu(0), F) \sqsubset (\delta(0), A)$, there exist i', j' such that $\gamma_{i'_E}(0) \sqsupseteq (\nu(0), F)$, $\nu_{j'_E}(0) \sqsupseteq (\nu(0), F)$. Hence the proof. □

Proposition 3.8. *Let $(V_1,\ T_1)$, $(V_2,\ T_2)$, $(U_1,\ S_1)$, $(U_2,\ S_2)$ be any four soft fuzzy soft topological vector spaces. Suppose that $f : V_1 \to U_1$ and $g : V_2 \to U_2$ are soft fuzzy soft differentiable. Then $f \times g : V_1 \times V_2 \to U_1 \times U_2$ is soft fuzzy soft differentiable.*

Proof. By hypothesis

$$f(x+r)-f(x)=f^{'}(x)(r)+\sigma(r),\ x,r\in V_1,$$
$$g(y+s)-g(y)=g^{'}(y)(s)+\eta(s),\ y,s\in V_2,$$

where σ, η are soft fuzzy soft tangent to 0. By Proposition 3.7, each soft fuzzy soft neighbourhood μ_E of 0_{δ_E} where $0_{\delta_E}(0)=(\delta(0),A)$, $(0,\phi)\sqsubset(\delta(0),A)\sqsubseteq(1,E)$, in $U_1\times U_2$ such that $\mu_E\Supset\mu_{1_E}\times\mu_{2_E}$, where μ_{1_E} and μ_{2_E} are soft fuzzy soft neighbourhoods of 0_{ν_E} where $0_{\nu_E}(0)=(\nu(0),F)$ in U_1, U_2 for each $(\nu(0),F)$, $(0,\phi)\sqsubset(\nu(0),F)^{'}\sqsubset(\delta(0),A)$. By Proposition 2.27 that μ_{1_E}, μ_{2_E} are soft fuzzy soft balanced. Since f and g are soft fuzzy soft differentiable, there are for each 0_{β_E} where $0_{\beta_E}(0)=(\beta(0),G)$, $(0,\phi)\sqsubset(\beta(0),G)\sqsubset(\nu(0),F)$, in V_1, V_2, soft fuzzy soft neighbourhoods of λ_{1_E}, λ_{2_E} of 0_{β_E} in V_1, V_2 such that $\sigma(t\lambda_{1_E})\Subset\rho_1(t)\mu_{1_E}$, $\eta(t\lambda_{2_E})\Subset\rho_2(t)\mu_{2_E}$. Let $\rho=\max\{\rho_1,\rho_2\}$. The product $\lambda_{1_E}\times\lambda_{2_E}$ is a soft fuzzy soft neighbourhood 0_{β_E} in $V=V_1\times V_2$. Let $\lambda_E=\lambda_{1_E}\times\lambda_{2_E}$. Now $\sigma(t\lambda_{1_E})\times\eta(t\lambda_{2_E})$ $=(\sigma\times\eta)(t(\lambda_{1_E}\times\lambda_{2_E}))$. Hence $\sigma(t\lambda_{1_E})\times\eta(t\lambda_{2_E})=(\sigma\times\eta)(t\lambda_E)$. Let $\sigma\times\eta=\zeta$. Then,

$$\begin{aligned}\rho(t)\mu_E(z)&\sqsupseteq(\rho(t)\mu_{1_E}(z_1)\sqcap\rho(t)\mu_{2_E}(z_2))\\&\sqsupseteq(\sigma(t\lambda_{1_E})(z_1)\sqcap\eta(t\lambda_{2_E})(z_2))\\&=\zeta(t\lambda_E)(z).\end{aligned}$$

where $z=<z_1,z_2>$, $z_1\in U_1$, $z_2\in U_2$. That is $\zeta(t\lambda_E)\sqsubseteq\rho(t)\mu_E$, where $\lambda_E\in V_1\times V_2$ and $\mu_E\in U_1\times U_2$. Hence ζ is soft fuzzy soft tangent to 0. Furthermore $f^{'}(x)\times g^{'}(y)$ is soft fuzzy soft continuous and linear by Proposition 3.5. □

Definition 3.9. Let $(V_1,\ T_1)$, $(V_2,\ T_2)$ be two soft fuzzy soft topological vector spaces. A bijection f of V_1 onto V_2 is said to be a soft fuzzy soft $\mathcal{C}^1$ diffeomorphism if f and its inverse f^{-1} are soft fuzzy soft differentiable, and $f^{'}$ and $(f^{-1})^{'}$ are soft fuzzy soft continuous.

Proposition 3.10. *Let $(U,\ T)$, $(V,\ S)$, $(W,\ R)$ be any three soft fuzzy soft topological vector spaces. If f is a soft fuzzy soft $\mathcal{C}^1$ diffeomorphism of U onto V and g is a soft fuzzy soft $\mathcal{C}^1$ diffeomorphism of V onto W, then $g\circ f$ is a soft fuzzy soft $\mathcal{C}^1$ diffeomorphism of U onto W.*

Proof. Proof is obtained from Proposition 2.31, Proposition 3.3. □

Proposition 3.11. *Let $(V_1,\ T_1)$, $(V_2,\ T_2)$, $(U_1,\ S_1)$, $(U_2,\ S_2)$ be any four soft fuzzy soft topological vector spaces. Suppose that $f:V_1\to U_1$ and $g:V_2\to U_2$ are soft fuzzy soft $\mathcal{C}^1$ diffeomorphism. Then $f\times g:V_1\times V_2\to U_1\times U_2$ is a soft fuzzy soft $\mathcal{C}^1$ diffeomorphism.*

Proof. Proof is obtained from Proposition 3.5, Proposition 3.6, Proposition 3.8. □

4. Soft fuzzy soft $\mathcal{C}^1$ manifolds

Definition 4.1. Let X be a nonempty set and E be the set of all parameters. A soft fuzzy soft $\mathcal{C}^1$ atlas $\mathfrak{S}$ on X is a collection of pairs $(\lambda_{j_E},\varphi_j)$ where j ranging over some index set, which satisfies the following conditions.

(i) Each λ_{j_E} is a soft fuzzy soft set in X and $\sqcup_{j\in J}\{\lambda_{j_E}(x)\} = (1, E)$, for all $x \in X$.
(ii) Each φ_j is a bijection, defined on $\widetilde{\lambda_{j_E}}$, which maps λ_{j_E} onto a soft fuzzy soft open set in some soft fuzzy soft topological vector space $(V_j,\ T_j)$, and for each l in the index set, $\varphi_j(\lambda_{j_E} \Cap \lambda_{l_E})$ is a soft fuzzy soft open set in $(V_j,\ T_j)$.
(iii) The function $\varphi_l \circ \varphi_j^{-1}$, which maps $\varphi_j(\lambda_{j_E} \Cap \lambda_{l_E})$ onto $\varphi_l(\lambda_{j_E} \Cap \lambda_{l_E})$ is a soft fuzzy soft $\mathcal{C}^1$ diffeomorphism for each pair of indices j, l.

Each pair $(\lambda_{j_E}, \varphi_j)$ is called a soft fuzzy soft chart of the soft fuzzy soft $\mathcal{C}^1$ atlas. If a point x lies in $\widetilde{\lambda_{j_E}}$, then $(\lambda_{j_E}, \varphi_j)$ is said to be a soft fuzzy soft chart at x.

Notation 4.2. $\widetilde{\lambda_{j_E}} = \{x \in X; \lambda_{j_E}(x) \sqsupset (0, \phi)\}$.

Proposition 4.3. *Let X_1 , X_2 be any two nonempty sets and E be the set of all parameters. Let $\mathfrak{S}$ be a soft fuzzy soft $\mathcal{C}^1$ atlas, with charts $(\lambda_{j_E}, \varphi_j)$ on a set X_1, and let $\mathfrak{M}$ be a soft fuzzy soft $\mathcal{C}^1$ atlas, with charts (μ_{l_E}, ψ_l) on a set X_2. Then the collection of pairs $(\lambda_{j_E} \times \mu_{l_E}, \varphi_j \times \psi_l)$ forms a soft fuzzy soft $\mathcal{C}^1$ atlas on $X_1 \times X_2$.*

Proof. (i) For all $x_1 \in X_1$, $x_2 \in X_2$, $\sqcup_j\{\lambda_{j_E}(x_1)\} = \sqcup_l\{\mu_{l_E}(x_2)\} = (1, E)$, and $\lambda_{j_E} \times \mu_{l_E}(x) = (\lambda_{j_E}(x_1) \sqcap \mu_{l_E}(x_2))$, $x =< x_1, x_2 >$. Therefore

$$\begin{aligned}\sqcup_{j,l}(\lambda_{j_E} \times \mu_{l_E}(x)) &= \sqcup_{j,l}(\lambda_{j_E}(x_1) \sqcap \mu_{l_E}(x_2)) \\ &= (\sqcup_j(\lambda_{j_E}(x_1))) \sqcap (\sqcup_l(\mu_{l_E}(x_2))) \\ &= (1, E)\end{aligned}$$

(ii) $\varphi_j(\lambda_{j_E})$ and $\psi(\mu_{l_E})$ are soft fuzzy soft open sets in a soft fuzzy soft topological vector spaces $(V_j,\ T_j)$, $(V_l,\ T_l)$ respectively. It follows that $\varphi_j(\lambda_{j_E}) \times \psi(\mu_{l_E})$ is a soft fuzzy soft open set in $V_j \times V_l$ and since $\varphi_j(\lambda_{j_E}) \times \psi_l(\mu_{l_E}) = (\varphi_j \times \psi_l)(\lambda_{j_E} \times \mu_{l_E})$, $(\varphi_j \times \psi_l)(\lambda_{j_E} \times \mu_{l_E})$ is a soft fuzzy soft open set in $V_j \times V_l$. Next consider the soft fuzzy soft sets $\lambda_{j_E} \Cap \lambda_{q_E}$ and $\mu_{l_E} \Cap \mu_{r_E}$ for each q, r. It is obvious that,

$$(\lambda_{j_E} \Cap \lambda_{q_E}) \times (\mu_{l_E} \Cap \mu_{r_E}) = (\lambda_{j_E} \times \mu_{l_E}) \Cap (\lambda_{q_E} \times \mu_{r_E}).$$

But it has proved that $(\varphi_j \times \psi_l)([\lambda_{j_E} \Cap \lambda_{q_E}] \times [\mu_{l_E} \sqcap \mu_{r_E}])$ is a soft fuzzy soft open set in $V_j \times V_l$ because $\varphi_j(\lambda_{j_E} \Cap \lambda_{q_E})$ and $\psi_l(\mu_{l_E} \Cap \mu_{r_E})$ are soft fuzzy soft open sets in $(V_j,\ T_j)$ and $(V_l,\ T_l)$ respectively. Hence $(\varphi_j \times \psi_l)([\lambda_{j_E} \times \mu_{l_E}] \Cap [\lambda_{q_E} \times \mu_{,r_E}])$ is a soft fuzzy soft open set in $V_j \times V_l$.
(iii) The last condition of Definition 4.1 is satisfied by Proposition 3.11. □

Proposition 4.4. *Let $\mathfrak{S}$ be a soft fuzzy soft $\mathcal{C}^1$ atlas with charts $(\lambda_{j_E}, \varphi_j)$ and suppose that for any soft fuzzy soft open set μ_E in soft fuzzy soft topological vector space $(V_j,\ T_j)$, $({\varphi_j}^{-1}(\mu_E), \varphi_j)$ is a soft fuzzy soft chart of $\mathfrak{S}$. The family $\{\lambda_{j_E}\}$ of soft fuzzy soft sets forms a base for a soft fuzzy soft topology on X, and φ_j are soft fuzzy soft continuous.*

Proof. Since $(\lambda_{j_E}, \varphi_j)$ is a soft fuzzy soft chart, $\sqcup_j\{\lambda_{j_E}(x)\} = (1, E)$, for all $x \in X$. Next if $(\lambda_{l_E}, \varphi_l)$, $(\lambda_{m_E}, \varphi_m)$ are soft fuzzy soft charts then $((\lambda_{l_E} \Cap \lambda_{m_E}), \varphi_l)$ is a soft fuzzy soft chart, since $\varphi_l(\lambda_{l_E} \Cap \lambda_{m_E})$ is a soft fuzzy soft open set. This shows that if λ_{l_E}, $\lambda_{m_E} \in \{\lambda_{j_E}\}$ then $\lambda_{l_E} \Cap \lambda_{m_E} \in \{\lambda_{j_E}\}$. Thus the conditions (i) and

(ii) of Proposition 3.2 are satisfied. Finally, for each l and constant membership function $\mathfrak{K}_c$ in X, let $\lambda'_{l_E}= {\varphi_l}^{-1}(\mathfrak{K}_{c_E} \Cap \varphi_l(\lambda_{l_E}))$, then $(\lambda'_{l_E}, \varphi_l)$ is a soft fuzzy soft chart, and hence λ'_{l_E} belongs to $\{\lambda_{j_E}\}$. The membership function of λ'_{l_E} is $\lambda'_{l_E}(x)$ $= \mathfrak{K}_{c_E}(x) \sqcap \lambda_{l_E}(x)$, $x \in X$. Thus for every constant membership function $\mathfrak{K}_c$ in X, $\mathfrak{K}_{c_E} \Cap \lambda_{l_E} \in \{\lambda_{j_E}\}$. By Proposition 3.2 $\{\lambda_{j_E}\}$ is a base. Each function φ_j is soft fuzzy soft continuous since ${\varphi_j}^{-1}$ maps a soft fuzzy soft open set onto a soft fuzzy soft open set. □

Definition 4.5. Let X be a soft fuzzy soft topological space and λ_E be a soft fuzzy soft open set, (λ_E, φ) is said to be SFScompatible with the soft fuzzy soft $\mathcal{C}^1$ atlas $\{(\lambda_{j_E}, \varphi_j)\}$ if each mapping $\varphi_j \circ \varphi^{-1}$ of $\varphi(\lambda_E \Cap \lambda_{j_E})$ onto $\varphi_j(\lambda_E \Cap \lambda_{j_E})$ is a soft fuzzy soft $\mathcal{C}^1$ diffeomorphism. Two soft fuzzy soft $\mathcal{C}^1$ atlases are SFScompatible if each soft fuzzy soft chart of one soft fuzzy soft $\mathcal{C}^1$ atlas is SFScompatible with each soft fuzzy soft chart of other soft fuzzy soft $\mathcal{C}^1$ atlas.

Definition 4.6. The relation of SFS compatibility between soft fuzzy soft $\mathcal{C}^1$ atlases is an equivalence relation. Set of all equivalence class of soft fuzzy soft $\mathcal{C}^1$ atlases on X is said to be a soft fuzzy soft $\mathcal{C}^1$ manifold.

Proposition 4.7. *Let X, Y be soft fuzzy soft $\mathcal{C}^1$ manifolds, then the product $X \times Y$ is a soft fuzzy soft $\mathcal{C}^1$ manifold.*

Proof. Proof follows from Proposition 4.3. □

Definition 4.8. Let X, Y be soft fuzzy soft $\mathcal{C}^1$ manifolds and let f be a function from X into Y. Then f is said to be soft fuzzy soft differentiable at a point $x \in X$ if there is a soft fuzzy soft chart (μ_E, φ) at $x \in X$ and a soft fuzzy soft chart (ν_E, ψ) at $f(x) \in Y$ such that the mapping $\psi \circ f \circ \varphi^{-1}$, which maps $\varphi(\mu_E \Cap f^{-1}(\nu_E))$ into $\psi(\nu_E)$ is soft fuzzy soft differentiable at $\varphi(x)$. The function f is soft fuzzy soft differentiable if it is soft fuzzy soft differentiable at every point of X, it is a soft fuzzy soft $\mathcal{C}^1$ diffeomorphism if $\psi \circ f \circ \varphi^{-1}$ is a soft fuzzy soft $\mathcal{C}^1$ diffeomorphism.

Proposition 4.9. *Let X, Y, Z be soft fuzzy soft $\mathcal{C}^1$ manifolds, f be a function from X into Y and g be a function from Y into Z. If f and g are soft fuzzy soft differentiable then $g \circ f$ is soft fuzzy soft differentiable.*

Proof. Let (λ_E, φ), (μ_E, ψ), (ν_E, χ) be soft fuzzy soft charts at $x \in X$, $f(x) \in Y$, $g(f(x)) \in Z$, respectively. Then $\psi \circ f \circ \varphi^{-1}$ which maps $\varphi(\lambda_E \Cap f^{-1}(\mu_E))$ into $\psi(\mu_E)$, and $\chi \circ g \circ \psi^{-1}$ which maps $\psi((\mu_E) \Cap g^{-1}(\nu_E))$ into $\chi(\nu_E)$, are soft fuzzy soft differentiable. Hence $\chi \circ g \circ \psi^{-1} \circ \psi \circ f \circ \varphi^{-1} = \chi \circ (g \circ f) \circ \varphi^{-1}$, which maps $\varphi(\lambda_E \Cap f^{-1}(\mu_E) \Cap f^{-1}(g^{-1}(\nu_E)))$ into $\chi(\nu_E)$, is soft fuzzy soft differentiable, by Proposition 2.31. □

Corollary 4.10. *If f and g are soft fuzzy soft $\mathcal{C}^1$ diffeomorphisms then the composition $g \circ f$ is a soft fuzzy soft $\mathcal{C}^1$ diffeomorphism.*

Proof. Proof is clear. □

5. Soft fuzzy soft tangent vector space

Definition 5.1. Let X be a soft fuzzy soft $\mathcal{C}^1$ manifold and let $x \in X$. Consider the triples $(\lambda_E, \varphi, v_{\delta_E})$, where (λ_E, φ) is a soft fuzzy soft chart at x and v_{δ_E} is a soft fuzzy soft point of the soft fuzzy soft topological vector spaces in which $\varphi(\lambda_E)$ lies. Two such triples $(\lambda_E, \varphi, v_{\delta_E})$, $(\mu_E, \psi, w_{\beta_E})$ are said to be related, denoted by $(\lambda_E, \varphi, v_{\delta_E}) \sim (\mu_E, \psi, w_{\beta_E})$, if the soft fuzzy soft derivative of $\psi \circ \varphi^{-1}$ at $\varphi(x)$ maps v_{δ_E} into w_{β_E}. That is $(\psi \circ \varphi^{-1})'(\varphi(x))v_{\delta_E} = w_{\beta_E}$.

Proposition 5.2. *The relation $(\lambda_E, \varphi, v_{\delta_E}) \sim (\mu_E, \psi, w_{\beta_E})$ is an equivalence relation.*

Proof. Proof is easily obtained by Definition 5.1. □

Definition 5.3. An equivalence class of triples $(\lambda_E, \varphi, v_{\delta_E})$ is called a soft fuzzy soft tangent vector of the soft fuzzy soft $\mathcal{C}^1$ manifold X at x, and the soft fuzzy soft tangent space at x, denoted by $SFS\mathcal{T}_x(X)$, is defined as the set of all soft fuzzy soft tangent vectors at x.

Definition 5.4. The set $SFS\mathcal{T}_x(X)$ can be given the structure of a vector space. Define the sum of two soft fuzzy soft tangent vectors at $x \in X$ as $(\lambda_{1_E}, \varphi_1, v_{1_{\delta_E}}) + (\lambda_{2_E}, \varphi_2, v_{2_{\beta_E}}) = (\lambda_{2_E}, \varphi_2, (\varphi_2 \circ {\varphi_1}^{-1})'(\varphi_1(x))v_{1_{\delta_E}} + v_{2_{\beta_E}})$. Define the product of a soft fuzzy soft tangent vector with a scalar k as $k(\lambda_E, \varphi, v_{\delta_E}) = (\lambda_E, \varphi, kv_{\delta_E})$.

Proposition 5.5. *If*

$$(\lambda_{1_E}, \varphi_1, v_{1_{\delta_E}}) \sim (\mu_{1_E}, \psi_1, w_{1_{\delta_E}}) \text{ and } (\lambda_{2_E}, \varphi_2, v_{2_{\beta_E}}) \sim (\mu_{2_E}, \psi_2, w_{2_{\beta_E}}),$$

then $(\lambda_{1_E}, \varphi_1, v_{1_{\delta_E}}) + (\lambda_{2_E}, \varphi_2, v_{2_{\beta_E}}) \sim (\mu_{1_E}, \psi_1, w_{1_{\delta_E}}) + ((\mu, M)_2, \psi_2, w_{2_{\beta_E}})$.

Proof. From the sums $(\lambda_{2_E}, \varphi_2, (\varphi_2 \circ {\varphi_1}^{-1})'(\varphi_1(x))v_{1_{\delta_E}} + v_{2_{\beta_E}})$,

$$(\mu_{2_E}, \psi_2, (\psi_2 \circ {\psi_1}^{-1})'(\psi(x))w_{1_{\delta_E}} + w_{2_{\beta_E}}).$$

From the related triples, $(\psi_2 \circ \varphi_2^{-1})'(\varphi_2(x))((\varphi_2 \circ {\varphi_1}^{-1})'(\varphi_1(x))v_{1_{\delta_E}} + v_{2_{\beta_E}})$

$$\begin{aligned}
&= (\psi_2 \circ {\varphi^{-1}}_1)'(\varphi_1(x))v_{1_{\delta_E}} + (\psi_2 \circ {\varphi_2}^{-1})'(\varphi_2(x))v_{2_{\beta_E}} \\
&= ((\psi_2 \circ {\varphi_1}^{-1})'(\varphi_1(x)) \circ (\varphi_1 \circ {\psi_1}^{-1})'(\psi_1(x)))w_{1_{\delta_E}} + w_{2_{\beta_E}} \\
&= (\psi_2 \circ {\psi_1}^{-1})'(\psi_1(x))w_{1_{\delta_E}} + w_{2_{\beta_E}}
\end{aligned}$$

□

Proposition 5.6. *If* $(\lambda_E, \varphi, v_{\delta_E}) \sim ((\mu, M), \psi, w_{\beta_E})$ *then*

$$k(\lambda_E, \varphi, v_{\delta_E}) \sim k((\mu, M), \psi, w_{\beta_E}).$$

Proof. Proof is easily obtained from Definition 5.4. □

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