Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 8, No. 2, (August 2014), pp. 177–192
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Topological structure of imprecise soft set

Tridiv Jyoti Neog, Manoj Bora, Dusmanta Kumar Sut

Received 19 July 2013; Revised 7 October 2013; Accepted 23 November 2013

Abstract. The purpose of this work is to study the topological structure of imprecise soft sets. Our work is an attempt to introduce the concepts on imprecise soft point, imprecise soft neighborhood, imprecise soft closure, imprecise soft interior imprecise soft subspace topology and basis for an imprecise soft topological space.

2010 AMS Classification: 03E72

Keywords: Imprecise set, Imprecise soft set, Imprecise soft topological space, Imprecise soft point, Imprecise soft neighborhood, Imprecise soft closure, Imprecise soft interior, Imprecise soft subspace topology, Basis for an imprecise soft topological space.

Corresponding Author: Tridiv Jyoti Neog (tridivjyoti@gmail.com)

1. Introduction

In the theory of fuzzy sets put forward by Zadeh in 1965, it has been observed that the operation of complementation of a normal fuzzy set does not explain the principles of exclusion and contradiction followed by the classical sets. This is due to the fact that in the Zadehian definition of complementation, membership value and membership function had been taken to be of the same meaning [1]. In order to link fuzziness with probability, Zadeh then forwarded Probability-Possibility Consistency Principle. But no consistency between probability and fuzziness was reflected by that principle and two more Probability-Possibility Consistency Principles were forwarded thereafter by others. Recently Baruah [2] has introduced the theory of imprecise sets where these two mistakes in fuzzy sets are absent. Most of the concepts we meet in our day to day life contain uncertainty. Mathematical modeling of such day to day problems involving uncertainties is of great importance now a days. The important existing theories e.g. Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. that deal with uncertainties have their own difficulties. In fact, the inadequacy of the parameterization tool do not allow these theories to handle vagueness properly. In 1999 Molodtsov [3] introduced the concept of Soft Sets and established the fundamental results of the new theory. Soft Set Theory is free from parameterization inadequacy syndrome of Fuzzy Set Theory, Rough Set Theory, Probability Theory etc. Maji [3] studied the theory of soft sets and initiated some new results. In 2011, Neog and Sut [4] put forward a new definition of complement of a soft set and showed that the axioms of exclusion and contradiction are satisfied by the soft sets also. Recently, combining imprecise sets with soft sets, Neog and Sut [5] have put forward a new model known as imprecise soft sets and successfully applied the notion of imprecise soft sets in a decision problem [7]. Topological structure of soft sets have been studied by many researchers in present times, e.g. [9, 11]. Moreover many researchers have contributed to the development of fuzzy soft topological spaces, e.g. [6, 8, 10]. In this work, we have studied the topological structure of imprecise soft sets and some results have been established. Our work is an attempt to introduce the concepts on imprecise soft point, imprecise soft neighborhood, imprecise soft closure, imprecise soft interior imprecise soft subspace topology and basis for an imprecise soft topological space.

2. Preliminaries

In this section, we first recall some basic concepts which would be used in the sequel. The following definitions regarding imprecise sets are due to Baruah [2].

Definition 2.1. An imprecise number $[\alpha, \beta, \gamma]$ is an interval around the real number β with the elements in the interval being partially present.

Definition 2.2. Partial presence of an element in an imprecise real number $[\alpha, \beta, \gamma]$ is described by the presence level indicator function $p(x)$ which is counted from the reference function $r(x)$ such that the presence level for any $x, \alpha \leq x \leq \gamma$ is $(p(x) - r(x))$, where $0 \leq r(x) \leq p(x) \leq 1$.

Definition 2.3. A normal imprecise number $N = [\alpha, \beta, \gamma]$ is associated with a presence level indicator function $\mu_N(x)$, where

$$\mu_N(x) = \begin{cases} \Psi_1(x) & if \alpha \leq x \leq \beta \\ \Psi_2(x) & if \beta \leq x \leq \gamma \\ 0, & otherwise \end{cases}$$

with a constant reference function 0 in the entire real line. Here $\Psi_1(x)$ is continuous and non-decreasing in the interval $[\alpha, \beta]$ and $\Psi_2(x)$ is continuous and non-increasing in the interval $[\beta, \gamma]$, with

$$\Psi_1(\alpha) = \Psi_2(\gamma) = 0$$
$$\Psi_1(\beta) = \Psi_2(\beta) = 1$$

Here, the imprecise number would be characterized by $\{x, \mu_N(x), 0 : x \in R\}$, R being the real line.

Definition 2.4. For a normal imprecise number $N = [\alpha, \beta, \gamma]$ with a presence level indicator function $\mu_N(x)$, where

$$\mu_N(x) = \begin{cases} \Psi_1(x) & if \alpha \le x \le \beta \\ \Psi_2(x) & if \beta \le x \le \gamma \\ 0, & otherwise \end{cases}$$

such that

$$\Psi_1(\alpha) = \Psi_2(\gamma) = 0,$$
$$\Psi_1(\beta) = \Psi_2(\beta) = 1$$

with constant reference function equal to $0, \Psi_1(x)$ is the distribution function of a random variable defined in the interval $[\alpha, \beta]$ and $\Psi_2(x)$ is the complementary distribution function of another random variable defined in the interval $[\beta, \gamma]$. We are using the term random variable here in the broader measure theoretic sense which does not require that the notion of probability need to appear in defining randomness.

Definition 2.5. For a normal imprecise number $N = \{x, \mu_N(x), 0 : x \in R\}$, the complement $N^c = \{x, 1, \mu_N(x) : x \in R\}$ will have constant presence level indicator function equal to 1, the reference function being $\mu_n(x)$ for $- \in fty < x <\in fty$.

Definition 2.6. If a normal imprecise number $N = [\alpha, \beta, \gamma]$ is defined with a presence level indicator function $\mu_N(x)$, where

$$\mu_N(x) = \begin{cases} \Psi_1(x) & if \alpha \le x \le \beta \\ \Psi_2(x) & if \beta \le x \le \gamma \\ 0, & otherwise \end{cases}$$

with

$$\Psi_1(\alpha) = \Psi_2(\gamma) = 0,$$
$$\Psi_1(\beta) = \Psi_2(\beta) = 1$$

the complement N^c should have the presence level indicator function $\mu_{N^c}(x)$, with $\mu_{N^c}(x) = 1, - \in fty < x <\in fty$, where $\mu_{N^c}(x)$ is to be counted from $\Psi_1(x)$ if $\alpha \le x \le \beta$, from $\Psi_2(x)$ if $\eta \le x \le \gamma$, and from 0, otherwise. Molodtsov [3] defined soft set in the following way-

Definition 2.7. A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\epsilon), \epsilon \in E$,from this family may be considered as the set of ϵ - elements of the soft set (F, E), or as the set of ϵ-approximate elements of the soft set. The following definitions regarding imprecise soft sets are due to Neog and Sut [5].

Definition 2.8. Let U be the initial universe and E be the set of parameters. Let $A \subseteq E$ and $\widetilde{P}(U)$ denote the set of all imprecise subsets over the universe U. Then a pair (F, A) is called an imprecise soft set over U where $F : A \to \widetilde{P}(U)$ is a mapping from A into $\widetilde{P}(U)$.

Definition 2.9. Let U be the initial universe, E be the set of parameters and $\widetilde{P}(U)$ denote the set of all imprecise subsets over the universe U. Then the pair (F, E) is called a total imprecise soft set over U where $F : A \to \widetilde{P}(U)$ is a mapping from E into $\widetilde{P}(U)$.

Definition 2.10. An imprecise soft set (F, A) over U is said to be null imprecise soft set (with respect to the parameter set A), denoted by $(\widetilde{\varphi}, A)$ if $\forall \epsilon \in A, F(\epsilon)$ is the null set φ. The Null Imprecise Soft Set is not unique, it depends upon the set of parameters under consideration.

Definition 2.11. An imprecise soft set (F, A) over U is said to be absolute imprecise soft set (with respect to the parameter set A), denoted by $(\widetilde{U}, A)$ if $\forall \epsilon \in A, F(\epsilon)$ is the absolute set U. The Absolute Imprecise Soft Set is not unique, it depends upon the set of parameters under consideration.

Definition 2.12. Union of two imprecise soft sets (F, A) and (G, B) over (U, E) is an imprecise soft set (H, C) where $C = A \cup B$ and $\forall \epsilon \in C$,

$$H(\epsilon) = \begin{cases} F(\epsilon) & if \epsilon \in A - B \\ G(\epsilon) & if \epsilon \in B - A \\ F(\epsilon) \cup G(\epsilon) & if \epsilon \in A \cap B \end{cases}$$

and is written as $(F, A)\widetilde{\cup}(G, B) = (H, C)$

Definition 2.13. Let (F, A) and (G, B) be two imprecise soft sets over (U, E). Then intersection of the imprecise soft sets (F, A) and (G, B) is an imprecise soft set (H, C) where $C = A \cap B$ and $\forall \epsilon \in C, H(\epsilon) = F(\epsilon) \cap G(\epsilon)$. We write $(F, A)\widetilde{\cap}(G, B) = (H, C)$.

Definition 2.14. For two imprecise soft sets (F, A) and (G, B) over (U, E), we say that (F, A) is an imprecise soft subset of (G, B), if

(1) $A \subseteq B$
(2) For all $\epsilon \in A, F(\epsilon) \subseteq G(\epsilon)$and is written as$(F, A)\widetilde{\subseteq}(G, B)$

Definition 2.15. For two imprecise soft sets (F, A)and(G, B)over (U, E), we say that (F, A) is equal to (G, B), if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$.

Definition 2.16. The complement of an imprecise soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to \widetilde{P}(U)$ is a mapping given by, $F^c(\epsilon) = (F(\epsilon))^c, \forall \epsilon \in A$. In other words, $\forall \epsilon \in A$, if $F(\epsilon) = \{x, \mu_{F(\epsilon)}(x), 0; x \in U\}$, then $F^c(\epsilon) = \{x, 1\mu_{F(\epsilon)}(x); x \in U\}$.

3. Topological structure of imprecise soft sets

In order to deal with the topological structure of imprecise soft sets properly, we would consider the total imprecise soft set (F, E) over U, where E is the set of parameters under consideration. First we define an imprecise soft topological space as follows:

Definition 3.1. An imprecise soft topology $\Im$ on (U, E) is a family of imprecise soft sets over (U, E) satisfying the following properties.

(1) $\widetilde{\varphi}, \widetilde{E} \in \Im$
(2) If (F,E),(G,E)$\in \Im$ then $(F, E)\widetilde{\cap}(G, E) \in \Im$
(3) If $(F_\alpha, E) \in \Im$ for all $\alpha \in \Delta$, an index set, then $\underset{\alpha \in \Delta}{\cup}(F_\alpha, E) \in \Im$.

Definition 3.2. If $\Im$ is an imprecise soft topology on (U, E), the triple $(U, E, \Im)$ is said to be an imprecise soft topological space. Also each member of $\Im$ is called an imprecise soft open set in $(U, E, \Im)$.

Example 3.3. Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$

$$(F, E) = \Big\{ F(e_1) = \{(a, 0.4, 0), (b, 0.1, 0), (c, 0, 0)\} \\ F(e_2) = \{(a, 0.6, 0), (b, 0.5, 0), (c, 0.2, 0)\} \\ F(e_3) = \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ F(e_4) = \{(a, 0.2, 0), (b, 0.6, 0), (c, 0.3, 0)\} \Big\}$$

$$(G, E) = \Big\{ G(e_1) = \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ G(e_2) = \{(a, 0.5, 0), (b, 0.2, 0), (c, 0.1, 0)\} \\ G(e_3) = \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ G(e_4) = \{(a, 0.1, 0), (b, 0.3, 0), (c, 0.2, 0)\} \Big\}$$

We consider a collection $\Im$ of imprecise soft sets over (U, E) as

$$\Im = \Big\{ (\widetilde{\varphi}, E), (\widetilde{U}, E), (F, E), (G, E) \Big\}.$$

We see that $\Im$ is an imprecise soft topology on (U, E) and the triple $(U, E, \Im)$ is an imprecise soft topological space. The open sets in $(U, E, \Im)$ are $(\widetilde{\varphi}, E)$, $(\widetilde{U}, E)$, (F, E), (G, E).

Definition 3.4. An imprecise soft set over U is called an imprecise soft closed set in $(U, E, \Im)$ if its complement (F^c, E) is an imprecise soft open set in $(U, E, \Im)$.

Definition 3.5. An imprecise soft topological space $(U, E, \Im)$ is called indiscrete if it contains only $(\widetilde{\varphi}, E)$ and $(\widetilde{U}, E)$ while the discrete imprecise soft topology consists of all imprecise soft sets over (U, E).

Example 3.6. Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$

$$(F, E) = \Big\{ \begin{array}{lcl} F(e_1) & = & \{(a, 0.4, 0), (b, 0.1, 0), (c, 0, 0)\} \\ F(e_2) & = & \{(a, 0.6, 0), (b, 0.5, 0), (c, 0.2, 0)\} \\ F(e_3) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ F(e_4) & = & \{(a, 0.2, 0), (b, 0.6, 0), (c, 0.3, 0)\} \end{array} \Big\}$$

$$(G, E) = \Big\{ \begin{array}{lcl} G(e_1) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ G(e_2) & = & \{(a, 0.7, 0), (b, 0.2, 0), (c, 0.1, 0)\} \\ G(e_3) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ G(e_4) & = & \{(a, 0.5, 0), (b, 0.3, 0), (c, 0.9, 0)\} \end{array} \Big\}$$

We consider the imprecise soft topologies $\Im_1$ and $\Im_2$ on (U, E) as
$\Im_1 = \Big\{ ((\widetilde{\varphi}, E), (\widetilde{U}, E), (F, E) \Big\}$ and $\Im_2 = \Big\{ ((\widetilde{\varphi}, E), (\widetilde{U}, E), (G, E) \Big\}$

$\Im_1 \cup \Im_2 = \left\{((\widetilde{\varphi}, E), (\widetilde{U}, E), (G, E)\right\}$

$$(F,E)\widetilde{\cap}(G,E) = (H,E) = \left\{\begin{array}{lcl} H(e_1) & = & \{(a,0,0),(b,0,0),(c,0,0)\} \\ H(e_2) & = & \{(a,0.6,0),(b,0.2,0),(c,0.1,0)\} \\ H(e_3) & = & \{(a,0,0),(b,0,0),(c,0,0)\} \\ H(e_4) & = & \{(a,0.2,0),(b,0.3,0),(c,0.3,0)\} \end{array}\right\}$$

Thus, $(F,E),(G,E) \in \Im_1 \cup \Im_2$ but $(F,E)\widetilde{\cap}(G,E) \notin \Im_1 \cup \Im_2$. Thus $\Im_1 \cup \Im_2$ is not an imprecise soft topology on (U, E). However $\Im_1 \cap \Im_2 = \left\{(\widetilde{\varphi}, E), (\widetilde{U}, E)\right\}$ is an imprecise soft topology on (U, E). We thus arrive at the following proposition -

Proposition 3.7. *If $\Im_1$ and $\Im_2$ are two imprecise soft topologies on (U, E) then $\Im_1 \cap \Im_2$ is an imprecise soft topology on (U, E) but $\Im_1 \cup \Im_2$ is not necessarily an imprecise soft topology on (U, E).*

Proof. Since $\Im_1$ and $\Im_2$ are fuzzy soft topologies on (U, E). $(\widetilde{\varphi}, E), (\widetilde{U}, E) \in \Im_1$ and $(\widetilde{\varphi}, E), (\widetilde{U}, E) \in \Im_2$. Thus $(\widetilde{\varphi}, E), (\widetilde{U}, E) \in \Im_1 \cap \Im_2$. Let $(F,E),(G,E) \in \Im_1 \cap \Im_2$, then $(F,E),(G,E) \in \Im_1$ and $(F,E),(G,E) \in \Im_2$. As $\Im_1$ and $\Im_2$ are fuzzy soft topologies on (U, E), it follows that $(F,E)\widetilde{\cap}(G,E) \in \Im_1$ and $(F,E)\widetilde{\cap}(G,E) \in \Im_2$
$\Rightarrow (F,E)\widetilde{\cap}(G,E) \in \Im_1 \cap \Im_2$
Let $(F_\alpha, E) \in \Im_1 \cap \Im_2, \alpha \in \Delta$ where Δ is an arbitrary set. Then $(F_\alpha, E) \in \Im_1, \alpha \in \Delta$ and $(F_\alpha, E) \in \Im_2, \alpha \in \Delta$ As $\Im_1$ and $\Im_2$ are fuzzy soft topologies on (U, E), it follows that, $\underset{\alpha\in\Delta}{\cup}(F_\alpha, E) \in \Im_1$ and $\underset{\alpha\in\Delta}{\cup}(F_\alpha, E) \in \Im_2$. Thus $\underset{\alpha\in\Delta}{\cup}(F_\alpha, E) \in \Im_1 \widetilde{\cap} \Im_1$.
It follows that $\Im_1 \widetilde{\cap} \Im_1$ is a fuzzy soft topology on (U, E). Also, it is clear from Example 3.6 that union of $\Im_1$ and $\Im_2$ may not be a fuzzy soft topology on (U, E). □

Proposition 3.8. *If $\{\Im_\lambda : \lambda \in I\}$ is a family of imprecise soft topologies on (U, E), then $\underset{\lambda}{\cap}\{\Im_\lambda : \lambda \in I\}$ is also an imprecise soft topology on (U, E).*

Proof. The proof follows similar lines to proposition 3.7. □

Proposition 3.9. *Let $(U, E, \Im)$ be an imprecise soft topological space and Γ denote the collection of all closed imprecise soft sets in $(U, E, \Im)$. Then*

(1) $(\widetilde{\varphi}, E), (\widetilde{U}, E) \in \Gamma$
(2) *If $(F,E),(G,E) \in \Gamma$ then $(F,E)\widetilde{\cup}(G,E) \in \Gamma$*
(3) *If $(F_\alpha, E) \in \Gamma$ for all $\alpha \in \Delta$, an index set, then $\underset{\alpha\in\Delta}{\cap}(F_\alpha, E) \in \Gamma$*

Proof. (1) We have $(\widetilde{\varphi}, E), (\widetilde{U}, E) \in \Im$
$\Rightarrow (\widetilde{\varphi}, E)^c, (\widetilde{U}, E)^c \in \Gamma$
$\Rightarrow (\widetilde{U}, E), (\widetilde{\varphi}, E) \in \Gamma$
(2) $(F,E),(G,E) \in \Gamma$
$\Rightarrow (F,E)^c, (G,E)^c \in \Im$
$\Rightarrow (F,E)^c \widetilde{\cap} (G,E)^c \in \Im$
$\Rightarrow \left((F,E)\widetilde{\cup}(G,E)\right)^c \in \Im$
$\Rightarrow (F,E)\widetilde{\cup}(G,E) \in \Gamma$

(3) $(F_\alpha, E) \in \Gamma, \alpha \in \Delta$
$\Rightarrow (F_\alpha, E)^c \in \Im, \alpha \in \Delta$
$\overset{\Rightarrow \cup}{\alpha \in \Delta} (F_\alpha, E)^c \in \Im$
$\Rightarrow \left(\underset{\alpha \in \Delta (F_\alpha, E)}{\cap} \right)^c \in \Im$
$\overset{\Rightarrow \cap}{\alpha \in \Delta} (F_\alpha, E) \in \Gamma$

□

Definition 3.10. Let $\Im_1$ and $\Im_2$ be two imprecise soft topologies on (U, E). We say that $\Im_1$ is coarser than $\Im_2$ or that $\Im_2$ is finer than $\Im_1$ if $\Im_1 \subseteq \Im_2$ i.e. every $\Im_1$ imprecise soft open set is $\Im_2$ imprecise soft open set. If either $\Im_1 \subseteq \Im_2$ or $\Im_2 \subseteq \Im_1$, we say that the topologies $\Im_1$ and $\Im_2$ are comparable. If $\Im_1 \not\subset \Im_2$ and $\Im_2 \not\subset \Im_1$, we say that the topologies $\Im_1$ and $\Im_2$ are not comparable.

Example 3.11. Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$

$$(F, E) = \Big\{ \begin{array}{lcl} F(e_1) & = & \{(a, 0.4, 0), (b, 0.1, 0), (c, 0, 0)\} \\ F(e_2) & = & \{(a, 0.6, 0), (b, 0.5, 0), (c, 0.2, 0)\} \\ F(e_3) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ F(e_4) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \Big\} \end{array}$$

$$(G, E) = \Big\{ \begin{array}{lcl} G(e_1) & = & \{(a, 0.6, 0), (b, 0.1, 0), (c, 0, 0)\} \\ G(e_2) & = & \{(a, 0.7, 0), (b, 0.9, 0), (c, 0.5, 0)\} \\ G(e_3) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ G(e_4) & = & \{(a, 0.5, 0), (b, 0.3, 0), (c, 0.9, 0)\} \Big\} \end{array}$$

We consider the imprecise soft topologies $\Im_1$ and $\Im_2$ on (U, E) as $\Im_1 = \Big\{ (\widetilde{\varphi}, E), (\widetilde{U}, E), (F, E) \Big\}$ and $\Im_2 = \Big\{ (\widetilde{\varphi}, E), (\widetilde{U}, E), (F, E), (G, E) \Big\}$. We see that $\Im_1 \subseteq \Im_2$ and hence $\Im_1$ is coarser than $\Im_2$.

Definition 3.12. The imprecise soft set (F, E) over (U, E) is called an imprecise soft point in (U, E) denoted by $e(F, E)$, if for the element $e \in E, F(e) \neq \overline{0}$, and $F(\acute{e}) = \overline{0}$ for all $\acute{e} \in E - e$.

Example 3.13. Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$. We consider the imprecise soft set (F,E) over (U, E) as

$$(F, E) = \Big\{ \begin{array}{lcl} F(e_1) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ F(e_2) & = & \{(a, 0.1, 0), (b, 0.2, 0), (c, 0.7, 0)\} \\ F(e_3) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ F(e_4) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \Big\} \end{array}$$

Here $e_2 \in E$ and $F(e_2) \neq \overline{0}$ and for $\acute{e} \in E - e_2, F(\acute{e}) = \overline{0}$. Thus (F,E) is an imprecise soft point in (U, E) denoted by $e_2(F, E)$.

Definition 3.14. The imprecise soft point $e(F,E)$ is said to be in the imprecise soft set (G,E) if for the element $e \in E, F(e) \subseteq G(e)$. Symbolically we write $e(F,E)\widetilde{\in}(G,E)$.

Example 3.15. Let $U = \{a,b,c\}$ and $E = \{e_1,e_2,e_3,e_4\}$. We consider the imprecise soft point cited in Example 3.4 and an imprecise soft set (G,E) over (U, E) as

$$(G,E) = \Big\{ \begin{array}{lcl} G(e_1) & = & \{(a,0.1,0),(b,0.7,0),(c,0.9,0)\} \\ G(e_2) & = & \{(a,0.1,0),(b,0.3,0),(c,0.8,0)\} \\ G(e_3) & = & \{(a,0,0),(b,0,0),(c,0,0)\} \\ G(e_4) & = & \{(a,0.6,0),(b,0.5,0),(c,0.2,0)\} \Big\} \end{array}$$

Here $e_2 \in E$ and $F(e_2) \subseteq G(e_2)$. Thus by our definition,$e_2(F,E)\widetilde{\in}(G,E)$.

Definition 3.16. An imprecise soft set (H,E) in an imprecise soft topological space $(U,E,\Im)$ is called an imprecise soft neighborhood (nbd) of the imprecise soft point $e(F,E)$ if there is an imprecise soft open set (G,E) such that $e(F,E)\widetilde{\in}(G,E)\widetilde{\subseteq}(H,E)$.

Example 3.17. Let $U = \{a,b,c\}$ and $E = \{e_1,e_2,e_3,e_4\}$. We consider the imprecise soft point $e_2(F,E)$ given in Example 3.4 and the imprecise soft set (G,E) over (U, E) given in Example 3.5 above. Consider the imprecise soft topology $\Im = \Big\{(\widetilde{\varphi},E),(\widetilde{U},E),(G,E)\Big\}$. Now let us consider an imprecise soft set (H,E) over (U, E) as

$$(H,E) = \Big\{ \begin{array}{lcl} H(e_1) & = & \{(a,0.1,0),(b,0.8,0),(c,0.9,0)\} \\ H(e_2) & = & \{(a,0.2,0),(b,0.3,0),(c,0.8,0)\} \\ H(e_3) & = & \{(a,0,0),(b,0,0),(c,0,0)\} \\ H(e_4) & = & \{(a,0.7,0),(b,0.6,0),(c,0.2,0)\} \Big\} \end{array}$$

We see that $e(F,E)\widetilde{\in}(G,E)\widetilde{\subseteq}(H,E)$. Thus by our definition, the imprecise soft set (H,E) is an imprecise soft neighborhood of the imprecise soft point $e_2(F,E)$.

Definition 3.18. The neighborhood system of an imprecise soft point $e(F,E)\widetilde{\in}(U,E)$ is the family of all neighborhoods of $e(F,E)$ and is written as $N_{\Im}\left(e(F,E)\right)$.

Definition 3.19. An imprecise soft set (H,E) in an imprecise soft topological space $(U,E,\Im)$ is called an imprecise soft neighborhood (nbd) of the imprecise soft set (F,E) if there is an imprecise soft open set (G,E) such that $(F,E)\widetilde{\subseteq}(G,E)\widetilde{\subseteq}(H,E)$.

Example 3.20. Let $U = \{a,b,c\}$ and $E = \{e_1,e_2,e_3,e_4\}$. We consider the imprecise soft set (G,E) over (U, E) given in Example 3.5 and the imprecise soft set (H,E) over (U, E) given in Example 3.6 above. Consider the imprecise soft topology $\Im =$

$\left\{(\widetilde{\varphi}, E), (\widetilde{U}, E), (G, E)\right\}$. Let us consider an imprecise soft set (I,E) over (U, E) as

$$(I, E) = \left\{\begin{array}{lcl} I(e_1) & = & \{(a, 0.1, 0), (b, 0.6, 0), (c, 0.7, 0)\} \\ I(e_2) & = & \{(a, 0, 0), (b, 0.1, 0), (c, 0.8, 0)\} \\ I(e_3) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \\ I(e_4) & = & \{(a, 0, 0), (b, 0, 0), (c, 0, 0)\} \end{array}\right\}$$

We see that $(I, E)\widetilde{\subseteq}(G, E)\widetilde{\subseteq}(H, E)$. It follows that the imprecise soft set (H,E) is an imprecise soft neighborhood of the the imprecise soft set (I,E).

Proposition 3.21. *In an imprecise soft topological space $(U, E, \Im)$ the following hold -*

(1) $(\widetilde{U}, E) \in N_{\Im}(e(F, E))\, \forall e(F, E)$ *and* $(G, E) \in N_{\Im}(e(F, E)) \Rightarrow e(F, E)\widetilde{\in}(G, E)$
(2) *If* $(G, E) \in N_{\Im}(e(F, E))$ *and* $(G, E)\widetilde{\subseteq}(H, E)$ *then* $(H, E) \in N_{\Im}(e(F, E))$
(3) *If* $(G, E), (H, E) \in N_{\Im}(e(F, E))$ *then* $(G, E)\widetilde{\cap}(H, E) \in N_{\Im}(e(F, E))$
(4) *If* $(G, E) \in N_{\Im}(e(F, E))$ *then there is a* $(H, E) \in N_{\Im}(e(F, E))$ *such that* $(G, E) \in N_{\Im}(\acute{e}(M, E))$ *for each* $\acute{e}(M, E)\widetilde{\in}(H, E)$.

Proof. (1) Let $(\widetilde{U}, E) = (H, E)$. Then $\forall \epsilon \in E, H(\epsilon) = \overline{1}$ Now $F(e) \subseteq H(e)$. It follows that $e(F, E)\widetilde{\in}(H, E)\widetilde{\subseteq}(H, E)$ i.e. $e(F, E)\widetilde{\in}(\widetilde{U}, E)\widetilde{\subseteq}(\widetilde{U}, E)$. Thus $(\widetilde{U}, E) \in N_{\Im}(e(F, E))\, \forall e(F, E)$
For the second part, we have $(G, E) \in N_{\Im}(e(F, E))$. So there is a fuzzy soft open set(H, E) such that $e(F, E)\widetilde{\in}(H, E)\widetilde{\subseteq}(G, E)$. It follows that $e(F, E)$ $\widetilde{\in}(G, E)$.
(2) We have $(G, E) \in N_{\Im}(e(F, E))$. So there is a fuzzy soft open set (I, E) such that $e(F, E)\widetilde{\in}(I, E)\widetilde{\subseteq}(G, E)$. Also $(G, E)\widetilde{\subseteq}(H, E)$.
Thus $e(F, E)\widetilde{\in}(I, E)\widetilde{\subseteq}(G, E)\widetilde{\subseteq}(H, E)$ and hence $(H, E) \in N_{\Im}(e(F, E))$
(3) We have $(G, E), (H, E) \in N_{\Im}(e(F, E))$. So there are fuzzy soft open sets (I, E), (J, E) such that $e(F, E)\widetilde{\in}(I, E)\widetilde{\subseteq}(G, E)$ and $e(F, E)\widetilde{\in}(J, E)\widetilde{\subseteq}(H, E)$. Thus $e(F, E)\widetilde{\in}(I, E)\widetilde{\cap}(J, E)\widetilde{\subseteq}(G, E)\widetilde{\cap}(H, E)$ and since $(I, E)\widetilde{\cap}(J, E) \in \Im$, it follows that $(G, E)\widetilde{\cap}(H, E) \in N_{\Im}(e(F, E))$.
(4) We have $(G, E) \in N_{\Im}(e(F, E))$. So there is an $(I, E) \in \Im$ such that $e(F, E)\widetilde{\in}(I, E)\widetilde{\subseteq}(G, E)$. We take $(H, E) = (I, E)$.
Then for each $\acute{e}(M, E)\widetilde{\in}(H, E), \acute{e}(M, E)\widetilde{\in}(H, E)\widetilde{\subseteq}(G, E)$. It follows that $(G, E) \in N_{\Im}(\acute{e}(M, E))$.

□

Definition 3.22. Let $(U, E, \Im)$ be an imprecise soft topological space. Let (F, E) be an imprecise soft set over (U, E). The imprecise soft closure of (F,E) is defined as the intersection of all imprecise soft closed sets which contain (F,E) and is denoted by $\overline{(F, E)}$. We write

$$\overline{(F, E)} = \widetilde{\cap}\left\{(G, E) : (G, E) \text{is imprecise soft closed and} (F, E)\widetilde{\subseteq}(G, E)\right\}.$$

It is obvious that

(1) $\overline{(F, E)}$ is imprecise soft closed and

(2) $(F,E)\widetilde{\subseteq}\overline{(F,E)}$

Example 3.23. Let $U=\{a,b,c\}$ and $E=\{e_1,e_2,e_3,e_4\}$.

$$(F,E)=\Big\{\begin{array}{lcl} F(e_1) &=& \{(a,0.4,0),(b,0.1,0),(c,0,0)\} \\ F(e_2) &=& \{(a,0.6,0),(b,0.5,0),(c,0.2,0)\} \\ F(e_3) &=& \{(a,0,0),(b,0,0),(c,0,0)\} \\ F(e_4) &=& \{(a,0,0),(b,0,0),(c,0,0)\}\Big\} \end{array}$$

$$(G,E)=\Big\{\begin{array}{lcl} G(e_1) &=& \{(a,0.6,0),(b,0.1,0),(c,0,0)\} \\ G(e_2) &=& \{(a,0.7,0),(b,0.9,0),(c,0.5,0)\} \\ G(e_3) &=& \{(a,0,0),(b,0,0),(c,0,0)\} \\ G(e_4) &=& \{(a,0.5,0),(b,0.3,0),(c,0.9,0)\}\Big\} \end{array}$$

We consider the imprecise soft topology $\Im$ on (U, E) as

$$\Im=\Big\{(\widetilde{\varphi},E),(\widetilde{U},E),(F,E),(G,E)\Big\}.$$

Then imprecise soft closed sets are

$$(F,E)^c=\Big\{\begin{array}{lcl} F^c(e_1) &=& \{(a,1,0.4),(b,1,0.1),(c,1,0)\} \\ F^c(e_2) &=& \{(a,1,0.6),(b,1,0.5),(c,1,0.2)\} \\ F^c(e_3) &=& \{(a,1,0),(b,1,0),(c,1,0)\} \\ F^c(e_4) &=& \{(a,1,0),(b,1,0),(c,1,0)\}\Big\} \end{array}$$

$$(G,E)^c=\Big\{\begin{array}{lcl} G^c(e_1) &=& \{(a,1,0.6),(b,1,0.1),(c,1,0)\} \\ G^c(e_2) &=& \{(a,1,0.7),(b,1,0.9),(c,1,0.5)\} \\ G^c(e_3) &=& \{(a,1,0),(b,1,0),(c,1,0)\} \\ G^c(e_4) &=& \{(a,1,0.5),(b,1,0.3),(c,1,0.9)\}\Big\} \end{array}$$

We consider an imprecise soft set (H,E) over (U, E) as

$$(H,E)=\Big\{\begin{array}{lcl} H(e_1) &=& \{(a,1,0.5),(b,1,0.3),(c,1,0.2)\} \\ H(e_2) &=& \{(a,1,0.7),(b,1,0.5),(c,1,0.4)\} \\ H(e_3) &=& \{(a,0,0),(b,0,0),(c,0,0)\} \\ H(e_4) &=& \{(a,0,0),(b,0,0),(c,0,0)\}\Big\} \end{array}$$

Then

$$\begin{aligned}
\overline{(H,E)} &= \text{Intersection of all imprecise soft closed sets containing (H, E)}\\
&= (F,E)^c \widetilde{\cap} (\widetilde{U},E)\\
&= (F,E)^c\\
&= \Big\{ F^c(e_1) = \{(a,1,0.4),(b,1,0.1),(c,1,0)\},\\
&\quad F^c(e_2) = \{(a,1,0.6),(b,1,0.5),(c,1,0.2)\}\\
&\quad F^c(e_3) = \{(a,1,0),(b,1,0),(c,1,0)\},\\
&\quad F^c(e_4) = \{(a,1,0),(b,1,0),(c,1,0)\} \Big\}
\end{aligned}$$

Definition 3.24. Let $(U,E,\Im)$ be an imprecise soft topological space. Let (F,E) be an imprecise soft set over (U, E). The imprecise soft interior of (F,E) is defined as the union of all imprecise soft open sets contained in (F,E) and is denoted by $(F,E)^\circ$. We write

$$(F,E)^\circ = \widetilde{\cup} \Big\{ (G,E) : (G,E) \text{is imprecise soft open and} (G,E) \widetilde{\subseteq} (F,E) \Big\}.$$

It is obvious that

(1) $(F,E)^\circ$ is imprecise soft open.
(2) $(F,E)^\circ \widetilde{\subseteq} (F,E)$
(3) $(F,E)^\circ$ is the largest imprecise soft open set contained in (F, E).

Example 3.25. Let $U = \{a,b,c\}$ and $E = \{e_1,e_2,e_3,e_4\}$.

$$(F,E) = \Big\{ \begin{aligned}
F(e_1) &= \{(a,0.4,0),(b,0.1,0),(c,0,0)\}\\
F(e_2) &= \{(a,0.6,0),(b,0.5,0),(c,0.2,0)\}\\
F(e_3) &= \{(a,0,0),(b,0,0),(c,0,0)\}\\
F(e_4) &= \{(a,0,0),(b,0,0),(c,0,0)\} \Big\}
\end{aligned}$$

$$(G,E) = \Big\{ \begin{aligned}
G(e_1) &= \{(a,0.6,0),(b,0.1,0),(c,0,0)\}\\
G(e_2) &= \{(a,0.7,0),(b,0.9,0),(c,0.5,0)\}\\
G(e_3) &= \{(a,0,0),(b,0,0),(c,0,0)\}\\
G(e_4) &= \{(a,0.5,0),(b,0.3,0),(c,0.9,0)\} \Big\}
\end{aligned}$$

We consider the imprecise soft topology $\Im$ on (U, E) as

$$\Im = \Big\{ (\widetilde{\varphi},E), (\widetilde{U},E), (F,E), (G,E) \Big\}.$$

We consider an imprecise soft set (H,E) over (U, E) as

$$(H,E) = \Big\{ \begin{aligned}
H(e_1) &= \{(a,0.6,0),(b,0.3,0),(c,0.2,0)\}\\
H(e_2) &= \{(a,0.7,0),(b,0.5,0),(c,0.4,0)\}\\
H(e_3) &= \{(a,0.2,0),(b,0.8,0),(c,0.6,0)\}\\
H(e_4) &= \{(a,0,0),(b,0,0),(c,0,0)\} \Big\}
\end{aligned}$$

Then

$$\begin{aligned}(H,E)^\circ &= \text{Union of all imprecise soft open sets contained in (H, E)}\\ &= (F,E)\widetilde{\cup}(\widetilde{\varphi},E)\\ &= (F,E)\\ &= \Big\{F(e_1)=\{(a,0.4,0),(b,0.1,0),(c,0,0)\},\\ &\quad F(e_2)=\{(a,0.6,0),(b,0.5,0),(c,0.2,0)\}\\ &\quad F(e_3)=\{(a,0,0),(b,0,0),(c,0,0)\},\\ &\quad F(e_4)=\{(a,0,0),(b,0,0),(c,0,0)\}\Big\}\end{aligned}$$

Proposition 3.26. *Let $(U,E,\Im)$ be an imprecise soft topological space. Let (F,E) be an imprecise soft set over (U, E). Then (F,E) is an imprecise soft closed set if* $\overline{(F,E)}=(F,E)$.

Proof. Let $(U,E,\Im)$ be an imprecise soft topological space. Let (F,E) be an imprecise soft set over (U, E) such that $\overline{(F,E)}=(F,E)$. To prove that (F,E) is imprecise soft closed. We have

$$\overline{(F,E)}=\widetilde{\cap}\Big\{(G,E):(G,E)\text{is imprecise soft closed and}(F,E)\widetilde{\subseteq}(G,E)\Big\}.$$

$\overline{(F,E)}$ is imprecise soft closed, being an arbitrary intersection of imprecise soft closed sets. Also $\overline{(F,E)}$ is imprecise soft closed and $\overline{(F,E)}=(F,E)\Rightarrow(F,E)$ is imprecise soft closed. Conversely, suppose that (F,E) is imprecise soft closed in $(U,E,\Im)$. To prove that $\overline{(F,E)}=(F,E)$.It is clear from definition that any imprecise soft closed set (G,E), $(F,E)\widetilde{\subseteq}(G,E)\Rightarrow\overline{(F,E)}\widetilde{\subseteq}\overline{(G,E)}$. Since $(F,E)\widetilde{\subseteq}(F,E)\Rightarrow\overline{(F,E)}\widetilde{\subseteq}(F,E)$ and $(F,E)\widetilde{\subseteq}\overline{(F,E)}$ (from definition), it follows that $\overline{(F,E)}=(F,E)$. □

Proposition 3.27. *Let $(U,E,\Im)$ be an imprecise soft topological space. Let (F,E) be an imprecise soft set over (U, E). Then is an imprecise soft open set if* $(F,E)^\circ=(F,E)$.

Proof. Let $(U,E,\Im)$ be an imprecise soft topological space. Let (F,E) be an imprecise soft set over (U, E) such that $(F,E)^\circ=(F,E)$. To prove that (F,E) is imprecise soft open. We have

$$(F,E)^\circ=\widetilde{\cup}\Big\{(G,E):(G,E)\text{is imprecise soft open and}(G,E)\widetilde{\subseteq}(F,E)\Big\}.$$

$(F,E)^\circ$ is imprecise soft open, being an arbitrary union of imprecise soft open sets. Also $(F,E)^\circ$ is imprecise soft open and $(F,E)^\circ=(F,E)\Rightarrow(F,E)$ is imprecise soft open.

Conversely, suppose that (F, E) is imprecise soft open in $(U,E,\Im)$. To prove that $(F,E)^\circ=(F,E)$. It is clear from definition that any imprecise soft open set $(G,E)\widetilde{\subseteq}(F,E)\Rightarrow(G,E)\widetilde{\subseteq}(F,E)^\circ$. Since $(F,E)\widetilde{\subseteq}(F,E)\Rightarrow(F,E)\widetilde{\subseteq}(F,E)^\circ$ and $(F,E)^\circ\widetilde{\subseteq}(F,E)$(from definition), it follows that $(F,E)^\circ=(F,E)$. □

Proposition 3.28. *Let $(U,E,\Im)$ be an imprecise soft topological space. Let (F,E) and (G,E) be two imprecise soft sets over (U, E). Then*

(1) $\overline{(\widetilde{\varphi},E)}=(\widetilde{\varphi},E)$

(2) $(F,E)\widetilde{\subseteq}\overline{(F,E)}$
(3) $(F,E)\widetilde{\subseteq}(G,E) \Rightarrow \overline{(F,E)}\widetilde{\subseteq}\overline{(G,E)}$
(4) $\overline{(F,E)\widetilde{\cup}(G,E)} = \overline{(F,E)}\widetilde{\cup}\overline{(G,E)}$
(5) $\overline{(F,E)\widetilde{\cap}(G,E)} = \overline{(F,E)}\widetilde{\cap}\overline{(G,E)}$
(6) $\overline{\overline{(F,E)}} = \overline{(F,E)}$

Proof. (1) $(\widetilde{\varphi},E)$ is an imprecise soft closed set $\Rightarrow \overline{(\widetilde{\varphi},E)} = (\widetilde{\varphi},E)$

(2) By definition, $\overline{(F,E)}$ is the smallest imprecise soft closed set containing (F,E). Thus $(F,E)\widetilde{\subseteq}\overline{(F,E)}$.

(3) Let $(F,E)\widetilde{\subseteq}(G,E)$. Then $(F,E)\widetilde{\subseteq}(G,E)\widetilde{\subseteq}\overline{(G,E)}$
$(F,E)\widetilde{\subseteq}\overline{(G,E)}$
Since closure of an imprecise soft set is closed and so $\overline{(G,E)}$ is an imprecise soft closed set containing (F,E). Also $\overline{(F,E)}$ is the smallest imprecise soft closed set containing (F,E). In what follows $\overline{(F,E)}\widetilde{\subseteq}\overline{(G,E)}$.

(4) $(F,E)\widetilde{\subseteq}(F,E)\widetilde{\cup}(G,E), (G,E)\widetilde{\subseteq}(F,E)\widetilde{\cup}(G,E)$
$\Rightarrow \overline{(F,E)}\widetilde{\subseteq}\overline{(F,E)\widetilde{\cup}(G,E)}, \overline{(G,E)}\widetilde{\subseteq}\overline{(F,E)\widetilde{\cup}(G,E)}$
$\Rightarrow \overline{(F,E)}\widetilde{\cup}\overline{(G,E)}\widetilde{\subseteq}\overline{(F,E)\widetilde{\cup}(G,E)}$
Also $(F,E)\widetilde{\subseteq}\overline{(F,E)}, (G,E)\widetilde{\subseteq}\overline{(G,E)}$. This $\Rightarrow (F,E)\widetilde{\cup}(G,E)\widetilde{\subseteq}\overline{(F,E)}\widetilde{\cup}\overline{(G,E)}$
Now $\overline{(F,E)}, \overline{(G,E)}$ are closed and hence $\overline{(F,E)}\widetilde{\cup}\overline{(G,E)}$ is an imprecise soft closed set containing $(F,E)\widetilde{\cup}(G,E)$. Also $\overline{(F,E)\widetilde{\cup}(G,E)}$ is the smallest imprecise soft closed set containing $(F,E)\widetilde{\cup}(G,E)$.
In what follows

$$\overline{(F,E)\widetilde{\cup}(G,E)}\widetilde{\subseteq}\overline{(F,E)}\widetilde{\cup}\overline{(G,E)} \text{ i.e. } \overline{(F,E)\widetilde{\cup}(G,E)} = \overline{(F,E)}\widetilde{\cup}\overline{(G,E)}$$

(5) $(F,E)\widetilde{\cap}(G,E)\widetilde{\subseteq}(F,E), (F,E)\widetilde{\cap}(G,E)\widetilde{\subseteq}(G,E)$
$\Rightarrow \overline{(F,E)\widetilde{\cap}(G,E)}\widetilde{\subseteq}\overline{(F,E)}, \overline{(F,E)\widetilde{\cap}(G,E)}\widetilde{\subseteq}\overline{(G,E)}$
$\Rightarrow \overline{(F,E)\widetilde{\cap}(G,E)}\widetilde{\subseteq}\overline{(F,E)}\widetilde{\cap}\overline{(G,E)}$

(6) We have, $\overline{(F,E)}$ is closed. Also (G,E) is closed iff $\overline{(G,E)} = (G,E)$. Putting $(G,E) = \overline{(F,E)}$ we have, $\overline{\overline{(F,E)}} = \overline{(F,E)}$.

□

Proposition 3.29. *Let $(U,E,\Im)$ be an imprecise soft topological space. Let (F,E), (G,E) be two imprecise soft sets over (U, E). Then*

(1) $(\widetilde{\varphi},E)^\circ = (\widetilde{\varphi},E)$
(2) $(\widetilde{U},E)^\circ = (\widetilde{U},E)$
(3) $(F,E)\widetilde{\subseteq}((G,E) \Rightarrow (F,E)^\circ\widetilde{\subseteq}((G,E)^\circ$
(4) $((F,E)^\circ)^\circ = (F,E)$

Proof. (1) $(\widetilde{\varphi},E)$ is an imprecise soft open set $\Rightarrow (\widetilde{\varphi},E)^\circ = (\widetilde{\varphi},E)$

(2) By definition, $(\widetilde{U},E)^\circ$ is an imprecise soft open set $\Rightarrow (\widetilde{U},E)^\circ = (\widetilde{U},E)$

(3) Let $(F,E)\widetilde{\subseteq}(G,E)$ Any imprecise soft point $e(H,E) \in (F,E)^\circ \Rightarrow e(H,E)$ is an interior point of (F,E)
$\Rightarrow$ There is an imprecise soft open set (I,E) such that$e(H,E)\widetilde{\in}(I,E)\widetilde{\subseteq}(F,E)$
$\Rightarrow e(H,E)\widetilde{\in}(I,E)\widetilde{\subseteq}(F,E)\widetilde{\subseteq}(G,E)$

$\Rightarrow e(H,E)\widetilde{\in}(I,E)\widetilde{\subseteq}(G,E)$ and (G,E) is an imprecise soft open.
$\Rightarrow e(H,E)\widetilde{\in}(G,E)^{\circ}$
Thus $(F,E)\widetilde{\subseteq}(G,E) \Rightarrow (F,E)^{\circ}\widetilde{\subseteq}(G,E)^{\circ}$

(4) $(F,E)^{\circ}$ an imprecise soft open set. Also (G,E) is an imprecise soft open set iff $(G,E)^{\circ} = (G,E)$
Putting, $(F,E)^{\circ} = (G,E)$, it follows that $\left((F,E)^{\circ}\right)^{\circ} = (F,E)$

□

Definition 3.30. Let $(U, E, \Im)$ be an imprecise soft topological space. Let Y be an ordinary subset of U and (H,E) be an imprecise soft set over (Y, E) such that $\forall \epsilon E$,

$$H(\epsilon) = \begin{cases} (x,1,0), & if x \in Y \\ (x,0,0), & if x \notin Y \end{cases}$$

Let $T_Y = \big\{(H,E)\widetilde{\cap}(G,E) : (G,E) \in \Im\big\}$.
It can be verified that T_Y is an imprecise soft topology on (Y, E). We would call T_Y the imprecise soft subspace topology for (Y, E).

Example 3.31. Let $U = \{a,b,c\}$ and $E = \{e_1, e_2, e_3, e_4\}$.

$$(F,E) = \Big\{ \begin{array}{lcl} F(e_1) & = & \{(a,0.4,0),(b,0.1,0),(c,0,0)\} \\ F(e_2) & = & \{(a,0.6,0),(b,0.5,0),(c,0.2,0)\} \\ F(e_3) & = & \{(a,0,0),(b,0,0),(c,0,0)\} \\ F(e_4) & = & \{(a,0,0),(b,0,0),(c,0,0)\} \end{array} \Big\}$$

$$(G,E) = \Big\{ \begin{array}{lcl} G(e_1) & = & \{(a,0.6,0),(b,0.1,0),(c,0,0)\} \\ G(e_2) & = & \{(a,0.7,0),(b,0.9,0),(c,0.5,0)\} \\ G(e_3) & = & \{(a,0,0),(b,0,0),(c,0,0)\} \\ G(e_4) & = & \{(a,0.5,0),(b,0.3,0),(c,0.9,0)\} \end{array} \Big\}$$

We consider the fuzzy soft topology $\Im$ on (U, E) as

$$\Im = \Big\{(\widetilde{\varphi},E),(\widetilde{U},E),(F,E),(G,E)\Big\}.$$

Let $Y = a,b \subseteq U$ We consider the fuzzy soft set (H,E) over (Y, E) as

$$(H,E) = \Big\{ \begin{array}{lcl} H(e_1) & = & \{(a,1,0),(b,1,0),(c,0,0)\} \\ H(e_2) & = & \{(a,1,0),(b,1,0),(c,0,0)\} \\ H(e_3) & = & \{(a,1,0),(b,1,0),(c,0,0)\} \\ H(e_4) & = & \{(a,1,0),(b,1,0),(c,0,0)\} \end{array} \Big\}$$

Then

(1) $(\widetilde{\varphi},E)\widetilde{\cap}(H,E) = (\widetilde{\varphi},E)$
(2) $(\widetilde{U},E)\widetilde{\cap}(H,E) = (\widetilde{Y},E)$

(3)

$$\begin{aligned}(F,E)\widetilde{\cap}(H,E) &= (J,E)\\ &= \Big\{J(e_1)=\{(a,0.4,0),(b,0.1,0),(c,0,0)\},\\ &\quad J(e_2)=\{(a,0.6,0),(b,0.5,0),(c,0,0)\}\\ &\quad J(e_3)=\{(a,0,0),(b,0,0),(c,0,0)\},\\ &\quad J(e_4)=\{(a,0,0),(b,0,0),(c,0,0)\}\Big\}\end{aligned}$$

(4)

$$\begin{aligned}(G,E)\widetilde{\cap}(H,E) &= (I,E)\\ &= \Big\{I(e_1)=\{(a,0.6,0),(b,0.1,0),(c,0,0)\},\\ &\quad I(e_2)=\{(a,0.7,0),(b,0.9,0),(c,0,0)\}\\ &\quad I(e_3)=\{(a,0,0),(b,0,0),(c,0,0)\},\\ &\quad I(e_4)=\{(a,0.5,0),(b,0.3,0),(c,0,0)\}\Big\}\end{aligned}$$

It is clear that the collection $T_Y = \big\{(H,E)\widetilde{\cap}(G,E) : (G,E) \in \Im\big\}$ is an imprecise soft topology on (Y, E).

Definition 3.32. Let $(U, E, \Im)$ be an imprecise soft topological space and $\beta \subseteq \Im$. Then β is a basis for $\Im$ if for each $(G,E) \in \Im$ there exists $\acute{\beta} \subseteq \beta$ such that $(G,E) = \widetilde{\cup}\acute{\beta}$.

Example 3.33. Let $U = \{a, b\}$ and $E = \{e_1, e_2\}$.
$(F,E) = \Big\{F(e_1)=\{(a,0.2,0),(b,0.5,0)\}, F(e_2)=\{(a,0.3,0),(b,0.9,0)\}\Big\}$
$(G,E) = \Big\{G(e_1)=\{(a,0.1,0),(b,0.5,0)\}, G(e_2)=\{(a,0.3,0),(b,0.8,0)\}\Big\}$
$(H,E) = \Big\{H(e_1)=\{(a,0.2,0),(b,0.5,0)\}, H(e_2)=\{(a,0.3,0),(b,0.9,0)\}\Big\}$
$(I,E) = \Big\{I(e_1)=\{(a,0.1,0),(b,0.5,0)\}, I(e_2)=\{(a,0.3,0),(b,0.8,0)\}\Big\}$

It can be verified that $\Im = \Big\{(\widetilde{\varphi},E),(\widetilde{U},E),(F,E),(G,E),(H,E),(I,E)\Big\}$ is an imprecise soft topology on (U, E). Here $\beta = \Big\{(\widetilde{\varphi},E),(\widetilde{U},E),(F,E),(G,E),(I,E)\Big\}$ is a basis for $\Im$.

4. Conclusions

In our work, we have studied some properties related to imprecise soft topological spaces. The notions of imprecise soft point, imprecise soft neighborhood, imprecise soft closure, imprecise soft interior, imprecise soft subspace topology and basis for an imprecise soft topological space have been introduced and some related properties along with examples have been established.

References

[1] H. K. Baruah, The theory of fuzzy sets: Beliefs and realities, International Journal of Energy, Information and Communications 2 (2011) 1–22.

[2] H. K. Baruah, An introduction to the theory of imprecise sets: the mathematics of partial presence, J. Math. Comput. Sci. 2(2) (2012) 110–124.
[3] P. K. Maji and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[4] T. J. Neog and D. K. Sut, A new approach to the theory of soft sets, International Journal of Computer Applications 32 (2011) 1–6.
[5] T. J. Neog and D. K. Sut, An introduction to the theory of imprecise soft sets, International Journal of Intelligent Systems and Applications 4 (2012) 75–83.
[6] T. J. Neog, D. K. Sut and G. C. Hazarika, Fuzzy soft topological spaces, International Journal of Latest Trends in Mathematics 2 (2012) 54–67.
[7] T. J. Neog, An application of imprecise soft sets in teaching evaluation, International Journal for Basic Sciences & Social Sciences 2 (2012) 54–67.
[8] S. Roy and T. K. Samanta, A note on fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 3(2) (2012) 305–311.
[9] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. App. 61 (2011) 1786–1799.
[10] B. Tanay and M. Burc Kandemir, Topological structure of fuzzy soft sets, Comput. Math. App. 61 (2011) 2952–2957.
[11] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3(2) (2012) 171–185.

Tridiv Jyoti Neog (tridivjyoti@gmail.com)
Department of Mathematics, D. K. High School, Nakachari, Jorhat-785635, India

Manoj Bora (manojbora08@gmail.com)
Department of Mathematics, Jorhat Institute of Science and Technology, Jorhat-785010, India

Dusmanta Kumar Sut (sutdk001@yahoo.com)
Department of Mathematics, N. N. Saikia College, Titabar, Jorhat-785630, India