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Properties of soft sets associated with new operations

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Abstract. In this paper, we discuss the properties of soft sets. We introduce the concept of soft point and its existence is illustrated by suitable examples. Also, we characterize some of their properties. We prove that the intersection of two soft topologies is a soft topology and justify that union of two soft topologies need not be a soft topology. Further, we characterize soft basis in terms of soft topology.

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1. Introduction

Molodtsov [14] introduced the concept of soft sets in 1999 as a mathematical tool for dealing with uncertain objects. Then the properties and applications of soft set theory have been studied increasingly in [3, 4, 7, 12, 23]. There are many approaches to handle the manipulation of imperfect knowledge. The most successful one is fuzzy set theory by Zadeh [22]. The aim of soft set theory is to provide a tool with enough parameters to deal with uncertainty associated with the given data, which is free of the difficulty, mainly inadequacy of parametrization. Soft set theory has a large scope for application in many directions, some of which are decision making, attribute reduction, smoothness of functions, game theory, operation research, Riemann integration and so on [10, 11, 15, 21].

Recently, in [2], Aktas and Cagman introduced the notion of soft groups and obtained some fundamental properties. Shabir and Ali [18], studied soft semigroups and soft ideals which characterize (generalized) fuzzy ideals with entrance of a semigroup. Further, in [1], Acar, et al. introduced the concept of soft ring over a ring. In [5], soft subrings, soft ideals over a ring and soft subfield over a field has been introduced. Celik, et. al [8] introduced a new binary relation and some new operations on soft sets, also they introduced the notion of a subrings (ideals) of a given ring. However, in [16], Nazmul and Samanta introduced the basic idea of a soft topological group, its subsystem and morphism over a topological group.

In [6], Ayg*ü*no*ğ*lu and Ayg*ü*n studied soft continuity, soft product topology, soft compactness and generalized Tychonoff theorem in soft topological spaces. Further, Min [13] gave some properties. Also, in [9], Hussain and Ahmad discussed the properties of soft interior, closure and boundary on a soft topological spaces. The concept of fuzzy topological spaces was introduced and studied by Tanay and Kandemir in [19]. Varol and Ayg*ü*n [20] introduced soft Hausdorff spaces. They proved that in a soft Hausdorff space compact soft set is closed. In [23], Zorlutuna et.al introduced new concepts in soft topological spaces such as interior point, interior, neighborhood. In 2013, Nazmul and Samanta [17] discussed the neighborhood properties of soft topological spaces.

In this paper, we study in detail about the theory and properties of soft sets and soft topological spaces. The concept of soft point is introduced. Its existence is verified with suitable examples and its properties are studied. Now we present the basic definitions and results of soft set theory which are studied earlier in [3, 4, 7, 12, 23]. Throughout this work, U refers to an initial universe, E is a set of parameters, $\wp(U)$ is the power set of U and $A \subseteq E$.

2. Preliminaries

A soft set F_A [7] on the universe U is defined by the set $F_A = \{(x, f_A(x)) : x \in E,\ f_A(x) \in \wp(U)\}$ where $A \subseteq E$ and $f_A : E \to \wp(U)$ such that $f_A(x) = \varnothing$ if $x \notin A$. Here f_A is called an *approximate function* of the set F_A. The set of all soft sets over U will be denoted by $S(U)$. $f_A(x) = \varnothing$ means that there are no elements in U related to the parameter $x \in E$. Therefore, we do not display such elements in the soft sets, as it is meaningless to consider such parameters. Let $F_A \in S(U)$. If $f_A(x) = \varnothing$ for all $x \in E$, then F_A is called an *empty soft set* [12], denoted by $F_\varnothing$. If $f_A(x) = U$ for all $x \in A$, then F_A is called an A*-universal soft set* [7], denoted by $F_{\widetilde{A}}$. If $A = E$, then the A-universal soft set is called a universal soft set [7], denoted by $F_{\widetilde{E}}$. Let F_A, $F_B \in S(U)$. Then F_A is a *soft subset* of F_B, denoted by $F_A \widetilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$. Also, F_B is called the *soft superset* of F_A [7]. Let F_A, $F_B \in S(U)$. Then F_A is a soft equal of F_B [7], denoted by $F_A = F_B$, if $f_A(x) = f_B(x)$ for all $x \in E$. The *soft union* [7], denoted by $F_A \widetilde{\cup} F_B$, the *soft intersection* [7], denoted by $F_A \widetilde{\cap} F_B$ and the *soft difference* [7], denoted by $F_A \widetilde{\setminus} F_B$ of F_A and F_B are defined by the approximate functions,

$f_{A\widetilde{\cup}B}(x) = f_A(x) \cup f_B(x)$,

$f_{A\widetilde{\cap}B}(x) = f_A(x) \cap f_B(x)$ and

$f_{A\widetilde{\setminus}B}(x) = f_A(x) \setminus f_B(x)$, respectively. The *soft complement* $F_A^{\tilde{c}}$ [3] of F_A is defined by the approximate function, $f_{A^{\tilde{c}}}(x) = f_A^c(x)$ where $f_A^c(x)$ is the complement of the set $f_A(x)$, that is, $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

Let I be an arbitrary index set and $F_A \in S(U)$. The soft power set [7] of F_A is defined by, $\widetilde{P}(F_A) = \{F_{A_i} : F_{A_i} \widetilde{\subseteq} F_A : i \in I \subseteq N\}$. Its cardinality is defined by, $\left|\widetilde{P}(F_A)\right| = 2^{\sum_{x \in E} |f_A(x)|}$, where $|f_A(x)|$ is the cardinality of $f_A(x)$. Let $\{F_{A_i}\}_{i \in I}$ be a subfamily of $S(U)$. Then the soft union [23] of these soft sets is the soft set

$F_A = \widetilde{\cup}_{i\in I} F_{A_i}$ where $f_A = \cup_{i\in I} f_{A_i}$ and the soft intersection [23] of these soft sets is the soft set $F_A = \widetilde{\cap}_{i\in I} F_{A_i}$ where $f_A = \cap_{i\in I} f_{A_i}$.

Let $F_A \in S(U)$. Then the *soft topology* [7] on F_A, denoted by $\widetilde{\tau}$, is the collection of soft subsets of F_A having the following properties:

(a) $F_\varnothing$, $F_A \in \widetilde{\tau}$.

(b) If $\{F_{A_i} \widetilde{\subseteq} F_A \mid i \in I \subseteq \widetilde{\tau}$, then $\widetilde{\cup}_{i\in I} F_{A_i} \in \widetilde{\tau}$.

(c) If $\{F_{A_i} \widetilde{\subseteq} F_A \mid 1 \leq i \leq n,\ n \in \mathbb{N}\} \subseteq \widetilde{\tau}$, then $\widetilde{\cap}_{i=1}^{n} F_{A_i} \in \widetilde{\tau}$.

The pair $(F_A, \widetilde{\tau})$ is called a soft topological space [7]. Every element of $\widetilde{\tau}$ is called a soft open set [7]. From the definition of soft topological space, $F_\varnothing$ and F_A are soft open sets. Let $(F_A, \widetilde{\tau})$ be a soft topological space and $\widetilde{B} \subseteq \widetilde{\tau}$. If every element of $\widetilde{\tau}$ can be written as the soft union of elements of $\widetilde{B}$, then $\widetilde{B}$ is called a *soft basis* [7] for the soft topology $\widetilde{\tau}$. The following lemmas will be useful in the sequel. We use some of the results in [7] and [4] without mentioning it, when the context is clear.

Lemma 2.1 ([3])**.** *Let $F_A \in S(U)$. Then $(F_A^{\widetilde{c}})^{\widetilde{c}} = F_A$.*

Lemma 2.2 ([7])**.** *Let $F_A \in S(U)$. Then the following hold.*

(a) $F_\varnothing^{\widetilde{c}} = F_{\widetilde{E}}$.

(b) $F_{\widetilde{E}}^{\widetilde{c}} = F_\varnothing$.

Lemma 2.3 ([23], Proposition 3.3)**.** *Let I be an arbitrary index set and $\{F_{A_i}\}_{i\in I}$ be a subfamily of $S(U)$. Then the following hold.*

(a) $(\widetilde{\cup}_{i\in I} F_{A_i})^{\widetilde{c}} = \widetilde{\cap}_{i\in I} F_{A_i}^{\widetilde{c}}$.

(b) $(\widetilde{\cap}_{i\in I} F_{A_i})^{\widetilde{c}} = \widetilde{\cup}_{i\in I} F_{A_i}^{\widetilde{c}}$.

Lemma 2.4 ([23], Proposition 3.5)**.** *Let F_A, $F_B \in S(U)$. Then the following hold.*

(a) $F_A \widetilde{\subseteq} F_B$ *if and only if* $F_A \widetilde{\cap} F_B = F_A$.

(b) $F_A \widetilde{\subseteq} F_B$ *if and only if* $F_A \widetilde{\cup} F_B = F_B$.

3. Properties on soft sets

The following Example 3.1 shows that for any soft sets F_A and F_B of U with $F_A \widetilde{\subseteq} F_B$, A need not be a subset of B. Example 3.2 below shows that for any soft sets F_A and F_B of U with $F_A = F_B$, A need not be equal to B. Let $F_A \in S(U)$. Then the *approximate set*, K_A of the soft set F_A is defined by $K_A = \{x \in E \mid f_A(x) \neq \varnothing\}$.

Example 3.1. Consider the sets $U = \{h_1, h_2, h_3\}$, $E = \{x_1, x_2, x_3, x_4, x_5\}$, $A = \{x_1, x_2, x_3, x_5\}$ and $B = \{x_3, x_4, x_5\}$. Clearly, $A \nsubseteq B$. Suppose $f_A(x_1) = f_A(x_2) = \varnothing$, $f_A(x_3) = \{h_1\}$, $f_A(x_5) = \{h_2\}$, $f_B(x_3) = \{h_1, h_3\}$, $f_B(x_4) = \{h_1\}$ and $f_B(x_5) = \{h_1, h_2\}$. Then $f_A(x) \subseteq f_B(x)$ for all $x \in E$ and so $F_A \widetilde{\subseteq} F_B$.

Example 3.2. Consider the sets $U = \{h_1, h_2, h_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\}$ and $B = \{x_2, x_3\}$. Suppose $f_A(x_1) = \varnothing$, $f_A(x_2) = \{h_1, h_3\}$ and $f_B(x_2) = \{h_1, h_3\}$, $f_B(x_3) = \varnothing$. Then $F_A = \{(x_2, \{h_1, h_3\}\}$ and $F_B = \{(x_2, \{h_1, h_3\})\}$. Therefore, $F_A = F_B$. But $A \neq B$.

Example 3.3. Let $U = \{h_1, h_2, h_3, h_4\}$, $E = \{x_1, x_2, x_3, x_4\}$ and $B = \{x_1, x_2, x_3\} \subseteq E$. Suppose that $f_B(x_1) = \{h_1\}$, $f_B(x_2) = \{h_2\}$ and $f_B(x_3) = \varnothing$. Then $K_B = \{x_1, x_2\}$.

Theorem 3.4. *Let $F_A \in S(U)$. Then $K_A \subseteq A$ and $K_A \subseteq E$.*

Proof. By the definition of K_A, $K_A \subseteq E$. If $x \notin A$, then $f_A(x) = \varnothing$. Therefore, $x \notin K_A$. Hence $K_A \subseteq A$. $\square$

Theorem 3.5. *Let F_A, $F_B \in S(U)$. Then the following hold.*
(a) *If $F_A \widetilde{\subseteq} F_B$, then $K_A \subseteq K_B$.*
(b) *If $F_A = F_B$, then $K_A = K_B$.*

Proof. (a) Assume that $F_A \widetilde{\subseteq} F_B$. Let $x \in K_A$. Then $f_A(x) \neq \varnothing$. Since $F_A \widetilde{\subseteq} F_B$, $f_A(y) \subseteq f_B(y)$ for all $y \in E$. Since $f_A(x) \neq \varnothing$, $f_B(x) \neq \varnothing$. Thus, $x \in K_B$. Therefore, $K_A \subseteq K_B$.

(b) Assume that $F_A = F_B$. Then $f_A(x) = f_B(x)$ for all $x \in E$. That is, $f_A(x) \subseteq f_B(x)$ and $f_B(x) \subseteq f_A(x)$ for all $x \in E$. Therefore, $F_A \widetilde{\subseteq} F_B$ and $F_B \widetilde{\subseteq} F_A$. By (a), $K_A \subseteq K_B$ and $K_B \subseteq K_A$. Hence $K_A = K_B$. $\square$

The following Example 3.6 shows that the converse of Theorem 3.5 need not be true in general.

Example 3.6. (a) Consider $U = \{u_1, u_2, u_3, u_4\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_3\}$ and $B = \{x_1, x_2, x_3\}$. Suppose $F_A = \{(x_3, \{u_2, u_3\})\}$ and $F_B = \{(x_2, \{u_4\}), (x_3, \{u_1\})\}$. Then $K_A = \{x_3\}$ and $K_B = \{x_2, x_3\}$. Therefore, $K_A \subseteq K_B$. But $F_A \widetilde{\nsubseteq} F_B$.

(b) Consider $U = \{u_1, u_2, u_3, u_4\}$, $E = \{x_1, x_2, x_3, x_4\}$, $A = \{x_1, x_2, x_3\}$ and $B = \{x_2, x_3, x_4\}$. Suppose that $F_A = \{(x_2, \{u_2\}), (x_3, \{u_3\})\}$ and $F_B = \{(x_2, \{u_4\}), (x_3, \{u_1\})\}$. Then $K_A = \{x_2, x_3\} = K_B$. Therefore, $K_A = K_B$. Clearly, $F_A \neq F_B$.

Theorem 3.7. *Let F_A, $F_B \in S(U)$. Then the following hold.*
(a) *If $F_A \widetilde{\subseteq} F_B$ and $K_A = A$, then $A \subseteq B$.*
(b) *If $F_A = F_B$, $K_A = A$ and $K_B = B$, then $A = B$.*

Proof. (a) Assume that $F_A \widetilde{\subseteq} F_B$ and $K_A = A$. Since $F_A \widetilde{\subseteq} F_B$, by Theorem 3.5, $K_A \subseteq K_B$ and so $A \subseteq K_B$. By Theorem 3.4, $K_B \subseteq B$. Hence $A \subseteq B$.

(b) follows from Theorem 3.5(b). $\square$

The following Example 3.8 shows that the converse of Theorem 3.7 need not be true in general.

Example 3.8. (a) Consider $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3, x_4\}$, $A = \{x_1, x_2, x_3\}$ and $B = E$. Clearly, $A \subseteq B$. Suppose that $F_A = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$ and $F_B = F_{\widetilde{E}}$. Then $K_A = \{x_1, x_2\}$. Here $F_A \widetilde{\subseteq} F_B$. But $K_A \neq A$.

(b) Consider $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3, x_4\}$ and $A = B = \{x_1, x_2, x_3\}$. Suppose that $F_A = F_B = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$. Then $K_A = \{x_1, x_2\} = K_B$. Here $F_A = F_B$ and $A = B$. But $K_A \neq A$ and $K_B \neq B$.

Theorem 3.9. *Let F_A, $F_B \in S(U)$ with $F_A \widetilde{\subseteq} F_B$. Then the following hold.*
(a) *If $A \cap B = \varnothing$, then $f_A(x) = \varnothing$ for all $x \in A$.*
(b) *If $A \cap B \neq \varnothing$, then $f_A(x) = \varnothing$ for all $x \notin A \cap B$.*

Proof. If $F_A \widetilde{\subseteq} F_B$, then $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

(a) Assume that $A \cap B = \varnothing$. If $x \in A$, then $x \notin B$ which implies $f_B(x) = \varnothing$ and so $f_A(x) = \varnothing$, since $f_A(x) \subseteq f_B(x)$ for all $x \in E$. Since $x \in A$ is arbitrary, $f_A(x) = \varnothing$ for all $x \in A$.

(b) Suppose that $A \cap B \neq \varnothing$. Let $x \notin A \cap B$. Then $x \notin A$ or $x \notin B$. If $x \notin A$, then $f_A(x) = \varnothing$. If $x \notin B$, then $f_B(x) = \varnothing$. Since $f_A(x) \subseteq f_B(x)$ for all $x \in E$, $f_A(x) = \varnothing$ for all $x \in E$. Therefore, $f_A(x) = \varnothing$ for all $x \notin A \cap B$. □

Theorem 3.10. *Let $F_A \in S(U)$. If $A = \varnothing$, then $F_A = F_\varnothing$.*

Proof. Let $F_A \in S(U)$. Since $A = \varnothing$, $f_A(x) = \varnothing$ for all $x \notin A$. That is, $f_A(x) = \varnothing$ for all $x \in E = A^c$, since $A = \varnothing$ and $A \subseteq E$. Therefore, by definition, $F_A = F_\varnothing$. □

The converse of Theorem 3.10 need not be true as shown by the following Example 3.11.

Example 3.11. Consider $U = \{h_1, h_2, h_3, h4\}$, $E = \{x_1, x_2, x_3\}$ and $A = \{x_1, x_2\} \subseteq E$. Suppose that $f_A(x_1) = f_A(x_2) = \varnothing$. Then $f_A(x) = \varnothing$ for all $x \in E$. By definition, $F_A = F_\varnothing$. But here $A \neq \varnothing$.

Theorem 3.12. *Let $S(U)$ be the collection of all soft sets over U. Then the following hold.*

(a) *$F_\varnothing$ is the soft subset of every soft set in $S(U)$.*

(b) *$F_{\widetilde{E}}$ is the soft superset of every soft set in $S(U)$.*

Proof. (a) Let F_A be any soft set in $S(U)$. Let f_A and f_B be the approximate functions of F_A and $F_\varnothing$, respectively. Then $f_B(x) = \varnothing$ for all $x \in E$. Since empty set is a subset of every set, $f_B(x) \subseteq f_A(x)$ for all $x \in E$. Therefore, $F_\varnothing \widetilde{\subseteq} F_A$. Since $F_A \in S(U)$ is arbitrary, $F_\varnothing$ is the soft subset of every soft set in $S(U)$.

(b) Let F_A be any soft set in $S(U)$ and f_A be its approximate function. Since $f_A(x) \in \wp(U)$, $f_A(x) \subseteq U$ for all $x \in E$. Let f_E be the approximate function of $F_{\widetilde{E}}$. Then $f_E(x) = U$ for all $x \in E$. This implies that $f_A(x) \subseteq f_E(x)$ for all $x \in E$. Hence $F_A \widetilde{\subseteq} F_{\widetilde{E}}$. Since $F_A \in S(U)$ is arbitrary, $F_{\widetilde{E}}$ is the soft superset of every soft set in $S(U)$. □

Theorem 3.13. *Let $F_{\widetilde{A}}, F_B \in S(U)$ where $B \subset A$. Then $F_B \widetilde{\subseteq} F_{\widetilde{A}}$.*

Proof. Suppose that f_A and f_B are the approximate functions for the soft sets $F_{\widetilde{A}}$ and F_B, respectively. Then $f_A(x) = U$ for all $x \in A$. Let $(x, f_B(x)) \in F_B$. Then $x \in E$ and $f_B(x) \in \wp(U)$. If $x \in B$, then $x \in A$ and hence $f_A(x) = U \supseteq f_B(x)$. Therefore, $f_B(x) \subseteq f_A(x)$ for all $x \in B$. Suppose that $x \notin B$. Then $f_B(x) = \varnothing \subseteq U = f_A(x)$. Therefore, $f_B(x) \subseteq f_A(x)$ for all $x \notin B$. Hence $f_B(x) \subseteq f_A(x)$ for all $x \in B \cup B^c = E$. Therefore, $F_B \widetilde{\subseteq} F_{\widetilde{A}}$. □

Theorem 3.14. *Let F_A be any soft set in $S(U)$. Then the following hold.*

(a) *If $F_A = F_\varnothing$, then every soft subset of F_A is also empty.*

(b) *If $F_A \neq F_\varnothing$, then every soft superset of F_A is also non-empty.*

Proof. (a) Suppose that $F_A = F_\varnothing$. Then $f_A(x) = \varnothing$ for all $x \in E$. Let F_B be any soft subset of F_A. Then $f_B(x) \subseteq f_A(x)$ for all $x \in E$. Since $f_A(x) = \varnothing$ for all $x \in E$, $f_B(x) = \varnothing$ for all $x \in E$. Therefore, by definition, $F_B = F_\varnothing$. Hence every soft subset of F_A is also empty.

(b) Suppose that $F_A \neq F_\varnothing$. Then $f_A(x) \neq \varnothing$ for some $x \in E$. Let F_B be any soft superset of F_A. Then $f_A(x) \subseteq f_B(x)$ for all $x \in E$. Since $f_A(x) \neq \varnothing$ for some $x \in E$, $f_B(x) \neq \varnothing$ for those $x \in E$. Therefore, by definition, $F_B \neq F_\varnothing$. Hence every soft superset of F_A is also non-empty. □

Theorem 3.15. *Let $F_A \in S(U)$. Then the following hold.*

(a) $F_A \widetilde{\setminus} F_\varnothing = F_A$
(b) $F_\varnothing \widetilde{\setminus} F_A = F_\varnothing$
(c) $F_{\widetilde{E}} \widetilde{\setminus} F_A = F_A^{\tilde{c}}$
(d) $F_A \widetilde{\setminus} F_{\widetilde{E}} = F_\varnothing$
(e) $F_A \widetilde{\setminus} F_A = F_\varnothing$
(f) $F_A \widetilde{\setminus} F_A^{\tilde{c}} = F_A$
(g) $F_A^{\tilde{c}} \widetilde{\setminus} F_A = F_A^{\tilde{c}}$.

Proof. Let $F_A \in S(U)$ and f_A be the approximate function of F_A.

(a) Let f_B be the approximate function of $F_\varnothing$. Then $f_B(x) = \varnothing$ for all $x \in E$. For every $x \in E$, $f_{A\widetilde{\setminus}B}(x) = f_A(x)\setminus f_B(x) = f_A(x)\setminus\varnothing = f_A(x)$. Therefore, $F_A \widetilde{\setminus} F_\varnothing = F_A$.

(b) For every $x \in E$, $f_{B\widetilde{\setminus}A}(x) = f_B(x)\setminus f_A(x) = \varnothing\setminus f_A(x) = \varnothing$. Therefore, $F_\varnothing \widetilde{\setminus} F_A = F_\varnothing$.

(c) Let f_E be the approximate function of $F_{\widetilde{E}}$. Then $f_E(x) = U$ for all $x \in E$. For every $x \in E$, $f_{E\widetilde{\setminus}A}(x) = f_E(x)\setminus f_A(x) = U\setminus f_A(x) = f_A^c(x) = f_{A^{\tilde{c}}}(x)$. Therefore, $F_{\widetilde{E}} \widetilde{\setminus} F_A = F_A^{\tilde{c}}$.

(d) For every $x \in E$, $f_{A\widetilde{\setminus}E}(x) = f_A(x)\setminus f_E(x) = f_A(x)\setminus U = \varnothing$, since $f_A(x) \subseteq U$. Therefore, $F_A \widetilde{\setminus} F_{\widetilde{E}} = F_\varnothing$.

(e) For every $x \in E$, $f_{A\widetilde{\setminus}A}(x) = f_A(x)\setminus f_A(x) = \varnothing$. Therefore, $F_A \widetilde{\setminus} F_A = F_\varnothing$.

(f) For every $x \in E$, $f_{A\widetilde{\setminus}A^{\tilde{c}}}(x) = f_A(x)\setminus f_{A^{\tilde{c}}}(x) = f_A(x)\setminus f_A^c(x) = f_A(x)$. Therefore, $F_A \widetilde{\setminus} F_A^{\tilde{c}} = F_A$.

(g) For every $x \in E$, $f_{A^{\tilde{c}}\widetilde{\setminus}A}(x) = f_{A^{\tilde{c}}}(x)\setminus f_A(x) = f_A^c(x)\setminus f_A(x) = f_A^c(x) = f_{A^{\tilde{c}}}(x)$. Therefore, $F_A^{\tilde{c}} \widetilde{\setminus} F_A = F_A^{\tilde{c}}$. □

Theorem 3.16. *Let F_A, $F_B \in S(U)$. Then the following hold.*

(a) $F_A \widetilde{\cap} F_B = F_\varnothing$ *if and only if* $F_A \widetilde{\setminus} F_B = F_A$.
(b) $F_A \widetilde{\cap} F_B = F_\varnothing$ *if and only if* $F_B \widetilde{\setminus} F_A = F_B$.
(c) $F_A \widetilde{\subseteq} F_B$ *if and only if* $F_A \widetilde{\setminus} F_B = F_\varnothing$.

Proof. Let F_A, $F_B \in S(U)$. Let f_A and f_B be the approximate functions for the soft sets F_A and F_B, respectively.

(a) $F_A \widetilde{\cap} F_B = F_\varnothing \Leftrightarrow f_{A\widetilde{\cap}B}(x) = \varnothing$ for all $x \in E \Leftrightarrow f_A(x) \cap f_B(x) = \varnothing$ for all $x \in E \Leftrightarrow f_A(x)\setminus f_B(x) = f_A(x)$ for all $x \in E \Leftrightarrow f_{A\widetilde{\setminus}B}(x) = f_A(x)$ for all $x \in E \Leftrightarrow F_A \widetilde{\setminus} F_B = F_A$.

(b) $F_A \widetilde{\cap} F_B = F_\varnothing \Leftrightarrow f_{A\widetilde{\cap}B}(x) = \varnothing$ for all $x \in E \Leftrightarrow f_A(x) \cap f_B(x) = \varnothing$ for all $x \in E \Leftrightarrow f_B(x)\setminus f_A(x) = f_B(x)$ for all $x \in E \Leftrightarrow f_{B\widetilde{\setminus}A}(x) = f_B(x)$ for all $x \in E \Leftrightarrow F_B \widetilde{\setminus} F_A = F_B$.

(c) $F_A \widetilde{\subseteq} F_B \Leftrightarrow f_A(x) \subseteq f_B(x)$ for all $x \in E \Leftrightarrow f_A(x)\setminus f_B(x) = \varnothing$ for all $x \in E \Leftrightarrow f_{A\widetilde{\setminus}B}(x) = \varnothing$ for all $x \in E \Leftrightarrow F_A \widetilde{\setminus} F_B = F_\varnothing$. □

Theorem 3.17. *Let F_A, $F_B \in S(U)$. Then the following hold.*

(a) $F_A \tilde{\setminus} F_B = F_A \tilde{\cap} F_B^{\tilde{c}}$
(b) $F_A = (F_A \tilde{\cap} F_B) \tilde{\cup} (F_A \tilde{\setminus} F_B)$.

Proof. Let F_A, $F_B \in S(U)$. Let f_A and f_B be the approximate functions for the soft sets F_A and F_B, respectively.

(a) For every $x \in E$, $f_{A \tilde{\setminus} B}(x) = f_A(x) \setminus f_B(x) = f_A(x) \cap f_B^c(x) = f_A(x) \cap f_{B^{\tilde{c}}}(x) = f_{A \tilde{\cap} B^{\tilde{c}}}(x)$. Therefore, $F_A \tilde{\setminus} F_B = F_A \tilde{\cap} F_B^{\tilde{c}}$.

(b) For every $x \in E$, $f_{(A \tilde{\cap} B) \tilde{\cup} (A \tilde{\setminus} B)}(x) = f_{(A \tilde{\cap} B)}(x) \cup f_{A \tilde{\setminus} B}(x) = [f_A(x) \cap f_B(x)] \cup [f_A(x) \setminus f_B(x)] = f_A(x)$. Therefore, $F_A = (F_A \tilde{\cap} F_B) \tilde{\cup} (F_A \tilde{\setminus} F_B)$. □

Theorem 3.18. *Let F_A, F_B, $F_C \in S(U)$. Then the following hold.*

(a) $F_A \tilde{\cap} (F_B \tilde{\setminus} F_C) = (F_A \tilde{\cap} F_B) \tilde{\setminus} (F_A \tilde{\cap} F_C)$
(b) $F_A \tilde{\setminus} (F_B \tilde{\cup} F_C) = (F_A \tilde{\setminus} F_B) \tilde{\cap} (F_A \tilde{\setminus} F_C)$
(c) $F_A \tilde{\setminus} (F_B \tilde{\cap} F_C) = (F_A \tilde{\setminus} F_B) \tilde{\cup} (F_A \tilde{\setminus} F_C)$.

Proof. Let F_A, F_B, $F_C \in S(U)$. Let f_A, f_B and f_C be the approximate functions for the soft sets F_A, F_B and F_C, respectively.

(a) For every $x \in E$, $f_{A \tilde{\cap} (B \tilde{\setminus} C)}(x) = f_A(x) \cap f_{B \tilde{\setminus} C}(x) = f_A(x) \cap [f_B(x) \setminus f_C(x)] = [f_A(x) \cap f_B(x)] \setminus [f_A(x) \cap f_C(x)] = f_{A \tilde{\cap} B}(x) \setminus f_{A \tilde{\cap} C}(x) = f_{(A \tilde{\cap} B) \tilde{\setminus} (A \tilde{\cap} C)}(x)$. Therefore, $F_A \tilde{\cap} (F_B \tilde{\setminus} F_C) = (F_A \tilde{\cap} F_B) \tilde{\setminus} (F_A \tilde{\cap} F_C)$.

(b) For every $x \in E$, $f_{A \tilde{\setminus} (B \tilde{\cup} C)}(x) = f_A(x) \setminus f_{B \tilde{\cup} C}(x) = f_A(x) \setminus [f_B(x) \cup f_C(x)] = [f_A(x) \setminus f_B(x)] \cap [f_A(x) \setminus f_C(x)] = f_{A \tilde{\setminus} B}(x) \cap f_{A \tilde{\setminus} C}(x) = f_{(A \tilde{\setminus} B) \tilde{\cap} (A \tilde{\setminus} C)}(x)$. Therefore, $F_A \tilde{\setminus} (F_B \tilde{\cup} F_C) = (F_A \tilde{\setminus} F_B) \tilde{\cap} (F_A \tilde{\setminus} F_C)$.

(c) For every $x \in E$, $f_{A \tilde{\setminus} (B \tilde{\cap} C)}(x) = f_A(x) \setminus f_{B \tilde{\cap} C}(x) = f_A(x) \setminus [f_B(x) \cap f_C(x)] = [f_A(x) \setminus f_B(x)] \cup [f_A(x) \setminus f_C(x)] = f_{A \tilde{\setminus} B}(x) \cup f_{A \setminus C}(x) = f_{(A \setminus B) \tilde{\cup} (A \tilde{\setminus} C)}(x)$. Therefore, $F_A \tilde{\setminus} (F_B \tilde{\cap} F_C) = (F_A \tilde{\setminus} F_B) \tilde{\cup} (F_A \tilde{\setminus} F_C)$. □

A soft set P on the universe U is called a *soft point* if and only if its approximate function p takes the value $\varnothing$ for all $y \in E$ except one, say $x \in E$. That is, $p(x) = \begin{cases} \{u\} & \text{if } x = y \\ \varnothing & \text{if } x \neq y \end{cases}$ where $p : E \to \wp(U)$, $x \in E$, and $\{u\} \in \wp(U)$. Therefore, $P = \{(x, \{u\})\}$. The class of all soft points in U is denoted by $\tilde{P}$. Clearly, $P \neq F_\varnothing$. A soft point P is said to be in F_A, denoted by, $P \tilde{\in} F_A$ if and only if $p(x) \subseteq f_A(x)$ for all $x \in E$. The following Example 3.19 shows the existence of a soft point.

Example 3.19. Let $U = \{u_1, u_2\}$ and $E = \{x_1, x_2\}$. Then the soft points are $\{(x_1, \{u_1\})\}$, $\{(x_1, \{u_2\})\}$, $\{(x_2, \{u_1\})\}$ and $\{(x_2, \{u_2\})\}$.

Example 3.20. Let $U = \{u_1, u_2, u_3\}$ and $E = \{x_1, x_2\}$. Then the soft points are $\{(x_1, \{u_1\})\}$, $\{(x_1, \{u_2\})\}$, $\{(x_1, \{u_3\})\}$, $\{(x_2, \{u_1\})\}$, $\{(x_2, \{u_2\})\}$ and $\{(x_2, \{u_3\})\}$. Let $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_3\})\}$. Then the soft points of F_A are $\{(x_1, \{u_1\})\}$, $\{(x_1, \{u_2\})\}$ and $\{(x_2, \{u_3\})\}$.

Theorem 3.21. *Every soft set F_A in $S(U)$ can be expressed as the soft union of all soft points which belongs to F_A. That is, if $F_A \neq F_\varnothing$, then $F_A = \tilde{\bigcup}_{P \tilde{\in} F_A} P$.*

Proof. Let F_A be any soft set in $S(U)$ and f_A be its approximate function. Assume that $F_A \neq F_\varnothing$. Then for every $x \in E$, $f_A(x) = \cup_{u_i \in f_A(x)}\{u_i\}$. That is, $f_A(x) = \cup_{\{u_i\} \subseteq f_A(x)}\{u_i\} = \cup_{p_i(x) \subseteq f_A(x)} p_i(x)$ for all $x \in E$, where $p_i(x) = \begin{cases} \{u_i\} & \text{if } x = y \\ \varnothing & \text{if } x \neq y \end{cases}$ and p_i's are the approximate function of the soft set P_i. Therefore, P_i' s are the soft points in F_A. Hence $F_A = \widetilde{\cup}_{P_i \widetilde{\in} F_A} P_i$. That is, $F_A = \widetilde{\cup}_{P \widetilde{\in} F_A} P$. □

Theorem 3.22. *Let F_A, $F_B \in S(U)$. Then $F_A \widetilde{\subseteq} F_B$ if and only if $P \widetilde{\in} F_A \Rightarrow P \widetilde{\in} F_B$ for every $P \in \widetilde{P}$.*

Proof. Let F_A, $F_B \in S(U)$. Assume that $F_A \widetilde{\subseteq} F_B$. Let $P \in \widetilde{P}$ be arbitrary such that $P \widetilde{\in} F_A$. Then $P \widetilde{\subseteq} F_A$. Since $F_A \widetilde{\subseteq} F_B$ and $P \widetilde{\subseteq} F_A$, $P \widetilde{\subseteq} F_B$. This implies that $P \widetilde{\in} F_B$. Since $P \in \widetilde{P}$ is arbitrary, $P \widetilde{\in} F_A$ implies that $P \widetilde{\in} F_B$ for every $P \in \widetilde{P}$. Conversely, let $x \in E$ and $u \in f_A(x)$. Then $\{u\} \subseteq f_A(x)$. If $p(y) = \begin{cases} \{u\} & \text{if } y = x \\ \varnothing & \text{if } y \neq x \end{cases}$ is the approximate function for the soft point $\widetilde{P}$, then $p(x) \subseteq f_A(x)$ for all $x \in E$ and hence $P \widetilde{\in} F_A$ which implies $P \widetilde{\in} F_B$, by hypothesis. Then $p(x) \subseteq f_B(x)$ for all $x \in E$. Thus, $\{u\} \subseteq f_B(x)$ for $x = y$ and so $u \in f_B(x)$. Hence $f_A(x) \subseteq f_B(x)$. Since $x \in E$ is arbitrary, $f_A(x) \subseteq f_B(x)$ for every $x \in E$. Therefore, $F_A \widetilde{\subseteq} F_B$. □

Corollary 3.23. *Let F_A, $F_B \in S(U)$. Then $F_A = F_B$ if and only if $P \widetilde{\in} F_A \Leftrightarrow P \widetilde{\in} F_B$ for every $P \in \widetilde{P}$.*

Theorem 3.24. *Let F_A, $F_B \in S(U)$. Then $F_A \widetilde{\subseteq} F_B$ if and only if $P \widetilde{\notin} F_B \Rightarrow P \widetilde{\notin} F_A$ for every $P \in \widetilde{P}$.*

Proof. Let F_A, $F_B \in S(U)$. Assume that $F_A \widetilde{\subseteq} F_B$. Let $P \in \widetilde{P}$ be arbitrary such that $P \widetilde{\notin} F_B$. Then $p(x) \nsubseteq f_B(x)$ for some $x \in E$. Now $p(x) = \begin{cases} \{u\} & \text{if } x = y \\ \varnothing & \text{if } x \neq y. \end{cases}$ Since empty set is a subset of every set, $\{u\} \nsubseteq f_B(x)$ for $x = y$ in E which implies $u \notin f_B(x)$ for $x = y$ in E which in turn implies that $u \notin f_A(x)$ for $x = y$ in E, since $f_A(x) \subseteq f_B(x)$ for all $x \in E$. Hence $\{u\} \nsubseteq f_A(x)$ for $x = y$ in E so that $p(x) \nsubseteq f_A(x)$ for $x = y$ in E. Hence $P \widetilde{\notin} F_A$. Since $P \in \widetilde{P}$ is arbitrary, $P \widetilde{\notin} F_B$ implies that $P \widetilde{\notin} F_A$ for every $P \in \widetilde{P}$. Conversely, assume that $P \widetilde{\notin} F_B$ implies $P \widetilde{\notin} F_A$ for every $P \in \widetilde{P}$. Let $x \in E$ and suppose that $u \notin f_B(x)$. Then $\{u\} \nsubseteq f_B(x)$. If $p(y) = \begin{cases} \{u\} & \text{if } y = x \\ \varnothing & \text{if } y \neq x \end{cases}$ is the approximate function for the soft point $\widetilde{P}$, then $p(x) \nsubseteq f_B(x)$ for all $x \in E$ and hence $P \widetilde{\notin} F_B$ which implies that $P \widetilde{\notin} F_A$, by hypothesis. Then $p(x) \nsubseteq f_A(x)$ for all $x \in E$. Thus, $\{u\} \nsubseteq f_A(x)$ for $x = y$ and so $u \notin f_A(x)$. Hence $f_A(x) \subseteq f_B(x)$. Since $x \in E$ is arbitrary, $f_A(x) \subseteq f_B(x)$ for every $x \in E$. Therefore, $F_A \widetilde{\subseteq} F_B$. □

Theorem 3.25. *Let $F_A \in S(U)$. If $P \widetilde{\notin} F_A$, then $P \widetilde{\in} F_A^{\tilde{c}}$ for every $P \in \widetilde{P}$.*

Proof. Let $F_A \in S(U)$. Let $P \in \widetilde{P}$ be arbitrary such that $P \widetilde{\notin} F_A$. Then $p(x) \nsubseteq f_A(x)$ for some $x \in E$ where $p(x) = \begin{cases} \{u\} & \text{if } x = y \\ \varnothing & \text{if } x \neq y. \end{cases}$ Since empty set is a subset of every set, $\{u\} \nsubseteq f_A(x)$ for $x = y$ in E. This implies $u \notin f_A(x)$ for $x = y$ in E which implies $u \in f^c_A(x)$ for $x = y$ in E which in turn implies that $u \in f_{A^{\tilde{c}}(x)}$ for $x = y$ in E. Hence $\{u\} \subseteq f_{A^{\tilde{c}}}(x)$, for $x = y$ in E. That is, $p(x) \subseteq f_{A^{\tilde{c}}}(x)$ for $x = y$ in E. Also, since empty set is a subset of every set, $p(x) \subseteq f_{A^{\tilde{c}}}(x)$ for all $x \in E$. Therefore, $P \widetilde{\in} F^{\tilde{c}}_A$. Since $P \in \widetilde{P}$ is arbitrary, if $P \widetilde{\notin} F_A$, then $P \widetilde{\in} F^{\tilde{c}}_A$ for every $P \in \widetilde{P}$. □

Theorem 3.26. *Let I be an arbitrary index set and let $\{F_{A_i} \mid i \in I\}$ be a family of soft sets in $S(U)$. Then $P \widetilde{\in} \widetilde{\cup}\{F_{A_i} \mid i \in I\}$ if and only if there exists $i \in I$ such that $P \widetilde{\in} F_{A_i}$.*

Proof. Assume that $P \widetilde{\in} \widetilde{\cup}\{F_{A_i} \mid i \in I\}$. Then $p(x) \subseteq \cup\{f_{A_i}(x) \mid i \in I\}$ for every $x \in E$. Now $p(x) = \begin{cases} \{u\} & \text{if } x = y \\ \varnothing & \text{if } x \neq y. \end{cases}$ Thus, for $x = y$, $\{u\} \subseteq \cup\{f_{A_i}(x) \mid i \in I\}$ which implies $u \in \cup\{f_{A_i}(x) \mid i \in I\}$ so that $u \in f_{A_i}(x)$ for some $i \in I$. Therefore, for $x = y$, $p(x) \subseteq f_{A_i}(x)$ for some $i \in I$. If $x \neq y$, then $p(x) = \varnothing$ and so $p(x) \subseteq f_{A_i}(x)$ for every $i \in I$. Hence $p(x) \subseteq f_{A_i}(x)$ for some $i \in I$ and for all $x \in E$. Therefore, $P \widetilde{\in} F_{A_i}$ for some $i \in I$. Converse follows from the fact that $F_{A_i} \widetilde{\subseteq} \widetilde{\cup} F_{A_i}$ for every $i \in I$. □

Theorem 3.27. *Let I be an arbitrary index set. Let $\{F_{A_i} \mid i \in I\}$ be a family of soft sets in $S(U)$. Then $P \widetilde{\in} \widetilde{\cap}\{F_{A_i} \mid i \in I\}$ if and only if $P \widetilde{\in} F_{A_i}$ for every $i \in I$.*

Proof. Let $P \widetilde{\in} F_{A_i}$ for every $i \in I$. Then $p(x) \subseteq f_{A_i}(x)$ for every $i \in I$ and for all $x \in E$. Thus, for $x = y$, $\{u\} \subseteq f_{A_i}(x)$ for every $i \in I$ so that $\{u\} \subseteq \cap\{f_{A_i}(x) \mid i \in I\}$ and so $p(x) \subseteq \cap\{f_{A_i}(x) \mid i \in I\}$. Also, $p(x) \subseteq \cap\{f_{A_i}(x) \mid i \in I\}$ for $x \neq y$. Hence $p(x) \subseteq \cap\{f_{A_i}(x) \mid i \in I\}$ for all $x \in E$. Therefore, $P \widetilde{\in} \widetilde{\cap}\{F_{A_i} \mid i \in I\}$. Converse follows from the fact $\widetilde{\cap} F_{A_i} \widetilde{\subseteq} F_{A_i}$ for every $i \in I$. □

Theorem 3.28. *If $(F_A, \widetilde{\tau_1})$ and $(F_A, \widetilde{\tau_2})$ are two soft topological spaces, then $(F_A, \widetilde{\tau_1} \cap \widetilde{\tau_2})$ is a soft topology.*

Proof. Let $(F_A, \widetilde{\tau_1})$ and $(F_A, \widetilde{\tau_2})$ be two soft topological spaces. Since $\widetilde{\tau_1}$ and $\widetilde{\tau_2}$ are soft topologies on F_A, $F_A \in \widetilde{\tau_1}$ and $F_A \in \widetilde{\tau_2}$ and so $F_A \in \widetilde{\tau_1} \cap \widetilde{\tau_2}$. Let $\{F_{A_i} \widetilde{\subseteq} F_A \mid i \in I \subseteq \mathbb{N}\}$ be arbitrary family of soft sets in $\widetilde{\tau_1} \cap \widetilde{\tau_2}$. Then for every $i \in I \subseteq \mathbb{N}, F_{A_i} \in \widetilde{\tau_1}$ and $F_{A_i} \in \widetilde{\tau_2}$. Since $\widetilde{\tau_1}$ and $\widetilde{\tau_2}$ are soft topologies, $\widetilde{\cup}_{i \in I} F_{A_i} \in \widetilde{\tau_1}$ and $\widetilde{\cup}_{i \in I} F_{A_i} \in \widetilde{\tau_2}$ and so $\widetilde{\cup}_{i \in I} F_{A_i} \in \widetilde{\tau_1} \cap \widetilde{\tau_2}$. Let $\{F_{A_i} \widetilde{\subseteq} F_A \mid 1 \leq i \leq n, n \in \mathbb{N}\}$ be a finite family of soft sets in $\widetilde{\tau_1} \cap \widetilde{\tau_2}$. Since $\widetilde{\tau_1}$ and $\widetilde{\tau_2}$ are soft topologies, $\widetilde{\cap}^n_{i=1} F_{A_i} \in \widetilde{\tau_1}$ and $\widetilde{\cap}^n_{i=1} F_{A_i} \in \widetilde{\tau_2}$. Thus, $\widetilde{\cap}^n_{i=1} F_{A_i} \in \widetilde{\tau_1} \cap \widetilde{\tau_2}$. Therefore, $\widetilde{\tau_1} \cap \widetilde{\tau_2}$ is a soft topology on F_A. □

The following Example 3.29 shows that union of two soft topologies on F_A need not be a soft topology on F_A.

Example 3.29. Consider $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\}$, $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$, $F_{A_1} = \{(x_1, \{u_2\})\}$, $F_{A_2} = \{(x_2, \{u_2\})\}$ and $F_{A_3} = \{(x_1, \{u_1, u_2\})\}$. If $\widetilde{\tau_1} = \{F_\varnothing, F_A, F_{A_2}\}$ and $\widetilde{\tau_2} = \{F_\varnothing, F_A, F_{A_1}, F_{A_3}\}$, then $(F_A, \widetilde{\tau_1})$ and $(F_A, \widetilde{\tau_2})$ are soft topological spaces on F_A. But $\widetilde{\tau_1} \cup \widetilde{\tau_2} = \{F_\varnothing, F_A, F_{A_1}, F_{A_2}, F_{A_3}\}$ is not a soft topology, since $F_{A_1} \widetilde{\cup} F_{A_2} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\} \notin \widetilde{\tau_1} \cup \widetilde{\tau_2}$.

Theorem 3.30. *Let $(F_A, \widetilde{\tau})$ be a soft topological space. Then $\widetilde{B}$ be its soft basis if and only if for every $F_G \in \widetilde{\tau}$ and for each $P \widetilde{\in} F_G$, there is some $F_B \in \widetilde{B}$ such that $P \widetilde{\in} F_B \widetilde{\subseteq} F_G$.*

Proof. Assume that $\widetilde{B}$ is a soft basis. Then for every $F_G \in \widetilde{\tau}$, there exists $\{F_{B_i}\}_{i\in I}$ in $\widetilde{B}$ such that $\widetilde{\cup}_{i\in I} F_{B_i} = F_G$. If $P \widetilde{\in} F_G$, then $P \widetilde{\in} \widetilde{\cup}_{i\in I} F_{B_i}$ and so $P \widetilde{\in} F_{B_i}$ for some $i \in I$. Since $\widetilde{\cup}_{i\in I} F_{B_i} = F_G, F_{B_i} \widetilde{\subseteq} F_G$ for every $i \in I$. Thus, for every $F_G \in \widetilde{\tau}$ with $P \widetilde{\in} F_G$, there is some $F_{B_i} \in \widetilde{B}$ such that $P \widetilde{\in} F_{B_i} \widetilde{\subseteq} F_G$. Conversely, let $F_G \in \widetilde{\tau}$ and $P \widetilde{\in} F_G$. Then by hypothesis, there exists some $F_{B_i} \in \widetilde{B}$ such that $P \widetilde{\in} F_{B_i} \widetilde{\subseteq} F_G$. Hence $P \widetilde{\in} \widetilde{\cup}_{i\in I} F_{B_i} \widetilde{\subseteq} F_G$. Since $P \widetilde{\in} F_G$ is arbitrary, $F_G \widetilde{\subseteq} \widetilde{\cup}_{i\in I} F_{B_i} \widetilde{\subseteq} F_G$. Thus, $F_G = \widetilde{\cup}_{i\in I} F_{B_i}$. □

Theorem 3.31. *Let F_A be a non-empty soft set. A family $\widetilde{B}$ of soft subsets of F_A is a soft base for a soft topology $\widetilde{\tau}$ on F_A if and only if (a)$F_A = \widetilde{\cup}\widetilde{B}$ and (b) for every F_C, F_D in $\widetilde{B}$ and for each P in $F_C \widetilde{\cap} F_D$, there exists F_H in $\widetilde{B}$ such that $P \widetilde{\in} F_H \widetilde{\subseteq} F_C \widetilde{\cap} F_D$.*

Proof. Suppose the family $\widetilde{B}$ is a soft base for the soft topology $\widetilde{\tau}$ on F_A. Since $F_A \in \widetilde{\tau}$, we have $F_A = \widetilde{\cup}\widetilde{B}$. This establishes (a). Let F_C, $F_D \in \widetilde{B}$ and $P \widetilde{\in} F_C \widetilde{\cap} F_D$. Since $\widetilde{B} \subseteq \widetilde{\tau}$, F_C, $F_D \in \widetilde{\tau}$. Hence $F_C \widetilde{\cap} F_D \in \widetilde{\tau}$, since $\widetilde{\tau}$ is a soft topology. By Theorem 3.30, there exists P in $\widetilde{B}$ such that $P \widetilde{\in} F_H \widetilde{\subseteq} F_C \widetilde{\cap} F_D$. This establishes (b). Conversely, suppose that conditions (a) and (b) hold. By (a) $F_A \in \widetilde{\tau}$ and clearly, $F_\varnothing \in \widetilde{\tau}$. Let F_C, $F_D \in \widetilde{\tau}$ and $P \widetilde{\in} F_C \widetilde{\cap} F_D$. Then $F_C = \widetilde{\cup}_{F_G \in \widetilde{B}} F_G$ so that $F_G \widetilde{\subseteq} F_C$ for every $F_G \in \widetilde{B}$. Since $P \widetilde{\in} F_C, P \widetilde{\in} \widetilde{\cup}_{F_G \in \widetilde{B}} F_G$ and so $P \widetilde{\in} F_G$ for some $F_G \in \widetilde{B}$. Hence there exists some $F_G \in \widetilde{B}$ such that $P \widetilde{\in} F_G \widetilde{\subseteq} F_C$. Similarly, there exists some $F_H \in \widetilde{B}$ such that $P \widetilde{\in} F_H \widetilde{\subseteq} F_D$. Hence $P \widetilde{\in} F_G \widetilde{\cap} F_H \widetilde{\subseteq} F_C \widetilde{\cap} F_D$. By ($b$), we can find F_{W_P}in $\widetilde{B}$ such that $P \widetilde{\in} F_{W_P} \widetilde{\subseteq} F_G \widetilde{\cap} F_H$. Thus, $P \widetilde{\in} F_{W_P} \widetilde{\subseteq} F_C \widetilde{\cap} F_D$ for all $P \widetilde{\in} F_C \widetilde{\cap} F_D$. Let $P \widetilde{\in} \widetilde{\cup}_{P \widetilde{\in} F_C \widetilde{\cap} F_D} F_{W_P}$. Then $P \widetilde{\in} F_{W_P}$ with $P \widetilde{\in} F_C \widetilde{\cap} F_D$. Hence $\widetilde{\cup}_{P \widetilde{\in} F_C \widetilde{\cap} F_D} F_{W_P} \widetilde{\subseteq} F_C \widetilde{\cap} F_D$. Now $P \widetilde{\in} F_{W_P}$ for $P \widetilde{\in} F_C \widetilde{\cap} F_D$ implies that $P \widetilde{\subseteq} F_{W_P}$ for $P \widetilde{\in} F_C \widetilde{\cap} F_D$ which implies that $\widetilde{\cup}_{P \widetilde{\in} F_C \widetilde{\cap} F_D} P \widetilde{\subseteq} \widetilde{\cup}_{P \widetilde{\in} F_C \widetilde{\cap} F_D} F_{W_P}$. Hence by Theorem 3.21, $F_C \widetilde{\cap} F_D \widetilde{\subseteq} \widetilde{\cup}_{P \widetilde{\in} F_C \widetilde{\cap} F_D} F_{W_P}$. Hence $F_C \widetilde{\cap} F_D = \widetilde{\cup}_{P \widetilde{\in} F_C \widetilde{\cap} F_D} F_{W_P}$ and so $F_C \widetilde{\cap} F_D \in \widetilde{\tau}$. Suppose $\{F_{A_i} \mid i \in I\}$ be an arbitrary subfamily of elements in $\widetilde{\tau}$. Then for every $i \in I, F_{A_i} \in \widetilde{\tau}$ and so F_{A_i} can be expressed as a soft union of members of $\widetilde{B}$. Hence $\widetilde{\cup}_{i\in I} F_{A_i} \in \widetilde{\tau}$. Thus, $\widetilde{\tau}$ satisfies the condition for being a soft topology on F_A. Hence $\widetilde{B}$ is a soft base for the soft topology $\widetilde{\tau}$. □

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