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(r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping

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Abstract. The purpose of this paper is to introduce the (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping and to study the interrelations among the intuitionistic fuzzy quasi uniform continuous mappings introduced with suitable examples. Also, (r,s) intuitionistic fuzzy quasi uniform RG_δ compact space, (r,s)intuitionistic fuzzy quasi uniform regular G_δ connected set are introduced and studied with reference to (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh[11]. Since then the concept has invaded nearly all branches of Mathematics. Chang[4] introduced and developed the theory of fuzzy topological spaces and since then various notions in classical topology have been extended to fuzzy topological spaces. Fuzzy sets have applications in many fields such as information [10] and control [9]. Atanassov [3] generalised fuzzy sets to intuitionistic fuzzy sets. Cocker [6] introduced the notions of an intuitionistic fuzzy topological space, intuitionistic fuzzy continuous mapping and some related concepts. Anjan Mukherjee ([1] and [2]) introduced and studied the concept of fuzzy faintly continuous functions and fuzzy faintly α-continuous functions. G.K.Revathi,E.Roja and M. K. Uma ([7] and [8]) studied intuitionistic fuzzy regular G_δ set and also introduced and studied (r,s) intuitionistic fuzzy quasi uniform topological space, (r,s) intuitionistic fuzzy quasi uniform regular G_δ set and (r,s) intuitionistic fuzzy quasi uniform regular G_δ compactness.The purpose of this paper is to introduce (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping and to study the interrelations among the intuitionistic fuzzy quasi uniform continuous mappings introduced with suitable examples. Also, (r,s) intuitionistic fuzzy quasi uniform RG_δ compact space, (r,s)intuitionistic fuzzy quasi uniform regular G_δ connected set are introduced and studied with reference to (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping.

2. Preliminaries

Definition 2.1 ([3])**.** Let X be a non empty fixed set and I the closed interval [0,1]. An intuitionistic fuzzy set (IFS) A is an object of the following form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ where the mapping $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each x $\in$ X.Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A(x), \gamma_A(x)\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x) : x \in X\rangle\}$. For a given non empty set X, denote the family of all intuitionistic fuzzy sets in X by the symbol ζ^X.

Definition 2.2 ([3])**.** Let A and B be IFS's of the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x) :\rangle x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$
(ii) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$
(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$

Definition 2.3 ([4])**.** An intuitionistic fuzzy topology (IFT) in Coker's sense on a non empty set X is a family τ of IFSs in X satisfying the following axioms.

(T_1) $0_\sim$,$1_\sim \in \tau$
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(T_3) $\cup G_i \in \tau$ for arbitrary family $\{G_i / i \in I\} \subseteq \tau$

In this paper by (X, τ) or simply by X we will denote the Cocker's intuitionistic fuzzy topological space (IFTS). Each IFSs in τ is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4 ([5])**.** Let a and b be two real numbers in [0,1] satisfying the inequality $a + b \leq 1$. Then the pair $\langle a, b\rangle$ is called an intuitionistic fuzzy pair. Let $\langle a_1, b_1\rangle$,$\langle a_2, b_2\rangle$ be any two intuitionistic fuzzy pairs. Then define

(1) $\langle a_1, b_1\rangle \leq \langle a_2, b_2\rangle$ if and only if $a_1 \leq a_2$ and $b_1 \geq b_2$.
(2) $\langle a_1, b_1\rangle = \langle a_2, b_2\rangle$ if and only if $a_1 = a_2$ and $b_1 = b_2$.
(3) If $\{\langle a_i, b_i / i \in J\rangle\}$ is a family of intuitionistic fuzzy pairs, then $\vee\langle a_i, b_i\rangle = \langle \vee a_i, \wedge b_i\rangle$ and $\wedge\langle a_i, b_i\rangle = \langle \wedge a_i, \vee b_i\rangle$.
(4) The complement of an intuitionistic fuzzy pair $\langle a, b\rangle$ is the intuitionistic fuzzy pair defined by $\overline{\langle a, b\rangle} = \langle b, a\rangle$
(5) $1^\sim = \langle 1, 0\rangle$ and $0^\sim = \langle 0, 1\rangle$.

Definition 2.5 ([8])**.** Let Ω_X denotes the family of all intuitionistic fuzzy mappings $f : \zeta^X \to \zeta^X$ with the following properties.

(1) $f(0_\sim) = 0_\sim$,
(2) $A \subseteq f(A)$ for every $A \in \zeta^X$
(3) $f(\cup A_i) = \cup f(A_i)$ for every $A_i \in \zeta^X, i \in J$

For $f \in \Omega_X$, the function $f^{-1} \in \Omega_X$ is defined by $f^{-1}(A) = \cap\{B/f(\overline{B}) \subseteq \overline{A}\}$
For $f, g \in \Omega_X$, we define, for all $A \in \zeta^X$,
$f \sqcap g(A) = \cap\{f(A_1) \cup g(A_2)/A_1 \cup A_2 = A\}$,$(f \circ g)(A) = f(g(A))$

Definition 2.6 ([8])**.** Let $\mathcal{U} : \Omega_X \to I_0 \times I_1$ be an intuitionistic fuzzy mapping. Then $\mathcal{U}$ is called an intuitionistic fuzzy quasi uniformity on X if it satisfies the following conditions.

(1) $\mathcal{U}(f_1 \sqcap f_2) \supseteq \mathcal{U}(f_1) \cap \mathcal{U}(f_2)$ for $f_1, f_2 \in \Omega_X$
(2) For $f \in \Omega_X$ we have $\cup\{\mathcal{U}(f_1)/f_1 \circ f_1 \subseteq f\} \supseteq \mathcal{U}(f)$
(3) If $f_1 \supseteq f$ then $\mathcal{U}(f_1) \supseteq \mathcal{U}(f)$.
(3) There exists $f \in \Omega_X$ such that $\mathcal{U}(f) = (1, 0)$

Then the pair $(X,\mathcal{U})$ is said to be an intuitionistic fuzzy quasi uniform space.

Definition 2.7 ([8])**.** Let $(X,\mathcal{U})$ be an intuitionistic fuzzy quasi uniform space. Define, for each $r \in (0, 1] = I_0$, $s \in [0, 1) = I_1$ with $r + s \leq 1$ and $A \in \zeta^X$

$$(r, s)IFQI_{\mathcal{U}}(A) = \cup\{B/f(B) \subseteq A \text{ for some } f \in \Omega_X \text{ with } \mathcal{U}(f) > (r, s)\}$$

Definition 2.8 ([8])**.** Let $(X,\mathcal{U})$ be an intuitionistic fuzzy quasi uniform space. Then the mapping $T_{\mathcal{U}} : \zeta^X \to I_0 \times I_1$ is defined by

$$T_{\mathcal{U}}(A) = \cup\{(r, s)/(r, s)IFQI_{\mathcal{U}}(A) = A,\ r \in I_0,\ s \in I_1 \text{ with } r + s \leq 1\}.$$

The the pair $(X, T_{\mathcal{U}})$ is called an intuitionistic fuzzy quasi uniform topological space. The members of $(X, T_{\mathcal{U}})$ are called (r,s) intuitionistic fuzzy quasi uniform open sets.

Note 2.9 ([8])**.** The complement of (r,s) intuitionistic fuzzy quasi uniform open set is called (r,s) intuitionistic fuzzy quasi uniform closed set.

Definition 2.10 ([8])**.** Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set. Then the intuitionistic fuzzy set A is said to be a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set if there exists a (r,s) intuitionistic fuzzy quasi uniform regular open set U such that $U \subseteq A \subseteq (r, s)IFQ\sigma cl_{\mathcal{U}}(U)$ where $r \in I_0$,$s \in I_1$ with $r + s \leq 1$. The complement of (r,s) intuitionistic fuzzy quasi uniform regular G_δ set is said to be an (r,s) intuitionistic fuzzy quasi uniform regular F_σ set.

Definition 2.11 ([8])**.** Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and $\{V_i/i \in J\}$ be a family of (r,s) intuitionistic fuzzy quasi uniform regular G_δ sets in $(X, T_{\mathcal{U}})$. Then $\{V_i/i \in J\}$ is said to be a (r,s) intuitionistic fuzzy quasi uniform regular G_δ cover of $(X, T_{\mathcal{U}})$ if $\cup_{i\in J} V_i = 1_\sim$

Definition 2.12 ([8])**.** Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space .Then $(X, T_{\mathcal{U}})$ is said to be a (r,s)intuitionistic fuzzy quasi uniform regular G_δ $T_{\mathcal{U}_{1/2}}$ space if every (r,s) intuitionistic fuzzy quasi uniform regular G_δ set is a (r,s) intuitionistic fuzzy quasi uniform regular open set.

Definition 2.13 ([8]). Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set. Then the intuitionistic fuzzy quasi uniform interior of A is denoted and defined by
$(r,s)IFQint_{\mathcal{U}}(A) = \cup\{B/B \subseteq A$ and B is an(r,s) intuitionistic fuzzy quasi uniform open set where $r \in I_0$,$s \in I_1$ with $r+s \leq 1\}$.

Definition 2.14 ([8]). Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set. Then the intuitionistic fuzzy quasi uniform closure of A is denoted and defined by
$(r,s)IFQcl_{\mathcal{U}}(A) = \cap\{B/B \supseteq A$ and B is an (r,s) intuitionistic fuzzy quasi uniform closed set where $r \in I_0$,$s \in I_1$ with $r+s \leq 1$ $\}$.

3. (r,s) Intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping

Definition 3.1. Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be any intuitionistic fuzzy set in $(X, T_{\mathcal{U}})$. Then the (r,s) intuitionistic fuzzy quasi uniform θ interior * of A is denoted and defined as
$(r,s)IFQ\theta int^\star_{\mathcal{U}}(A) = \cup\{B/$ C is a (r,s) intuitionistic fuzzy quasi uniform regular open set, $C \neq 0_\sim$,$C \subseteq B \subseteq (r,s)IFQint_{\mathcal{U}}(A)$ and $(r,s)IFQcl_{\mathcal{U}}(C) \subseteq A\}$

Definition 3.2. Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be any intuitionistic fuzzy set in $(X, T_{\mathcal{U}})$. Then the (r,s) intuitionistic fuzzy quasi uniform θ closure * of A is denoted and defined as
$(r,s)IFQ\theta cl^\star_{\mathcal{U}}(A) = \cap\{B/$ C is a (r,s) intuitionistic fuzzy quasi uniform regular closed set, $C \neq 1_\sim$, $(r,s)IFQcl_{\mathcal{U}}(A) \subseteq B \subseteq C$ and $A \subseteq (r,s)IFQint_{\mathcal{U}}(C)\}$

Definition 3.3. Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space, A be an intuitionistic fuzzy set in $(X, T_{\mathcal{U}})$ and $\xi = \{(r,s)IFQ\theta int^\star_{\mathcal{U}}(A)\} \cup 0_\sim$ be a collection of intuitionistic fuzzy sets.Then A is said to be (r,s) intuitionistic fuzzy quasi uniform $\theta^\star$ open set (inshort, $(r,s)IFQU\theta^\star$OS) if $A \in \xi$.

Definition 3.4. Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set. Then the intuitionistic fuzzy quasi uniform regular $\theta^\star$ interior of A is denoted and defined by
$(r,s)IFQr\theta^\star int_{\mathcal{U}}(A) = \cup\{B/B \subseteq A$ and B is a(r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set where $r \in I_0$,$s \in I_1$ with $r+s \leq 1\}$.

Definition 3.5. Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set. Then the intuitionistic fuzzy quasi uniform regular $\theta^\star$ closure of A is denoted and defined by
$(r,s)IFQr\theta^\star cl_{\mathcal{U}}(A) = \cap\{B/B \supseteq A$ and B is a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ closed set where $r \in I_0$,$s \in I_1$ with $r+s \leq 1$ $\}$.

Note 3.6. Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set. Then, $(r,s)IFQr\theta^\star int_{\mathcal{U}}(A) \subseteq A \subseteq (r,s)IFQr\theta^\star cl_{\mathcal{U}}$ (A).

Proposition 3.7. *Every (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set is a (r,s) intuitionistic fuzzy quasi uniform open set.*

Proof. Proof is straight forward. □

Remark 3.8. The converse of the Proposition 3.7 need not be true. See Example 3.9.

Example 3.9. Let r=0.03 and s=0.06.Let $X = \{a, b\}$ be a non empty set, $B = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.5})\rangle$, $C = \langle x, (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5})\rangle$. Let $\Omega_X : \zeta^X \to \zeta^X$ be an intuitionistic fuzzy mapping. Now f_1, f_2 and $f_3 \in \Omega_X$ are defined as follows.

$$f_1(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ B & \text{if } A \subseteq B \\ 1_\sim & \text{otherwise} \end{cases}$$

$$f_2(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ C & \text{if } A \subseteq C \\ 1_\sim & \text{otherwise} \end{cases}$$

$$f_3(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ 1_\sim & \text{otherwise} \end{cases}$$

$$\mathcal{U}(f) = \begin{cases} 1^\sim & \text{if } f = f_3 \\ \langle 0.51, 0.21\rangle & \text{if } f = f_1 \\ \langle 0.66, 0.2\rangle & \text{if } f = f_2 \\ \langle 0.66, 0.2\rangle & \text{if } f = f_1 \sqcap f_2 \\ 0^\sim & \text{otherwise} \end{cases}$$

and $(X, \mathcal{U})$ be an intuitionistic fuzzy quasi uniform space. Define $T_\mathcal{U} : \zeta^X \to I_0 \times I_1$ an intuitionistic fuzzy mapping as

$$T_\mathcal{U}(A) = \begin{cases} 1^\sim & \text{if } A = 0_\sim \\ \langle 0.51, 0.21\rangle & \text{if } A = B \\ \langle 0.66, 0.2\rangle & \text{if } A = C \\ 0^\sim & \text{otherwise} \end{cases}$$

Here C is a (r,s) intuitionistic fuzzy quasi uniform open set but it is not a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set. Hence every (r,s) intuitionistic fuzzy quasi uniform open set need not be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set.

Definition 3.10. Let $(X, T_\mathcal{U})$ and $(Y, S_\mathcal{U})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_\mathcal{U}) \to (Y, S_\mathcal{U})$ be an intuitionistic fuzzy mapping. Then f is said to be a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping if for each intuitionistic fuzzy set A of $(X, T_\mathcal{U})$ and each (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set B containing $f(A)$, there exists a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set C containing A such that $f(C) \subseteq B$.

Definition 3.11. Let $(X, T_\mathcal{U})$ be an intuitionistic fuzzy quasi uniform topological space, A and B be any two intuitionistic fuzzy sets. Then A is said to be a (r,s) intuitionistic fuzzy quasi uniform regular G_δ neighbourhood of B if and only if there is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set C such that $B \subseteq C \subseteq A$.

Proposition 3.12. *Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{U}})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{U}})$ be an intuitionistic fuzzy mapping. Then the following are equivalent.*

(1) f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping .

(2) $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set in $(X, T_{\mathcal{U}})$ for each (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set A in $(Y, S_{\mathcal{U}})$.

(3) $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular F_σ set in $(X, T_{\mathcal{U}})$ for each (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ closed set A in $(Y, S_{\mathcal{U}})$.

(4) $(r, s)IFrG_\delta cl_{\mathcal{U}}(f^{-1}(A)) \subseteq f^{-1}((r, s)IFQr\theta^\star cl_{\mathcal{U}}(A))$ for every intuitionistic fuzzy set A in $(Y, S_{\mathcal{U}})$.

(5) $f^{-1}((r, s)IFQr\theta^\star int_{\mathcal{U}}(A)) \subseteq (r, s)IFQrG_\delta int_{\mathcal{U}}(f^{-1}A))$ for every intuitionistic fuzzy set A in $(Y, S_{\mathcal{U}})$.

Proof. (1) ⇒ (2) Let A be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set and B be an intuitionistic fuzzy set in $(X, T_{\mathcal{U}})$ such that $B \subseteq f^{-1}(A)$.Since f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping, there exists a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set C in $(X, T_{\mathcal{U}})$ with $B \subseteq C$ such that $f(C) \subseteq A$. Then $B \subseteq C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(A)$. Here, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ neighbourhood of B. Hence, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set in $(X, T_{\mathcal{U}})$.

(2) ⇒ (3) Let A be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ closed set in $(Y, S_{\mathcal{U}})$. Then $\overline{A}$ is a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set in $(Y, S_{\mathcal{U}})$. Then by(2),$f^{-1}(\overline{A})$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set in $(X, T_{\mathcal{U}})$. That is, $\overline{f^{-1}(\overline{A})}$ is a (r,s) intuitionistic fuzzy quasi uniform regular F_σ set. Therefore $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular F_σ set in $(X, T_{\mathcal{U}})$.

(3) ⇒ (4) Let A be an intuitionistic fuzzy set in $(Y, S_{\mathcal{U}})$. Then $A \subseteq (r, s)IFQr\theta^\star cl_{\mathcal{U}}(A)$.
Hence $f^{-1}(A) \subseteq f^{-1}((r, s)IFQr\theta^\star cl_{\mathcal{U}}(A))$. Now, $(r, s)IFQr\theta^\star cl_{\mathcal{U}}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ closed set. Then by(3), $f^{-1}((r, s)IFQr\theta^\star cl_{\mathcal{U}}(A))$ is a (r,s) intuitionistic fuzzy quasi uniform regular $F_\sigma$set in $(X, T_{\mathcal{U}})$.Thus,

$$\begin{aligned}(r, s)IFrG_\delta cl_{\mathcal{U}}(f^{-1}A)) &\subseteq (r, s)IFrG_\delta cl_{\mathcal{U}} f^{-1}((r, s)IFQr\theta^\star cl_{\mathcal{U}}(A)) \\ &= f^{-1}((r, s)IFQr\theta^\star cl_{\mathcal{U}}(A))\end{aligned}$$

$$That is, (r, s)IFrG_\delta cl_{\mathcal{U}}(f^{-1}(A)) \subseteq f^{-1}((r, s)IFQr\theta^\star cl_{\mathcal{U}}(A)).$$

(4) ⇒ (5) Taking complement of (4) we get,

$$\overline{f^{-1}((r, s)IFQr\theta^\star cl_{\mathcal{U}}(A))} \subseteq \overline{(r, s)IFQrG_\delta cl_{\mathcal{U}}(f^{-1}(A))}$$

$$f^{-1}(\overline{((r, s)IFQr\theta^\star cl_{\mathcal{U}}(A))}) \subseteq (r, s)IFQrG_\delta int_{\mathcal{U}} f^{-1}(\overline{(A)})$$

$$f^{-1}((r, s)IFQr\theta^\star int_{\mathcal{U}}(\overline{A}) \subseteq (r, s)IFQrG_\delta int_{\mathcal{U}} \overline{f^{-1}(A)}.$$

$$Put \overline{A} = B, Then, f^{-1}((r, s)IFQr\theta^\star int_{\mathcal{U}}(B)) \subseteq (r, s)IFQrG_\delta int_{\mathcal{U}}(f^{-1}(B)).$$

(5) ⇒ (1) Let A be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set in $(Y, S_{\mathcal{U}})$. Now by (5), $f^{-1}((r, s)IFQr\theta^\star int_{\mathcal{U}}(A)) = f^{-1}(A) \subseteq (r, s)IFQrG_\delta int_{\mathcal{U}}(f^{-1}(A))$.

But $(r,s)IFQrG_\delta int_{\mathcal{U}}(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence, $f^{-1}(A) = (r,s)IFQrG_\delta int_{\mathcal{U}}(f^{-1}(A))$.
Hence, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set. Let B be an intuitionistic fuzzy set in $(X, T_{\mathcal{U}})$ such that $B \subseteq f^{-1}(A)$. Then, $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Thus for any intuitionistic fuzzy set B in $(X, T_{\mathcal{U}})$ and for each (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set A containing $f(B)$, there exists a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set $f^{-1}(A)$ containing B such that $f(f^{-1}(A)) \subseteq A$. Thus, f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping. □

Definition 3.13. Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{U}})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{U}})$ be an intuitionistic fuzzy mapping. Then f is said to be a

(1) (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping if for every (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set A in $(Y, S_{\mathcal{U}})$, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular open set in $(X, T_{\mathcal{U}})$.

(2) (r,s) intuitionistic fuzzy quasi uniform weakly regular $\theta^\star$ continuous mapping if for every (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set A in $(Y, S_{\mathcal{U}})$,$f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform open set in $(X, T_{\mathcal{U}})$.

Proposition 3.14. *Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{U}})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{U}})$ be an intuitionistic fuzzy mapping. If f is a (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping then f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping.*

Proof. Let A be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set in $(Y, S_{\mathcal{U}})$ .Since f is a (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping,$f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular open set in $(X, T_{\mathcal{U}})$. Since every (r,s) intuitionistic fuzzy quasi uniform regular open set is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set in $(X, T_{\mathcal{U}})$.Hence, f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping. □

Remark 3.15. The converse of the Proposition 3.14 need not be true. See Example 3.16.

Example 3.16. Let r=0.03 and s=0.06. Let $X = \{a, b\}$ be a non empty set ,$B = \langle x, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.4}, \frac{b}{0.4})\rangle$,$C = \langle x, (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4})\rangle$. Let $\Omega_X : \zeta^X \to \zeta^X$ be an intuitionistic fuzzy mapping. Now f_1, f_2 and $f_3 \in \Omega_X$ are defined as follows.

$$f_1(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ B & \text{if } A \subseteq B \\ 1_\sim & \text{otherwise} \end{cases}$$

$$f_2(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ C & \text{if } A \subseteq C \\ 1_\sim & \text{otherwise} \end{cases}$$

$$f_3(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ 1_\sim & \text{otherwise} \end{cases}$$

$$\mathcal{U}(f) = \begin{cases} 1^{\sim} & \text{if } f = f_3 \\ \langle 0.5, 0.21\rangle & \text{if } f = f_1 \\ \langle 0.66, 0.2\rangle & \text{if } f = f_2 \\ \langle 0.66, 0.2\rangle & \text{if } f = f_1 \sqcap f_2 \\ 0^{\sim} & \text{otherwise} \end{cases}$$

and $(X, \mathcal{U})$ be an intuitionistic fuzzy quasi uniform space. Define $T_{\mathcal{U}} : \zeta^X \to I_0 \times I_1$ an intuitionistic fuzzy mapping as

$$T_{\mathcal{U}}(A) = \begin{cases} 1^{\sim} & \text{if } A = 0_{\sim} \\ \langle 0.51, 0.21\rangle & \text{if } A = B \\ \langle 0.66, 0.2\rangle & \text{if } A = C \\ 0^{\sim} & \text{otherwise} \end{cases}$$

Let $Y = \{c, d\}$ be a non empty set $E = \langle y, (\frac{c}{0.3}, \frac{d}{0.4}), (\frac{c}{0.5}, \frac{d}{0.5})\rangle$, $F = \langle y, (\frac{c}{0.5}, \frac{d}{0.5}), (\frac{c}{0.4}, \frac{d}{0.4})\rangle$. Let $\Omega_Y : \zeta^Y \to \zeta^Y$ be an intuitionistic fuzzy mapping. Now g_1, g_2 and $g_3 \in \Omega_Y$ are defined as follows.

$$g_1(D) = \begin{cases} 0_{\sim} & \text{if } D = 0_{\sim} \\ E & \text{if } D \subseteq E \\ 1_{\sim} & \text{otherwise} \end{cases}$$

$$g_2(D) = \begin{cases} 0_{\sim} & \text{if } D = 0_{\sim} \\ F & \text{if } D \subseteq F \\ 1_{\sim} & \text{otherwise} \end{cases}$$

$$g_3(D) = \begin{cases} 0_{\sim} & \text{if } D = 0_{\sim} \\ 1_{\sim} & \text{otherwise} \end{cases}$$

$$\mathcal{V}(g) = \begin{cases} 1^{\sim} & \text{if } g = g_3 \\ \langle 1/2, 1/4\rangle & \text{if } g = g_1 \\ \langle 2/3, 1/4\rangle & \text{if } g = g_2 \\ \langle 2/3, 1/4\rangle & \text{if } g = g_1 \sqcap g_2 \\ 0^{\sim} & \text{otherwise} \end{cases}$$

and $(Y, \mathcal{V})$ be an intuitionistic fuzzy quasi uniform space. Define $T_{\mathcal{V}} : \zeta^Y \to I_0 \times I_1$ an intuitionistic fuzzy mapping as

$$T_{\mathcal{V}}(D) = \begin{cases} 1^{\sim} & \text{if } D = 0_{\sim} \\ \langle 1/2, 1/4\rangle & \text{if } D = E \\ \langle 2/3, 1/4\rangle & \text{if} D = F \\ 0^{\sim} & \text{otherwise} \end{cases}$$

Then $(Y, S_{\mathcal{V}})$ is an intuitionistic fuzzy quasi uniform topological space. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ be an intuitionistic fuzzy mapping defined as $f(a) = c$ and $f(b) = d$. Now f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping. Here, $f^{-1}(E) = E$ and E is not a (r,s) intuitionistic fuzzy quasi uniform regular open set. Hence, f is not a (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping.

Proposition 3.17. *Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{U}})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{U}})$ be an intuitionistic fuzzy mapping. If f is a (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping then f is a (r,s) intuitionistic fuzzy quasi uniform weakly regular $\theta^\star$ continuous mapping.*

Proof. Let A be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set in $(Y, S_{\mathcal{U}})$. Since f is a (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping,$f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular open set in $(X, T_{\mathcal{U}})$. Since every (r,s) intuitionistic fuzzy quasi uniform regular open set is a (r,s) intuitionistic fuzzy quasi uniform open set, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform open set in $(X, T_{\mathcal{U}})$. Hence, f is a (r,s) intuitionistic fuzzy quasi uniform weakly regular $\theta^\star$ continuous mapping. □

Remark 3.18. The converse of the Proposition 3.17 need not be true. See Example 3.19.

Example 3.19. Let r=0.03 and s=0.06. Let $X = \{a, b\}$ be a non empty set, $B = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5})\rangle$,$C = \langle x, (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5})\rangle$. Let $\Omega_X : \zeta^X \to \zeta^X$ be an intuitionistic fuzzy mapping. Now f_1, f_2 and $f_3 \in \Omega_X$ are defined as follows.

$$f_1(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ B & \text{if } A \subseteq B \\ 1_\sim & \text{otherwise} \end{cases}$$

$$f_2(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ C & \text{if } A \subseteq C \\ 1_\sim & \text{otherwise} \end{cases}$$

$$f_3(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ 1_\sim & \text{otherwise} \end{cases}$$

$$\mathcal{U}(f) = \begin{cases} 1^\sim & \text{if } f = f_3 \\ \langle 0.41, 0.21\rangle & \text{if } f = f_1 \\ \langle 0.55, 0.2\rangle & \text{if } f = f_2 \\ \langle 0.55, 0.2\rangle & \text{if } f = f_1 \sqcap f_2 \\ 0^\sim & \text{otherwise} \end{cases}$$

and $(X, \mathcal{U})$ be an intuitionistic fuzzy quasi uniform space. Define $T_{\mathcal{U}} : \zeta^X \to I_0 \times I_1$ an intuitionistic fuzzy mapping as

$$T_{\mathcal{U}}(A) = \begin{cases} 1^\sim & \text{if } A = 0_\sim \\ \langle 0.41, 0.21\rangle & \text{if } A = B \\ \langle 0.55, 0.2\rangle & \text{if } A = C \\ 0^\sim & \text{otherwise} \end{cases}$$

Let $Y = \{c, d\}$ be a non empty set $E = \langle y, (\frac{c}{0.5}, \frac{d}{0.4}), (\frac{c}{0.5}, \frac{d}{0.6})\rangle$, $F = \langle y, (\frac{c}{0.5}, \frac{d}{0.2}), (\frac{c}{0.5}, \frac{d}{0.6})\rangle$. Let $\Omega_Y : \zeta^Y \to \zeta^Y$ be an intuitionistic fuzzy mapping. Now g_1, g_2 and $g_3 \in \Omega_Y$ are defined as follows.

$$g_1(D) = \begin{cases} 0_\sim & \text{if } D = 0_\sim \\ E & \text{if } D \subseteq E \\ 1_\sim & \text{otherwise} \end{cases}$$

$$g_2(D) = \begin{cases} 0_\sim & \text{if } D = 0_\sim \\ F & \text{if } D \subseteq F \\ 1_\sim & \text{otherwise} \end{cases}$$

$$g_3(D) = \begin{cases} 0_\sim & \text{if } D = 0_\sim \\ 1_\sim & \text{otherwise} \end{cases}$$

$$\mathcal{V}(g) = \begin{cases} 1^\sim & \text{if } g = g_3 \\ \langle 2/3, 1/4\rangle & \text{if } g = g_1 \\ \langle 1/2, 1/4\rangle & \text{if } g = g_2 \\ \langle 2/3, 1/4\rangle & \text{if } g = g_1 \sqcap g_2 \\ 0^\sim & \text{otherwise} \end{cases}$$

and $(Y, \mathcal{V})$ be an intuitionistic fuzzy quasi uniform space. Define $T_\mathcal{V} : \zeta^Y \to I_0 \times I_1$ an intuitionistic fuzzy mapping as

$$T_\mathcal{V}(D) = \begin{cases} 1^\sim & \text{if } D = 0_\sim \\ \langle 2/3, 1/4\rangle & \text{if } D = E \\ \langle 1/2, 1/4\rangle & \text{if} D = F \\ 0^\sim & \text{otherwise} \end{cases}$$

Then $(Y, S_\mathcal{V})$ is an intuitionistic fuzzy quasi uniform topological space. Let $f : (X, T_\mathcal{U}) \to (Y, S_\mathcal{V})$ be an intuitionistic fuzzy mapping defined as $f(a) = d$ and $f(b) = c$. Now f is a (r,s) intuitionistic fuzzy quasi uniform weakly regular $\theta^\star$ continuous mapping. Here, $f^{-1}(E) = B$ and B is not a (r,s) intuitionistic fuzzy quasi uniform regular open set. Hence, f is not a (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping.

Remark 3.20. (r,s) intuitionistic fuzzy quasi uniform weakly regular $\theta^\star$ continuous mapping and (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping are independent to each other as shown in Example 3.21 and 3.22.

Example 3.21. Let r=0.03 and s=0.06.Let $X = \{a, b\}$ be a non empty set ,$B = \langle x, (\frac{a}{0.4}, \frac{b}{0.3}), (\frac{a}{0.4}, \frac{b}{0.5})\rangle$,$C = \langle x, (\frac{a}{0.4}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.5})\rangle$ .Let $\Omega_X : \zeta^X \to \zeta^X$ be an intuitionistic fuzzy mapping. Now f_1, f_2 and $f_3 \in \Omega_X$ are defined as follows.

$$f_1(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ B & \text{if } A \subseteq B \\ 1_\sim & \text{otherwise} \end{cases}$$

$$f_2(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ C & \text{if } A \subseteq C \\ 1_\sim & \text{otherwise} \end{cases}$$

$$f_3(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ 1_\sim & \text{otherwise} \end{cases}$$

$$\mathcal{U}(f) = \begin{cases} 1^\sim & \text{if } f = f_3 \\ \langle 0.42, 0.21\rangle & \text{if } f = f_1 \\ \langle 0.54, 0.2\rangle & \text{if } f = f_2 \\ \langle 0.54, 0.2\rangle & \text{if } f = f_1 \sqcap f_2 \\ 0^\sim & \text{otherwise} \end{cases}$$

and $(X,\mathcal{U})$ be an intuitionistic fuzzy quasi uniform space. Define $T_{\mathcal{U}} : \zeta^X \to I_0 \times I_1$ an intuitionistic fuzzy mapping as

$$T_{\mathcal{U}}(A) = \begin{cases} 1^{\sim} & \text{if } A = 0_{\sim} \\ \langle 0.42, 0.21\rangle & \text{if } A = B \\ \langle 0.54, 0.2\rangle & \text{if } A = C \\ 0^{\sim} & \text{otherwise} \end{cases}$$

Let $Y = \{c, d\}$ be a non empty set and $E = \langle y, (\frac{c}{0.4}, \frac{d}{0.4}), (\frac{c}{0.4}, \frac{d}{0.5})\rangle$ $F = \langle y, (\frac{c}{0.6}, \frac{d}{0.5}), (\frac{c}{0.4}, \frac{d}{0.5})\rangle$. Let $\Omega_Y : \zeta^Y \to \zeta^Y$ be an intuitionistic fuzzy mapping. Now g_1, g_2 and $g_3 \in \Omega_Y$ are defined as follows.

$$g_1(D) = \begin{cases} 0_{\sim} & \text{if } D = 0_{\sim} \\ E & \text{if } D \subseteq B \\ 1_{\sim} & \text{otherwise} \end{cases}$$

$$g_2(D) = \begin{cases} 0_{\sim} & \text{if } D = 0_{\sim} \\ F & \text{if } D \subseteq F \\ 1_{\sim} & \text{otherwise} \end{cases}$$

$$g_3(D) = \begin{cases} 0_{\sim} & \text{if } D = 0_{\sim} \\ 1_{\sim} & \text{otherwise} \end{cases}$$

$$\mathcal{V}(g) = \begin{cases} 1^{\sim} & \text{if } g = g_3 \\ \langle 1/2, 1/3\rangle & \text{if } g = g_1 \\ \langle 2/3, 1/3\rangle & \text{if } g = g_2 \\ \langle 2/3, 1/3\rangle & \text{if } g = g_1 \sqcap g_2 \\ 0^{\sim} & \text{otherwise} \end{cases}$$

and $(Y, \mathcal{V})$ be an intuitionistic fuzzy quasi uniform space. Define $T_{\mathcal{V}} : \zeta^Y \to I_0 \times I_1$ an intuitionistic fuzzy mapping as

$$T_{\mathcal{V}}(D) = \begin{cases} 1^{\sim} & \text{if } D = 0_{\sim} \\ \langle 1/2, 1/3\rangle & \text{if } D = E \\ \langle 2/3, 1/3\rangle & \text{if} D = F \\ 0^{\sim} & \text{otherwise} \end{cases}$$

Then $(Y, S_{\mathcal{V}})$ is an intuitionistic fuzzy quasi uniform topological space. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ be an intuitionistic fuzzy mapping defined as $f(a) = c$ and $f(b) = d$. Now f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping. Here, $f^{-1}(E) = E$ but E is not a (r,s) intuitionistic fuzzy quasi uniform open set in $(X, T_{\mathcal{U}})$. Hence, f is not a (r,s) intuitionistic fuzzy quasi uniform weakly regular $\theta^\star$ continuous mapping.

Example 3.22. Let r=0.02 and s=0.061. Let $X = \{a, b\}$ be a non empty set ,$B = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5})\rangle$,$C = \langle x, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.4})\rangle$. Let $\Omega_X : \zeta^X \to \zeta^X$ be an intuitionistic fuzzy mapping. Now f_1, f_2 and $f_3 \in \Omega_X$ are defined as follows.

$$f_1(A) = \begin{cases} 0_{\sim} & \text{if } A = 0_{\sim} \\ B & \text{if } A \subseteq B \\ 1_{\sim} & \text{otherwise} \end{cases}$$

$$f_2(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ C & \text{if } A \subseteq C \\ 1_\sim & \text{otherwise} \end{cases}$$

$$f_3(A) = \begin{cases} 0_\sim & \text{if } A = 0_\sim \\ 1_\sim & \text{otherwise} \end{cases}$$

$$\mathcal{U}(f) = \begin{cases} 1^\sim & \text{if } f = f_3 \\ \langle 0.32, 0.22\rangle & \text{if } f = f_1 \\ \langle 0.52, 0.2\rangle & \text{if } f = f_2 \\ \langle 0.52, 0.2\rangle & \text{if } f = f_1 \sqcap f_2 \\ 0^\sim & \text{otherwise} \end{cases}$$

and $(X,\mathcal{U})$ be an intuitionistic fuzzy quasi uniform space. Define $T_{\mathcal{U}} : \zeta^X \to I_0 \times I_1$ an intuitionistic fuzzy mapping as

$$T_{\mathcal{U}}(A) = \begin{cases} 1^\sim & \text{if } A = 0_\sim \\ \langle 0.32, 0.22\rangle & \text{if } A = B \\ \langle 0.52, 0.2\rangle & \text{if } A = C \\ 0^\sim & \text{otherwise} \end{cases}$$

Let $Y = \{c, d\}$ be a non empty set and $E = \langle y, (\frac{c}{0.3}, \frac{d}{0.4}), (\frac{c}{0.4}, \frac{d}{0.5})\rangle$ $F = \langle y, (\frac{c}{0.4}, \frac{d}{0.4}), (\frac{c}{0.4}, \frac{d}{0.5})\rangle$. Let $\Omega_Y : \zeta^Y \to \zeta^Y$ be an intuitionistic fuzzy mapping. Now g_1, g_2 and $g_3 \in \Omega_Y$ are defined as follows.

$$g_1(D) = \begin{cases} 0_\sim & \text{if } D = 0_\sim \\ E & \text{if } D \subseteq B \\ 1_\sim & \text{otherwise} \end{cases}$$

$$g_2(D) = \begin{cases} 0_\sim & \text{if } D = 0_\sim \\ F & \text{if } D \subseteq F \\ 1_\sim & \text{otherwise} \end{cases}$$

$$g_3(D) = \begin{cases} 0_\sim & \text{if } D = 0_\sim \\ 1_\sim & \text{otherwise} \end{cases}$$

$$\mathcal{V}(g) = \begin{cases} 1^\sim & \text{if } g = g_3 \\ \langle 1/4, 1/3\rangle & \text{if } g = g_1 \\ \langle 2/3, 1/3\rangle & \text{if } g = g_2 \\ \langle 2/3, 1/3\rangle & \text{if } g = g_1 \sqcap g_2 \\ 0^\sim & \text{otherwise} \end{cases}$$

and $(Y, \mathcal{V})$ be an intuitionistic fuzzy quasi uniform space. Define $T_{\mathcal{V}} : \zeta^Y \to I_0 \times I_1$ an intuitionistic fuzzy mapping as

$$T_{\mathcal{V}}(D) = \begin{cases} 1^\sim & \text{if } D = 0_\sim \\ \langle 1/4, 1/3\rangle & \text{if } D = E \\ \langle 2/3, 1/3\rangle & \text{if} D = F \\ 0^\sim & \text{otherwise} \end{cases}$$

Then $(Y, S_{\mathcal{V}})$ is an intuitionistic fuzzy quasi uniform topological space. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ be an intuitionistic fuzzy mapping defined as $f(a) = f(b) = d$. Now, f is a (r,s) intuitionistic fuzzy quasi uniform weakly regular $\theta^\star$ continuous mapping. Here, $f^{-1}(F) = B$ but B is not a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set in $(X, T_{\mathcal{U}})$. Hence, f is not a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping.

Definition 3.23. Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space. Then $(X, T_{\mathcal{U}})$ is said to be an (r,s) intuitionistic fuzzy quasi uniform RG_δ compact space if for every (r,s) intuitionistic fuzzy quasi uniform regular G_δ cover $\{V_i/i \in J\}$ of $(X, T_{\mathcal{U}})$ there is a finite subset J_0 of J such that $\cup_{i\in J_0} V_i = 1_\sim$.

Definition 3.24. Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space and $\{V_i/i \in J\}$ be a family of (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open sets in $(X, T_{\mathcal{U}})$. Then $\{V_i/i \in J\}$ is said to be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open cover of $(X, T_{\mathcal{U}})$ if $\cup_{i\in J} V_i = 1_\sim$

Definition 3.25. Let $(X, T_{\mathcal{U}})$ be a (r,s) intuitionistic fuzzy quasi uniform topological space. Then $(X, T_{\mathcal{U}})$ is said to be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ compact space if for every (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open cover $\{V_i/i \in J\}$ of $(X, T_{\mathcal{U}})$ there exists a finite subset J_0 of J such that $\cup_{i\in J_0} V_i = 1_\sim$

Proposition 3.26. *Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ be a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous surjection mapping from a (r,s) intuitionistic fuzzy quasi uniform RG_δ compact space $(X, T_{\mathcal{U}})$ to an intuitionistic fuzzy quasi uniform topological space $(Y, S_{\mathcal{V}})$.Then $(Y, S_{\mathcal{V}})$ is a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ compact space.*

Proof. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ be a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous surjection mapping and $\{V_i/i \in J\}$ be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open cover of $(Y, S_{\mathcal{V}})$.Then $\{f^{-1}(V_i)/i \in J\}$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ cover of $(X, T_{\mathcal{V}})$. Since $(X, T_{\mathcal{U}})$ is a (r,s) intuitionistic fuzzy quasi uniform RG_δ compact space, there exists a finite subset J_0 of J such that $\cup_{i\in J_0} f^{-1}(V_i) = 1_\sim$.

$$\begin{aligned} 1_\sim &= f(1_\sim) = f(\cup_{i\in J_0} f^{-1}(V_i)) \\ &\subseteq \cup_{i\in J_0} f(f^{-1}(V_i)) \\ &= \cup_{i\in J_0} V_i \end{aligned}$$

Hence, $(Y, S_{\mathcal{V}})$ is a(r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ compact space. □

Definition 3.27. Let $(X, T_{\mathcal{U}})$ be a (r,s) intuitionistic fuzzy quasi uniform topological space, A and B be any two intuitionistic fuzzy sets of $(X, T_{\mathcal{U}})$. Then A and B are said to be (r,s) intuitionistic fuzzy quasi uniform weakly* regular G_δ separeted if there are (r,s) intuitionistic fuzzy quasi uniform regular G_δ sets C and D such that $A \subseteq C$, $B \subseteq D$, AqD and BqC.

Definition 3.28. Let $(X, T_{\mathcal{U}})$ be a (r,s) intuitionistic fuzzy quasi uniform topological space ,A and B be any two intuitionistic fuzzy sets of $(X, T_{\mathcal{U}})$.Then A and B are said to be (r,s) intuitionistic fuzzy quasi uniform weakly* regular $\theta^\star$ separeted if there are (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open sets C and D such that $A \subseteq C$,$B \subseteq D$, AqD and BqC.

Definition 3.29. Let $(X, T_{\mathcal{U}})$ be a (r,s) intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set in $(X, T_{\mathcal{U}})$. Then A is said to be (r,s) intuitionistic fuzzy quasi uniform regular G_δ connected if and only if A cannot be expressed as the union of two (r,s) intuitionistic fuzzy quasi uniform weakly* regular G_δ separeted sets.

Definition 3.30. Let $(X, T_{\mathcal{U}})$ be a (r,s) intuitionistic fuzzy quasi uniform topological space and A be an intuitionistic fuzzy set in $(X, T_{\mathcal{U}})$. Then A is said to be (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ connected if and only if A cannot be expressed as the union of two (r,s) intuitionistic fuzzy quasi uniform weakly* regular $\theta^\star$ separeted sets.

Proposition 3.31. *Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{U}})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{U}})$ be a surjective (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping. If an intuitionistic fuzzy set A is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ connected set in $(X, T_{\mathcal{U}})$ then $f(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ connected set in $(Y, S_{\mathcal{U}})$.*

Proof. Suppose $f(A)$ is not a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ connected set in $(Y, S_{\mathcal{U}})$. Then there exists (r,s) intuitionistic fuzzy quasi uniform weakly* regular $\theta^\star$ separeted sets B and C in $(Y, S_{\mathcal{U}})$ such that $f(A) = B \cup C$. Since f is (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping,$f^{-1}(B)$ and $f^{-1}(C)$ are (r,s) intuitionistic fuzzy quasi uniform regular G_δ sets in $(X, T_{\mathcal{U}})$. Now $A = f^{-1}(f(A)) = f^{-1}(B \cup C) = f^{-1}(B) \cup f^{-1}(C)$. Since $f^{-1}(B)$ and $f^{-1}(C)$ are (r,s) intuitionistic fuzzy quasi uniform weakly* regular G_δ separeted sets in $(X, T_{\mathcal{U}})$. A is not a (r,s) intuitionistic fuzzy quasi uniform regular G_δ connected sets in $(X, T_{\mathcal{U}})$ which is a contradiction.Hence $f(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ connected set in $(Y, S_{\mathcal{U}})$. □

Definition 3.32. Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space. Then $(X, T_{\mathcal{U}})$ is called a (r,s) intuitionistic fuzzy quasi uniform regular G_δ T_2 space if and only if for every pair of non zero intuitionistic fuzzy sets A and B with $A \cap B = 0_\sim$, there exist a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set C and D such that $A \cap C \supseteq 0_\sim$,$B \cap D \supseteq 0_\sim$ and $C \cap D = 0_\sim$.

Definition 3.33. Let $(X, T_{\mathcal{U}})$ be an intuitionistic fuzzy quasi uniform topological space. Then $(X, T_{\mathcal{U}})$ is called a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ T_2 space if and only if for every pair of non zero intuitionistic fuzzy sets A and B with $A \cap B = 0_\sim$,there exist a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set C and D such that $A \cap C \supseteq 0_\sim$, $B \cap D \supseteq 0_\sim$ and $C \cap D = 0_\sim$.

Proposition 3.34. *Let $(X, T_{\mathcal{U}})$, $(Y, S_{\mathcal{U}})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{U}})$ be a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping. If $(Y, S_{\mathcal{U}})$ is a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ T_2 space then $(X, T_{\mathcal{U}})$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ T_2 space.*

Proof. Let A and B be any two intuitionistic fuzzy sets in $(X, T_{\mathcal{U}})$ such that $A \cap B = 0_\sim$. Hence $f(A) \cap f(B) = 0_\sim$. Since $(Y, S_{\mathcal{U}})$ is a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ T_2 space,there exists (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open sets C and D such that $f(A) \cap C \supseteq 0_\sim$, $f(B) \cap D \supseteq 0_\sim$ and $C \cap D = 0_\sim$.

Since f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping, $f^{-1}(C)$ and $f^{-1}(D)$ are (r,s) intuitionistic fuzzy quasi uniform regular G_δ sets in $(X, T_\mathcal{U})$.Therefore,

$$f^{-1}(f(A) \cap C) \supseteq f^{-1}(0_\sim)$$
$$f^{-1}(f(A)) \cap f^{-1}(C) \supseteq 0_\sim$$
$$A \cap f^{-1}(C) \supseteq 0_\sim$$

Similarly, $B \cap f^{-1}(D) \supseteq 0_\sim$.Also,$f^{-1}(C \cap D) = f^{-1}(0_\sim) = 0_\sim$. Hence, $f^{-1}(C) \cap f^{-1}(D) = 0_\sim$. Therefore, $(X, T_\mathcal{U})$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ T_2 space. □

Proposition 3.35. *Let $(X, T_\mathcal{U})$ and $(Y, S_\mathcal{U})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_\mathcal{U}) \to (Y, S_\mathcal{U})$ be a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping.If $(X, T_\mathcal{U})$ is a (r,s)intuitionistic fuzzy quasi uniform regular G_δ $T_{\mathcal{U}_{1/2}}$ space then f is a (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping.*

Proof. Let A be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set in $(Y, S_\mathcal{U})$. Since f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set in $(X, T_\mathcal{U})$. Also, $(X, T_\mathcal{U})$ is a (r,s)intuitionistic fuzzy quasi uniform regular G_δ $T_{\mathcal{U}_{1/2}}$ space. Hence, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular open set in $(X, T_\mathcal{U})$. Therefore f is a (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping. □

Definition 3.36. Let $(X, T_\mathcal{U})$ be an intuitionistic fuzzy quasi uniform topological space. Then $(X, T_\mathcal{U})$ is said to be a (r,s)intuitionistic fuzzy quasi uniform $T_{\mathcal{U}_R}$ space if every (r,s) intuitionistic fuzzy quasi uniform open set $(X, T_\mathcal{U})$ is a (r,s) intuitionistic fuzzy quasi uniform regular open set.

Proposition 3.37. *Let $(X, T_\mathcal{U})$ and $(Y, S_\mathcal{U})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_\mathcal{U}) \to (Y, S_\mathcal{U})$ be a (r,s) intuitionistic fuzzy quasi uniform weakly $\theta^\star$ continuous mapping. If $(X, T_\mathcal{U})$ is a (r,s)intuitionistic fuzzy quasi uniform $T_{\mathcal{U}_R}$ space then f is a (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping.*

Proof. Let A be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set in $(Y, S_\mathcal{U})$. Since f is a (r,s) intuitionistic fuzzy quasi uniform weakly regular $\theta^\star$ continuous mapping, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform open set in $(X, T_\mathcal{U})$. Also, $(X, T_\mathcal{U})$ is a (r,s)intuitionistic fuzzy quasi uniform $T_{\mathcal{U}_R}$ space. Hence, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular open set in $(X, T_\mathcal{U})$. Therefore f is a (r,s) intuitionistic fuzzy quasi uniform faintly r continuous mapping. □

Proposition 3.38. *Let $(X, T_\mathcal{U})$ and $(Y, S_\mathcal{U})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_\mathcal{U}) \to (Y, S_\mathcal{U})$ be a (r,s) intuitionistic fuzzy* *quasi uniform weakly $\theta^\star$ continuous mapping. If $(X, T_\mathcal{U})$ is a (r,s)intuitionistic fuzzy quasi uniform $T_{\mathcal{U}_R}$ space then f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping.*

Proof. Proof follows from the Propositions 3.14 and 3.37. □

Proposition 3.39. *Let $(X, T_\mathcal{U})$ and $(Y, S_\mathcal{U})$ be any two intuitionistic fuzzy quasi uniform topological spaces and $f : (X, T_\mathcal{U}) \to (Y, S_\mathcal{U})$ be a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping. If $(X, T_\mathcal{U})$ is a (r,s)intuitionistic fuzzy quasi uniform $T_{\mathcal{U}_{1/2}}$ space then f is a (r,s) intuitionistic fuzzy quasi uniform weakly regular $\theta^\star$ continuous mapping.*

Proof. Let A be a (r,s) intuitionistic fuzzy quasi uniform regular $\theta^\star$ open set in $(Y, S_\mathcal{U})$. Since f is a (r,s) intuitionistic fuzzy quasi uniform faintly regular G_δ continuous mapping, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular G_δ set in $(X, T_\mathcal{U})$. Also, $(X, T_\mathcal{U})$ is a (r,s)intuitionistic fuzzy quasi uniform regular G_δ $T_{\mathcal{U}_{1/2}}$ space. Hence, $f^{-1}(A)$ is a (r,s) intuitionistic fuzzy quasi uniform regular open set in $(X, T_\mathcal{U})$. Hence every (r,s) intuitionistic fuzzy quasi uniform regular open set is a (r,s) intuitionistic fuzzy quasi uniform open set. Therefore f is a (r,s) intuitionistic fuzzy quasi uniform weakly regular $\theta^\star$ continuous mapping. □

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