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(λ, μ)-fuzzy prime ideals in ternary semirings

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Abstract. In this paper we introduce the notion of (λ, μ)-Fuzzy prime ideals in ternary semirings. We have shown that if P is a (λ, μ)-fuzzy ideal of a ternary semiring S and $ImP \cap (\lambda, \mu) \neq \phi$, then P is a (λ, μ)-fuzzy prime ideal of S if and only if (i) $ImP \cap [0, \lambda] = \phi$, (ii) $ImP \cap [\mu, 1] \neq \phi$, (iii) $|ImP \cap (\lambda, \mu)| = 1$, (iv) P_μ is a prime ideal of S.

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1. Introduction

The notion of ternary algebraic system was introduced by Lehmer [16] in 1932. He investigated certain ternary algebraic systems called triplexes. In 1971, Lister [17] characterized additive semigroups of rings which are closed under the triple ring product and he called this algebraic system a ternary ring. Dutta and Kar [1] introduced a notion of ternary semirings which is a generalization of ternary rings and semirings, and they studied some properties of ternary semirings [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The theory of fuzzy subsets was first studied by Zadeh [19] in 1965. Many papers on fuzzy subsets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory, etc. Kavikumar et al.[11] and [12] studied fuzzy ideals, fuzzy bi-ideals and fuzzy quasi-ideals in ternary semirings. In [13, 14, 15] we introduced the notions of fuzzy prime ideals, (λ, μ)-Fuzzy ideals, (λ, μ)-Fuzzy quasi-ideals and (λ, μ)-Fuzzy bi-ideals in ternary semirings. In this paper we introduce the notion of (λ, μ)-Fuzzy prime ideals in ternary semirings and find the relationship with other prime ideals.

2. Preliminaries

In this section, we refer to some elementary aspects of the theory of ternary semirings and fuzzy algebraic systems that are necessary for this paper.

Definition 2.1. A nonempty set S together with a binary operation called, addition $+$ and a ternary multiplication, denoted by juxtaposition, is said to be a ternary semiring if $(S, +)$ is a commutative semigroup satisfying the following conditions:
(i) $(abc)de = a(bcd)e = ab(cde)$,
(ii) $(a + b)cd = acd + bcd$,
(iii) $a(b + c)d = abd + acd$ and
(iv) $ab(c + d) = abc + abd$ for all $a, b, c, d, e \in S$.

Definition 2.2. Let S be a ternary semiring. If there exists an element $0 \in S$ such that $0 + x = x = x + 0$ and $0xy = x0y = xy0 = 0$ for all $x, y \in S$, then 0 is called the zero element or simply the zero of the ternary semiring S. In this case we say that S is a ternary semiring with zero.

Throughout this paper λ and μ $(0 \leq \lambda < \mu \leq 1)$, are arbitrary, but fixed and S denotes a ternary semiring with zero.

Definition 2.3. An additive subsemigroup T of S is called a ternary subsemiring of S if $t_1t_2t_3 \in T$ for all $t_1, t_2, t_3 \in T$.

Definition 2.4. An additive subsemigroup I of S is called a left [resp. right, lateral] ideal of S if $s_1s_2i \in I$ [resp. $is_1s_2 \in I, s_1is_2 \in I$] for all $s_1, s_2 \in S$ and $i \in I$. If I is a left, right and lateral ideal of S, then I is called an ideal of S.

It is obvious that every ideal of a ternary semiring with zero contains the zero element.

Definition 2.5. A proper ideal P of a ternary semiring S is called a prime ideal of S, if $ABC \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$ for any three ideals A, B, C of S.

Let X be a non-empty set. A map $A : X \to [0, 1]$ is called a fuzzy subset in X.

Definition 2.6. Let A, B and C be any three fuzzy subsets of a ternary semiring S. Then $A \cap B$, $A \cup B$, $A + B$, $A \cdot B \cdot C$ are fuzzy subsets of S defined by

$$(A \cap B)(x) = min\{A(x),\ B(x)\}$$
$$(A \cup B)(x) = max\{A(x),\ B(x)\}$$

$$(A + B)(x) = \begin{cases} sup\{min\{A(y), B(z)\}\} & if \quad x = y + z \\ 0 & otherwise \end{cases}$$

$$(A \cdot B \cdot C)(x) = \begin{cases} sup\{min\{A(u), B(v), C(w)\}\} & if \quad x = uvw, \\ 0 & otherwise \end{cases}$$

Definition 2.7. Let X be a nonempty set and let A be a fuzzy subset of X. Let $0 \leq t \leq 1$. Then the set $A_t = \{x \in X \ / \ A(x) \geq t\}$ is called a level set of X with respect to A.

Definition 2.8. Let A be a fuzzy subset of a ternary semiring S. Then A is called a fuzzy ternary subsemiring of S if

1. $A(x+y) \geq min\{A(x), A(y)\}$
2. $A(xyz) \geq min\{A(x), A(y), A(z)\}$ for all $x, y, z \in S$.

Definition 2.9. A fuzzy subset A of a ternary semiring S is called a fuzzy ideal of S if

(i) $A(x+y) \geq min\{A(x), A(y)\}$
(ii) $A(xyz) \geq A(x)$
(iii) $A(xyz) \geq A(z)$ and
(iv) $A(xyz) \geq A(y)$ for all $x, y, z \in S$.

A fuzzy subset A with conditions (i) and (ii) is called a fuzzy right ideal of S. If A satisfies (i)and(iii), then it is called a fuzzy left ideal of S. Also if A satisfies (i)and(iv), then it is called a fuzzy lateral ideal of S. It is clear that A is a fuzzy ideal of a ternary semiring S if and only if $A(xyz) \geq max\{A(x), A(y), A(z)\}$ for all $x, y, z \in S$.

Definition 2.10 ([13])**.** A fuzzy ideal P of a ternary semiring S is said to be a fuzzy prime ideal of S if

(i) P is not a constant function and
(ii) for any fuzzy ideals A, B, C in S if $A \cdot B \cdot C \subseteq P$, then $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$.

Based on the concept of (λ, μ)-fuzzy subrings and (λ, μ)-fuzzy ideals introduced by B.Yao [18], we introduce the following concepts which are the generalization of fuzzy subsets. Throughout this paper λ and μ $(0 \leq \lambda < \mu \leq 1)$, are arbitrary, but fixed.

Definition 2.11. Let A be a fuzzy subset of a ternary semiring S. A is called a (λ, μ)-fuzzy right(resp. left, lateral) ideal of S if

1. $A(x+y) \vee \lambda \geq A(x) \wedge A(y) \wedge \mu$
2. $A(xyz) \vee \lambda \geq A(x) \wedge \mu$ [resp. $A(xyz) \vee \lambda \geq A(z) \wedge \mu$, $A(xyz) \vee \lambda \geq A(y) \wedge \mu$] for all $x, y, z \in S$.

Every fuzzy right (resp. left, lateral) ideal is a (λ, μ)-fuzzy right (resp. left, lateral) ideal by taking $\lambda = 0$ and $\mu = 1$. But the converse is not true as the following example shows.

Example 2.12. Let S be a ternary semiring consists of non-positive integers. Let

$$A(x) = \begin{cases} 0.9 & if \;\; x = -4 \\ 0.8 & if \;\; x \in \langle -2 \rangle + \langle -5 \rangle \;\; and \;\; x \neq -4 \\ 0.3 & if \;\; x \notin \langle -2 \rangle + \langle -5 \rangle \;\; and \;\; x \neq -3 \\ 0.2 & if \;\; x = -3 \end{cases}$$

Clearly A is a (0.3, 0.8)-fuzzy right ideal. But A is not a fuzzy right ideal since $A(-64 = -4 \cdot -4 \cdot -4) < A(-4)$.

Definition 2.13. Let A be a fuzzy subset of a ternary semiring S. The (λ, μ)-characteristic function of A denoted by χ_A^* is defined as

$$\chi_A^* = \begin{cases} \mu & if \;\; x \in A \\ \lambda & otherwise \end{cases}$$

Theorem 2.14 ([14])**.** *Let A be a fuzzy subset of a ternary semiring S. Then A is a (λ, μ)-fuzzy right (resp. left, lateral) ideal of S if and only if A_t is a right (resp. left, lateral) ideal of S for all $t \in (\lambda, \mu]$ whenever nonempty.*

Theorem 2.15 ([14])**.** *χ_I^* is a (λ, μ)-fuzzy ideal of S if and only if I is an ideal in S.*

3. (λ, μ)- Fuzzy prime ideals

In this section we introduce the notion of (λ, μ)-fuzzy prime ideals in ternary semiring S.

Definition 3.1. Let A, B and C be any three fuzzy subsets of a ternary semiring S. Then the $(\lambda, \mu) -$" $\circ$ " product is defined by

$$(A \circ B \circ C)(x) = \begin{cases} sup\{min\{A(u), B(v), C(w), \mu\}\} \vee \lambda & if \;\; x = uvw, \\ 0 & otherwise. \end{cases}$$

Definition 3.2. A (λ, μ)-fuzzy ideal P of a ternary semiring S is called (λ, μ)-fuzzy prime ideal if $A \circ B \circ C \subseteq P \cup \overline{\lambda}$ implies $A \cap \overline{\mu} \subseteq P \cup \overline{\lambda}$ or $B \cap \overline{\mu} \subseteq P \cup \overline{\lambda}$ or $C \cap \overline{\mu} \subseteq P \cup \overline{\lambda}$ for all (λ, μ)-fuzzy ideals A, B, C of S where $\overline{\lambda}(x) = \lambda, \overline{\mu}(x) = \mu$ for all $x \in S$.

Note. (1) If $\lambda = 0$ and $\mu = 1$, then (λ, μ)-fuzzy prime ideal coincides with fuzzy prime ideal.
(2) Here after we denote $\overline{\lambda}$ and $\overline{\mu}$ simply as λ and μ.

Lemma 3.3. *Let I be an ideal of a ternary semiring S and let P be a (λ, μ)-fuzzy ideal of S. Then $\chi_I^* \cap \mu \subseteq P \cup \lambda$ if and only if $I \subseteq P_\mu$.*

Proof. Suppose $\chi_I^* \cap \mu \subseteq P \cup \lambda$. Then $\chi_I^*(a) \wedge \mu \leq P(a) \vee \lambda$ for all $a \in S$. Let $a \in I$ then $P(a) \geq \mu$ which implies $a \in P_\mu$. Thus $I \subseteq P_\mu$. Conversely assume $I \subseteq P_\mu$. Let $x \in I$ then $P(x) \geq \mu$. It follows that $P(x) \vee \lambda \geq \chi_I^*(x) \wedge \mu$. Thus $\chi_I^* \cap \mu \subseteq P \cup \lambda$. □

Theorem 3.4. *Let P be a (λ, μ)-fuzzy ideal of S and $ImP \cap (\lambda, \mu) = \phi$. Then P is a (λ, μ)-fuzzy prime ideal of S if and only if the level set P_μ is a prime ideal in S whenever nonempty.*

Proof. Let P be a (λ, μ)-fuzzy prime ideal. Let I_1, I_2 and I_3 be ideals in S such that $I_1 I_2 I_3 \subseteq P_\mu$. Then $\chi_{I_1}^*, \chi_{I_2}^*$ and $\chi_{I_3}^*$ are (λ, μ)-fuzzy ideals of S. Clearly $\chi_{I_1}^* \circ \chi_{I_2}^* \circ \chi_{I_3}^* \subseteq P \cup \lambda$. Since P is a (λ, μ)-fuzzy prime ideal, $\chi_{I_1}^* \cap \mu \subseteq P \cup \lambda$ or $\chi_{I_2}^* \cap \mu \subseteq P \cup \lambda$ or $\chi_{I_3}^* \cap \mu \subseteq P \cup \lambda$. Now, $\chi_{I_1}^* \cap \mu \subseteq P \cup \lambda$ implies $I_1 \subseteq P_\mu$, $\chi_{I_2}^* \cap \mu \subseteq P \cup \lambda$ implies $I_2 \subseteq P_\mu$ and $\chi_{I_3}^* \cap \mu \subseteq P \cup \lambda$ implies $I_3 \subseteq P_\mu$. Hence P_μ is a prime ideal in S. Conversely, if there exist (λ, μ)-fuzzy ideals A, B and C of S such that $A \circ B \circ C \subseteq P \cup \lambda$ but $A \cap \mu \nsubseteq P \cup \lambda$, $B \cap \mu \nsubseteq P \cup \lambda$ and $C \cap \mu \nsubseteq P \cup \lambda$ then there exist $x, y, z \in S$ such that $A(x) \wedge \mu = t > P(x) \vee \lambda$, $B(y) \wedge \mu = s > P(y) \vee \lambda$ and $C(z) \wedge \mu = r > P(z) \vee \lambda$. Then $t, s, r \in (\lambda, \mu]$ and A_t, B_s, C_r are ideals in S. Clearly $x \in A_t$ but $x \notin P_\mu$, $y \in B_s$ but $y \notin P_\mu$ and $z \in C_r$ but $z \notin P_\mu$. Since P_μ is a prime ideal, $A_tB_sC_r \nsubseteq P_\mu$. Then there exist $x_1 \in A_t$, $y_1 \in B_s$ and $z_1 \in C_r$ such that $x_1y_1z_1 \notin P_\mu$. Therefore $P(x_1y_1z_1) \leq \lambda$. Now $(A \circ B \circ C)(x_1y_1z_1) \geq A(x_1) \wedge B(y_1) \wedge C(z_1) \wedge \mu = A(x_1) \wedge \mu \wedge B(y_1) \wedge \mu \wedge C(z_1) \wedge \mu > \lambda = P(x_1y_1z_1) \vee \lambda$. This is a contradiction to $A \circ B \circ C \subseteq P \cup \lambda$. Therefore P is a (λ, μ)-fuzzy prime ideal of S. □

Corollary 3.5. *χ_P^* is a (λ, μ)-fuzzy prime ideal of S if and only if P is a prime ideal of S.*

Proof. The proof follows from Theorem 3.4, since $Im\chi_P^* = \{\lambda, \mu\}$. □

Corollary 3.6. *χ_P is a fuzzy prime ideal of S if and only if P is a prime ideal of S.*

Proof. By taking $\lambda = 0$ and $\mu = 1$ in Theorem 3.4, we get the result. □

Example 3.7. Let S be a ternary semiring consists of non-positive integers Z_0^-. Let

$$P(x) = \begin{cases} 0.9 & if \quad x = -16 \\ 0.8 & if \quad x \in \langle -2 \rangle \ \ and \ \ x \neq -16 \\ 0.2 & if \quad x = -3 \\ 0 & otherwise. \end{cases}$$

Clearly P is a (0.3, 0.8)-fuzzy prime ideal. But P is not a fuzzy prime ideal as $|ImP| > 2$.

Lemma 3.8. *Let P be a non-constant (λ, μ)-fuzzy prime ideal of S and $ImP \cap (\lambda, \mu) \neq \phi$. Then $ImP \cap [0, \lambda] = \phi$.*

Proof. Since $ImP \cap (\lambda, \mu) \neq \phi$ then there exist $a \in S$ such that $P(a) = t$ where $\lambda < t < \mu$. Suppose $ImP \cap [0, \lambda] \neq \phi$ then there exists $c \in S$ such that $P(c) \leq \lambda$. Choose $t_1, t_2 \in [0, 1]$ such that $\lambda < t_1 < t_2 < t < \mu$. By Theorem 2.14 P_t is an ideal in S. Now, we define the fuzzy subsets A, B and C as follows:

$$A(x) = \begin{cases} \mu & if \quad x \in P_t \\ \lambda & otherwise, \end{cases}$$

$B(x) = t_1$, $C(x) = t_2$, for all $x \in S$. Then

$$(A \circ B \circ C)(x) = \begin{cases} t_1 & if \quad x = uvw, u \in P_t \\ \lambda & otherwise. \end{cases}$$

Clearly A, B and C are (λ, μ)-fuzzy ideals of S and $A \circ B \circ C \subseteq P \cup \lambda$. But $A(a) \wedge \mu = \mu > t = P(a) \vee \lambda$, $B(c) \wedge \mu = t_1 > \lambda = P(c) \vee \lambda$ and $C(c) \wedge \mu = t_2 > \lambda = P(c) \vee \lambda$, this contradicts that P is a (λ, μ)-fuzzy prime ideal. Hence $ImP \cap [0, \lambda] = \phi$. □

Lemma 3.9. *Let P be a non-constant (λ, μ)-fuzzy prime ideal of S and $ImP \cap (\lambda, \mu) \neq \phi$. Then $ImP \cap [\mu, 1] \neq \phi$.*

Proof. Suppose $ImP \cap [\mu, 1] = \phi$ then $P_\mu = \phi$. By Lemma 3.8, $ImP \cap [0, \lambda] = \phi$. Since P is non constant, then there exists t_1, t_2 such that $\lambda < t_1 < t_2 < \mu$ with $P(a) = t_1; P(b) = t_2$ for some $a, b \in S$. Choose $s_1, s_2 \in [0, 1]$ such that $t_1 < s_1 < s_2 < t_2 < \mu$. By Theorem 2.14 P_t is an ideal in S. Now we define the fuzzy subsets as follows:

$$A(x) = \begin{cases} \mu & if \;\; x \in P_{t_2} \\ \lambda & otherwise, \end{cases}$$

$B(x) = s_1$, $C(x) = s_2$, for all $x \in S$. Then

$$(A \circ B \circ C)(x) = \begin{cases} s_1 & if \;\; x = uvw, u \in P_{t_2} \\ \lambda & otherwise. \end{cases}$$

Clearly A, B and C are (λ, μ)-fuzzy ideals of S and $A \circ B \circ C \subseteq P \cup \lambda$ but $A(b) \wedge \mu = \mu > t_2 = P(b) \vee \lambda$, $B(a) \wedge \mu = s_1 > t_1 = P(a) \vee \lambda$ and $C(a) \wedge \mu = s_2 > t_1 = P(a) \vee \lambda$. This contradicts that P is a (λ, μ)-fuzzy prime ideal. Therefore $ImP \cap [\mu, 1] \neq \phi$. □

Lemma 3.10. *Let P be a non-constant (λ, μ)-fuzzy prime ideal of S and $ImP \cap (\lambda, \mu) \neq \phi$. Then $|ImP \cap (\lambda, \mu)| = 1$.*

Proof. Suppose $ImP \cap (\lambda, \mu) > 1$ then there exists $t_1, t_2 \in (\lambda, \mu)$ such that $t_1 < t_2 < \mu$ with $P(a) = t_1; P(b) = t_2$ for some $a, b \in S$. Choose $s_1, s_2 \in [0, 1]$ such that $t_1 < s_1 < s_2 < t_2 < \mu$. By Theorem 2.14 P_t is an ideal in S. Now we define the fuzzy subsets as follows:

$$A(x) = \begin{cases} \mu & if \;\; x \in P_{t_2} \\ \lambda & otherwise, \end{cases}$$

$B(x) = s_1$, $C(x) = s_2$, for all $x \in S$. Then

$$(A \circ B \circ C)(x) = \begin{cases} s_1 & if \;\; x = uvw, u \in P_{t_2} \\ \lambda & otherwise. \end{cases}$$

Clearly A, B and C are (λ, μ)-fuzzy ideals of S and $A \circ B \circ C \subseteq P \cup \lambda$ but $A(b) \wedge \mu = \mu > t_2 = P(b) \vee \lambda$, $B(a) \wedge \mu = s_1 > t_1 = P(a) \vee \lambda$ and $C(a) \wedge \mu = s_2 > t_1 = P(a) \vee \lambda$. This contradicts that P is a (λ, μ)-fuzzy prime ideal. Therefore $|ImP \cap (\lambda, \mu)| = 1$. □

Lemma 3.11. *Let P be a non-constant (λ, μ)-fuzzy prime ideal of S and $ImP \cap (\lambda, \mu) \neq \phi$. Then P_μ is a prime ideal in S.*

Proof. Clearly P_μ is an ideal of S. Suppose P_μ is not a prime ideal in S then there are ideals I_1, I_2 and I_3 in S such that $I_1 I_2 I_3 \subseteq P_\mu$ but $I_1 \nsubseteq P_\mu$, $I_2 \nsubseteq P_\mu$ and $I_3 \nsubseteq P_\mu$. Then $\chi^*_{I_1}, \chi^*_{I_2}$ and $\chi^*_{I_3}$ are (λ, μ)-fuzzy ideals of S. Clearly $\chi^*_{I_1} \circ \chi^*_{I_2} \circ \chi^*_{I_3} \subseteq P \cup \lambda$. Since $I_1 \nsubseteq P_\mu$ implies $\chi^*_{I_1} \cap \mu \nsubseteq P \cup \lambda$. Similarly $\chi^*_{I_2} \cap \mu \nsubseteq P \cup \lambda$ and $\chi^*_{I_3} \cap \mu \nsubseteq P \cup \lambda$. This contradicts that P is a (λ, μ)-fuzzy prime ideal. Hence P_μ is a prime ideal in S. □

Now we prove our main Theorem.

Theorem 3.12. *Let P be a (λ, μ)-fuzzy ideal of a ternary semiring S and $ImP \cap (\lambda, \mu) \neq \phi$. Then P is a (λ, μ)-fuzzy prime ideal of S if and only if*
(i) $ImP \cap [0, \lambda] = \phi$
(ii) $ImP \cap [\mu, 1] \neq \phi$
(iii) $|ImP \cap (\lambda, \mu)| = 1$
(iv) P_μ *is a prime ideal of S.*

Proof. Let P be a (λ, μ)-fuzzy prime ideal of S. Conditions (i) to (iv) follows from Lemmas 3.8,3.9,3.10,3.11. Conversely, if there exist (λ, μ)-fuzzy ideals A, B and C of S such that $A \circ B \circ C \subseteq P \cup \lambda$ with $A \cap \mu \nsubseteq P \cup \lambda$, $B \cap \mu \nsubseteq P \cup \lambda$ and $C \cap \mu \nsubseteq P \cup \lambda$, then there exist $a, b, c \in S$ such that $A(a) \wedge \mu = t_1 > P(a) \vee \lambda$, $B(b) \wedge \mu = t_2 > P(b) \vee \lambda$ and $C(c) \wedge \mu = t_3 > P(c) \vee \lambda$. Let $ImP \cap (\lambda, \mu) = t$. Then $t_1, t_2, t_3 \in (\lambda, \mu]$, $t_1 > t, t_2 > t, t_3 > t$, $a \notin P_\mu$, $b \notin P_\mu$, $c \notin P_\mu$. Therefore A_{t_1}, B_{t_2} and C_{t_3} are ideals in S and $A_{t_1} B_{t_2} C_{t_3} \nsubseteq P_\mu$. Then there exist $a_1 \in A_{t_1}$, $b_1 \in B_{t_2}$ and $c_1 \in C_{t_3}$ such that $a_1 b_1 c_1 \notin P_\mu$. Thus $P(a_1 b_1 c_1) = t$. Now $(A \circ B \circ C)(a_1 b_1 c_1) \geq A(a_1) \wedge B(b_1) \wedge C(c_1) \wedge \mu = A(a_1) \wedge \mu \wedge B(b_1) \wedge \mu \wedge C(c_1) \wedge \mu > t = P(a_1 b_1 c_1) \vee \lambda$. This is a contradiction to $A \circ B \circ C \subseteq P \cup \lambda$. Therefore P is a (λ, μ)-fuzzy prime ideal of S. □

Corollary 3.13. *Let P be a non-constant fuzzy ideal of a ternary semiring S. Then P is a fuzzy prime ideal of S if and only if*
(i) $ImP = \{1, t\}$ *where* $0 < t < 1$.
(ii) P_1 *is a prime ideal of S.*

Proof. By taking $\lambda = 0$ and $\mu = 1$ in Theorem 3.12, we get the result. □

Example 3.14. Let S be a ternary semiring consists of non-positive integers Z_0^-. Let

$$P(x) = \begin{cases} 0.92 & if \quad x = -18 \\ 0.87 & if \quad x = -6 \\ 0.8 & if \quad x \in \langle -2 \rangle \quad and \quad x \notin \{-6, -18\} \\ 0.5 & otherwise. \end{cases}$$

Clearly P is a (0.3, 0.8)-fuzzy prime ideal. But P is not a fuzzy prime ideal as $|ImP| > 2$.

Example 3.15. Let S be a ternary semiring consists of non-positive integers Z_0^-. Let

$$P(x) = \begin{cases} 0.9 & if \quad x \in \langle -3 \rangle \\ 0.6 & otherwise. \end{cases}$$

Clearly P is a (0.3, 0.8)-fuzzy prime ideal. But P is not a fuzzy prime ideal as $P_1 = \{\phi\}$.

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