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Fuzzy e-continuity and fuzzy e-open sets

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Abstract. In this paper the concept of fuzzy e-open set is introduced and its properties are studied in fuzzy topological spaces. Moreover, we introduce the fuzzy e-continuous mapping and other mapping and establish their various characteristic properties. Further fuzzy e-separation axioms have been introduced and investigated with the help of fuzzy e-open sets.

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1. Introduction

The concept of fuzzy has invaded almost all branches of mathematics with the introduction of fuzzy sets by Zadeh [12] of 1965. The theory of fuzzy topological spaces was introduced and developed by Chang[5]. In 2008, Erdal Ekici[7], has introduced the concept of e-open sets in general topology. In this paper, we extend the notion of e-open sets to fuzzy topological space in the name fuzzy e-open sets and study some properties based on this new concept. We further study the relation between fuzzy e-open sets with other types of fuzzy open sets. We also introduce the concepts of fuzzy e-continuous mappings and study their nature with separation axioms.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, γ) (or simply X, Y and Z) represent non-empty fuzzy topological spaces. Let A be a fuzzy subset of a space X. The fuzzy closure of A, fuzzy interior of A, fuzzy δ-closure of A and the fuzzy δ-interior of A are denoted by cl(A), int(A), $cl_\delta(A)$ and $\text{int}_\delta(A)$ respectively. A fuzzy subset A of space X is called fuzzy regular open[2] (resp.fuzzy regular closed) if $A = \text{int}(cl(A))$ (resp. $A = cl(\text{int}(A))$. The fuzzy δ-interior of fuzzy subset A of X is the union of all fuzzy regular open sets contained in A. A fuzzy subset A is called fuzzy δ -open[11] if $A = \text{int}_\delta(A)$. The complement of fuzzy δ-open set is called fuzzy δ-closed (i.e, $A = cl_\delta(A)$).

A fuzzy subset A of a space X is called fuzzy semi open[2] (resp. fuzzy α-open set[10], fuzzy β-open set[3], fuzzy pre-open set[4], fuzzy γ-open[9], fuzzy δ-preopen [1], fuzzy δ-semi open)[8] if $A \leq\ cl\ \text{int}\, A$ (resp. $A \leq int\ (cl\ (\text{int}(A)))$, $A \leq\ cl\ (int\ (cl(A)))$, $A \leq int\ (cl(A))$, $A \leq cl\ (int(A)) \vee int(\ cl(A))$, $A \leq int\ (cl_\delta(A))$, $A \leq\ \ cl\ (\text{int}_\delta(A))$). The complement of a fuzzy δ-semiopen set (resp. fuzzy δ-preopen set) is called fuzzy δ-semiclosed (resp.fuzzy δ-preclosed). The union of all fuzzy δ-semi open (resp. fuzzy δ-preopen) sets contained in a fuzzy set A in a fuzzy topological space X is called the fuzzy δ-semi interior [8] (resp. fuzzy δ-pre interior [1]) of A and it is denoted by $s\text{int}_\delta(A)$ (resp. $p\text{int}_\delta(A)$). The intersection of all fuzzy δ-semi closed (resp. fuzzy δ-preclosed) sets containing a fuzzy set A in a fuzzy topological space X is called the fuzzy δ-semiclosure [8] (resp. fuzzy δ-preclosure [1]) of A and it is denoted by $scl_\delta(A)$ (resp. $pcl_\delta(A)$).

A function $f : X\ \rightarrow\ Y$ is called fuzzy δ-pre continuous[1] (resp. fuzzy δ-semi continuous [6] if $f^{-1}\,(\lambda)$ is fuzzy δ-pre open(resp. fuzzy δ-semi open) in X for every fuzzy open set λ of Y.

3. Fuzzy e-open set

Definition 3.1. A fuzzy subset μ of a space X is called fuzzy e-open(briefly, fe-open) if

$$\mu \leq cl(\text{int}_\delta \mu) \vee \text{int}(cl_\delta \mu),$$

fuzzy e-closed(briefly, fe-closed) if

$$\mu \geq cl(\text{int}_\delta \mu) \wedge \text{int}(cl_\delta \mu).$$

From the definitions we obtain the following diagram

$$\begin{tikzcd}
\text{fuzzy regular open} \arrow[d, Rightarrow] \\
\text{fuzzy } \delta\text{-open} \arrow[d, Rightarrow] \\
\text{fuzzy open} \arrow[r, Rightarrow] \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & \text{fuzzy semi open} \arrow[r, Rightarrow] & \text{fuzzy } \gamma\text{-open} \arrow[r, Rightarrow] & \text{fuzzy } \beta\text{-open} \\
\text{fuzzy } \delta\text{-preopen} \arrow[d, Rightarrow] & \text{fuzzy } \delta\text{-semiopen} \arrow[dl, Rightarrow] \\
\text{fuzzy } e\text{-open}
\end{tikzcd}$$

None of these implications are reversible as shown in the following example.

Example 3.2. Let $X = \{a, b, c\}$ and $\upsilon_1, \upsilon_2, \upsilon_3$ be fuzzy sets of X defined as

$$\upsilon_1(a) = 0.2,\ \upsilon_2(a) = 0.1,\ \upsilon_3(a) = 0.2$$
$$\upsilon_1(b) = 0.3,\ \upsilon_2(b) = 0.1,\ \upsilon_3(b) = 0.4$$
$$\upsilon_1(c) = 0.4,\ \upsilon_2(c) = 0.4,\ \upsilon_3(c) = 0.4$$

Let $\tau = \{0, \upsilon_1, \upsilon_2, 1\}$, then the fuzzy set υ_3 is fuzzy e-open set. But it is not fuzzy δ-preopen.

Example 3.3. Let $X = \{a, b, c\}$ and $\nu_1, \nu_2, \nu_3, \nu_4$ be fuzzy sets of X defined as

$$\nu_1(a) = 0.3,\ \nu_2(a) = 0.4,\ \nu_3(a) = 0.4,\ \nu_4(a) = 0.3$$
$$\nu_1(b) = 0.5,\ \nu_2(b) = 0.2,\ \nu_3(b) = 0.5,\ \nu_4(b) = 0.5$$
$$\nu_1(c) = 0.5,\ \nu_2(c) = 0.6,\ \nu_3(c) = 0.6,\ \nu_4(c) = 0.4$$

Let $\tau = \{0, \nu_1, \nu_2, \nu_3, \nu_1 \wedge \nu_2, 1\}$. Then the fuzzy set ν_4 is fuzzy e-open but not fuzzy δ-semi open and also not a fuzzy β-open, fuzzy γ-open and fuzzy semi open.

Example 3.4. Let $X = \{a, b, c\}$ and u_1, u_2, u_3, u_4 be fuzzy sets of X defined as

$$u_1(a) = 0.3, u_2(a) = 0.6, u_3(a) = 0.6, u_4(a) = 0.3$$
$$u_1(b) = 0.4, u_2(b) = 0.5, u_3(b) = 0.5, u_4(b) = 0.4$$
$$u_1(c) = 0.5, u_2(c) = 0.5, u_3(c) = 0.4, u_4(c) = 0.4$$

Let $\tau = \{0, u_1, u_2, u_3, u_4, 1\}$ and let λ be fuzzy set defined as $\lambda(a) = 0.7,\ \lambda(b) = 0.6,\ \lambda(c) = 0.4$. Then λ is not fuzzy e-open set but it is fuzzy β-open, fuzzy γ-open and fuzzy semi open.

Lemma 3.5. [1, 8] *Let μ be a fuzzy subset of X, then*

(i) $pcl_\delta(\mu) = \mu \vee cl(\text{int}_\delta(\mu))$ *and* $\text{pint}_\delta(\mu) = \mu \wedge \text{int}(cl_\delta(\mu))$
(ii) $scl_\delta(\mu) = \mu \vee \text{int}(cl_\delta(\mu))$ *and* $\text{sint}_\delta(\mu) = \mu \wedge cl(\text{int}_\delta(\mu))$

Theorem 3.6. *For any fuzzy subset μ of a space X, μ is fuzzy e-open if and only if $\mu = \text{pint}_\delta(\mu) \vee \text{sint}_\delta(\mu)$.*

Proof. Let μ be fuzzy e-open. Then $\mu \leq cl(\text{int}_\delta\mu) \vee \text{int}(cl_\delta\mu)$. By lemma 3.5, we have $\text{pint}_\delta(\mu) \vee \text{sint}_\delta(\mu)= (\mu \wedge \text{int}(cl_\delta(\mu)) \vee (\mu \wedge cl(\text{int}_\delta(\mu)) = \mu \wedge (\text{int}(cl_\delta(\mu)) \vee cl(\text{int}_\delta(\mu))) = \mu$.
Conversely, if $\mu = \text{pint}_\delta(\mu) \vee \text{sint}_\delta(\mu)$ then, by lemma 3.5, $\mu = \text{pint}_\delta(\mu) \vee \text{sint}_\delta(\mu)= \mu \wedge \text{int}(cl_\delta(\mu)) \vee \mu \wedge cl(int_\delta(\mu)) = \mu \wedge (\text{int}(cl_\delta(\mu)) \vee cl(\text{int}_\delta(\mu))) \leq \text{int}(cl_\delta(\mu)) \vee cl(\text{int}_\delta(\mu))$ and hence μ is fuzzy e-open. □

Theorem 3.7. *In a fuzzy topological space X,*

(i) *Any union of fuzzy e-open sets is a fuzzy e-open set, and*
(ii) *Any intersection of fuzzy e-closed sets is a fuzzy e-closed set.*

Proof. (i) Let λ_α be a collection of fuzzy e-open sets. Then for each α, $\lambda_\alpha \leq (cl(\text{int}_\delta(\lambda_\alpha))) \vee (\text{int}(cl_\delta(\lambda_\alpha))) \leq (cl(\text{int}_\delta(\vee\lambda_\alpha))) \vee (\text{int}(cl_\delta(\vee\lambda_\alpha)))$. Thus $\vee\lambda_\alpha$ is a fuzzy e-open set.
(ii) Since $\mu_\alpha = 1 - \lambda_\alpha$ is fuzzy closed set, from (i) we have $\mu_\alpha = 1 - \lambda_\alpha \geq 1 - [(cl(\text{int}_\delta(\vee\lambda_\alpha))) \vee (\text{int}(cl_\delta(\vee\lambda_\alpha)))]$. From this we have $\mu_\alpha \geq [1 - (cl(\text{int}_\delta(\vee\lambda_\alpha)))] \wedge [1-(\text{int}(cl_\delta(\vee\lambda_\alpha)))]$. This implies $\mu_\alpha \geq [(int(\text{cl}_\delta(1-(\vee\lambda_\alpha))))] \wedge [(\text{cl}(int_\delta(1-(\vee\lambda_\alpha))))]$.

As $1-(\vee\lambda_\alpha)=\wedge(1-\lambda_\alpha)$ we get $\mu_\alpha \geq [(int(\mathrm{cl}_\delta(\wedge(\mu_\alpha))))] \wedge [(\mathrm{cl}(int_\delta(\wedge(\mu_\alpha))))]$. Thus $\wedge\mu_\alpha$ is a fuzzy e-closed set. □

Definition 3.8. Let μ be any fuzzy set. Then
(i)$fe\text{-}cl(\mu)=\wedge\{\lambda:\lambda\geq\mu,\ \lambda$ is a fuzzy e-closed set of X$\}$
(ii)$fe\text{-}\mathrm{int}(\mu)=\vee\{\lambda:\lambda\leq\mu,\ \lambda$ is a fuzzy e-open set of X$\}$

Theorem 3.9. *In a fuzzy topological space X, λ be a fuzzy e-closed (resp. fuzzy e-open) if and only if $\lambda=fe\text{-}cl(\lambda)$ (resp. $\lambda=fe\text{-}\mathrm{int}(\lambda)$).*

Proof. Suppose $\lambda=fe\text{-}cl(\lambda)=\wedge\{\mu:\mu$ is a fuzzy e-closed set and $\mu\geq\lambda\}$. This means $\lambda\in\wedge\{\mu:\mu$ is a fuzzy e-closed set and $\mu\geq\lambda\}$ and hence λ is fuzzy e-closed set.
Conversely, suppose λ be a fuzzy e-closed in X. Then we have $\lambda\in\{\mu:\mu$ is a fuzzy e-closed set and $\mu\geq\lambda\}$. Hence, $\lambda\leq\mu$ implies $\lambda=\wedge\{\mu:\mu$ is a fuzzy e-closed set and $\mu\geq\lambda\}=fe\text{-}cl(\lambda)$.
Similarly for $\lambda=fe\text{-}\mathrm{int}(\lambda)$. □

Theorem 3.10. *In a fuzzy topological space X the following holds for fuzzy e-closure sets. (i) $fe\text{-}cl(0)=0$.*
(ii) $fe\text{-}cl(\lambda)$ is a fuzzy e-closed set in X.
(iii) $fe\text{-}cl(\lambda)\leq fe\text{-}cl(\mu)$ if $\lambda\leq\mu$.
(iv) $fe\text{-}cl(fe\text{-}cl(\lambda))=fe\text{-}cl(\lambda)$.
Similar results hold for fuzzy e-interiors.

Theorem 3.11. *In a fuzzy topological space X, we have*
(i)$fe\text{-}cl(\lambda\vee\mu)\geq fe\text{-}cl(\lambda)\vee fe\text{-}cl(\mu)$
(ii) $fe\text{-}cl(\lambda\wedge\mu)\leq fe\text{-}cl(\lambda)\wedge fe\text{-}cl(\mu)$.

Proof. (i) $\lambda\leq\lambda\vee\mu$ or $\mu\leq\lambda\vee\mu$ this implies $fe\text{-}cl\lambda\leq fe\text{-}cl(\lambda\vee\mu)$ or$fe\text{-}cl\mu\leq fe\text{-}cl(\lambda\vee\mu)$. Therefore $fe\text{-}cl(\lambda\vee\mu)\geq fe\text{-}cl(\lambda)\vee fe\text{-}cl(\mu)$.
(ii) Similar proof of (i). □

Theorem 3.12. *In a fuzzy topological space X, we have*
(i) $fe\text{-}\mathrm{int}(\lambda\vee\mu)\geq fe\text{-}\mathrm{int}(\lambda)\vee fe\text{-}\mathrm{int}(\mu)$ *and*
(ii) $fe\text{-}\mathrm{int}(\lambda\wedge\mu)\leq fe\text{-}\mathrm{int}(\lambda)\wedge fe\text{-}\mathrm{int}(\mu)$.

Theorem 3.13. *Let u be fuzzy e-open set, we have*
(i) *If* $\mathrm{int}_\delta(u)=0$, *then u is fuzzy δ-preopen.*
(ii) *If* $cl_\delta(u)=0$, *then u is fuzzy δ-semiopen.*

Theorem 3.14. *Let μ be a fuzzy subset of a space X, then, $fe\text{-}cl(\mu)=fpcl_\delta(\mu)\wedge fscl_\delta(\mu)$.*

Proof. It is obvious that, $fe\text{-}cl(\mu)\leq fpcl_\delta(\mu)\wedge fscl_\delta(\mu)$.
Conversely, from definition we have $fe\text{-}cl(\mu)\geq cl(\mathrm{int}_\delta(e\text{-}cl(\mu)))\wedge int(cl_\delta(e\text{-}cl(\mu)))$ $\geq cl(\mathrm{int}_\delta(\mu))\wedge\mathrm{int}(cl_\delta(\mu))$. Since $fe\text{-}cl(\mu)$ is fuzzy e- closed, by lemma 3.5, we have $fpcl_\delta(\mu)\wedge fscl_\delta(\mu)=(\mu\vee cl(int_\delta(\mu)))\wedge(\mu\vee int(cl_\delta(\mu)))=\mu\vee(cl(int_\delta(\mu))\wedge int(cl_\delta(\mu)))=\mu\leq fe\text{-}cl(\mu)$.
□

Theorem 3.15. *Let μ be a fuzzy subset of a space X, then* $fe\text{-int}(\mu) = fp\text{int}_\delta(\mu) \wedge fs\text{int}_\delta(\mu)$.

Proof is similar to the above theorem.

Theorem 3.16. *Let λ be any fuzzy set in X, then*

(i) $fe\text{-}cl(1-\lambda) = 1 - fe\text{-int}(\lambda)$
(ii) $fe\text{-int}(1-\lambda) = 1 - fe\text{-}cl(\lambda)$.

Proof. (i) Let υ be fuzzy e-open set. Then for a fuzzy e-open set $\upsilon \leq \lambda$, $\upsilon \geq 1-\lambda$. Then $fe\text{-int}(\lambda) = \vee\{1-\upsilon,\ \upsilon$ is a fuzzy e-closed set and $\upsilon \geq 1\text{-}\lambda\} = 1 - \wedge\{\upsilon :$ υ is a fuzzy e-closed set and $\upsilon \geq 1-\lambda\} = 1 - fe\text{-}cl(1-\lambda)$. Thus $fe\text{-}cl(1-\lambda) = 1 - fe\text{-int}(\lambda)$.
(ii) Let μ be fuzzy e-open set. Then for a fuzzy e-closed set $\mu \geq \lambda$, $\mu \leq 1-\lambda$. Then $fe\text{-}cl(\lambda) = \wedge\{1-\mu,\ \mu$ is a fuzzy e-open set and $\mu \leq 1\text{-}\lambda\} = 1 - \vee\{\mu :$ μ is a fuzzy e-open set and $\mu \leq 1-\lambda\} = 1 - fe\text{-int}(1-\lambda)$. Thus $fe\text{-int}(1-\lambda) = 1 - fe\text{-}cl(\lambda)$. □

4. Fuzzy e-continuity and separation axioms

Definition 4.1. A mapping $f : X \to Y$ is said to be a fuzzy e-continuous if $f^{-1}(\lambda)$ is fuzzy e-open in X for every fuzzy open set λ in Y.

Definition 4.2. A mapping $f : X \to Y$ is said to be a fuzzy e-irresolute if $f^{-1}(\lambda)$ is fuzzy e-open in X for every fuzzy e-open set λ in Y.

Theorem 4.3. *For a mapping $f : (X, T) \to (Y, S)$, the following statements are equivalent*

(i) *f is a fuzzy e- continuous.*
(ii) *For every fuzzy singleton $x_p \in X$ and every fuzzy open set v in Y such that $f(x_p) \leq v$, there exit fuzzy e- open set $u \leq X$ such that $x_p \leq u$ and $f(u) \leq v$.*
(iii) $f^{-1}(\lambda) = \text{int}(cl_\delta f^{-1}(\lambda)) \vee cl(int_\delta f^{-1}(\lambda))$ *for each fuzzy open set λ in Y.*
(iv) *The inverse image of each fuzzy closed set in Y is fuzzy e-closed.*
(v) $cl(\text{int}_\delta f^{-1}(v) \wedge \text{int}(cl_\delta f^{-1}(v) \leq f^{-1}(cl(\nu))$ *for each fuzzy set $\nu \leq Y$.*
(vi) $f(cl\text{int}_\delta(u) \wedge \text{int}\, cl_\delta(u)) \leq cl(f(u))$ *for every fuzzy set $u \leq X$.*

Proof. $(i) \Rightarrow (ii)$: Let the singleton set x_p in X and every fuzzy open set v in Y such that $f(x_p) \leq v$. Since f is fuzzy e-continuous. Then $x_p \in f^{-1}(f(x_p)) \leq f^{-1}(v)$. Let $u = f^{-1}(v)$ which is a fuzzy e-open set in X. So, we have $x_p \leq u$. Now $f(u) = f(f^{-1}(v)) \leq v$.

$(ii) \Rightarrow (iii)$: Let λ be any fuzzy open set in Y. Let x_p be any fuzzy point in X such that $f(x_p) \leq \lambda$. Then $x_p \in f^{-1}(\lambda)$. By(ii), there exists a fuzzy e-open set $u \leq X$ such that $x_p \leq u$ and $f(u) \leq \lambda$. Therefore, $x_p \in u \leq f^{-1}(f(u)) \leq f^{-1}(\lambda) \leq \text{int}(\text{cl}_\delta f^{-1}(\lambda)) \vee cl(int_\delta f^{-1}(\lambda))$.

$(iii) \Rightarrow (iv)$: Let λ be any fuzzy closed set in Y. Then $1-\lambda$ be a fuzzy open set in Y. By (iii), $f^{-1}(1-\lambda) \leq \text{int}(cl_\delta f^{-1}(1-\lambda)) \vee cl(\text{int}_\delta f^{-1}(1-\lambda))$. This implies $1 - f^{-1}(\lambda) \leq \text{int}(cl_\delta(1 - f^{-1}(\lambda))) \vee cl(\text{int}_\delta(1 - f^{-1}(\lambda))) \leq \text{int}(1 - \text{int}_\delta f^{-1}(\lambda)) \vee cl(1 - cl_\delta f^{-1}(\lambda)) = 1 - cl(\text{int}_\delta f^{-1}(\lambda)) \vee 1 - \text{int}(cl_\delta f^{-1}(\lambda))$ and hence $1 - f^{-1}(\lambda) =$

$1-(cl(\text{int}_\delta f^{-1}(\lambda))\wedge \text{int}(cl_\delta f^{-1}(\lambda)))$. Hence $f^{-1}(\lambda)\geq cl(\text{int}_\delta f^{-1}(\lambda))\wedge \text{int}(cl_\delta f^{-1}(\lambda))$ and this implies $f^{-1}(\lambda)$ is fuzzy e-closed in X.

$(iv)\Rightarrow(v)$: Let $\nu\leq Y$. Then $f^{-1}(cl(\nu))$ is fuzzy e-closed in X. (i.e)$\text{int}(cl_\delta f^{-1}(\nu))\wedge cl(\text{int}_\delta f^{-1}(\nu))\leq \text{int}(cl_\delta f^{-1}(cl(\nu)))\wedge cl(\text{int}_\delta f^{-1}(cl(\nu)))\leq f^{-1}(cl(\nu))$.

$(v)\Rightarrow(vi)$: Let $u\leq X$. Put $\nu=f(u)$ in (v). Then, $\text{int}(cl_\delta f^{-1}(f(u)))\wedge cl(\text{int}_\delta f^{-1}(f(u)))\leq f^{-1}(cl(f(u)))$. This implies that $\text{int}(cl_\delta(u))\wedge cl(\text{int}_\delta(u))\leq f^{-1}(cl(f(u)))$ $f(\text{int}(cl_\delta(u))\wedge cl(\text{int}_\delta(u)))\leq cl(f(u))$

$(vi)\Rightarrow(i)$: Let $\upsilon\leq Y$ be fuzzy open set. Put $u=I_Y-\upsilon$ and $u=f^{-1}(\upsilon)$ then $f(\text{int}(cl_\delta(f^{-1}(\upsilon)))\wedge cl(\text{int}_\delta(f^{-1}(\upsilon))))\leq cl(f(f^{-1}(\upsilon)))\leq cl(\upsilon)=\upsilon$. That is, $f^{-1}(\upsilon)$ is fuzzy e-closed in X, so f is fuzzy e-continuous. □

we obtain the following diagram hold:

$$\begin{tikzcd}[column sep=small] \text{fuzzy } \delta\text{-pre continuous} \arrow[dr, Rightarrow] & & \text{fuzzy } \delta\text{-semi continuous} \arrow[dl, Rightarrow] \\ & \text{fuzzy } e\text{-continuous} & \end{tikzcd}$$

These implications are not reversible as shown in the following example.

Example 4.4. Let $X=\{a,b,c\}$ and $\upsilon_1,\upsilon_2,\upsilon_3$ and υ_4 be fuzzy sets of X defined as

$$\upsilon_1(a)=0.4,\ \upsilon_2(a)=0.6,\ \upsilon_3(a)=0.6,\ \upsilon_4(a)=0.4$$
$$\upsilon_1(b)=0.6,\ \upsilon_2(b)=0.4,\ \upsilon_3(b)=0.4,\ \upsilon_4(b)=0.5$$
$$\upsilon_1(c)=0.5,\ \upsilon_2(c)=0.4,\ \upsilon_3(c)=0.5,\ \upsilon_4(c)=0.5$$

Let $\tau_1=\{0,\ \upsilon_1,\ \upsilon_2,\ \upsilon_1\vee\upsilon_2,\ \upsilon_1\wedge\upsilon_2,\ 1\}$, $\tau_2=\{0,\upsilon_3,1\}$, and $\tau_3=\{0,\upsilon_4,1\}$ and the mapping $f:(X,\tau_1)\to(X,\tau_2)$ and $g:(X,\tau_1)\to(X,\tau_3)$ defined as $f(a)=a, f(b)=b, f(c)=c$. It is clear that f is fuzzy e-continuous, but it is not fuzzy δ-pre continuous and g is fuzzy e-continuous, but it is not fuzzy δ-semi continuous.

Theorem 4.5. *Let X,Y and Z be fuzzy topological spaces.*

(i) *If $f:X\to Y$ fuzzy e-continuous and $g:Y\to Z$ is fuzzy continuous. Then $g\circ f:X\to Z$ is fuzzy e-continuous.*

(ii) *If $f:X\to Y$ fuzzy e-irresolute and $g:Y\to Z$ is fuzzy e-continuous. Then $g\circ f:X\to Z$ is fuzzy e-continuous.*

Proof. Obvious. □

Definition 4.6. A fuzzy topological space (X,τ) is said to be fuzzy e-T_1 if for each pair of distinct points x and y of X, there exists fuzzy e-open sets U_1 and U_2 such that $x\in U_1$ and $y\in U_2$, $x\notin U_2$ and $y\notin U_1$.

Theorem 4.7. *If $f:(X,\tau)\to(Y,\sigma)$ is fuzzy e-continuous injective function and Y is fuzzy T_1 then X is fuzzy e-T_1.*

Proof. Suppose that Y is fuzzy T_1. For any two distinct points x and y of X, there exists fuzzy open sets F_1 and F_2 in Y such that $f(x) \in F_1$, $f(y) \in F_2$, $f(x) \notin F_2$ and $f(y) \notin F_1$. Since f is injective fuzzy e-continuous function, we have $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are fuzzy e-open sets in X. Hence by definition X is fuzzy e-T_1. □

Definition 4.8. A fuzzy topological space (X, τ) is said to be fuzzy e-T_2 (i.e., fuzzy e-Hausdorff) if for each pair of distinct points x and y of X, there exists disjoint fuzzy e-open sets U and V such that $x \in U$ and $y \in V$.

Theorem 4.9. *If $f : (X, \tau) \to (Y, \sigma)$ is fuzzy e-continuous injective function and Y is fuzzy T_2 then X is fuzzy e-T_2.*

Proof. Suppose that Y is fuzzy T_2 space. For any two distinct points x and y of X, there exists fuzzy open sets U and V in Y such that $f(x) \in U$, $f(y) \in V$, $f(x) \notin V$ and $f(y) \notin U$. Since f is injective fuzzy e-continuous function, we have $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy e-open sets in X. Hence by definition, X is fuzzy e-T_2. □

Definition 4.10. A fuzzy topological space (X, τ) is said to be fuzzy e-normal if for every two disjoint fuzzy closed sets A and B of X, there exist two disjoint fuzzy e-open sets U and V such that $A \leq U$ and $B \leq V$ and $U \wedge V = 0$.

Theorem 4.11. *If $f : (X, \tau) \to (Y, \sigma)$ is fuzzy e-continuous closed injective function and Y is fuzzy normal then X is fuzzy e-normal.*

Proof. Suppose that Y fuzzy normal. Let A and B be closed fuzzy sets in X such that $A \wedge B = 0$. Since f is fuzzy closed injection $f(A)$ and $f(B)$ are fuzzy closed in Y and $f(A) \wedge f(B) = 0$. Since Y is normal, there exists fuzzy open sets U and V in Y such that $f(A) \leq U$, $f(B) \leq V$ and $U \wedge V = 0$. Therefore we obtain, $A \leq f^{-1}(U)$ and $B \leq f^{-1}(V)$ and $f^{-1}(U \wedge V) = 0$. Since f is fuzzy e- continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy e-open sets. Hence by definition X is fuzzy e-normal. □

Definition 4.12. A space X is said to be fuzzy e-regular if for each closed set F of X and each $x \in X - F$, there exist disjoint fuzzy e-open sets U and V such that $x \in U$ and $F \leq V$.

Theorem 4.13. *If $f : (X, \tau) \to (Y, \sigma)$ is fuzzy e-continuous closed injective function and Y is fuzzy regular then X is fuzzy e-regular.*

Proof. Let F be fuzzy closed set in Y with $y \notin F$. Take $y = f(x)$. Since Y is fuzzy regular, there exists disjoint fuzzy open sets U and V such that $x \in U$ and $y = f(x) \in f(U)$ and $F \leq f(V)$ such that $f(U)$ and $f(V)$ are disjoint fuzzy open sets. Therefore we obtain that, $f^{-1}(F) \leq V$. Since f is fuzzy e-continuous, $f^{-1}(F)$ is fuzzy e-closed set in X and $x \notin f^{-1}(F)$. Hence by definition X is fuzzy e- regular. □

Definition 4.14. A fuzzy set v in a fuzzy topological spaces (X, τ) is said to be fuzzy e-connected if and only if v cannot be expressed as the union of two fuzzy e-open sets.

Theorem 4.15. *Let $f : X \to Y$ be a fuzzy e-continuous surjective mapping. If v is a fuzzy e-connected subset in X then, $f(v)$ is fuzzy connected in Y.*

Proof. Suppose that $f(d)$ is not fuzzy connected in Y. Then, there exist fuzzy open sets u and v in Y such that $f(d) = u \vee v$. Since f is fuzzy e-continuous surjective mapping, $f^{-1}(u)$ and $f^{-1}(v)$ are fuzzy e-open set in X and $d = f^{-1}(u \vee v) = f^{-1}(u) \vee f^{-1}(v)$. It is clear that $f^{-1}(u)$ and $f^{-1}(v)$ are fuzzy e-open set in X. Therefore, d is not fuzzy e-connected in X, which is a contradiction. Hence, Y is fuzzy connected. $\square$

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