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Proximity of soft sets

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Abstract. In this paper the notion of proximity of soft sets is introduced and its properties are studied. The notion of Lodato proximities of soft sets is also defined. A topology of soft sets has been introduced using Kuratowski closure operator of soft sets. The notion of proximally soft continuity and proximally soft neighbourhood are also introduced.

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1. Introduction

Uncertainty is present in almost every sphere of our daily life. Traditional mathematical tools are not sufficient to handle all the practical problems in fields such as medical science, social science, engineering, economics etc involving uncertainty of various types. Zadeh, in 1965, was the first to come up with his remarkable theory of fuzzy set for dealing these types of uncertainties where conventional tools fail. His theory brought a grand paradigmatic change in mathematics. Later there are theories namely the theory of intuitionistic fuzzy sets, vague sets, rough sets, interval mathematics etc to name a few, all are intended to become a tool for handling the uncertainty. All these theories are successful to some extent in dealing with the problems arising due to the vagueness present in the real world. But there are also cases where these theories failed to give satisfactory results, possibly due to the inadequacy of the parameterization tool in them. Then in 1999, Molodtsov [28] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainty. Possible applications of soft set in various problems such as smoothness of functions, game theory, operation research, Rieman integration, Perron integration, probability theory, measurement theory, economics, medical science etc are shown by Molodtsov [28] and others [19, 20, 24, 25, 26, 27]. Many authors [12, 13] have also combined soft sets and rough sets to form hybrid soft sets. H. Aktas and N. Cagman [1] have shown that every fuzzy set and every rough set can be considered as a soft set. In that sense we can say that this theory is much more general than its predecessors. Later other authors like Maji et. al. [21, 22, 23, 24, 25] have further studied the theory of soft sets, fuzzy soft sets and used this theory to solve some decision making problems. Feng et. al. [14] studied soft product operations in 2013. Research in soft set theory (SST) has been done in many areas like algebra, topology, real life applications etc [2, 9, 10, 11, 12, 17, 18, 19, 26, 33]. Several authors like Shabir & Naz [32], Hazra, Majumdar and Samanta [15] have studied the notion of soft topological spaces. Also Aygunoglu & Aygun [3] have studied soft product topologies and soft compactness. The notion of fuzzy soft topologies has been also studied by few authors [30, 31, 34]. On the other hand proximities have been studied by several authors [4, 5, 7, 8, 29] in crisp sense as well as in fuzzy sense. In [16], we have introduced a notion of proximity in soft setting for the first time, which is termed as 'soft proximity'. In the present paper, we have further defined a different notion of proximity and we call it proximity of soft sets. We have studied the underlying topology of soft sets and established a relation between Kuratowski closure operator and Lodato proximity in soft setting. The rest of the paper is constructed as follows: In Section 2, some preliminary definitions and results regarding soft sets, soft topology, crisp proximity and soft proximity are given which will be used in the rest of the paper. In Section 3 the notion of proximities of soft sets is introduced and some of their important properties are studied. Section 4 concludes the paper.

2. Preliminaries

In this section some definitions, results and examples regarding soft sets are given which will be used in the rest of this paper.

Definition 2.1 ([28])**.** A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.
In other words, a soft set over U is a parameterized family of subsets of the universe U. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A).

Definition 2.2 ([1])**.** For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a *soft subset* of (G, B) if (i) $A \subset B$, (ii) $\forall e \in A, F(e) \subset G(e)$.

Definition 2.3 ([25])**.** Two soft sets (F, A) and (G, B) over a common universe U are said to be *soft equal* if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

In 2008, Majumdar & Samanta have given a new definition of complement of soft sets as follows:

Definition 2.4 ([27])**.** The *complement of a soft set* (F, A) is denoted by $(F, A)'$ and is defined by $(F, A)' = (F', A)$, where $F' : A \to P(U)$ is a mapping given by $F'(e) = U - F(e), \forall e \in A$.

Definition 2.5 ([25])**.** A soft set (F, A) over U is said to be *null soft set* denoted by Φ if $\forall e \in A, F(e) = \varnothing$.

Definition 2.6 ([25])**.** A soft set (F, A) over U is said to be *absolute soft set* denoted by $\tilde{A}$ if , if $\forall e \in A, F(e) = U$.

Definition 2.7 ([25])**.** The *union of two soft sets* (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$ and $\forall e \in C$,

$$\begin{aligned} H(e) &= F(e) \text{ if } e \in A - B \\ &= G(e) \text{ if } e \in B - A \\ &= F(e) \cup G(e) \text{ if } e \in A \cap B. \end{aligned}$$

We write $(F, A)\tilde{\cup}(G, B)$.

Definition 2.8 ([25])**.** The *intersection of two soft sets* (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cap B$ and

$$\forall e \in C,\ H(e) = F(e) \cap G(e).$$

We write $(F, A)\tilde{\cap}(G, B)$.

Definition 2.9 ([19])**.** Let $\tilde{f} : U_1 \to U_2$ and $\hat{f} : E_1 \to E_2$ be two mappings. Then the pair $f = (\tilde{f}, \hat{f})$ is said to be a *soft mapping* from $P(U_1)^{E_1}$ to $P(U_2)^{E_2}$ and the *image* $f(F)$ of any $F \in P(U_1)^{E_1}$ is defined as:

$$\begin{aligned} f(F)(e') &= \tilde{f}\Big(\bigcup_{e \in \hat{f}^{-1}(e')} F(e)\Big) \text{ if } \hat{f}^{-1}(e') \neq \varnothing \\ &= \varnothing \text{ if } \hat{f}^{-1}(e') = \varnothing, \forall e' \in E_2. \end{aligned}$$

Definition 2.10 ([15])**.** Let τ be a family of soft sets over (U, E).
Define $\tau(e) = \{F(e) : F \in \tau\}$ for $e \in E$.
Then τ is said to be a *topology of soft subsets* over (U, E) if $\tau(e)$ is a crisp topology on $U, \forall e \in E$.
In this case $((U, E), \tau)$ is said to be a *topological space of soft subsets*.
If τ is a topology of soft subsets over (U, E) , then the members of τ are called *open soft sets* and a soft set F over (U, E) is said to be *closed* if $F' \in \tau$.

Theorem 2.11 ([15])**.** *Let Ω be the family of all closed soft sets over (U, E) , then (i) $\tilde{\Phi}, \tilde{A} \in \Omega$ (ii) $F_i \in \Omega \Rightarrow \tilde{\cap}_i F_i \in \Omega$ and (iii) $F_1, F_2 \in \Omega \Rightarrow F_1 \tilde{\cup} F_2 \in \Omega$.*

Note 2.12. The family of all open soft sets over (U, E) will form a soft topology in the sense of Shabir & Naz [32] .

Definition 2.13 ([15])**.** Let $\mathfrak{T}_1$ and $\mathfrak{T}_2$ be two soft topologies over (U_1, E_1) and (U_2, E_2) respectively. A soft mapping $f = (\tilde{f}, \hat{f})$ from $P(U_1)^{E_1}$ to $P(U_2)^{E_2}$ is said to be *soft continuous* if the inverse image of every $e-$open set of $\mathfrak{T}_2$ under f is $\hat{f}^{-1}(e)$-open in $\mathfrak{T}_1$ $\forall e \in E_2$.

Theorem 2.14 ([15])**.** *$f = (\tilde{f}, \hat{f})$ is soft continuous if and only if inverse of each e-closed set in $\mathfrak{T}_2$ under f is $\hat{f}^{-1}(e)$-closed set in $\mathfrak{T}_1$, $\forall e \in E_2$.*

Next we give some basic definitions regarding proximity of ordinary sets, i.e. crisp sets.

Definition 2.15 ([29])**.** A *basic proximity* Π on X is a binary relation on $P(X)$ satisfying the following conditions:
(i) $\Pi = \Pi^{-1}$

(ii) $\forall A, B, C \subset X, (A \cup B, C) \in \Pi \Leftrightarrow (A, C) \in \Pi$ or $(B, C) \in \Pi$
(iii) $\forall A, B \subset X, A \cap B \neq \varnothing \Rightarrow (A, B) \in \Pi$
(iv) $(A, \varnothing) \notin \Pi \ \forall A \subset X$.

If Π is a basic proximity on X, then the pair (X, Π) is called a *basic proximity space.*

Definition 2.16 ([29])**.** A basic proximity Π on X is called *separated* if for every $x, y \in X$, $(\{x\}, \{y\}) \in \Pi$ implies $x = y$.

Definition 2.17 ([29])**.** A basic proximity Π on X is called *Lodato proximity* if for all $A, B, C \subset X, (A, B) \in \Pi$ and $(b, C) \in \Pi \forall b \in B \Rightarrow (A, C) \in \Pi$.

We now give the definitions of filters and grills. Filters were introduced by Carton [4] and grills were introduced by Choquet [7].

Definition 2.18 ([4])**.** A *filter* $\mathcal{F}$ on X is a non-empty family of subsets of X satisfying
(i) $\forall A, B \subset X,\ B \in \mathcal{F}$ and $B \subset A \Rightarrow A \in \mathcal{F}$.
(ii)$A, B \in \mathcal{F}$ implies $A \cap B \in \mathcal{F}$.

A filter $\mathcal{F}$ is called a *proper filter* if $\varnothing \notin \mathcal{F}$. A proper filter which is not contained in any other filter is called an *ultrafilter*.

Definition 2.19 ([7])**.** A *grill* $\mathcal{G}$ on X is a collection of subsets of X satisfying
(i) $\varnothing \notin \mathcal{G}$.
(ii) $\forall A, B \subset X,\ B \in \mathcal{G}$ and $B \subset A \Rightarrow A \in \mathcal{G}$.
(iii) $\forall A, B \subset X,\ A \cup B \in \mathcal{G}$ implies $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

A grill $\mathcal{G}$ is called a *proper grill* if $\mathcal{G} \neq \varnothing$.

Definition 2.20. Let Π be a basic proximity on X. Then for each $A \subset X$ define $\Pi(A) = \{B \subset X : (A, B) \in \Pi\}$.

Definition 2.21 ([16])**.** Let E be a set of parameters and X be a nonempty set and $\mathcal{A}$ be a set of basic proximities on X. Then the pair (π, E) is called a *basic soft proximity* on (X, E) if π is a mapping given by $\pi : E \to \mathcal{A}$.
The set of all basic soft proximities on (X, E) will be denoted by $M_S(X, E)$.
If $(\pi, E) \in M_S(X, E)$, then $((X, E), \pi)$ is called a basic soft proximity space.

Definition 2.22 ([16])**.** The pair $(\mathcal{G}, E)$ is said to be a *soft grill* on X if $\mathcal{G}$ is a mapping given by $\mathcal{G} : E \to \mathcal{B}$, where $\mathcal{B}$ is the set of all grills on X.

Definition 2.23 ([16])**.** Let (π, E) be a basic soft proximity on X and (F, E) be a soft set. Define $\pi(F) : E \to \mathcal{B}$ by
$\pi(F)(e) = \pi(e)(F(e))$.
Clearly $(\pi(F), E)$ is a soft grill on X.

Definition 2.24 ([16])**.** A mapping $c : P(X)^E \to P(X)^E$ is said to be a *Čech closure operator of soft sets* on (X, E) if
(i) $c(\tilde{\Phi}) = \tilde{\Phi}$,
(ii) $c(F) \supset F,\ \forall F \in P(X)^E$,
(iii) $c(F \tilde{\cup} G) = c(F) \tilde{\cup} c(G),\ \forall F, G \in P(X)^E$.

Moreover if c satisfies the additional condition $c(c(F)) = c(F),\ \forall F \in P(X)^E$, then c said to be a *Kuratowski closure operator of soft sets* on (X, E). If c is a Čech closure operator of soft sets, then $((X, E), c)$ is called *closure space of soft sets.*

Theorem 2.25 ([16]). *Let c be a Kuratowski closure operator of soft sets on (X, E). Let us define $\tau_c = \{F \in P(X)^E : c(F') = F'\}$, where F' is the complement of the soft set F.*
Then τ_c forms a topology of soft sets (in the sense of [32]) and the closure operator induced by τ_c coincides with c.

Hence without any loss of generality, where c is a Kuratowski closure operator of soft sets on (X, E), the triple $((X, E), c)$ will be called a *topological space of soft sets*.

Theorem 2.26 ([16]). *A soft set F is closed if and only if $c(F) = F$.*

Definition 2.27 ([16]). Let $(\pi, E) \in M_S(X, E)$. For $F \in P(X)^E$, and $e \in E$ define $c_\pi(F)(e) = \{x \in X : \{x\} \in \pi(e)(F(e))\}$ i.e., $c_\pi(F)(e) = c_{\pi(e)}(F(e))$.

Definition 2.28 ([16]). Let $x \in X$ and $e \in E$. A *soft point* $\{x\}_e$ is a soft set on E such that $\{x\}_e(e_1) = \{x\}$ if $e_1 = e$ and $= \varnothing$ if $e_1 \neq e$.

Definition 2.29 ([16]). Let E be a set of parameters, X be a nonempty set and $\mathcal{L}$ be the set of all Lodato proximities on X. Then the pair (π, E) is called a *Lodato soft proximity* on (X, E) if π is a mapping given by $\pi : E \to \mathcal{L}$.
The set of all Lodato soft proximities will be denoted by $M_S^{LO}(X, E)$.

Theorem 2.30 ([16]). *If $(\pi, E) \in M_S^{LO}(X, E)$, then c_π is a Kuratowski closure operator of soft sets on (X, E) i.e., $((X, E), c_\pi)$ is a topological space of soft sets.*

Definition 2.31 ([16]). A soft mapping $f = (\tilde{f}, \hat{f}) : ((X_1, E_1), \pi_1) \to ((X_2, E_2), \pi_{\mathbf{2}})$ is said to be *soft proximally continuous* if for each $F, G \in P(X_1)^{E_1}$,
$(F(e), G(e)) \in \pi_1(e) \Rightarrow (\tilde{f}(F(e)), \tilde{f}(G(e))) \in \pi_2(\hat{f}(e)), \forall e \in E_1$.

Definition 2.32 ([16]). Let $(\pi, E) \in M_S(X, E)$ and $F, G \in P(X)^E$. Then G is said to be a *soft proximal neighbourhood* of F, denoted by $G \gg F$ if $X - G(e) \notin \pi(e)(F(e))$ $\forall e \in E$.
The set of all soft proximal neighbourhoods of F w.r.t π will be denoted by $\mathcal{N}(\pi, F)$

3. Proximity of soft sets

In this section we introduce a notion of proximity of soft sets and study its properties.

Definition 3.1. A subset π of $P(X)^E \times P(X)^E$ is said to be a *basic proximity of soft sets* on (X, E) if
(i) $\pi = \pi^{-1}$
(ii) $(F, G\tilde{\cup}H) \in \pi \Leftrightarrow (F, G) \in \pi$ or $(F, H) \in \pi$
(iii) $F\tilde{\cap}G \neq \tilde{\Phi} \Rightarrow (F, G) \in \pi$
(iv) $(F, \tilde{\Phi}) \notin \pi$.
The set of all basic proximities of soft sets on (X, E) will be denoted by $M^S(X, E)$.

Definition 3.2. A subfamily $\mathcal{G}$ of $P(X)^E$ is said to be a *grill of soft sets* if (i) $\tilde{\Phi} \notin \mathcal{G}$,
(ii) for each $F, G \in P(X)^E$, $G \in \mathcal{G}$ and $G \subset F \Rightarrow F \in \mathcal{G}$,
(iii) for each $F, G \in P(X)^E$, $F\tilde{\cup}G \in \mathcal{G}$ implies $F \in \mathcal{G}$ or $G \in \mathcal{G}$.
A grill $\mathcal{G}$ is called a *proper grill* if $\mathcal{G} \neq \varnothing$

Definition 3.3. Let $\pi \in M^S(X,E)$ and $F \in P(X)^E$. Define $\pi(F)$ by
$\pi(F) = \{G \in P(X)^E : (F,G) \in \pi\}$.

Theorem 3.4. *If $\pi \in M^S(X,E)$ and $F \in P(X)^E$, then $\pi(F)$ is a grill of soft sets on (X,E).*

Proof. Since $(F,\tilde{\Phi}) \notin \pi, \tilde{\Phi} \notin \pi(F)$.
Now $G\tilde{\cup}H \in \pi(F) \Leftrightarrow (F, G\tilde{\cup}H) \in \pi \Leftrightarrow (F,G) \in \pi$ or $(F,H) \in \pi$
$\Leftrightarrow G \in \pi(F)$ or $H \in \pi(F)$.
Therefore $\pi(F)$ is a grill of soft sets on (X,E). □

Definition 3.5. Let $\pi \in M^S(X,E)$, $F \in P(X)^E$ and $e \in E$. Define
$c_\pi(F)(e) = \{x \in X : (\{x\}_e, F) \in \pi\}$.

Definition 3.6. Let c be a Čech closure operator of soft sets on (X,E). Then c is said to be R_0' if for all $x,y \in X$ and for all $e_1,e_2 \in E, x \in c(\{y\}_{e_2})(e_1) \Leftrightarrow y \in c(\{x\}_{e_1})(e_2)$.

Theorem 3.7. *If $\pi \in M^S(X,E)$, then c_π is an $R_0'-$Čech closure operator of soft sets on (X,E).*

Proof. Clearly $c_\pi(\tilde{\Phi})(e) = \varnothing$ for each $e \in E$. Therefore $c_\pi(\tilde{\Phi}) = \tilde{\Phi}$.
Let $F \in P(X)^E$ and $e \in E$. Then
$x \in F(e) \Rightarrow \{x\}_e \tilde{\cap} F \neq \tilde{\Phi} \Rightarrow (\{x\}_e, F) \in \pi \Rightarrow x \in c_\pi(F)(e)$.
Therefore $F(e) \subset c_\pi(F)(e)$. Thus $F \subset c_\pi(F)$.
Let $F,G \in P(X)^E$ and $e \in E$. Then
$c_\pi(F\tilde{\cup}G)(e) = \{x \in X : (\{x\}_e, F\tilde{\cup}G) \in \pi\}$.
$= \{x \in X : (\{x\}_e, F) \in \pi$ or $(\{x\}_e, G) \in \pi\}$.
$= \{x \in X : (\{x\}_e, F) \in \pi\} \cup \{x \in X : (\{x\}_e, G) \in \pi\}$.
$= c_\pi(F)(e) \cup c_\pi(G)(e) = (c_\pi(F)\tilde{\cup}c_\pi(G))(e)$.
Therefore $c_\pi(F\tilde{\cup}G) = c_\pi(F)\tilde{\cup}c_\pi(G)$.
Thus c_π is a Čech closure operator of soft sets on (X,E).
Let $x,y \in X$ and $e_1,e_2 \in E$. Then
$x \in c_\pi(\{y\}_{e_2})(e_1) \Leftrightarrow (\{x\}_{e_1}, \{y\}_{e_2}) \in \pi \Leftrightarrow (\{y\}_{e_2}, \{x\}_{e_1}) \in \pi \Leftrightarrow y \in c_\pi(\{x\}_{e_1})(e_2)$.
Therefore c_π is an $R_0'-$Čech closure operator of soft sets on (X,E). □

Theorem 3.8. *If c is an $R_0'-$Čech closure operator of soft sets on (X,E), then there is a basic proximity π of soft sets on (X,E) such that $c_\pi = c$.*

Proof. Define $\pi = \{(F,G) : F,G \in P(X)^E, (c(F)\tilde{\cap}G)\tilde{\cup}(F\tilde{\cap}c(G)) \neq \tilde{\Phi}\}$.
Clearly $\pi = \pi^{-1}$.
Let $F,G,H \in P(X)^E$. Then
$(F, G\tilde{\cup}H) \in \pi \Leftrightarrow (c(F)\tilde{\cap}(G\tilde{\cup}H))\tilde{\cup}(F\tilde{\cap}c(G\tilde{\cup}H)) \neq \tilde{\Phi}$
$\Leftrightarrow \{(c(F)\tilde{\cap}G)\tilde{\cup}(c(F)\tilde{\cap}H)\}\tilde{\cup}\{(F\tilde{\cap}c(G))\tilde{\cup}(F\tilde{\cap}c(H))\} \neq \tilde{\Phi}$
$\Leftrightarrow (c(F)\tilde{\cap}G)\tilde{\cup}(F\tilde{\cap}c(G))\tilde{\cup}(c(F)\tilde{\cap}H)\tilde{\cup}(F\tilde{\cap}c(H)) \neq \tilde{\Phi}$
$\Leftrightarrow (c(F)\tilde{\cap}G)\tilde{\cup}(F\tilde{\cap}c(G)) \neq \tilde{\Phi}$ or $(c(F)\tilde{\cap}H)\tilde{\cup}(F\tilde{\cap}c(H)) \neq \tilde{\Phi}$
$\Leftrightarrow (F,G) \in \pi$ or $(F,H) \in \pi$.
Let $F,G \in P(X)^E$ such that $F\tilde{\cap}G \neq \tilde{\Phi}$.
Therefore $(c(F)\tilde{\cap}G)\tilde{\cup}(F\tilde{\cap}c(G)) \neq \tilde{\Phi}$ and hence $(F,G) \in \pi$.
Clearly $(F,\tilde{\Phi}) \notin \pi$ for each $F \in P(X)^E$ [since $(c(F)\tilde{\cap}\tilde{\Phi})\tilde{\cup}(F\tilde{\cap}c(\tilde{\Phi})) = \tilde{\Phi}$].

Therefore π is a basic proximity of soft sets on (X, E).
Let $F \in P(X)^E$ and $e \in E$.
Therefore $c_\pi(F)(e) = \{x \in X : (\{x\}_e, F) \in \pi\}$
$= \{x \in X : (c(\{x\}_e)\tilde{\cap}F)\tilde{\cup}(\{x\}_e\tilde{\cap}c(F)) \neq \tilde{\Phi}\}$ ....(i)
$\{x\}_e\tilde{\cap}c(F) \neq \tilde{\Phi} \Rightarrow x \in c(F)(e)$.
$c(\{x\}_e)\tilde{\cap}F \neq \tilde{\Phi} \Rightarrow \exists e_1 \in E$ such that $c(\{x\}_e)(e_1) \cap F(e_1) \neq \varnothing$.
Therefore $\exists y \in X$ such that $y \in c(\{x\}_e)(e_1) \cap F(e_1)$.
Therefore $y \in c(\{x\}_e)(e_1)$ and $y \in F(e_1)$.
Since c is an R_0' closure operator of soft sets on $(X, E), x \in c(\{y\}_{e_1})(e)$.
Since $y \in F(e_1), \{y\}_{e_1} \subset F$. Therefore $c(\{y\}_{e_1}) \subset c(F)$. Thus $x \in c(F)(e)$.
Thus from (i) we have, $x \in c_\pi(F)(e) \Rightarrow x \in c(F)(e)$.
Therefore $c_\pi(F)(e) \subset c(F)(e)$......... (ii)
Also $x \in c(F)(e) \Rightarrow \{x\}_e\tilde{\cap}c(F) \neq \tilde{\Phi} \Rightarrow (\{x\}_e, F) \in \pi \Rightarrow x \in c_\pi(F)(e)$.
Thus $c(F)(e) \subset c_\pi(F)(e)$. .... (iii)
From (ii) and (iii) we have $c_\pi(F)(e) = c(F)(e)$. Since e is arbitrary point of E, $c_\pi(F) = F$. This is true for each $F \in P(X)^E$. Thus $c_\pi = c$.
Therefore there is a basic proximity π of soft sets on (X, E) such that $c_\pi = c$. □

Definition 3.9. A basic proximity π of soft sets on (X, E) is said to be *Lodato proximity of soft sets* on (X, E) if for each $F, G \in P(X)^E$,

$$(F, G) \in \pi \Leftrightarrow (F, c_\pi(G)) \in \pi.$$

The set of all Lodato proximities of soft sets on (X, E) will be denoted by $M^S_{LO}(X, E)$.

Theorem 3.10. *If $\pi \in M^S_{LO}(X, E)$, then c_π is a Kuratowski closure operator of soft sets on (X, E).*

Proof. Let $\pi \in M^S_{LO}(X, E)$. Then c_π is a Čech closure operator of soft sets on (X, E).
Let $F \in P(X)^E$. Then for each $e \in E$,
$c_\pi(F)(e) = \{x \in X : (\{x\}_e, F) \in \pi\}$
$= \{x \in X : (\{x\}_e, c_\pi(F)) \in \pi\}$
$= c_\pi(c_\pi(F))(e)$.
Therefore $c_\pi(F) = c_\pi(c_\pi(F))$.
Thus c_π is a Kuratowski closure operator of soft sets on (X, E). □

Definition 3.11. Let $\pi \in M^S(X, E)$ and $F, G \in P(X)^E$. Then G is said to be a *proximal soft neighbourhood* of F, denoted by $G \Supset F$ if $G' \notin \pi(F)$.
The set of all proximal soft neighbourhoods of F with respect to π will be denoted by $N(\pi, F)$.

Remark 3.12. Let $\pi \in M^S(X, E), F \in P(X)^E, x \in X$ and $e \in E$.
Then $F \Supset \{x\}_e \Leftrightarrow F' \notin \pi(\{x\}_e) \Leftrightarrow (\{x\}_e, F') \notin \pi \Leftrightarrow x \notin c_\pi(F')(e) \Leftrightarrow \{x\}_e\tilde{\cap}c_\pi(F') = \tilde{\Phi} \Leftrightarrow F$ is a neighbourhood of $\{x\}_e$ in the closure space $((X, E), c_\pi)$.

Theorem 3.13. *The following results hold :*
(i) For each $F, G \in P(X)^E, G \in N(\pi, F) \Rightarrow F' \in N(\pi, G')$.
(ii) For each $F \in P(X)^E, c_\pi(F) = \tilde{\cap}\{G : G \in N(\pi, F)\}$.

Proof. (i) Let $F, G \in P(X)^E$ such that $G \in N(\pi, F)$.
Therefore $G' \notin \pi(F) \Rightarrow (F, G') \notin \pi \Rightarrow (G', (F')') \notin \pi \Rightarrow (F')' \notin \pi(G') \Rightarrow F' \in$

$N(\pi, G')$.
(ii) Let $F \in P(X)^E$ and $e \in E$. Then
$c_\pi(F)(e) = \{x \in X : (\{x\}_e, F) \in \pi\}$
$\qquad = \{x \in X : (\{x\}_e)' \notin N(\pi, F)\}$.
Therefore $x \in c_\pi(F)(e)$ if and only if $(\{x\}_e)' \notin N(\pi, F)$ if and only if for each $G \in N(\pi, F), G \not\subset (\{x\}_e)'$ if and only if for each $G \in N(\pi, F), x \in G(e)$.
Therefore $c_\pi(F)(e) = \tilde{\cap}\{G : G \in N(\pi, F)\}(e)$.
Thus $c_\pi(F) = \tilde{\cap}\{G : G \in N(\pi, F)\}$. □

Definition 3.14. A soft mapping $f = (\tilde{f}, \hat{f}) : ((X_1, E_1), \pi_1) \to ((X_2, E_2), \pi_2)$ is said to be *proximally soft continuous* if for each $F, G \in P(X_1)^{E_1}$,
$(F, G) \in \pi_1 \Rightarrow (f(F), f(G)) \in \pi_2$.

Theorem 3.15. *Every proximally soft continuous mapping is soft continuous.*

Proof. Let $f = (\tilde{f}, \hat{f}) : ((X_1, E_1), \pi_1) \to ((X_2, E_2), \pi_2)$ be soft proximally continuous.
Let $F \in P(X_1)^{E_1}, e' \in E_2$ such that $\hat{f}^{-1}(e') \neq \varnothing$.
Therefore
$$\begin{aligned} f(c_{\pi_1}(F))(e') &= \tilde{f}\big(\bigcup_{e \in \hat{f}^{-1}(e')} c_{\pi_1}(F)(e)\big) \\ &= \tilde{f}\big(\bigcup_{e \in \hat{f}^{-1}(e')} \{x \in X_1 : (\{x\}_e, F) \in \pi_1\}\big) \\ &\subset \tilde{f}\big(\bigcup_{e \in \hat{f}^{-1}(e')} \{x \in X_1 : (f(\{x\}_e), f(F)) \in \pi_2\}\big) \\ &= \{\tilde{f}(x) \in X_2 : (\{\tilde{f}(x)\}_{e'}, f(F)) \in \pi_2\} \\ &= (c_{\pi_2} f(F))(e'). \end{aligned}$$
Thus $f(c_{\pi_1}(F)) \subset c_{\pi_2} f(F)$. Therefore f is soft continuous. □

Theorem 3.16. *If c is an R_0'-Kuratowski closure operator of soft sets on (X, E), then there is a Lodato proximity of soft sets π on (X, E) such that $c_\pi = c$.*

Proof. Define $\pi = \{(F, G) : F, G \in P(X)^E, c(F)\tilde{\cap}c(G) \neq \tilde{\Phi}\}$.
Clearly $\pi = \pi^{-1}$.
Let $F, G, H \in P(X)^E$. Then
$$\begin{aligned} (F, G\tilde{\cup}H) \in \pi &\Leftrightarrow c(F)\tilde{\cap}c(G\tilde{\cup}H) \neq \tilde{\Phi} \\ &\Leftrightarrow c(F)\tilde{\cap}(c(G)\tilde{\cup}c(H)) \neq \tilde{\Phi} \\ &\Leftrightarrow (c(F)\tilde{\cap}c(G))\tilde{\cup}(c(F)\tilde{\cap}c(H)) \neq \tilde{\Phi} \\ &\Leftrightarrow c(F)\tilde{\cap}c(G) \neq \tilde{\Phi} \text{ or } c(F)\tilde{\cap}c(H) \neq \tilde{\Phi} \\ &\Leftrightarrow (F, G) \in \pi \text{ or } (F, H) \in \pi. \end{aligned}$$
Let $F, G \in P(X)^E$ such that $F\tilde{\cap}G \neq \tilde{\Phi}$.
Therefore $c(F)\tilde{\cap}c(G) \neq \tilde{\Phi}$ and hence $(F, G) \in \pi$.
Clearly $(F, \tilde{\Phi}) \notin \pi$ for each $F \in P(X)^E$ [since $c(F)\tilde{\cap}c(\tilde{\Phi}) = \tilde{\Phi}$].
Let $F \in P(X)^E, e \in E$.
Therefore $c_\pi(F)(e) = \{x \in X : (\{x\}_e, F) \in \pi\}$
$\qquad = \{x \in X : c(\{x\}_e)\tilde{\cap}c(F) \neq \tilde{\Phi}\}$.
$c(\{x\}_e)\tilde{\cap}c(F) \neq \tilde{\Phi} \Rightarrow \exists e_1 \in E$ such that $c(\{x\}_e)(e_1) \cap c(F)(e_1) \neq \varnothing$.
Therefore $\exists y \in X$ such that $y \in c(\{x\}_e)(e_1) \cap c(F)(e_1)$.

Therefore $y \in c(\{x\}_e)(e_1)$ and $y \in c(F)(e_1)$.
Since c is an R_0'-Kuratowski closure operator of soft sets on (X, E), $x \in c(\{y\}_{e_1})(e)$.
Since $y \in c(F)(e_1)$, $\{y\}_{e_1} \subset c(F)$. Therefore $c(\{y\}_{e_1}) \subset c(c(F)) = c(F)$.
Therefore $x \in c(F)(e)$.
Thus $x \in c_\pi(F)(e) \Rightarrow x \in c(F)(e)$. Therefore $c_\pi(F)(e) \subset c(F)(e)$ .....(i)
Also $x \in c(F)(e) \Rightarrow \{x\}_e \tilde{\cap} c(F) \neq \tilde{\Phi} \Rightarrow c(\{x\}_e) \tilde{\cap} c(F) \neq \tilde{\Phi} \Rightarrow (\{x\}_e, F) \in \pi \Rightarrow x \in c_\pi(F)(e)$.
Thus $c(F)(e) \subset c_\pi(F)(e)$. ....(ii)
From (i) and (ii) we conclude that $c_\pi(F)(e) = c(F)(e)$.
Thus $c_\pi(F) = c(F)$ and hence $c_\pi = c$.
Therefore $(F, c_\pi(G)) \in \pi \Rightarrow (F, c(G)) \in \pi \Rightarrow c(F) \tilde{\cap} c(c(G)) \neq \tilde{\Phi} \Rightarrow c(F) \tilde{\cap} c(G) \neq \tilde{\Phi} \Rightarrow (F, G) \in \pi$.
Also $(F, G) \in \pi \Rightarrow (F, c_\pi(G)) \in \pi$.
Thus π is a Lodato proximity of soft sets on (X, E) such that $c_\pi = c$. □

4. Conclusion

In recent years, soft set has been emerged as a new tool for modeling uncertainty based problems. Mathematical structures like topology on soft sets, plays a crucial role in understanding the applicability of soft sets in many practical problems. Many authors have already studied the notion of topology on soft sets in recent years. On the other hand, proximity is structure which has a very important role in many problems of topological spaces like compactification and extension problems etc. So softification of the concept of proximity is highly desirable. In one of our earlier papers we have introduced a notion of proximity of soft sets which is called soft proximity. In this paper we have presented a new approach of proximity of soft sets. We are confident that this study will open up new areas of research in soft topology. We shall try to investigate the relations between the soft proximity and soft uniformity [6].

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