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A study on intuitionistic fuzzy topological* groups

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Abstract. The purpose of this paper is to introduce the concepts of an fully stratified space, intuitionistic fuzzy topological* group and intuitionistic fuzzy topological semigroup are introduced and studied. Some interesting properties are discussed. We also investigate some interesting properties of an intuitionistic fuzzy subgroup and intuitionistic fuzzy normal subgroup.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh [15]. Since then the concept has invaded nearly all branches of Mathematics. Chang [3] was introduced and developed the theory of fuzzy topological spaces and since then various notions in classical topology have been extended to fuzzy topological spaces. Atanassov [1] generalised fuzzy sets to intuitionistic fuzzy sets. On the otherhand, Coker [5] was introduced the notions of an intuitionistic fuzzy topological space and some other concepts. The concepts of quasi-coincidence for intuitionistic fuzzy point was introduced and developed by Francisco Gallego Lupianez [12]. Inheung chon [4] introduced and studied the concepts of some properties of fuzzy topological groups. Ma Ji Liang and Yu Chun Hai [13] changed the definition of a fuzzy topological group in order to make sure that an ordinary topological group is a special case of a fuzzy topological group. K.Hur, H.W.Kang and H.K.Song [9] studied the definition of an intuitionistic fuzzy subgroup. P.K.Sharma [14] discussed the concept of an intuitionistic fuzzy left and right coset and also introduced the notions of an intuitionistic fuzzy normal subgroup. In this paper, fully stratified space, intuitionistic fuzzy topological* group, intuitionistic fuzzy topological semigroup are introduced and studied. Some interesting properties are discussed. We also investigate some interesting properties of an intuitionistic fuzzy subgroup and intuitionistic fuzzy normal subgroup.

2. Preliminaries

Definition 2.1 ([1]). Let X be a nonempty fixed set and I be the closed interval [0,1]. An intuitionistic fuzzy set(IFS) A is an object of the following form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, where the mappings $\mu_A : X \longrightarrow I$ and $\gamma_A : X \longrightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. Obviously,every fuzzy set A on a nonempty set X is an IFS of the following form, $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.2 ([1]). Let X be a nonempty set. An intuitionistic fuzzy set (IFS, in short) A is an ordered pair (μ_A, γ_A) of fuzzy sets in X. Here $(\mu_A, \gamma_A)(x) = (\mu_A(x), \gamma_A(x))$ and $\mu_A(x)$, $\gamma_A(x)$ respectively denote the degree of membership and the degree of non membership of $x \in X$ to the set A and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.3 ([1]). Let X be a nonempty set and the IFSs A and B in the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(i) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;
(ii) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;
(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$.

Definition 2.4 ([1]). The IFSs $0_\sim$ and $1_\sim$ are defined by $0_\sim$=$\{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim$=$\{\langle x, 1, 0\rangle : x \in X\}$.

Definition 2.5 ([1]). Let X and Y be two nonempty sets and $f : X \to Y$ be a function.

(i) If $B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$ is an IFS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$), is the IFS in X defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}.$$

(ii) If $A = \{\langle x, \lambda_A(x), \vartheta_A(x)\rangle : x \in X\}$ is an IFS in X, then the image of A under f, denoted by $f(A)$, is the IFS in X defined by

$$f(A) = \{\langle y, f(\lambda_A(y)), (1 - f(1 - \vartheta_A))(y)\rangle : y \in Y\}.$$

$$f(\lambda_A)(y) = \begin{cases} sup_{x \epsilon f^{-1}(y)} \lambda_A(x) & \text{if } f^{-1}(y) \neq 0 \text{ ,} \\ 0, & \text{otherwise,} \end{cases}$$

$$(1 - f(1 - \vartheta_A))(y) = \begin{cases} inf_{x \epsilon f^{-1}(y)} \vartheta_A(x) & \text{if } f^{-1}(y) \neq 0 \text{ ,} \\ 1, & \text{otherwise.} \end{cases}$$

For the sake of simplicity, let us use the symbol $f_-(\vartheta_A)$ for $1 - f(1 - \vartheta_A)$.

Definition 2.6 ([5]). An intuitionistic fuzzy topology (IFT) in Coker's sense on a non empty set X is a family τ of IFSs in X satisfying the following axioms.

(T_1) $0_\sim$,$1_\sim \in \tau$
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(T_3) $\cup G_i \in \tau$ for arbitrary family $\{G_i/i \in I\} \subseteq \tau$

In this paper by (X, τ) or simply by X we will denote the Cocker's intuitionistic fuzzy topological space (IFTS). Each IFSs in τ is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.7 ([5]). Let A be an IFS in IFTS X. Then

$int(A) = \bigcup\{G \mid G$ is an IFOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy interior of A;

$clA = \bigcap\{G \mid G$ is an IFCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy closure of A.

Proposition 2.8 ([11]). *Let $f : (X, T) \to (Y, U)$ be a map. Then the following statements are equivalent:*
(i) *f is a continuous map.*
(ii) *$f(cl(A)) \subseteq cl(f(A))$ for each IFS A of X.*
(iii) *$cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each IFS B of Y .*
(iv) *$f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for each IFS B of Y .*

Definition 2.9 ([2]). Let (X_i, τ_i), $i = 1, 2$ be two IFTSs, and then the product IFT $\tau_1 \times \tau_2$ on $X_1 \times X_2$ is defined as the IFT generated by $\{p_i^{-1}(U_i) : U_i \in \tau_i, i = 1, 2\}$ where $p_i : X_1 \times X_2 \to X_i$, $i = 1, 2$ are the projection maps, and the IFTS $(X_1 \times X_2, \tau_1 \times \tau_2)$ is called the product IFTS.

This definition can be extended to an arbitrary family of IFTSs as follows:

Let $\{(X_i, \tau_i) : i \in J\}$ be a family of IFTSs. Then the product intuitionistic fuzzy topology τ on $X = \prod X_i$ is the one having $\{p_j^{-1}(U_j) : U_j \in \tau_j, j \in J\}$ as a subbase where $p_j : X \to X_j$ is the jth projection map. (X, τ) is called the product IFTS of the family $\{(X_i, \tau_i) : i \in J\}$.

Definition 2.10 ([7]). Let X, Y be nonempty sets and $A = \langle x, \mu_A(x), \gamma_A(x)\rangle$, $B = \langle y, \mu_B(y), \gamma_B(y)\rangle$ IFS's of X and Y , respectively. Then $A \times B$ is an IFS of $X \times Y$ defined by $(A \times B)(x, y) = \langle (x, y), min(\mu_A(x), \mu_B(y)), max(\gamma_A(x), \gamma_B(y))\rangle$

Definition 2.11 ([12]). Let X be a nonempty set and $x \in X$ a fixed element in X. If $r \in I_0$, $s \in I_1$ are fixed real numbers such that $r + s \leq 1$, then the IFS $x_{r,s} = \langle y, x_r, 1 - x_{1-s}\rangle$ is called an intuitionistic fuzzy point(IFP) in X, where r denotes the degree of membership of $x_{r,s}$, s denotes the degree of nonmembership of $x_{r,s}$ and $x \in X$ the support of $x_{r,s}$. The IFP $x_{r,s}$ is contained in the IFS $A(x_{r,s} \in A)$ if and only if $r < \mu_A(x), s > \gamma_A(x)$.

Definition 2.12 ([10]). Let $c(\alpha, \beta)$ be an IFP in X and let $A = \langle x, \mu_A, \gamma_A\rangle$ be an IFS in X. Then $c(\alpha, \beta)$ is said to belong to A, written $c(\alpha, \beta) \in A$, if $\mu_A(x) \geq \alpha$ and $\gamma_A(x) \leq \beta$. We say that $c(\alpha, \beta)$ is quasi-coincident with A, written $c(\alpha, \beta)qA$, if $\mu_A(c) + \alpha > 1$ and$\gamma_A(c) + \beta < 1$.

Proposition 2.13 ([8]). *Let U and V be two IFS's and $c(a, b)$ an IFP in X. Then:*
(i) *$U\tilde{q}\overline{V}$ iff $U \leq V$,*
(ii) *UqV iff $U \nleq \overline{V}$,*

(iii) $c(a,b) \leq U$ *iff* $c(a,b)\tilde{q}\overline{U}$,
(iv) $c(a,b)qU$ *iff* $c(a,b) \nleq \overline{U}$.

Definition 2.14 ([12]). Let (X,τ) be an IFTS, and let p be an IFP of X. Say that an IFS N of X is a Q-neighbourhood of p if there exists an IFOS A of (X,τ) such that pqA and $A \subseteq N$.

Proposition 2.15 ([12]). *Let X, Y be two nonempty sets, let $f : X \to Y$ be a map, let τ be an IFT in X, and let s be an IFT in Y. Then, $f : (X,\tau) \to (Y,s)$ is continuous if, and only if, for each IFP p of X, and for each Q-neighbourhood V of $f(p)$, there exists a Q-neighbourhood U of p such that $f(U) \subseteq V$.*

Proposition 2.16 ([5]). *Let (X,τ) and (Y,ϕ) be ITS's and $f : X \to Y$ a continuous function. If A is compact in (X,τ), then so is $f(A)$ in (Y,ϕ).*

Proposition 2.17 ([6]). *Let the IFTSs (X_1,τ_1) and (X_2,τ_2) be fuzzy compact. Then the product IFTS on $X = X_1 \times X_2$ is fuzzy compact, too.*

Definition 2.18 ([9]). An IFS $A = (\mu_A, \gamma_A)$ of a group G is said to be intuitionistic fuzzy subgroup of G (in short IFSG) of G if
(i) $\mu_A(xy) \geq Min(\mu_A(x), \mu_A(y))$
(ii) $\mu_A(x^{-1}) \geq \mu_A(x)$
(iii) $\gamma_A(xy) \leq Max(\gamma_A(x), \gamma_A(y))$
(iv) $\gamma_A(x^{-1}) \leq \gamma_A(x)$, for all $x, y \in G$.
In other words, An IFS A of X is IFSG of G iff $\mu_A(xy^{-1}) \geq Min(\mu_A(x), \mu_A(y))$ and $\gamma_A(xy^{-1}) \leq Max(\gamma_A(x), \gamma_A(y))$ holds for all $x, y \in G$.

Definition 2.19 ([14]). Let G be a group and A be IFSG of group G . Let $x \in G$ be a fixed element. Then for every element $g \in G$, we define
(i) $(xA)(g) = (\mu_{xA}(g), \gamma_{xA}(g))$,where $\mu_{xA}(g) = \mu_A(x^{-1}g)$ and $\gamma_{xA}(g) = \gamma_A(x^{-1}g)$. Then xA is called intuitionistic fuzzy left coset of G determined by A and x.
(ii) $Ax(g) = (\mu_{Ax}(g), \gamma_{Ax}(g))$, where $\mu_{Ax}(g) = \mu_A(gx^{-1})$ and $\gamma_{Ax}(g) = \gamma_A(gx^{-1})$ Then Ax is called the intuitionistic fuzzy right coset of G determined by A and x.

Definition 2.20 ([14]). An IFSG A of a group G is IFNSG of G if and only if $xA = Ax$ for all $x \in G$.

3. Intuitionistic fuzzy topological* groups

Notation 3.1. Let $(X,\cdot)$ be a group and $A = \langle x_1, \mu_A, \gamma_A\rangle$, $B = \langle x_2, \mu_B, \gamma_B\rangle$ are two intuitionistic fuzzy sets in X. We define AB and B^{-1} by the respective formula,
(i) $\mu_{AB}(x) = sup_{x=x_1x_2} min(\mu_A(x_1), \mu_B(x_2))$ and $\gamma_{AB}(x) = inf_{x=x_1x_2} max(\gamma_A(x_1), \gamma_B(x_2))$.
(ii) $\mu_{B^{-1}}(x) = \mu_B(x^{-1})$ and $\gamma_{B^{-1}}(x) = \gamma_B(x^{-1})$

Definition 3.2. Let $(X,\cdot)$ be a group and let (X,τ) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A\rangle$, $B = \langle x, \mu_B, \gamma_B\rangle$ and $C = \langle x, \mu_C, \gamma_C\rangle$ be an intuitionistic fuzzy sets in X. $(X,\cdot,\tau)$ is called an intuitionistic fuzzy topological* group or IFT^*G for short if and only if
(i) For all $x, y \in X$ and any Q-neighbourhood C of an intuitionistic fuzzy point $(xy)_{r,s}$, there are Q-neighbourhoods A of $x_{r,s}$ and B of $y_{r,s}$ such that $AB \subseteq C$.

(ii) For all $x \in X$ and any Q-neighbourhood B of an intuitionistic fuzzy point $(x^{-1})_{r,s}$, there exists a Q-neighbourhoods A of $x_{r,s}$ such that $A^{-1} \subseteq B$.

Definition 3.3. Let $(S, \cdot)$ be a semigroup and let (S, ϕ) be an intuitionistic fuzzy topological space. Let $B = \langle x, \mu_B, \gamma_B \rangle$, $C = \langle x, \mu_C, \gamma_C \rangle$ and $D = \langle x, \mu_D, \gamma_D \rangle$ be an intuitionistic fuzzy sets in X. $(S, \cdot, \phi)$ is called an intuitionistic fuzzy topological semigroup or $IFTSG$ for short if and only if for all $x, y \in S$ and any Q-neighbourhood D of an intuitionistic fuzzy point $(xy)_{r,s}$, there are Q-neighbourhoods B of $x_{r,s}$ and C of $y_{r,s}$ such that $BC \subseteq D$.

Definition 3.4. Let (X, τ) be an intuitionistic fuzzy topological space. Let $\alpha, \beta \in [0, 1]$. An intuitionistic fuzzy set $(\alpha\beta)^* = \{\langle x, \mu_{(\alpha\beta)^*}, \gamma_{(\alpha\beta)^*} \rangle : x \in X\}$, where $\mu_{(\alpha\beta)^*}(x) = \alpha$ and $\gamma_{(\alpha\beta)^*}(x) = \beta$, for every $x \in X$ such that $\mu_{(\alpha\beta)^*}(x) + \gamma_{(\alpha\beta)^*}(x) = 1$. Then (X, τ) is called a fully stratified space if for every $\alpha, \beta \in [0, 1]$, $(\alpha\beta)^* \in \tau$.

Proposition 3.5. *Let $\{(X_j, \tau_j)\}$, $j \in J$ be a family of an intuitionistic fuzzy topological spaces and (X, τ) be an product intuitionistic fuzzy topological space. The product intuitionistic fuzzy topology τ on X has as a base the set of finite intersections of an intuitionistic fuzzy sets of the form $p_j^{-1}[A_j]$, where $A_j \in \tau_j, j \in J$.*

Notation 3.6. An intuitionistic fuzzy set $A \times B$ is denoted by $(A, B)(x, y) = \langle (x, y), \mu_{(A,B)}(x, y), \gamma_{(A,B)}(x, y) \rangle$, where $\mu_{(A,B)}(x, y) = min(\mu_A(x), \mu_B(y))$ and $\gamma_{(A,B)}(x, y) = max(\gamma_A(x), \gamma_B(y))$, $(A, B) \in (X, \tau) \times (X, \tau)$.

Proposition 3.7. *If there is no intuitionistic fuzzy set in τ or intuitionistic fuzzy topological space (X, τ) is a fully stratified space. Let $A = \langle x, \mu_A, \gamma_A \rangle$, $B = \langle x, \mu_B, \gamma_B \rangle$ and $C = \langle x, \mu_C, \gamma_C \rangle$ be an intuitionistic fuzzy sets of X. Then, for all $x, y \in X$ and any Q-neighbourhood C of an intuitionistic fuzzy point $(xy)_{r,s}$, there are Q-neighbourhoods A of $x_{r,s}$ and B of $y_{r,s}$ such that $AB \subset C$, then the map $g : (X, \tau) \times (X, \tau) \to (X, \tau)$ defined by $g(x, y) = xy$ is an intuitionistic fuzzy continuous function.*

Proof. Assume that for any Q-neighbourhood C of an intuitionistic fuzzy point $(xy)_{r,s}$, there are Q-neighbourhoods A of $x_{r,s}$ and B of $y_{r,s}$ such that $AB \subseteq C$ and $A, B \in \tau$. For an intuitionistic fuzzy point $(x, y)_{r,s} \in (X, \tau) \times (X, \tau)$, from

$$\begin{aligned}\mu_{(A,B)}(x, y) + r &= min(\mu_A(x), \mu_B(y)) + r \\ &= min(\mu_A(x) + r, \mu_B(y) + r) \\ &> 1\end{aligned}$$

and

$$\begin{aligned}\gamma_{(A,B)}(x, y) + s &= max(\gamma_A(x), \gamma_B(y)) + s \\ &= max(\gamma_A(x) + s, \gamma_B(y) + s) \\ &< 1\end{aligned}$$

This implies that, $(x, y)_{r,s} q(A, B)$, since (A, B) belongs to the product intuitionistic fuzzy topology. Then By Proposition(3.1), there must be $A, B \in \tau$ such that $(A, B) \subset A$ and (A, B) is a Q-neighbourhood of $(x, y)_{r,s}$. Hence $g(A, B) = AB \subset C$. Then g is an intuitionistic fuzzy continuous function at the intuitionistic fuzzy point

$(x, y)_{r,s}$. Hence By Proposition(2.3), it follows that g is an intuitionistic fuzzy continuous function. □

Proposition 3.8. *Suppose there is no intuitionistic fuzzy set in τ or intuitionistic fuzzy topological space (X, τ) is a fully stratified space. let $A = \langle x, \mu_A, \gamma_A\rangle$ and $B = \langle x, \mu_B, \gamma_B\rangle$ be an intuitionistic fuzzy sets of X. Then, for all $x \in X$ and any Q-neighbourhood B of an intuitionistic fuzzy point $(x^{-1})_{r,s}$, there exists a Q-neighbourhood A of $x_{r,s}$ such that $A^{-1} \subset B$ if and only if the map $h : (X, \tau) \to (X, \tau)$ defined by $h(x) = x^{-1}$ is an intuitionistic fuzzy continuous function.*

Proof. **Necessity:** Assume that for any Q-neighbourhood B of an intuitionistic fuzzy point $(x^{-1})_{r,s}$, there exists a Q-neighbourhood A of $x_{r,s}$ such that $A^{-1} \subset B$ and $A, B \in \tau$. For an intuitionistic fuzzy point $x_{r,s} \in (X, \tau)$, from

$$\begin{aligned}\mu_{A^{-1}}(x) + r &= \mu_A(x^{-1}) + r\\ &> \mu_A(x) + r > 1\end{aligned}$$

and

$$\begin{aligned}\gamma_{A^{-1}}(x) + s &= \gamma_A(x^{-1}) + s\\ &< \gamma_A(x) + s < 1\end{aligned}$$

This implies that, $x_{r,s}qA^{-1}$. Hence A^{-1} is a Q-neighbourhood of $x_{r,s}$. Thus $h(A) = A^{-1} \subset B$. This implies that, $h(A) \subset B$. Then h is an intuitionistic fuzzy continuous function at the intuitionistic fuzzy point $x_{r,s}$. Hence By Proposition(2.3), h is an intuitionistic fuzzy continuous function.
Sufficiency: Assume that h is an intuitionistic fuzzy continuous function. Then h is an intuitionistic fuzzy continuous function at the intuitionistic fuzzy point $x_{r,s}$. Therefore for any Q-neighbourhood C of $h(x_{r,s}) = x^{-1}_{r,s}$, there exists a Q-neighbourhood A of $x_{r,s}$ such that $h(A) \subset C$. This implies that, $A^{-1} \subset C$. Hence for any Q-neighbourhood C of $x^{-1}_{r,s}$, there exists a Q-neighbourhood A of $x_{r,s}$ such that $A^{-1} \subset C$. □

Proposition 3.9. *Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $(S, \cdot, \phi)$ be an intuitionistic fuzzy topological semigroup.*

(i) *If U and V are intuitionistic fuzzy compact subsets of S, then UV is an intuitionistic fuzzy compact.*

(ii) *If W is an intuitionistic fuzzy compact subsets of X, then W^{-1} is an intuitionistic fuzzy compact.*

Notation 3.10. Let (X, τ) be a fully stratified space. Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $\alpha^* = \langle x, \mu_{\alpha^*}, \gamma_{\alpha^*}\rangle$ and $\alpha^*_\circ = \langle x, \mu_{\alpha^*_\circ}, \gamma_{\alpha^*_\circ}\rangle$ are an intuitionistic fuzzy set in X.

Note 3.11. Let (X, τ) be a fully stratified space, if $\mathcal{A} = \{A\}$ is a Q-neighbourhood base of $x_{r,s}$, and $\alpha^* \in \tau$ for any $\alpha \in [0, 1]$, then $\mathcal{A}_{\alpha_\circ} = \{A_{\alpha_\circ} | A_{\alpha_\circ} = A \cap \alpha^*_\circ, \alpha_\circ = sup[\mu_A(x)]$ and $\alpha^\circ = inf[\gamma_A(x)] | A \in \mathcal{A}\}$ is also a Q-neighbourhood base of $x_{r,s}$

Proposition 3.12. *Suppose there is no intuitionistic fuzzy set in τ or (X, τ) is a fully stratified space. Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $A = \langle x, \mu_A, \gamma_A\rangle$, $B = \langle x, \mu_B, \gamma_B\rangle$, $C = \langle x, \mu_C, \gamma_C\rangle$ and $D = \langle x, \mu_D, \gamma_D\rangle$ be an* *intuitionistic fuzzy sets of X .Then the mappings $f : x \to xa$, $g : x \to ax$ and $h : x \to x^{-1}$ are all intuitionistic fuzzy homeomorphic functions of (X, τ) onto itself, where $a \in X$ is a definite point.*

Proposition 3.13. *Suppose there is no intuitionistic fuzzy set in τ or (X, τ) is a fully stratified space. Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $A = \langle x, \mu_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \gamma_B \rangle$ be an intuitionistic fuzzy sets of X. Then*

(i) *If $IFcl(B)$ is an intuitionistic fuzzy closed set in τ, then $pIFcl(B), IFcl(B)p$, $IFcl(B)^{-1}$ are all intuitionistic fuzzy closed sets, where $p \in X$ is a definite point.*

(ii) *If A is an intuitionistic fuzzy open set in τ, then PA, AP, A^{-1} are all intuitionistic fuzzy open sets, where P is an non intuitionistic fuzzy set in X.*

Proposition 3.14. *Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $A = \langle x, \mu_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \gamma_B \rangle$ be an intuitionistic fuzzy sets of X. Suppose there is no intuitionistic fuzzy set in τ or (X, τ) is a fully stratified space. Then $IFcl(pAp^{-1}) = pIFcl(A)p^{-1}$, where $p \in X$ is a definite point.*

Notation 3.15.

(i) An intuitionistic fuzzy set $IFcl(A) \times IFcl(B)$ is denoted by $(IFcl(A), IFcl(B))(x, y) = \langle (x, y), min(\mu_{IFcl(A)}(x), \mu_{IFcl(B)}(y)), max(\gamma_{IFcl(A)}(x), \gamma_{IFcl(B)}(y)) \rangle$.

(ii) An intuitionistic fuzzy set $IFcl(A \times B)$ is denoted by $IFcl(A, B)(x, y) = \langle (x, y), min(\mu_{IFcl(A,B)}(x), \mu_{IFcl(A,B)}(y)), max(\gamma_{IFcl(A,B)}(x), \gamma_{IFcl(A,B)}(y)) \rangle$.

Proposition 3.16. *Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $A = \langle x, \mu_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \gamma_B \rangle$ be an intuitionistic fuzzy sets of X. Suppose there is no intuitionistic fuzzy set in τ or (X, τ) is a fully stratified space. If $(IFcl(A), IFcl(B)) \subseteq IFcl(A, B)$,*

(i) *$IFcl(A)IFcl(B^{-1}) \subseteq IFcl(AB^{-1})$*

(ii) *$IFcl(A)IFcl(B) \subseteq IFcl(AB)$*

Proof. **(i)** Let $f : (X, \tau) \times (X, \tau) \to (X, \tau)$ be a map defined by $f(x, y) = xy^{-1}$. By Proposition(3.2), it follows that f is an intuitionistic fuzzy continuous function. Since $(IFcl(A), IFcl(B)) \subseteq IFcl(A, B)$, $f(IFcl(A), IFcl(B)) \subseteq f(IFcl(A, B))$. Since f is an intuitionistic fuzzy continuous function, By Proposition(2.1),

$$f(IFcl(A, B)) \subseteq IFcl(f(A, B))$$

Thus $f(IFcl(A), IFcl(B)) \subseteq IFcl(f(A, B))$

$$IFcl(A)(IFcl(B))^{-1} \subseteq IFcl(AB^{-1}) \tag{3.1}$$

For $x \in X$,

$$\begin{aligned} IFcl(B^{-1})(x) &= (\bigcap_{K_i \supseteq B^{-1}} K_i)(x) \\ &= inf_{K_i^{-1} \supseteq B} K_i^{-1}(x^{-1}) \\ &= IFcl(B)(x^{-1}) \\ &= (IFcl(B))^{-1}(x) \end{aligned}$$

(3.2) $$IFcl(B^{-1})(x) = (IFcl(B))^{-1}(x)$$

From (3.3), it follows that $IFcl(A)IFcl(B^{-1}) \subseteq IFcl(AB^{-1})$.

(ii) Similarly we may prove $IFcl(A)IFcl(B) \subseteq IFcl(AB)$. □

Proposition 3.17. *Suppose there is no intuitionistic fuzzy set in τ or (X, τ) is a fully stratified space. Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set of X and $(IFcl(A), IFcl(A)) \subseteq IFcl(A, A)$. If A is an intuitionistic fuzzy subgroup of an intuitionistic fuzzy topological* group, $IFcl(A)$ is an intuitionistic fuzzy subgroup of an intuitionistic fuzzy topological* group.*

Proof. **(i)** Let $z, p \in X$. Then

$$\begin{aligned} \mu_A(z) &= \mu_A(pp^{-1}z) \\ &\geq min(\mu_A(p), \mu_A(p^{-1}z)) \end{aligned}$$

Thus

$$\begin{aligned} \mu_A(z) &\geq sup_{z=ab} min(\mu_A(a), \mu_A(b)) \\ &\geq \mu_{AA}(z) \end{aligned}$$

Hence $\mu_A(z) \geq \mu_{AA}(z)$. Thus

(3.3) $$\mu_{IFcl(A)}(z) \geq \mu_{IFcl(AA)}(z)$$

By Proposition(3.9(ii)), $IFcl(A)IFcl(A) \subseteq IFcl(AA)$. That is, $\mu_{IFcl(A)IFcl(A)}(z) \leq \mu_{IFcl(AA)}(z)$
From (3.4), $\mu_{IFcl(A)}(z) \geq \mu_{IFcl(A)IFcl(A)}(z)$

$$\begin{aligned} \mu_{IFcl(A)}(xy) &\geq \mu_{IFcl(A)IFcl(A)}(xy) \\ &\geq sup_{xy=ab} min(\mu_{IFcl(A)}(a), \mu_{IFcl(A)}(b)) \\ &\geq min(\mu_{IFcl(A)}(x), \mu_{IFcl(A)}(y)) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(z) &= \gamma_A(pp^{-1}z) \\ &\leq max(\gamma_A(p), \gamma_A(p^{-1}z)) \end{aligned}$$

$$\begin{aligned} \gamma_A(z) &\leq inf_{z=ab} max(\gamma_A(a), \gamma_A(b)) \\ &\leq \gamma_{AA}(z) \end{aligned}$$

Hence $\gamma_A(z) \leq \gamma_{AA}(z)$. Thus

(3.4) $$\gamma_{IFcl(A)}(z) \leq \gamma_{IFcl(AA)}(z)$$

By Proposition(3.9(ii)), $IFcl(A)IFcl(A) \subseteq IFcl(AA)$. That is, $\gamma_{IFcl(A)IFcl(A)}(z) \geq \gamma_{IFcl(AA)}(z)$

From (3.4), $\gamma_{IFcl(A)}(z) \leq \gamma_{IFcl(A)IFcl(A)}(z)$

$$\begin{aligned}\gamma_{IFcl(A)}(xy) &\leq \gamma_{IFcl(A)IFcl(A)}(xy)\\ &\leq inf_{xy=ab}max(\gamma_{IFcl(A)}(a), \gamma_{IFcl(A)}(b))\\ &\leq max(\gamma_{IFcl(A)}(x), \gamma_{IFcl(A)}(y))\end{aligned}$$

(ii) Since A is an intuitionistic fuzzy subgroup of X, $\mu_A(x) \geq \mu_A(x^{-1}) = \mu_{A^{-1}}(x)$ for all $x \in X$. This implies that $\mu_A(x) \geq \mu_{A^{-1}}(x)$. Hence $\mu_{IFcl(A)}(x) \geq \mu_{IFcl(A^{-1})}(x)$. By Equation(3.4), $\mu_{IFcl(A^{-1})}(x) = \mu_{(IFcl(A))^{-1}}(x)$. This implies that $\mu_{IFcl(A)}(x) \geq \mu_{(IFcl(A))^{-1}}(x)$ and Hence $IFcl(A)$ and $\gamma_A(x) \leq \gamma_A(x^{-1}) = \gamma_{A^{-1}}(x)$ for all $x \in X$. This implies that $\gamma_A(x) \leq \gamma_{A^{-1}}(x)$. Hence $\gamma_{IFcl(A)}(x) \leq \gamma_{IFcl(A^{-1})}(x)$. By Equation(3.4), $\gamma_{IFcl(A^{-1})}(x) = \gamma_{(IFcl(A))^{-1}}(x)$. This implies that $\gamma_{IFcl(A)}(x) \leq \gamma_{(IFcl(A))^{-1}}(x)$. Hence $IFcl(A)$ is an intuitionistic fuzzy subgroup of X. □

Proposition 3.18. *Suppose there is no intuitionistic fuzzy set in τ or (X, τ) is a fully stratified space. Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $B = \langle x, \mu_B, \gamma_B \rangle$ be an intuitionistic fuzzy set of X and B is an intuitionistic fuzzy subgroup of X. If B is an intuitionistic fuzzy normal subgroup of an intuitionistic fuzzy topological* group, then $IFcl(B)$ is an intuitionistic fuzzy normal subgroup of an intuitionistic fuzzy topological* group.*

Proof. Let B be an intuitionistic fuzzy normal subgroup. Then $xB = Bx$ for $x \in X$.

$$\mu_{xIFcl(B)}(z) = \mu_{IFcl(B)}(x^{-1}z)(\text{since the left coset } \mu_{xB}(z) = \mu_B(x^{-1}z)) \quad (3.7)$$

$$\begin{aligned}\mu_{IFcl(B)x}(z) &= \mu_{IFcl(B)}(zx^{-1})(\text{since the right coset } \mu_{Bx}(z) = \mu_B(zx^{-1}))\\ &= \mu_{IFcl(B)}(x^{-1}z)(\text{since the associative law } xy = yx) \quad (3.8)\end{aligned}$$

From (3.7) and (3.8), $\mu_{xIFcl(B)}(z) = \mu_{IFcl(B)x}(z)$ and

$$\gamma_{xIFcl(B)}(z) = \gamma_{IFcl(B)}(x^{-1}z)(\text{since the left coset } \gamma_{xB}(z) = \gamma_B(x^{-1}z)) \quad (3.9)$$

$$\begin{aligned}\gamma_{IFcl(B)x}(z) &= \gamma_{IFcl(B)}(zx^{-1})(\text{since the right coset } \gamma_{Bx}(z) = \gamma_B(zx^{-1}))\\ &= \gamma_{IFcl(B)}(x^{-1}z)(\text{since the associative law } xy = yx) \quad (3.10)\end{aligned}$$

From (3.9) and (3.10), $\gamma_{xIFcl(B)}(z) = \gamma_{IFcl(B)x}(z)$. This implies that, $xIFcl(B) = IFcl(B)x$. Hence $IFcl(B)$ is an intuitionistic fuzzy normal subgroup of an intuitionistic fuzzy topological* group. □

Notation 3.19. Let $(X, \cdot)$ be a group and let e be an identity of X. Let $A = \langle y, \mu_A, \gamma_A\}$ be an intuitionistic fuzzy set in X. Let $x_{r,s}$ be an intuitionistic fuzzy point in X. We define $x_{r,s}A$ by the respective formula,

$x_{r,s}A = \langle x, \mu_{rA}, \gamma_{sA}\rangle$, where $\mu_{rA}(z) = sup_{z=xy}min(r(x), \mu_A(y))$ and
$\gamma_{sA}(z) = inf_{z=xy}max(s(x), \gamma_A(y))$ for every $x \in X$.

Proposition 3.20. *Let $(X, \cdot)$ be a group and let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group. Then $xA = x_{r,s}A$, where $a \in X$ is a definite point.*

Proof. Let $A = \{x \in X : \mu_A(x) = \mu_A(e), \gamma_A(x) = \gamma_A(e)\}$ be an intuitionistic fuzzy set in X and let $x_{r,s}$ be an intuitionistic fuzzy point in X. Let $r = 1$, $s = 0$ and $r + s \leq 1$. □

Remark 3.21. Suppose there is no intuitionistic fuzzy set in τ or (X, τ) is a fully stratified space. Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $A = \langle x, \mu_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \gamma_B \rangle$ be an intuitionistic fuzzy sets of X. Then

(i) If $IFcl(B)$ is an intuitionistic fuzzy closed set in τ, then $x_{r,s}IFcl(B)$, $IFcl(B)x_{r,s}, IFcl(B)^{-1}$ are all intuitionistic fuzzy closed sets.

(ii) If A is an intuitionistic fuzzy open set in τ, then PA, AP, A^{-1} are all intuitionistic fuzzy open sets, where P is an non intuitionistic fuzzy set in X.

Notation 3.22. Let $(X, .)$ be a group and let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy subgroup of group X. Let $x_{r,s}$ be an intuitionistic fuzzy point. Then for every element $z \in X$,

(i) $x_{r,s}A(z) = A(x^{-1}z)$.

(ii) $Ax_{r,s}(z) = A(zx^{-1})$

Proposition 3.23. *Suppose there is no intuitionistic fuzzy set in τ or (X, τ) is a fully stratified space. Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $C = \langle x, \mu_C, \gamma_C \rangle$ be an intuitionistic fuzzy set of X. Let e be an identity of X. If C is a Q-neighbourhood of $e_{r,s}$, then $x_{1,0}C$ is a Q-neighbourhood of $x_{r,s}$.*

Proof. Since C is a Q-neighbourhood of $e_{r,s}$, there exists an intuitionistic fuzzy open set $A = \{x \in X : \mu_A(x) = \mu_A(e), \gamma_A(x) = \gamma_A(e)\}$ such that $r + \mu_A(e) > 1$ and $s + \gamma_A(e) < 1$, $A \subseteq C$. Let $r = 1$, $s = 0$ and $r + s \leq 1$

$$\mu_{rA}(x) = sup_{x=xe} min(r(x), \mu_A(e)) = min(1, \mu_A(e)) = \mu_A(e) \text{ and}$$
$$\gamma_{sA}(x) = inf_{x=xe} max(s(x), \gamma_A(e)) = max(0, \gamma_A(e)) = \gamma_A(e)$$

Since $\mu_A(e) > 1 - r$ and $\gamma_A(e) < 1 - s$.

$$r + \mu_{rA}(x) = r + \mu_A(e) \geq r + 1 - r = 1 \text{ and}$$
$$s + \gamma_{sA}(x) = s + \gamma_A(e) \leq s + 1 - s = 1$$

Thus $x_{1,0}C(z) = C(x^{-1}z) \supseteq A(x^{-1}z) = x_{1,0}A(z)$ for all $z \in X$, for all $z \in X$. Hence $x_{1,0}C$ is a Q-neighbourhood of $x_{r,s}$ and $x_{1,0}A \subseteq x_{1,0}C$. Therefore By Remark(3.1), $x_{1,0}A$ is an intuitionistic fuzzy open set. Thus there exists an intuitionistic fuzzy open set $x_{1,0}A$ such that $x_{1,0}C$ is a open Q-neighbourhood of $x_{r,s}$ and $x_{1,0}A \subseteq x_{1,0}C$. Hence $x_{1,0}C$ is a Q-neighbourhood of $x_{r,s}$. □

Proposition 3.24. *An intuitionistic fuzzy point $x_{r,s} \subseteq IFcl(A)$ if and only if each Q-neighbourhood of $x_{r,s}$ is quasi-coincident with A.*

Notation 3.25. Let X be a nonempty set. An intuitionistic fuzzy set AC is an ordered pair (μ_{AC}, γ_{AC}) of fuzzy sets in X. Here $(\mu_{AC}, \gamma_{AC})(x) = (\mu_{AC}(x), \gamma_{AC}(x))$. That is, $AC(x) = \langle \mu_{AC}(x), \gamma_{AC}(x) \rangle$ for every $x \in X$.

Theorem 3.26. *Suppose there is no intuitionistic fuzzy set in τ or (X, τ) is a fully stratified space. Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy subset of X. Let e be an identity of X.*

If $x_{r,s} \subseteq IFcl(A)$, then $(\bigcap_{C\in\{C\}} AC)(x) = (\bigcap_{c\in\{C\}} CA)(x) \supset 0$, where $\{C\}$ is the system of all Q-neighbourhoods off $e_{a,b}$ in X with $a \leq r$ and $b \geq s$.

Proof. Since $x_{r,s} \subseteq IFcl(A)$, By Proposition (3.13), each Q-neighbourhood of $x_{r,s}$ is quasi-coincident with A . For any $C \in \{C\}$, there exists an intuitionistic fuzzy open set $B = \{x \in X : \mu_B(x) = \mu_B(e), \gamma_B(x) = \gamma_B(e)\}$ such that $\mu_B(e) + a > 1$ and $\gamma_B(e) + b < 1$ and $B \subseteq C$. Let $r = 1$, $s = 0$ and $r + s \leq 1$. By Remark(3.1), $x_{1,0}B^{-1}$ is an intuitionistic fuzzy open set.

$$\begin{aligned}\mu_{rB^{-1}}(x) &= sup_{x=xe} min(r(x), \mu_{B^{-1}}(e))\\ &= min(1, \mu_B(e^{-1}))\\ &= min(1, \mu_B(e))\\ &= \mu_B(e)\end{aligned}$$

and

$$\begin{aligned}\gamma_{sB^{-1}}(x) &= inf_{x=xe} max(s(x), \gamma_{B^{-1}}(e))\\ &= max(0, \gamma_B(e^{-1}))\\ &= max(0, \gamma_B(e))\\ &= \gamma_B(e)\end{aligned}$$

Thus

$$\mu_{rB^{-1}}(x) + r = r + \mu_B(e) \geq a + \mu_B(e) > 1$$

and

$$\gamma_{sB^{-1}}(x) + s = s + \gamma_B(e) \leq b + \gamma_B(e) < 1$$

Hence $x_{1,0}C^{-1}(z) = C^{-1}(x^{-1}z) \supseteq B^{-1}(x^{-1}z) = x_{r,s}B^{-1}(z)$. This implies that, $x_{1,0}B^{-1} \subseteq x_{1,0}C^{-1}$. That is, $x_{1,0}C^{-1}$ is a Q-neighbourhood of $x_{r,s}$. Since $x_{r,s}C^{-1}$ and A are quasi-coincident, there exists $y \in X$ such that $\mu_{rC^{-1}}(y) + \mu_A(y) > 1$ and $\gamma_{sC^{-1}}(y) + \gamma_A(y) < 1$. Hence $x_{1,0}C^{-1}(y) > 0$ and $A(y) > 0$. Let $r = 1$, $s = 0$ and $r + s \leq 1$.

$$\begin{aligned}\mu_{rC^{-1}}(y) &= sup_{y=xx^{-1}y} min(r(x), \mu_{C^{-1}}(x^{-1}y))\\ &= min(1, \mu_{C^{-1}}(x^{-1}y))\\ &= \mu_{C^{-1}}(x^{-1}y)\\ &= \mu_C(y^{-1}x)\end{aligned}$$

and

$$\begin{aligned}\gamma_{sC^{-1}}(y) &= inf_{y=xx^{-1}y} max(s(x), \gamma_{C^{-1}}(x^{-1}y))\\ &= max(0, \mu_{C^{-1}}(x^{-1}y))\\ &= \gamma_{C^{-1}}(x^{-1}y)\\ &= \gamma_C(y^{-1}x)\end{aligned}$$

Thus

$$\begin{aligned}\mu_{AC}(x) &= sup_{x=de} min(\mu_A(d), \mu_C(e)) \\ &\geq min(\mu_A(y), \mu_C(y^{-1}x)) \\ &= min(\mu_A(y), \mu_{rC^{-1}}(y)) \\ &> 0\end{aligned}$$

and

$$\begin{aligned}\gamma_{AC}(x) &= inf_{x=de} max(\gamma_A(d), \gamma_C(e)) \\ &\leq max(\gamma_A(y), \gamma_C(y^{-1}x)) \\ &= max(\gamma_A(y), \gamma_{sC^{-1}}(y)) \\ &< 0\end{aligned}$$

That is, $AC(x) \supset 0$ for every $C \in \{C\}$. Hence $(\cap AC)(x) = inf_{C\in\{C\}} AC(x) \supset 0$. It is easy to prove $\cap AC = \cap CA$. □

Theorem 3.27. *Suppose there is no intuitionistic fuzzy set in τ or (X, τ) is a fully stratified space. Let $(X, \cdot, \tau)$ be an intuitionistic fuzzy topological* group and let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy subset of X. Let e be an identity of X. If $x_{r,s} \subseteq \cap_{C\in\{C\}} AC = \cap_{C\in\{C\}} CA$ and $r > 0.5$, $s < 0.5$, then $x_{r,s} \subseteq IFcl(A)$, where $\{C\}$ is the system of all Q-neighbourhoods of $e_{a,b}$ in X with $a \geq r$ and $b \leq s$.*

Proof. Let $x_{r,s} \subseteq AC$ for each $C \in \{C\}$. Then $\mu_{AC}(x) \geq r$ and $\gamma_{AC}(x) \leq s$ for each $C \in \{C\}$. Let D be an arbitrary Q-neighbourhood of $x_{r,s}$. Then there exists an intuitionistic fuzzy open set B such that $\mu_B(x) + r > 1$ and $\gamma_B(x) + s < 1$ and $B \subseteq D$. Thus $D(x) \supset 0$. Let $r = 0$, $s = 1$ and $r + s \leq 1$. By Remark(3.1), $B^{-1}x_{1,0}$ is an intuitionistic fuzzy open set.

$$\begin{aligned}\mu_{B^{-1}r}(e) &= sup_{e=x^{-1}x} min(\mu_{B^{-1}}(x^{-1}), r(x)) \\ &= min(\mu_B(x), 1) \\ &= \mu_B(x)\end{aligned}$$

and

$$\begin{aligned}\gamma_{B^{-1}s}(e) &= inf_{e=x^{-1}x} max(\gamma_{B^{-1}}(x^{-1}), s(x)) \\ &= max(\gamma_B(x), 0) \\ &= \gamma_B(x)\end{aligned}$$

$$\mu_{B^{-1}r}(e) + a = \mu_B(x) + a \geq \mu_B(x) + r > 1$$

and

$$\gamma_{B^{-1}s}(e) + b = \gamma_B(x) + b \leq \gamma_B(x) + s < 1$$

Therefore $B^{-1}x_{1,0}(z) = B^{-1}(zx^{-1}) \subseteq D^{-1}(zx^{-1}) = D^{-1}x_{r,s}(z)$. This implies that, $B^{-1}x_{1,0} \subseteq D^{-1}x_{1,0}$. That is, $D^{-1}x_{1,0}$ is a Q-neighbourhood of $e_{a,b}$. That is,

$D^{-1}x_{1,0} \in \{C\}$. Thus $\mu_{AD^{-1}r}(x) \geq r$ and $\gamma_{AD^{-1}s}(x) \leq s$.

$$\begin{aligned}\mu_{AD^{-1}r}(x) &= sup_{x=ex} min(\mu_{AD^{-1}}(e), r(x)) \\ &= min(\mu_{AD^{-1}}(e), 1) \\ &= \mu_{AD^{-1}}(e)\end{aligned}$$

and

$$\begin{aligned}\gamma_{AD^{-1}s}(x) &= inf_{x=ex} max(\gamma_{AD^{-1}}(e), s(x)) \\ &= max(\gamma_{AD^{-1}}(e), 1) \\ &= \gamma_{AD^{-1}}(e)\end{aligned}$$

$$\begin{aligned}\mu_{AD^{-1}}(e) &= sup_{e=kk^{-1}} min(\mu_A(k), \mu_{D^{-1}}(k^{-1})) \\ &= min(\mu_A(k), \mu_D(k))\end{aligned}$$

and

$$\begin{aligned}\gamma_{AD^{-1}}(e) &= inf_{e=kk^{-1}} max(\gamma_A(k), \gamma_{D^{-1}}(k^{-1})) \\ &= max(\gamma_A(k), \gamma_D(k))\end{aligned}$$

Thus there exists $z \in X$ such that, $\mu_{AD^{-1}}(e) = min(\mu_A(z), \mu_D(z))$ and $\gamma_{AD^{-1}}(e) = max(\gamma_A(z), \gamma_D(z))$. Since $\mu_{AD^{-1}}(e) \geq r$ and $\gamma_{AD^{-1}}(e) \leq s$, $\mu_A(z) \geq r$ and $\gamma_A(z) \leq s$ and $\mu_D(z) \geq r$ and $\gamma_D(z) \leq s$. Since $r > 0.5$, $s < 0.5$,

$$\mu_A(z) + \mu_D(z) \geq r + r = 2r > 1$$

and

$$\gamma_A(z) + \gamma_D(z) \leq s + s = 2s < 1$$

That is D is quasi-coincident with A. By Proposition(3.13), $x_{r,s} \subseteq IFcl(A)$. □

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