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Fuzzy EPQ model with dynamic demand under bi-level trade credit policy

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Abstract. An Economic Production Quantity (EPQ) model with dynamic demand is developed in an imprecise environment under bi-level trade credit policy. Supplier offers a delay period (M) to the retailer for payment. Due to this facility retailer also offers a trade credit period (N) to his customers to boost the demand. During trade credit period of customers, demand of the item increases with time at a decreasing rate. Different inventory parameters are assumed as fuzzy numbers. Average profit function is imprecise in nature and its possibilistic mean value is maximized for making optimal decision. Depending upon the values of M and N twelve scenarios may occur. All the scenarios are illustrated with numerical examples.

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1. Introduction

In classical inventory models it is normally assumed that payments will be made to the suppliers for the goods immediately after receiving the consignment. However, one can easily observe that in many cases suppliers provide credit periods for the retailers to stimulate demand. A considerable number of research papers have been published in this direction during last two decades ([1], [15], [5], [20], [11], [12], [13] etc.). Most of the above models assumed that suppliers supply the items to the retailers in a lot. But in many real life situations it is observed that suppliers replenish the item at a finite rate. For example, suppliers of rice, wheat etc., collect and supply the item to the retailers at a finite rate in general. As replenishment of the item is made at a finite rate, the corresponding model can be treated for a retailer as an EPQ model under finite replenishment rate. Very few research papers have been considered this real life situation ([12], [13]).

Again all the above models assumed that producer/supplier offers the retailer a delay period for payment after delivery of item and the retailer sells the goods and accumulate revenue and earn interest within the trade credit period. They implicitly assumed that the customer would pay for the items as soon as the items are received from the retailer. That is, they assumed that the supplier would offer the retailer a delay period for payment but the retailer would not offer the trade credit period to his/her customer. In most business transactions, this assumption is debatable and these situations can be defined as one level of trade credit. Huang [13] proposed two levels of trade credit policy where the supplier would offer the retailer a delay period for payment and the retailer also adopts the trade credit policy to stimulate his/her customer demand to develop the retailer's replenishment model. Furthermore, he also assumed that the retailer's trade credit period offered by supplier M is not shorter than the customer's trade credit period offered by the retailer N ($M \geq N$). Recently, Goswami [10] developed an optimal replenishment decisions in the EPQ model for deteriorating items with two levels of trade credit financing. Mahata [16] developed an EPQ-based inventory model for exponentially deteriorating items under trade credit policy.

One of the drawbacks of the models ([13], [10], [16]) is that the assumed demand during retailer's credit period is constant, which is not realistic. As in reality demand during credit period increases and depends on the credit period. Again as demand depends on credit period, the assumption of ($M \geq N$) is also debatable.

It has been recognized that one's ability to make precise statement concerning different parameters of an inventory model diminishes with increasing complexities of world economy throughout the year. As a result it is very difficult to estimate the parameters of an item precisely. Many authors have developed inventory models in imprecise environment ([21],[14],[17],[8],[9], [3], [2], [6], [18],[19] etc.).

Here an inventory model of an item is developed where the item is supplied to the retailer by the supplier at a finite replenishment rate. The supplier offers the retailer a delay period (M) for payment and the retailer also offers his/her customers a delay period (N) for payment to stimulate his/her customer's demand. i.e., item purchased by the customers during this period has to pay final payment at time N. As effective credit period of a customer purchasing item at t ($0 < t < N$) is (N-t), it is assumed that demand increases with decreasing rate during [0, N]. After the trade credit period [0, N], demand at t=N, prevails for the rest of the period.

Here the carrying cost rate, ordering cost, unit purchasing price and unit selling price are assumed as fuzzy number to fit the real world. Model is formulated as profit maximization principle. Here the average profit is fuzzy in nature. The possibilistic mean value of a fuzzy number ([4]) is used to rank fuzzy numbers for the optimal decision. The outline of this paper is as follows. Section 2 contains relevant notations and assumption connected to the model. Section 3, Section 4 and Section 5 present the mathematical formulation and determination of the optimal replenishment time of the proposed fuzzy EPQ model under two levels of trade credit policy. Section 6 contains the algorithm of the proposed model. Section 7 is the illustration with numerical example and Section 8 represent sensitivity analysis. Final Section 9 contains the concluding remarks.

2. Notations and assumptions

The following notations and assumptions are used in developing the model.

2.1. Notations.

(i) $q(t)$=inventory level at time t
(ii) P=replenishment rate per year
(iii) $\tilde{A}$=Fuzzy set up cost per order
(iv) $\widetilde{c}_p$=unit purchasing price which is fuzzy in nature
(v) $\widetilde{s}_p$=unit selling price which is fuzzy in nature
(vi) $\tilde{h}$=fuzzy unit stock holding cost per item per year excluding interest charges
(vii) I_e=interest earned per order quantity per year by the the retailer
(viii) I_k=interest charged per order quantity in stocks per year by the supplier
(ix) M=Retailer's trade credit period offered by supplier in years.
(x) N=Customers trade credit period offered by retailer
(xi) D=Demand rate per year
(xii) T=Cycle length in years
(xiii) t_1=Length of time up to which replenishment is being held. It is taken as decision variable
(xiv) $\widetilde{P_{ij}}(i$=1,2 ; j=1,2,...,6)=annual total profit, which is a function of t_1, where annual total Profit=Sales revenue - Purchasing cost - Ordering cost - Holding cost - Interest to be paid + Interest earned

2.2. Assumptions.

(i) Demand rate D increases with time during credit period of customers and is of the form

$$D = \begin{cases} a - be^{-ct} & \text{if } 0 < t \leq N \\ a - be^{-cN} & \text{if } N \leq t \leq T, \end{cases}$$

where a, b, c are positive constants.
(ii) Shortages are not allowed and lead time is negligible.
(iii) Time horizon is infinite.
(iv) Interest charged per order quantity (I_k) is greater than or equal to interest earned per order quantity (I_e) i.e., $I_k \geq I_e$. Selling price ($\widetilde{s}_p$) is greater than or equal to purchasing price($\widetilde{c}_p$) i.e., $\widetilde{s}_p \,\tilde{\geq}\, \widetilde{c}_p$.
(v) Replenishment rate, P, is known and constant.
(vi) Replenishment time (t_1) is taken as decision variable.
(vii) Selling price($\widetilde{s}_p$) is mark-up(m) of purchasing price($\widetilde{c}_p$) i.e., $\widetilde{s}_p$=$m\widetilde{c}_p$, $m > 1$.
(viii) When $N < M$, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit. When $N \geq M$, retailer will not earn any interest.

3. Mathematical formulation

In the development of the model it is assumed that supplier supplies the item to the retailer at a finite rate P, during $[0,\ t_1]$ and retailer sells the item to the customers during $[0,\ T]$.
Here the revenue from sales ($\widetilde{SP}$) is given by $\widetilde{SP}$=$Pt_1\widetilde{s}_p$=$Pt_1m\widetilde{c}_p$, m>1.

Purchasing cost ($\widetilde{PC}$) is given by $\widetilde{PC}$=$Pt_1\widetilde{c}_p$.

According to assumption ($viii$) depending on the values of M and N, there may be two cases (Case 1: $N < M$ and Case 2: $N \geq M$) to occur on interest charged and interest earned per year. Depending on the values of M and N, six subcases may arise for each of the above two cases.

Case-1: $N < M$

Depending on the values of M and N six subcases may arise which are presented below:

Subcase-1.1 : $N < M \leq t_1$

Let $q(t)$ be the inventory level at time t which is given by

$$\frac{dq(t)}{dt} = \begin{cases} P - D = P - a + be^{-ct} & \text{for } 0 < t \leq N \\ P - D = P - a + be^{-cN} & \text{for } N \leq t \leq t_1 \\ -D = -a + be^{-cN} & \text{for } t_1 \leq t \leq T, \end{cases} \tag{3.1}$$

where $q(0)$=0=$q(T)$ and maximum inventory occur at t=t_1.
On integration and using the above conditions and continuity at t=N

$$q(t) = \begin{cases} (P-a)t + \dfrac{b}{c}(1 - e^{-ct}) & \text{if } \ 0 < t \leq N \\ (P - a + be^{-cN})t + \dfrac{b}{c} - \left(\dfrac{b}{c} + bN\right)e^{-cN} & \text{if } \ N \leq t \leq t_1 \\ (a - be^{-cN})(T - t) & \text{if } \ t_1 \leq t \leq T \end{cases} \tag{3.2}$$

At t=t_1,

$$T = t_1 + \frac{1}{a - be^{-cN}}\left[(P - a + be^{-cN})t_1 + \frac{b}{c} - \left(\frac{b}{c} + bN\right)e^{-cN}\right] \tag{3.3}$$

Holding cost($\widetilde{HOC}_{11}$) is given by

$$\begin{aligned} \widetilde{HOC}_{11} &= \widetilde{h}\Big[\int_0^T q(t)dt\Big] = \widetilde{h}\Big[\int_0^N q(t)dt + \int_N^{t_1} q(t)dt + \int_{t_1}^T q(t)dt\Big] \\ &= \widetilde{h}[I_1 + I_2 + I_3], \end{aligned} \tag{3.4}$$

where

$$I_1 = \int_0^N \left[(P-a)t + \frac{b}{c}(1 - e^{-ct})\right]dt = \frac{(P-a)N^2}{2} + \frac{b}{c}N - \frac{b}{c^2}(1 - e^{-cN})$$

$$
\begin{aligned}
I_2 &= \int_N^{t_1} \Big[(P-a+be^{-cN})t+\frac{b}{c}-\Big(\frac{b}{c}+bN\Big)e^{-cN}\Big]dt \\
&= (P-a+be^{-cN})\frac{(t_1^2-N^2)}{2}+\Big[\frac{b}{c}-\Big(\frac{b}{c}+bN\Big)e^{-cN}\Big](t_1-N) \\
I_3 &= \int_{t_1}^{T} \Big[(a-be^{-cN})(T-t)\Big]dt=(a-be^{-cN})\frac{(T-t_1)^2}{2}
\end{aligned}
$$

Interest to be paid ($\widetilde{IP}_{11}$) is given by

$$
\begin{aligned}
\widetilde{IP}_{11} &= \widetilde{c}_p I_k\Big[\int_M^T q(t)dt\Big] \\
&= \widetilde{c}_p I_k\Big[\int_M^{t_1} q(t)dt+\int_{t_1}^{T} q(t)dt\Big] \\
&= \widetilde{c}_p I_k[I_4+I_3], \qquad (3.5)
\end{aligned}
$$

where

$$
\begin{aligned}
I_4 &= \int_M^{t_1} \Big[(P-a+be^{-cN})t+\frac{b}{c}-\Big(\frac{b}{c}+bN\Big)e^{-cN}\Big]dt \\
&= \frac{(P-a+be^{-cN})(t_1^2-M^2)}{2}+\Big[\frac{b}{c}-\Big(\frac{b}{c}+bN\Big)e^{-cN}\Big](t_1-M)
\end{aligned}
$$

Interest earned($\widetilde{IE}_{11}$) is given by

$$
\begin{aligned}
\widetilde{IE}_{11} &= \widetilde{s}_p I_e\Big[\int_0^N D(t)dt(M-N)+\int_N^M D(t)dt(M-t)\Big] \\
&= m\widetilde{c}_p I_e\Big[(M-N)\Big(aN-\frac{b}{c}(1-e^{-cN})\Big)+(a-be^{-cN})\frac{(M-N)^2}{2}\Big] \qquad (3.6)
\end{aligned}
$$

<u>Subcase-1.2</u>: <u>$0<N\le t_1$, $t_1<M\le T$</u>

In this subcase the inventory level at time t, $q(t)$, is given by the same differential equation and so we get the same expression of $q(t)$ as in subcase-1.1.
Here holding cost($\widetilde{HOC}_{12}$)=$\widetilde{HOC}_{11}$ and interest earned($\widetilde{IE}_{12}$)=$\widetilde{IE}_{11}$. Here interest to be paid ($\widetilde{IP}_{12}$) is given by

$$
\widetilde{IP}_{12}=\widetilde{c}_p I_k\int_M^T q(t)dt=\widetilde{c}_p I_k(a-be^{-cN})\frac{(T-M)^2}{2} \qquad (3.7)
$$

<u>Subcase-1.3</u> : <u>$0\le N\le t_1$, $M\ge T$</u>

In this case the inventory level at time t, $q(t)$, is given by the same differential equation and so we get the same expression of $q(t)$ as in subcase-1.1. Here holding

cost($\widetilde{HOC}_{13}$)=$\widetilde{HOC}_{11}$ and interest to be paid($\widetilde{IP}_{13}$)=0.
Interest earned ($\widetilde{IE}_{13}$) is given by

$$(3.8)\quad \widetilde{IE}_{13} = \widetilde{s}_p I_e\Big[\int_0^N D(t)dt(T-N) + \int_N^T D(t)dt(T-t) + \int_0^T D(t)dt(M-T)\Big]$$
$$= m\widetilde{c}_p I_e\Big[(M-N)(aN - \frac{b}{c}(1-e^{-cN})) + (a-be^{-cN})(T-N)\frac{(2M-N-T)}{2}\Big]$$

<u>Subcase-1.4</u> : <u>$t_1 \le N \le T$</u>, <u>$N < M \le T$</u>

Let $q(t)$ be the inventory level at time t which is given by

$$(3.9)\quad \frac{dq(t)}{dt} = \begin{cases} P - D = P - a + be^{-ct} & \text{for } 0 < t \le t_1 \\ -D = -a + be^{-ct} & \text{for } t_1 \le t \le N \\ -D = -a + be^{-cN} & \text{for } N \le t \le T, \end{cases}$$

where $q(0)$=0=$q(T)$ and maximum inventory occur at t=t_1.
On integration and using the above conditions

$$(3.10)\quad q(t) = \begin{cases} (P-a)t + \frac{b}{c}(1-e^{-ct}) & \text{if } 0 < t \le t_1 \\ -at - \frac{b}{c}e^{-ct} + Pt_1 + \frac{b}{c} & \text{if } t_1 \le t \le N \\ (a - be^{-cN})(T-t) & \text{if } N \le t \le T \end{cases}$$

At t=N,

$$(3.11)\quad T = \frac{1}{a - be^{-cN}}\Big[Pt_1 + \frac{b}{c} - \Big(\frac{b}{c} + bN\Big)e^{-cN}\Big]$$

Here, holding cost ($\widetilde{HOC}_{14}$) is given by

$$(3.12)\quad \widetilde{HOC}_{14} = \widetilde{h}\int_0^T q(t)dt = \widetilde{h}\Big[\int_0^{t_1} q(t)dt + \int_{t_1}^N q(t)dt + \int_N^T q(t)dt\Big]$$
$$= \widetilde{h}(I_5 + I_6 + I_7),$$

$$\text{where } I_5 = \int_0^{t_1}[(P-a)t + \frac{b}{c}(1-e^{-ct})]dt = (P-a)\frac{t_1^2}{2} + \frac{b}{c}t_1 + \frac{b}{c^2}(e^{-ct_1} - 1),$$
$$I_6 = \int_{t_1}^N [-at - \frac{b}{c}e^{-ct} + Pt_1 + \frac{b}{c}]dt$$
$$= a\frac{(t_1^2 - N^2)}{2} + \frac{b}{c^2}(e^{-cN} - e^{-ct_1}) + (Pt_1 + \frac{b}{c})(N - t_1),$$
$$I_7 = \int_N^T (a - be^{-cN})(T-t)dt = (a - be^{-cN})\frac{(T-N)^2}{2}$$

Interest to be paid ($\widetilde{IP}_{14}$) is given by

$$\begin{aligned}\widetilde{IP}_{14} &= \widetilde{c}_p I_k \int_M^T q(t)dt = \widetilde{c}_p I_k \int_M^T (a-be^{-cN})(T-t)dt \\ &= \widetilde{c}_p I_k (a-be^{-cN})\frac{(T-M)^2}{2}\end{aligned} \tag{3.13}$$

Interest earned ($\widetilde{IE}_{14}$) is given by

$$\begin{aligned}\widetilde{IE}_{14} &= \widetilde{s}_p I_e \Big[\int_0^N D(t)dt(M-N) + \int_N^M D(t)dt(M-t)\Big] \\ &= m\widetilde{c}_p I_e \Big[(M-N)(aN - \frac{b}{c}(1-e^{-cN})) \\ &\quad +(a-be^{-cN})\frac{(M-N)^2}{2}\Big]\end{aligned} \tag{3.14}$$

Subcase-1.5: $t_1 \le N \le T$, $M > T$

In this subcase the inventory level at time t, $q(t)$ is given by the same differential equation and so we get the same expression of $q(t)$ as in subcase-1.4. Here holding cost($\widetilde{HOC}_{15}$)=$\widetilde{HOC}_{14}$ and interest to be paid($\widetilde{IP}_{15}$)=0 and interest earned ($\widetilde{IE}_{15}$) is given by

$$\begin{aligned}\widetilde{IE}_{15} &= \widetilde{s}_p I_e \Big[\int_0^N D(t)dt(T-N) + \int_N^T D(t)dt(T-t) + \int_0^T D(t)dt(M-T)\Big] \\ &= m\widetilde{c}_p I_e \Big[\int_0^N (a-be^{-ct})dt(T-N) + \int_N^T (a-be^{-cN})dt(T-t) + (M-T) \\ &\quad \Big[\int_0^N (a-be^{-ct})dt + \int_N^T (a-be^{-cN})dt\Big]\Big] \\ &= m\widetilde{c}_p I_e \Big[(M-N)(aN - \frac{b}{c}(1-e^{-cN})) + (a-be^{-cN})(T-N) \\ &\quad \times \frac{(2M-N-T)}{2}\Big]\end{aligned} \tag{3.15}$$

Subcase-1.6: $T \le N < M$

In this subcase, the inventory level at time t, $q(t)$, during the time interval $(0 \le t \le T)$ is given by

$$\frac{dq}{dt} = \begin{cases} P - D = P - a + be^{-ct} & \text{if } 0 < t \le t_1 \\ -D = -a + be^{-ct} & \text{if } t_1 \le t \le T, \end{cases} \tag{3.16}$$

where $q(0)$=0=$q(T)$.

On integration and using the above conditions, we get

$$q(t) = \begin{cases} (P-a)t + \frac{b}{c}(1-e^{-ct}) & \text{if } 0 < t \le t_1 \\ a(T-t) + \frac{b}{c}(e^{-cT} - e^{-ct}) & \text{if } t_1 \le t \le T \end{cases} \tag{3.17}$$

At $t = t_1, Pt_1 + \frac{b}{c} = aT + \frac{b}{c}e^{-cT}$.

Here holding cost ($\widetilde{HOC}_{16}$) is given by

$$\text{(3.19)}\ \widetilde{HOC}_{16} = \widetilde{h}\int_0^T q(t)dt = \widetilde{h}\Big[\int_0^{t_1} q(t)dt + \int_{t_1}^T q(t)dt\Big] = \widetilde{h}(I_8 + I_9),$$

where

$$\begin{aligned} I_8 &= \int_0^{t_1}[(P-a)t + \frac{b}{c}(1-e^{-ct})]dt = \frac{(P-a)t_1^2}{2} + \frac{b}{c}t_1 + \frac{b}{c^2}(e^{-ct_1}-1) \\ I_9 &= \int_{t_1}^T \Big[a(T-t) + \frac{b}{c}(e^{-cT}-e^{-ct})\Big]dt \\ &= \frac{a(T-t_1)^2}{2} + \frac{b}{c}e^{-cT}(T-t_1) + \frac{b}{c^2}(e^{-cT}-e^{-ct_1}) \end{aligned}$$

Here interest to be paid($\widetilde{IP}_{16}$)=0.
Interest earned ($\widetilde{IE}_{16}$) is given by

$$\begin{aligned} \widetilde{IE}_{16} &= \widetilde{s}_p I_e \int_0^T D(t)dt(M-N) \\ &= m\widetilde{c}_p I_e\Big[(M-N)(aT + \frac{b}{c}(e^{-cT}-1))\Big] \end{aligned} \tag{3.20}$$

<u>Case-2</u>: <u>$N \geq M$</u>

As in this case, the customer's trade credit period N is equal to or larger than the supplier credit period M, there is no interest earned for the retailer. In this case also depending on values of M and N, six subcases may arise which are presented below:

<u>Subcase-2.1</u>: <u>$M \leq N \leq t_1 \leq T$</u>

In this subcase the inventory level at time t, $q(t)$, is given by the same differential equation and so we get the same expression of $q(t)$ as in subcase-1.1. Here holding cost($\widetilde{HOC}_{21}$)=$\widetilde{HOC}_{11}$.
Here interest to be paid ($\widetilde{IP}_{21}$)is given by

$$\begin{aligned} \widetilde{IP}_{21} &= \widetilde{c}_p I_k\Big[PM(N-M) + \int_M^N Pdt(N-t) + \int_N^T q(t)dt\Big] \\ &= \widetilde{c}_p I_k\Big[PM(N-M) + \int_M^N Pdt(N-t) + \Big\{\int_N^{t_1}\Big[(P-a+be^{-cN})t \\ &\quad + \frac{b}{c} - \Big(\frac{b}{c}+bN\Big)e^{-cN}\Big]dt + \int_{t_1}^T (a-be^{-cN})(T-t)dt\Big\}\Big] \\ &= \widetilde{c}_p I_k\Big[\frac{P(N^2-M^2)}{2} + \frac{(P-a+be^{-cN})(t_1^2-N^2)}{2} \\ &\quad + \Big\{\frac{b}{c} - \Big(\frac{b}{c}+bN\Big)e^{-cN}\Big\}(t_1-N) + \frac{(a-be^{-cN})(T-t_1)^2}{2}\Big] \end{aligned} \tag{3.21}$$

Subcase-2.2: $M \leq t_1 \leq N \leq T$

In this subcase the inventory level at time t, $q(t)$ is given by the same differential equation and so we get the same expressions of $q(t)$ and holding cost(i.e.,$\widetilde{HOC}_{22}$ $=\widetilde{HOC}_{14}$) as in subcase-1.4.
Here interest to pay ($\widetilde{IP}_{22}$) is given by

$$
\begin{aligned}
\widetilde{IP}_{22} &= \widetilde{c}_p I_k\Big[PM(N-M)+\int_M^{t_1} Pdt(t_1-t) \\
&\quad +P(t_1-M)(N-t_1)+\int_N^T q(t)dt\Big] \\
&= \widetilde{c}_p I_k\left[\frac{(a-be^{-cN})(T-N)^2}{2}+PNt_1-\frac{PM^2}{2}-\frac{Pt_1^2}{2}\right]
\end{aligned}
\tag{3.22}
$$

Subcase-2.3: $0 \leq M \leq t_1$, $N \geq T$

In this subcase the inventory level at time t, $q(t)$ is given by the same differential equation and so we get the same expressions of $q(t)$ and holding cost ($\widetilde{HOC}_{23}$) as in subcase-1.6. Here interest to pay ($\widetilde{IP}_{23}$) is given by

$$
\begin{aligned}
\widetilde{IP}_{23} &= \widetilde{c}_p I_k\left[PM(N-M)+\int_M^{t_1} Pdt(t_1-t)+P(t_1-M)(N-t_1)\right] \\
&= \widetilde{c}_p I_k\left[PNt_1-\frac{PM^2}{2}-\frac{Pt_1^2}{2}\right]
\end{aligned}
\tag{3.23}
$$

Subcase-2.4: $t_1 \leq M \leq N \leq T$

In this subcase the inventory level at time t, $q(t)$ is given by the same differential equation and so we get the same expressions of $q(t)$ and holding cost ($\widetilde{HOC}_{24}$) as in subcase-1.4. Here interest to pay(IP_{24}) is given by

$$
\begin{aligned}
\widetilde{IP}_{24} &= \widetilde{c}_p I_k\left[Pt_1(N-M)+\int_N^T q(t)dt\right] \\
&= \widetilde{c}_p I_k\left[Pt_1(N-M)+\frac{(a-be^{-cN})(T-N)^2}{2}dt\right]
\end{aligned}
\tag{3.24}
$$

Subcase-2.5: $t_1 \leq M \leq T \leq N$

In this subcase the inventory level at time t, $q(t)$ is given by the same differential equation and so we get the same expressions of $q(t)$ and holding cost ($\widetilde{HOC}_{25}$) as in subcase-1.6. Here interest to be paid ($\widetilde{IP}_{25}$) is given by

$$
\widetilde{IP}_{25} = \widetilde{c}_p I_k P t_1(N-M) \tag{3.25}
$$

Subcase-2.6: $T \leq M \leq N$

In this subcase the inventory level at time t, $q(t)$ is given by the same differential equation and so we get the same expressions of $q(t)$ and holding cost ($\widetilde{HOC}_{26}$) as in subcase-1.6. Here interest to be paid ($\widetilde{IP}_{26}$)is given by

$$\widetilde{IP}_{26} = \widetilde{c}_p I_k P t_1 (N-M) \tag{3.26}$$

4. Mathematical form of the model

From the above discussion the average profit $\widetilde{P}_{ij}(t_1)$ in j-th subcase of i-th case is given by

$$\begin{aligned}\widetilde{P}_{ij}(t_1) &= \text{Sales revenue} - \text{Purchasing cost} - \text{Ordering cost} - \text{Holding cost} \\ &\quad -\text{Interest to be paid} + \text{Interest earned} \\ &= \left[\widetilde{SP} - \widetilde{PC} - \widetilde{A} - \widetilde{HOC}_{ij} - \widetilde{IP}_{ij} + \widetilde{IE}_{ij}\right] \Big/ T (i=1,2; j=1,2,...,6) \qquad (4.1)\end{aligned}$$

and now our problem is to determine optimal value of t_1 to maximize the average profit $\widetilde{P}_{ij}(t_1)$ (i=1,2; j=1,2,...,6).

5. Determination of optimal replenishment time (t_1^*)

Let us consider the fuzzy numbers $\widetilde{A}$, $\widetilde{h}$ and $\widetilde{c}_p$ as triangular fuzzy numbers (TFN) $\widetilde{A}=(\underline{A}, A, \overline{A})$, $\widetilde{h}=(\underline{h}, h, \overline{h})$ and $\widetilde{c}_p=(\underline{c}_p, c_p, \overline{c}_p)$.
Then the α-cuts [7] of the above fuzzy numbers are

$$\begin{aligned} A(\alpha) &= [\underline{A} + \alpha(A - \underline{A}), \overline{A} - \alpha(\overline{A} - A)], \\ h(\alpha) &= [\underline{h} + \alpha(h - \underline{h}), \overline{h} - \alpha(\overline{h} - h)], \\ c_p(\alpha) &= [\underline{c}_p + \alpha(c_p - \underline{c}_p), \overline{c}_p - \alpha(\overline{c}_p - c_p)]. \end{aligned}$$

For determining the optimal production time t_1^* in different cases, let us first derive the possibilistic mean value $\overline{M}(\widetilde{P}_{ij})$ (i=1,2; j=1,2,...,6) of fuzzy profit functions $\widetilde{P}_{ij}(t_1)$(i=1,2; j=1,2,...,6),
where

$$\overline{M}(\widetilde{P}_{ij}) = \int_0^1 \alpha(P_{ij,L}(\alpha) + P_{ij,R}(\alpha))d\alpha, (i=1,2; j=1,2,...,6). \tag{5.1}$$

Then we optimize this $\overline{M}(\widetilde{P}_{ij})$ (i=1,2 ; j=1,2,...,6) with respect to t_1, so that the optimal t_1^* can be obtained.
Values of $\overline{M}(\widetilde{P}_{ij})$'s ($i$=1,2; j=1,2,...,6) in different subcases are derived below:
Subcase-1.1: $N \le t_1$, $N < M \le t_1$ In this subcase average profit($\widetilde{P}_{11}$) is given by

$$\begin{aligned}\widetilde{P}_{11} &= \Big[Pt_1 m\widetilde{c}_p - Pt_1\widetilde{c}_p - \widetilde{A} - \widetilde{h}(I_1+I_2+I_3) - \widetilde{c}_p I_k(I_4+I_3) + \\ &\quad m\widetilde{c}_p I_e\Big[(M-N)\Big\{aN - \frac{b}{c}(1-e^{-cN})\Big\} + (a - be^{-cN})\frac{(M-N)^2}{2}\Big]\Big]\Big/T \qquad (5.2)\end{aligned}$$

Taking α-cut on both sides, we get,

$$
\begin{aligned}
[P_{11,L}(\alpha), P_{11,R}(\alpha)] = & \Big[\frac{1}{T}\{Pt_1(m-1)(\underline{c}_p+\alpha(c_p-\underline{c}_p)) - \overline{A}+\alpha(\overline{A}-A) \\
& -(\overline{h}-\alpha(\overline{h}-h))(I_1+I_2+I_3) \\
& -(\overline{c}_p-\alpha(\overline{c}_p-c_p))I_k(I_4+I_3)+(\underline{c}_p+\alpha(c_p-\underline{c}_p))I_eme_1\}, \\
& \frac{1}{T}\{Pt_1(m-1)(\overline{c}_p-\alpha(\overline{c}_p-c_p)) - \underline{A}-\alpha(A-\underline{A}) \\
& -(\underline{h}+\alpha(h-\underline{h}))(I_1+I_2+I_3) \\
& -(\underline{c}_p+\alpha(c_p-\underline{c}_p))I_k(I_4+I_3) \\
& +(\overline{c}_p-\alpha(\overline{c}_p-c_p))I_eme_1\}\Big], \qquad (5.3)
\end{aligned}
$$

where

$$e_1 = (M-N)[aN-\frac{b}{c}(1-e^{-cN})]+(a-be^{-cN})\frac{(M-N)^2}{2} \qquad (5.4)$$

This gives

$$
\begin{aligned}
P_{11,L}(\alpha) = & \frac{1}{T}\{Pt_1(m-1)(\underline{c}_p+\alpha(c_p-\underline{c}_p)) - \overline{A}+\alpha(\overline{A}-A)-(\overline{h}-\alpha(\overline{h}-h)) \\
& (I_1+I_2+I_3)-(\overline{c}_p-\alpha(\overline{c}_p-c_p))I_k(I_4+I_3) \\
& +(\underline{c}_p+\alpha(c_p-\underline{c}_p))I_eme_1\}, \qquad (5.5)
\end{aligned}
$$

$$
\begin{aligned}
P_{11,R}(\alpha) = & \frac{1}{T}\{Pt_1(m-1)(\overline{c}_p-\alpha(\overline{c}_p-c_p)) - \underline{A}-\alpha(A-\underline{A})-(\underline{h}+\alpha(h-\underline{h})) \\
& (I_1+I_2+I_3)-(\underline{c}_p+\alpha(c_p-\underline{c}_p))I_k(I_4+I_3) \\
& +(\overline{c}_p-\alpha(\overline{c}_p-c_p))I_eme_1\} \qquad (5.6)
\end{aligned}
$$

So the possibilistic mean value of the fuzzy profit function $\widetilde{P}_{11}$ is

$$\overline{M}(\widetilde{P}_{11}) = \int_0^1 \alpha(P_{1,L}(\alpha)+P_{1,R}(\alpha))d\alpha \qquad (5.7)$$

Substituting the above values of $P_{11,L}(\alpha)$ and $P_{11,R}(\alpha)$ and then after simplification, we get,

$$
\begin{aligned}
\overline{M}(\widetilde{P}_{11}) = & \frac{(m-1)Pt_1(\overline{c}_p+\underline{c}_p+4c_p)}{6T} - \left[\frac{(\overline{A}+\underline{A}+4A)}{6T}\right] \\
& -\left[\frac{(\overline{h}+\underline{h}+4h)(I_1+I_2+I_3)}{6T}\right] \\
& -\left[\frac{I_k(\overline{c}_p+\underline{c}_p+4c_p)(I_4+I_3)}{6T}\right]+\left[\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_eme_1}{6T}\right] \qquad (5.8)
\end{aligned}
$$

Differentiating, we get,

$$\begin{aligned}\frac{d(M(\widetilde{P}_{11}))}{dt_1} =& \frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6T} - \frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)t_1}{6T^2(a-be^{-cN})} \\ &+\frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cN})} + \frac{(\overline{h}+\underline{h}+4h)(I_1+I_2+I_3)P}{6T^2(a-be^{-cN})} \\ &+\frac{(\overline{h}+\underline{h}+4h)((\frac{b}{c}+bN)e^{-cN}-\frac{b}{c})}{6T} \\ &+\frac{(\overline{h}+\underline{h}+4h)(a-P-be^{-cN})}{6} + \frac{I_k(\overline{c}_p+\underline{c}_p+4c_p)(I_4+I_3)P}{6T^2(a-be^{-cN})} \\ &+\frac{I_k(\overline{c}_p+\underline{c}_p+4c_p)((\frac{b}{c}+bN)e^{-cN}-\frac{b}{c})}{6T} \\ &+\frac{I_k(\overline{c}_p+\underline{c}_p+4c_p)(a-P-be^{-cN})}{6} \\ &-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_e m e_1 P}{6T^2(a-be^{-cN})}\end{aligned} \tag{5.9}$$

Again differentiating, we get,

$$\begin{aligned}\frac{d^2(M(\widetilde{P}_{11}))}{dt_1^2} =& -\frac{(\overline{A}+\underline{A}+4A)P^2}{3T^3(a-be^{-cN})^2} - \frac{(\overline{h}+\underline{h}+4h)(I_1+I_2+I_3)P^2}{3T^3(a-be^{-cN})^2} \\ &-\frac{(\overline{h}+\underline{h}+4h)P((\frac{b}{c}+bN)e^{-cN}-\frac{b}{c})}{3T^2(a-be^{-cN})} \\ &-\frac{(\overline{h}+\underline{h}+4h)P(a-P-be^{-cN})}{6T(a-be^{-cN})} - \frac{I_k(\overline{c}_p+\underline{c}_p+4c_p)P^2(I_4+I_3)}{3T^3(a-be^{-cN})^2} \\ &-\frac{I_k(\overline{c}_p+\underline{c}_p+4c_p)((\frac{b}{c}+bN)e^{-cN}-\frac{b}{c})P}{3T^3(a-be^{-cN})} \\ &-\frac{I_k(\overline{c}_p+\underline{c}_p+4c_p)P(a-P-be^{-cN})}{6T(a-be^{-cN})} - \frac{I_e(\overline{c}_p+\underline{c}_p+4c_p)me_1P^2}{3T^3(a-be^{-cN})^2}\end{aligned} \tag{5.10}$$

Substituting the expressions of T, I_1, I_2, I_3 and I_4 in the equation $\frac{d(M(\widetilde{P}_{11}))}{dt_1} = 0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{11})$, as

$$\begin{aligned} t_1^* =& (-6v_1v_{10} - 12v_1v_2v_9 - v_1w_2 + \\ &\sqrt{(6v_1v_{10}+12v_1v_2v_9+v_1w_2)^2 - 4(6v_1^2v_9+v_1w_1)} \\ &(6v_{10}v_2+6v_2^2v_9+v_1w_3-v_1w_4))/(2(6v_1^2v_9+v_1w_1)),\end{aligned} \tag{5.11}$$

where

$$\begin{aligned}
v_1 &= \frac{P}{a-be^{-cN}}, v_2 = \frac{\frac{b}{c}-(\frac{b}{c}+bN)e^{-cN}}{a-be^{-cN}},\\
v_3 &= \frac{P-a+be^{-cN}}{2}, v_4 = \frac{b}{c}-\left(\frac{b}{c}+bN\right)e^{-cN},\\
v_5 &= \frac{N^2}{2}(P-a+be^{-cN})+N(\frac{b}{c}-(\frac{b}{c}+bN)e^{-cN}), v_6 = \frac{(P-a+be^{-cN})^2}{2(a-be^{-cN})},\\
v_7 &= (P-a+be^{-cN})v_2, v_8 = \frac{(P-a+be^{-cN})M^2}{2}+\{\frac{b}{c}-(\frac{b}{c}+bN)e^{-cN}\}M,\\
v_9 &= \frac{(\overline{h}+\underline{h}+4h)(a-P-be^{-cN})}{6}+\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_k(a-P-be^{-cN})}{6},\\
v_1^{'} &= (m-1)P(\overline{c}_p+\underline{c}_p+4c_p), v_{10} = v_1^{'}+\frac{(\frac{b}{c}+bN)e^{-cN}-\frac{b}{c})}{6}\{(\overline{h}+\underline{h}+4h)\\
&\quad +(\overline{c}_p+\underline{c}_p+4c_p)I_k\},\\
v_{11} &= \frac{(P-a)N^2}{2}+\frac{b}{c}N-\frac{b}{c^2}(1-e^{-cN})-v_5+v_2^2,\\
w_1 &= (v_3+v_6)\{(\overline{h}+\underline{h}+4h)+(\overline{c}_p+\underline{c}_p+4c_p)I_k\},\\
w_2 &= (v_4+v_7)\{(\overline{h}+\underline{h}+4h)+(\overline{c}_p+\underline{c}_p+4c_p)I_k\}-v_1^{'}P,\\
w_3 &= (\overline{A}+\underline{A}+4A)+v_{11}(\overline{h}+\underline{h}+4h)+(v_2^2-v_8)(\overline{c}_p+\underline{c}_p+4c_p)I_k,\\
w_4 &= me_1I_e(\overline{c}_p+\underline{c}_p+4c_p)
\end{aligned}$$

Similarly, in other subcases possibilistic mean values are calculated as below:
Subcase-1.2: $0 < N < t_1$, $t_1 < M < T$. In this case possibilistic mean value of the fuzzy profit function $\widetilde{P}_{12}$ is

$$\begin{aligned}
\overline{M}(\widetilde{P}_{12}) &= \frac{(m-1)Pt_1(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(\overline{A}+\underline{A}+4A)}{6T}\\
&\quad -\frac{(\overline{h}+\underline{h}+4h)(I_1+I_2+I_3)}{6T}\\
&\quad -\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_k(a-be^{-cN})(T-M)^2}{12T}+\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_eme_1}{6T}
\end{aligned} \tag{5.12}$$

Differentiating, we get,

$$\begin{aligned}
\frac{d(M(\widetilde{P}_{12}))}{dt_1} &= \frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)t_1}{6T^2(a-be^{-cN})}\\
&\quad +\frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cN})}+\frac{(\overline{h}+\underline{h}+4h)(I_1+I_2+I_3)P}{6T^2(a-be^{-cN})}\\
&\quad +\frac{(\overline{h}+\underline{h}+4h)((\frac{b}{c}+bN)e^{-cN}-\frac{b}{c})}{6T}\\
&\quad +\frac{(\overline{h}+\underline{h}+4h)(a-P-be^{-cN})}{6}+\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_k(T-M)^2P}{12T^2}
\end{aligned}$$

$$-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_k(T-M)P}{6T}-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_eme_1P}{6T^2(a-be^{-cN})} \tag{5.13}$$

Substituting the expressions of T, I_1, I_2, I_3 in the equation $\frac{d(M(\widetilde{P}_{12}))}{dt_1}=0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{12})$(since for t=t_1^*, $\frac{d^2(M(\widetilde{P}_{12}))}{dt_1^2}$<0), as

$$\begin{aligned} t_1^* &= \big(-6v_1v_{13}+v_1v_1'(\overline{h}+\underline{h}+4h)-12v_1v_{12}v_2-4hv_1v_4-\underline{h}v_1v_4-\overline{h}v_1v_4 \\ &-4hv_1v_7-\underline{h}v_1v_7-\overline{h}v_1v_7 \\ &+\sqrt{(-4(v_1v_{14}+6v_{13}v_2+6v_{12}v_2^2)(6v_1^2v_{12}+4hv_1v_3+\underline{h}v_1v_3+\overline{h}v_1v_3} \\ &+4hv_1v_6+\underline{h}v_1v_6+\overline{h}v_1v_6)+(6v_1v_{13}-v_1v_1'(\overline{h}+\underline{h}+4h)+12v_1v_{12}v_2 \\ &+4hv_1v_4+\underline{h}v_1v_4+\overline{h}v_1v_4+4hv_1v_7+\underline{h}v_1v_7+\overline{h}v_1v_7)^2\big)/(2(6v_1^2v_{12} \\ &+4hv_1v_3+\underline{h}v_1v_3+\overline{h}v_1v_3+4hv_1v_6+\underline{h}v_1v_6+\overline{h}v_1v_6)), \end{aligned} \tag{5.14}$$

where

$$\begin{aligned} v_{12} &= \frac{(\overline{h}+\underline{h}+4h)(a-P-be^{-cN})}{6}+\frac{P(\overline{c}_p+\underline{c}_p+4c_p)(1-2I_k)}{12}, \\ v_{13} &= (m-1)P(\overline{c}_p+\underline{c}_p+4c_p)+\frac{(\overline{h}+\underline{h}+4h)}{6}((\frac{b}{c}+bN)e^{-cN}-\frac{b}{c}) \\ &-\frac{I_kMP(\overline{c}_p+\underline{c}_p+4c_p)}{3}, \\ v_{14} &= (\overline{A}+\underline{A}+4A)+\frac{1}{2}(\overline{c}_p+\underline{c}_p+4c_p)M^2(a-be^{-cN})-mI_ee_1(\overline{c}_p+\underline{c}_p+4c_p) \\ &+(\overline{h}+\underline{h}+4h)v_{11}. \end{aligned}$$

Subcase-1.3: $0<N\le t_1$, $M>T$ In this case possibilistic mean value of the fuzzy profit function $\widetilde{P}_{13}$ is

$$\begin{aligned} \overline{M}(\widetilde{P}_{13}) &= \frac{(m-1)Pt_1(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(\overline{A}+\underline{A}+4A)}{6T} \\ &-\frac{(\overline{h}+\underline{h}+4h)(I_1+I_2+I_3)}{6T}+\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_eme_2}{6T}, \end{aligned} \tag{5.15}$$

where

$$e_2 = (M-N)\Big(aN-\frac{b}{c}(1-e^{-cN})\Big)+(a-be^{-cN})(T-N)\frac{(2M-N-T)}{2}$$

$$\begin{aligned} \frac{d(M(\widetilde{P}_{13}))}{dt_1} &= \frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)t_1}{6T^2(a-be^{-cN})} \\ &+\frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cN})}+\frac{(\overline{h}+\underline{h}+4h)(I_1+I_2+I_3)P}{6T^2(a-be^{-cN})} \\ &+\frac{(\overline{h}+\underline{h}+4h)((\frac{b}{c}+bN)e^{-cN}-\frac{b}{c})}{6T} \end{aligned}$$

$$+\frac{(\overline{h}+\underline{h}+4h)(a-P-be^{-cN})}{6}-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_e m e_2 P}{6T^2(a-be^{-cN})}. \tag{5.16}$$

Substituting the expressions of T, I_1, I_2, I_3 in the equation $\frac{d(M(\widetilde{P}_{13}))}{dt_1}=0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{13})$(since for $t=t_1^*$, $\frac{d^2(M(\widetilde{P}_{13}))}{dt_1^2}<0$), as

$$\begin{aligned}
t_1^* \;=\; & (-6v_1v_{16}+v_1v_1^{'}(\overline{h}+\underline{h}+4h)-12v_1v_{15}v_2-4hv_1v_4-\underline{h}v_1v_4-\overline{h}v_1v_4 \\
& -4hv_1v_7-\underline{h}v_1v_7-\overline{h}v_1v_7 \\
& +\sqrt{(-4(v_1v_{17}+6v_{16}v_2+6v_{15}v_2^2)(6v_1^2v_{15}+(\overline{h}+\underline{h}+4h)(v_1v_3+v_1v_6)) \\
& +(6v_1v_{16}-v_1v_1^{'}(\overline{h}+\underline{h}+4h)+12v_1v_{15}v_2+(\overline{h}+\underline{h}+4h)(v_1v_4+v_1v_7))^2)} \\
& /(2(6v_1^2v_{15}+(\overline{h}+\underline{h}+4h)(v_1v_3+v_1v_6))),
\end{aligned} \tag{5.17}$$

where

$$\begin{aligned}
v_{15} &= \frac{(\overline{h}+\underline{h}+4h)(a-P-be^{-cN})}{6}+\frac{1}{2}mI_eP(\overline{c}_p+\underline{c}_p+4c_p), \\
v_{16} &= \frac{1}{6}[(\overline{h}+\underline{h}+4h)((\frac{b}{c}+bN)e^{-cN}-\frac{b}{c})-I_emMP(\overline{c}_p+\underline{c}_p+4c_p), \\
v_{17} &= (\overline{A}+\underline{A}+4A)+(\overline{h}+\underline{h}+4h)v_{11}+mI_e(\overline{c}_p+\underline{c}_p+4c_p) \\
& \left\{\frac{N(2M-N)(a-be^{-cN})}{2}-(M-N)(aN-\frac{b}{c}(1-e^{-cN}))\right\}.
\end{aligned}$$

<u>Subcase-1.4</u>: <u>$t_1\le N\le T$, $N<M\le T$</u> In this case possibilistic mean value of the fuzzy profit function $\widetilde{P}_{14}$ is

$$\begin{aligned}
\overline{M}(\widetilde{P_{14}}) \;=\; & \frac{(m-1)Pt_1(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(\overline{A}+\underline{A}+4A)}{6T} \\
& -\frac{(\overline{h}+\underline{h}+4h)(I_5+I_6+I_7)}{6T}-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_k(a-be^{-cN})\frac{(T-M)^2}{2}}{6T} \\
& +\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_eme_3}{6T},
\end{aligned} \tag{5.18}$$

where

$$e_3=(M-N)(aN-\frac{b}{c}(1-e^{-cN}))+(a-be^{-cN})\frac{(M-N)^2}{2}$$

$$\begin{aligned}
\frac{d(M(\widetilde{P}_{14}))}{dt_1} \;=\; & \frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)t_1}{6T^2(a-be^{-cN})} \\
& +\frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cN})}+\frac{(\overline{h}+\underline{h}+4h)(a-be^{-cN})}{12} \\
& -\frac{(\overline{h}+\underline{h}+4h)e_7^2}{12T^2(a-be^{-cN})}+\frac{(\overline{h}+\underline{h}+4h)PNe_7}{6T^2(a-be^{-cN})} \\
& -\frac{(\overline{h}+\underline{h}+4h)P}{12}+\frac{(\overline{h}+\underline{h}+4h)e_8}{6T^2(a-be^{-cN})}-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_k(2P-1)}{12}
\end{aligned}$$

$$+\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_kPM^2}{12T^2}-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_eme_3P}{6T^2(a-be^{-cN})}, \tag{5.19}$$

where $e_7=\left(\frac{b}{c}+bN\right)e^{-cN}-\frac{b}{c},\ \ e_8=\frac{e^{-cN}-1}{c^2}+\frac{bN}{c}-\frac{bN^2e^{-cN}}{2}$.

Substituting the expressions of T in the equation $\frac{d(M(\widetilde{P}_{14}))}{dt_1}=0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{14})$(since for t=t_1^*, $\frac{d^2(M(\widetilde{P}_{14}))}{dt_1^2}$<0), as

$$\begin{aligned}t_1^* &= \Big(-v_1v_{18}^{'}+v_{18}^{'}v_1{''}-2v_1v_2v_{18}\\ &+\sqrt{(v_1v_{18}^{'}-v_{18}^{'}v_1{''}+2v_1v_2v_{18})^2-4v_1^2v_{18}(-v_{19}+v_2v_{18}^{'}+v_2^2v_{18})}\Big)/(2v_1^2v_{18}),\end{aligned} \tag{5.20}$$

where

$$\begin{aligned}v_{18} &= (m-1)Pc+\frac{1}{12}(\overline{h}+\underline{h}+4h)(a-P-be^{-cN})+\frac{1}{2}I_k(\overline{c}_p+\underline{c}_p+4c_p)(1-2P),\\ v_{19} &= \frac{1}{6(a-be^{-cN})}[(\overline{A}+\underline{A}+4A)P+(\overline{h}+\underline{h}+4h)(e_8+PNe_7-\frac{1}{2}e_7^2)+\frac{1}{2}I_k(\overline{c}_p\\ &\quad+\underline{c}_p+4c_p)PM^2(a-be^{-cN})-me_3P(\overline{c}_p+\underline{c}_p+4c_p)I_e)]\\ v_1^{''} &= \frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)}{6(a-be^{-cN})},v_{18}^{'}=\frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6}.\end{aligned}$$

Subcase-1.5: $t_1\le N\le T$, $M>T$ In this case possibilistic mean value of the fuzzy profit function $\widetilde{P}_{15}$ is

$$\begin{aligned}\overline{M}(\widetilde{P}_{15}) &= \frac{(m-1)Pt_1(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(\overline{A}+\underline{A}+4A)}{6T}\\ &\quad-\frac{(\overline{h}+\underline{h}+4h)(I_5+I_6+I_7)}{6T}+\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_eme_4}{6T},\end{aligned} \tag{5.21}$$

$$\text{where } e_4 = (M-N)(aN-\frac{b}{c}(1-e^{-cN}))+(a-be^{-cN})(T-N)\frac{(2M-N-T)}{2}$$

$$\begin{aligned}\frac{d(M(\widetilde{P}_{15}))}{dt_1} &= \frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)t_1}{6T^2(a-be^{-cN})}\\ &\quad+\frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cN})}+\frac{(\overline{h}+\underline{h}+4h)(a-be^{-cN})}{12}-\frac{(\overline{h}+\underline{h}+4h)e_7^2}{12T^2(a-be^{-cN})}\\ &\quad+\frac{(\overline{h}+\underline{h}+4h)PNe_7}{6T^2(a-be^{-cN})}-\frac{(\overline{h}+\underline{h}+4h)P}{12}+\frac{(\overline{h}+\underline{h}+4h)Pe_8}{6T^2(a-be^{-cN})}\\ &\quad-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_eme_4P}{6T^2(a-be^{-cN})},\end{aligned} \tag{5.22}$$

$$\text{where } e_7 = \left(\frac{b}{c}+bN\right)e^{-cN}-\frac{b}{c},\ \ e_8=\frac{e^{-cN}-1}{c^2}+\frac{bN}{c}-\frac{bN^2e^{-cN}}{2}$$

Substituting the expressions of T, I_5, I_6, I_7 in the equation $\frac{d(M(\widetilde{P}_{15}))}{dt_1}=0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{15})$(since for t=t_1^*, $\frac{d^2(M(\widetilde{P}_{15}))}{dt_1^2}$<0), as

$$\begin{aligned} t_1^* &= \big(-2v_1v_2v_{20}+v_1v_{21}-Pv_1^{'}v_{21}+\sqrt{(2v_1v_2v_{20}-v_1v_{21}+Pv_1^{'}v_{21})^2} \\ &\quad -4v_1^2v_{20}(v_2^2v_{20}+v_1^{'}v_{21}-v_2v_{21}+v_{22})\big)/(2v_1^2v_{20}), \end{aligned} \tag{5.23}$$

$$\begin{aligned} \text{where } v_{20} &= \frac{1}{12}(\overline{h}+\underline{h}+4h)(a-P-be^{-cN})+\frac{1}{12}mP(\overline{c}_p+\underline{c}_p+4c_p)I_e \\ v_{21} &= \frac{M}{6}mP(\overline{c}_p+\underline{c}_p+4c_p)I_e \\ v_{22} &= \frac{1}{6(a-be^{-cN})}[(\overline{A}+\underline{A}+4A)P+(\overline{h}+\underline{h}+4h)(-\frac{e_7^2}{2}+PNe_7+Pe_8) \\ &\quad +mPN(2M-N)(\overline{c}_p+\underline{c}_p+4c_p)I_e] \end{aligned}$$

<u>Subcase-1.6</u>: <u>$T \le N < M$</u> In this subcase possibilistic mean value of the fuzzy profit function $\widetilde{P}_{16}$ is

$$\begin{aligned} \overline{M}(\widetilde{P}_{16}) &= \frac{(m-1)Pt_1(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(\overline{A}+\underline{A}+4A)}{6T}-\frac{(\overline{h}+\underline{h}+4h)(I_8+I_9)}{6T} \\ &\quad +\frac{(\overline{I_e}+\underline{I_e}+4I_e)mce_5}{6T}, \end{aligned} \tag{5.24}$$

where e_5=$\Big[(M-N)(aT+\frac{b}{c}(e^{-cT}-1))\Big]$

$$\begin{aligned} \frac{d(M(\widetilde{P}_{16}))}{dt_1} &= \frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)t_1}{6T^2(a-be^{-cN})} \\ &\quad +\frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cT})}+\frac{(\overline{h}+\underline{h}+4h)P(I_8+I_9)}{6T^2(a-be^{-cT})}+\frac{(\overline{h}+\underline{h}+4h)(a-P)}{6} \\ &\quad +\frac{(\overline{h}+\underline{h}+4h)(e^{-cT}-1)b}{6cT}-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_eme_5P}{6T^2(a-be^{-cT})}. \end{aligned} \tag{5.25}$$

Substituting the expressions of T, I_8 and I_9 in the equation $\frac{d(M(\widetilde{P}_{16}))}{dt_1}=0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{16})$(since for t=t_1^*, $\frac{d^2(M(\widetilde{P}_{16}))}{dt_1^2}$<0), as

$$\begin{aligned} t_1^* &= ((P-1)v_1v_1^{'}-12(a-b)v_1v_2v_{27}-12bcv_1v_2^2v_{27}-12v_1v_2v_{28}+12v_1v_2v_{29} \\ &\quad +6v_1v_{30}-6v_{32}+6v_{34}+6cv_2v_{34} \\ &\quad +\sqrt{((P-1)v_1v_1^{'}-12(a-b)v_1v_2v_{27}-12bcv_1v_2^2v_{27}} \\ &\quad -12v_1v_2v_{28}+12v_1v_2v_{29}+6v_1v_{30}-6v_{32}+6v_{34}+6cv_2v_{34})^2-4(v_1^{'}v_2+ \\ &\quad 6av_2^2v_{27}-6bv_2^2v_{27}+6bcv_2^2v_{27}+6v_2^2v_{28}-6v_2^2v_{29}-6v_2v_{30}+6v_{31})(6av_1^2v_{27} \end{aligned}$$

$$-6bv_1^2v_{27}+6bcv_1^2v_2v_{27}+6v_1^2v_{28}-6v_1^2v_{29}+6v_{33}-6cv_1v_{34})))/(2(6av_1^2v_{27}$$
$$-6bv_1^2v_{27}+6bcv_1^2v_2v_{27}+6v_1^2v_{28}-6v_1^2v_{29}+6v_{33}-6cv_1v_{34})),$$

(5.26)

$$\begin{aligned}
\text{where } v_{27} &= (m-1)Pc+\frac{1}{6}(\overline{h}+\underline{h}+4h)(a-b-P)\\
v_{28} &= \frac{1}{12}(\overline{h}+\underline{h}+4h)Pc^2v_{26}, \quad v_{29}=\frac{1}{12}mP(M-N)c^2I_e\\
v_{30} &= \frac{1}{6}\{(\overline{h}+\underline{h}+4h)Pcv_{26}+mI_eP(a-b)(M-N)(\overline{c}_p+\underline{c}_p+4c_p)\}\\
v_{31} &= \frac{1}{6}\{(\overline{A}+\underline{A}+4A)P+(\overline{h}+\underline{h}+4h)P(v_{25}+v_{26})\}\\
v_{32} &= \frac{1}{6}(\overline{h}+\underline{h}+4h)Pv_{24}, \quad v_{33}=\frac{1}{6}(\overline{h}+\underline{h}+4h)Pv_{23}\\
v_{34} &= \frac{1}{6}(\overline{h}+\underline{h}+4h)P(v_1-1)\frac{b}{c}
\end{aligned}$$

<u>Subcase-2.1</u>: <u>$M \le N \le t_1 \le T$</u> In this subcase possibilistic mean value of the fuzzy profit function $\widetilde{P}_{21}$ is

$$\begin{aligned}
\overline{M}(\widetilde{P}_{21}) &= \frac{(m-1)Pt_1(\overline{c}_p+\underline{c}_p+4c_p)}{6T}\\
&-\frac{(\overline{A}+\underline{A}+4A)}{6T}-\frac{(\overline{h}+\underline{h}+4h)(I_1+I_2+I_3)}{6T}\\
&-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_k}{6T}\left[\frac{P(N^2-M^2)}{2}+\frac{(P-a+be^{-cN})(t_1^2-N^2)}{2}\right.\\
&\left.-\left\{\left(\frac{b}{c}+bN\right)e^{-cN}-\frac{b}{c}\right\}(t_1-N)+\frac{(a-be^{-cN})(T-t_1)^2}{2}\right]
\end{aligned} \tag{5.27}$$

$$\begin{aligned}
\frac{d(M(\widetilde{P}_{21}))}{dt_1} &= \frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)t_1}{6T^2(a-be^{-cN})}\\
&+\frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cN})}+\frac{(\overline{h}+\underline{h}+4h)P(I_1+I_2+I_3)}{6T^2(a-be^{-cN})}\\
&+\frac{(\overline{h}+\underline{h}+4h)}{6T}\left[\left(\frac{b}{c}+bN\right)e^{-cN}-\frac{b}{c}+(a-be^{-cN}-P)T\right]\\
&+\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_kP}{6T^2(a-be^{-cN})}\left[\frac{P(N^2-M^2)}{2}+\frac{(P-a+be^{-cN})}{2}\right.\\
&(t_1^2-N^2)-\left\{\left(\frac{b}{c}+bN\right)e^{-cN}-\frac{b}{c}\right\}(t_1-N)+(a-be^{-cN})\\
&\left.\frac{(T-t_1)^2}{2}\right]+\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_k}{6T}\left[\left\{\left(\frac{b}{c}+bN\right)e^{-cN}-\frac{b}{c}\right\}\right.\\
&\left.-(P-a+be^{-cN})T\right]
\end{aligned} \tag{5.28}$$

Substituting the expressions of T, I_1, I_2, I_3 in the equation $\frac{d(M(\widetilde{P}_{21}))}{dt_1} = 0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{21})$(since for t=t_1^*, $\frac{d^2(M(\widetilde{P}_{21}))}{dt_1^2}$<0), as

$$\begin{aligned} t_1^* &= \big(-v_1v_1^{'} + Pv_1v_1^{'} - 12v_1v_2v_{35} - 6v_1v_{36} + 6v_2v_{36}^{'} - v_1v_{38} + \sqrt{((v_1v_1^{'} - Pv_1v_1^{'}} \\ &\quad +12v_1v_2v_{35} + 6v_1v_{36} - 6v_2v_{36}^{'} + v_1v_{38})^2 - 4(v_1^2v_2 + 6v_2^2v_{35} + 6v_2v_{36} + v_1v_{37}) \\ &\quad (6v_1^2v_{35} - 6v_1v_{36}^{'} + v_1v_{39})))\big)/(2(6v_1^2v_{35} - 6v_1v_{36}^{'} + v_1v_{39})), \end{aligned} \tag{5.29}$$

$$\begin{aligned} \text{where } v_{35} &= (m-1)Pc + \frac{1}{6}(\overline{h} + \underline{h} + 4h)(a - P - be^{-cN}) \\ &\quad + \frac{1}{12}(\overline{c}_p + \underline{c}_p + 4c_p)I_k(a - P - 2be^{-cN}) \\ v_{36} &= \frac{1}{6}\{(\overline{h} + \underline{h} + 4h) + (\overline{c}_p + \underline{c}_p + 4c_p)I_k\}\{(\frac{b}{c} + bN)e^{-cN} - \frac{b}{c}\} \\ v_{36}^{'} &= -\frac{1}{6}(\overline{c}_p + \underline{c}_p + 4c_p)I_k \\ v_{37} &= (\overline{A} + \underline{A} + 4A) + v_{11}(\overline{h} + \underline{h} + 4h) + (\overline{c}_p + \underline{c}_p + 4c_p)I_k\{\frac{P(N^2 - M^2)}{2} \\ &\quad - \frac{N^2(P - a + be^{-cN})}{2}\}, \quad v_{38} = (\overline{h} + \underline{h} + 4h)(v_4 + v_7) \\ v_{39} &= (\overline{h} + \underline{h} + 4h)(v_3 + v_6) + \frac{I_k}{2}(\overline{c}_p + \underline{c}_p + 4c_p)(P - a + be^{-cN}). \end{aligned}$$

<u>Subcase-2.2</u>: <u>$0 < M \le t_1 \le N \le T$</u> In this subcase possibilistic mean value of the fuzzy profit function $\widetilde{P}_{22}$ is

$$\begin{aligned} \overline{M}(\widetilde{P}_{22}) &= \frac{(m-1)Pt_1(\overline{c}_p + \underline{c}_p + 4c_p)}{6T} - \frac{(\overline{A} + \underline{A} + 4A)}{6T} \\ &\quad - \frac{(\overline{h} + \underline{h} + 4h)(I_5 + I_6 + I_7)}{6T} - \frac{(\overline{c}_p + \underline{c}_p + 4c_p)I_k}{6T} \\ &\quad \left[PNt_1 + \frac{(a - be^{-cN})(T-N)^2}{2} - \frac{PM^2}{2} - \frac{Pt_1^2}{2}\right] \end{aligned} \tag{5.30}$$

$$\begin{aligned} \frac{d(M(\widetilde{P}_{22}))}{dt_1} &= \frac{(m-1)P(\overline{c}_p + \underline{c}_p + 4c_p)}{6T} - \frac{(m-1)P^2(\overline{c}_p + \underline{c}_p + 4c_p)t_1}{6T^2(a - be^{-cN})} \\ &\quad + \frac{(\overline{A} + \underline{A} + 4A)P}{6T^2(a - be^{-cN})} + \frac{(\overline{h} + \underline{h} + 4h)P(I_5 + I_6 + I_7)}{6T^2(a - be^{-cN})} \\ &\quad + \frac{(\overline{h} + \underline{h} + 4h)P(t_1 - T)}{6T} + \frac{(\overline{c}_p + \underline{c}_p + 4c_p)I_kP}{6T^2(a - be^{-cN})} \\ &\quad \left[PNt_1 - \frac{PM^2}{2} + \frac{(a - be^{-cN})(T-N)^2}{2} - \frac{Pt_1^2}{2}\right] \\ &\quad + \frac{(\overline{c}_p + \underline{c}_p + 4c_p)I_kP(t_1 - T)}{6T} \end{aligned} \tag{5.31}$$

Substituting the expressions of T, I_1, I_2 and I_3 in the equation $\frac{d(M(\widetilde{P}_{22}))}{dt_1} = 0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{22})$(since for t=t_1^*, $\frac{d^2(M(\widetilde{P}_{22}))}{dt_1^2}$<0), as

$$\begin{aligned} t_1^* &= (-v_1v_1^{'} + Pv_1v_1^{'} - 12v_1v_2v_{40} - 6v_1v_{41} - 6v_2v_{42} - 6v_{44} \\ &\quad + \sqrt{((v_1v_1^{'} - Pv_1v_1^{'} + 12v_1v_2v_{40}} + 6v_1v_{41} + 6v_2v_{42} + 6v_{44})^2 \\ &\quad -4(v_1^{'}v_2 + 6v2^2v_{40} + 6v_2v_{41} + 6v_{43})(6v_1^2v_{40} + 6v_1v_{42} \\ &\quad -6v_{45})))/(2(6v_1^2v_{40} + 6v_1v_{42} - 6v_{45})), \end{aligned} \tag{5.32}$$

$$\begin{aligned} \text{where } v_{40} &= (m-1)Pc - \frac{(\overline{h}+\underline{h}+4h)P}{12} - \frac{1}{6}P(\overline{c}_p + \underline{c}_p + 4c_p)I_k \\ v_{41} &= \frac{(\overline{h}+\underline{h}+4h)P(bNe^{-cN} - aN)}{6(a-be^{-cN})} + \frac{1}{12}(\overline{c}_p + \underline{c}_p + 4c_p)I_k \\ v_{42} &= \frac{1}{6}P(\overline{h}+\underline{h}+4h) + \frac{1}{6}P(\overline{c}_p + \underline{c}_p + 4c_p)I_k \\ v_{43} &= \frac{P}{6(a-be^{-cN})}[(\overline{A}+\underline{A}+4A) + (\overline{h}+\underline{h}+4h)\{\frac{b}{c}(e^{-cN}-1) + \frac{b}{c}N \\ &\quad -\frac{1}{2}bN^2e^{-cN}\} - (\overline{c}_p + \underline{c}_p + 4c_p)I_k\{\frac{N}{2P}(a-be^{-cN}) + \frac{1}{2}PM^2\}] \\ v_{44} &= \frac{P}{6(a-be^{-cN})}[(\overline{h}+\underline{h}+4h)PN + PN(\overline{c}_p + \underline{c}_p + 4c_p)I_k] \\ v_{45} &= \frac{P}{12(a-be^{-cN})}[(\overline{h}+\underline{h}+4h) + (\overline{c}_p + \underline{c}_p + 4c_p)I_k] \end{aligned}$$

Subcase-2.3: $0 < M \leq t_1$, $N \geq T$ In this subcase possibilistic mean value of the fuzzy profit function $\widetilde{P}_{23}$ is

$$\begin{aligned} \overline{M}(\widetilde{P}_{23}) &= \frac{(m-1)Pt_1(\overline{c}_p + \underline{c}_p + 4c_p)}{6T} - \frac{(\overline{A}+\underline{A}+4A)}{6T} - \frac{(\overline{h}+\underline{h}+4h)(I_8+I_9)}{6T} \\ &\quad - \frac{(\overline{c}_p + \underline{c}_p + 4c_p)I_k}{6T}\left[PNt_1 - \frac{PM^2}{2} - \frac{Pt_1^2}{2}\right] \end{aligned} \tag{5.33}$$

$$\begin{aligned} \frac{d(M(\widetilde{P}_{23}))}{dt_1} &= \frac{(m-1)P(\overline{c_p} + \underline{c_p} + 4c_p)}{6T} - \frac{(m-1)P^2(\overline{c}_p + \underline{c}_p + 4c_p)t_1}{6T^2(a-be^{-cN})} \\ &\quad + \frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cT})} + \frac{(\overline{h}+\underline{h}+4h)P(I_8+I_9)}{6T^2(a-be^{-cT})} \\ &\quad + \frac{(\overline{h}+\underline{h}+4h)}{6T}\{(a-P)T + \frac{b}{c}(e^{-cT}-1)\} \\ &\quad + \frac{(\overline{c}_p + \underline{c}_p + 4c_p)I_kP}{6T^2(a-be^{-cT})}\left[PNt_1 - \frac{PM^2}{2} - \frac{Pt_1^2}{2}\right] \\ &\quad + \frac{(\overline{c}_p + \underline{c}_p + 4c_p)I_kP(t_1-N)}{6T} \end{aligned} \tag{5.34}$$

Substituting the expressions of T, I_8 and I_9 in the equation $\frac{d(M(\widetilde{P}_{23}))}{dt_1} = 0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{23})$(since for t=t_1^*, $\frac{d^2(M(\widetilde{P}_{23}))}{dt_1^2}$<0), as

$$
\begin{aligned}
t_1^* \quad = \quad & ((P-1)v_1v1^{'} - 12(a-b)v_1v_2v_{46} - 12bcv_1v_2^2v_{46} - 12v_1v_2v_{47} + 6v_1v_{48} \\
& -6v_2v_{49} + 6v_1v_{50} - 6v_{52} - 6v_{52}^2 + 6cv_2v_{52}^2 \\
& +\sqrt{(-4(6av_1^2v_{46} - 6bv_1^2v_{46} + 6bcv_1^2v_2v_{46} + 6v_1^2v_{47}} \\
& +6v_1v_{49} + 6v_{52} - 6cv_1v_{52}^2)(v_1^{'}v_2 + 6av_2^2v_{46} - 6bv_2^2v_{46} + 6bcv_2^2v_{46} + 6v_2^2v_{47} \\
& -6v_2v_{48} - 6v_2v_{50} + 6v_{51}) + ((P-1)v_1v1^{'} - 12(a-b)v_1v_2v_{46} - 12bcv_1v_2^2v_{46} \\
& -12v_1v_2v_{47} + 6v_1v_{48} - 6v_2v_{49} + 6v_1v_{50} - 6v_{52} - 6v_{52}^2 + 6cv_2v_{52}^2)^2)) \\
& /(2(6av_1^2v_{46} - 6bv_1^2v_{46} + 6bcv_1^2v_2v_{46} + 6v_1^2v_{47} + 6v_1v_{49} + 6v_{52} - 6cv_1v_{52}^2)),
\end{aligned}
\tag{5.35}
$$

$$
\begin{aligned}
\text{where } v_{46} &= (m-1)Pc + \frac{1}{6}(\overline{h} + \underline{h} + 4h)(a-b-P) \\
v_{47} &= \frac{1}{12}(\overline{h} + \underline{h} + 4h)c^2v_{26}, \quad v_{48} = \frac{1}{6}(\overline{h} + \underline{h} + 4h)cv_{26} \\
v_{49} &= \frac{1}{6}(\overline{c}_p + \underline{c}_p + 4c_p)I_kP, \quad v_{50} = \frac{1}{6}N(\overline{c}_p + \underline{c}_p + 4c_p)I_k \\
v_{51} &= \frac{1}{6}\{P(\overline{A} + \underline{A} + 4A) + (\overline{h} + \underline{h} + 4h)(v_{25} + v_{26}) \\
& \quad -\frac{1}{2}PM^2(\overline{c}_p + \underline{c}_p + 4c_p)I_k\} \\
v_{52} &= \frac{1}{12}\{2(\overline{h} + \underline{h} + 4h)v_{23} - P(\overline{c}_p + \underline{c}_p + 4c_p)I_k\} \\
v_{52}^{'} &= \frac{1}{6}\{(\overline{h} + \underline{h} + 4h)v_{24} + PN(\overline{c}_p + \underline{c}_p + 4c_p)I_k\} \\
v_{52}^{''} &= \frac{b}{6c}\{(\overline{h} + \underline{h} + 4h)(v_1 - 1)\}
\end{aligned}
$$

<u>Subcase-2.4</u>: <u>$t_1 \leq M \leq N \leq T$</u> In this subcase possibilistic mean value of the fuzzy profit function $\widetilde{P}_{24}$ is

$$
\begin{aligned}
\overline{M}(\widetilde{P}_{24}) \quad = \quad & \frac{(m-1)Pt_1(\overline{c}_p + \underline{c}_p + 4c_p)}{6T} \\
& -\frac{(\overline{A} + \underline{A} + 4A)}{6T} - \frac{(\overline{h} + \underline{h} + 4h)(I_5 + I_6 + I_7)}{6T} \\
& -\frac{(\overline{c}_p + \underline{c}_p + 4c_p)I_k}{6T}\left[Pt_1(N-M) + \frac{(a - be^{-cN})(T-N)^2}{2}\right]
\end{aligned}
\tag{5.36}
$$

$$\begin{aligned}\frac{d(M(\widetilde{P}_{24}))}{dt_1} &= \frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6T} - \frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)t_1}{6T^2(a-be^{-cN})} \\ &+\frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cN})} + \frac{(\overline{h}+\underline{h}+4h)P(I_5+I_6+I_7)}{6T^2(a-be^{-cN})} \\ &+\frac{(\overline{h}+\underline{h}+4h)P(t_1-T)}{6T} + \frac{(\overline{c}_p+\underline{c}_p+4c_p)I_kP}{6T^2(a-be^{-cN})} \\ &\left[Pt_1(N-M)+\frac{(a-be^{-cN})(T-N)^2}{2}\right] \\ &+\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_kMP}{6T} - \frac{(\overline{c}_p+\underline{c}_p+4c_p)I_kP}{6}\end{aligned} \tag{5.37}$$

Substituting the expressions of T, I_5, I_6 and I_7 in the equation $\frac{d(M(\widetilde{P}_{24}))}{dt_1}=0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{24})$(since for t=t_1^*, $\frac{d^2(M(\widetilde{P}_{24}))}{dt_1^2}$<0), as

$$\begin{aligned} t_1^* &= \Big((P-1)v_1v_1{}'-12v_1v_2v_{53}+6v_1v_{54}-6v_2v_{55}-6v_{58} \\ &+\sqrt{(-4(v_1'v_2+6v_2^2v_{53}-6v_2v_{54}+6v_{56})} \\ &(6v_1^2v_{53}+6v_1v_{55}-6v_{57})+((P-1)v_1v_1{}'-12v_1v_2v_{53}+6v_1v_{54} \\ &-6v_2v_{55}-6v_{58})^2)\Big)/(2(6v_1^2v_{53}+6v_1v_{55}-6v_{57})),\end{aligned} \tag{5.38}$$

$$\begin{aligned} \text{where } v_{53} &= \frac{1}{12}(\overline{h}+\underline{h}+4h)(P-2)-\frac{I_kP}{12}(\overline{c}_p+\underline{c}_p+4c_p) \\ v_{54} &= \frac{NP}{6}(\overline{h}+\underline{h}+4h)+\frac{I_k}{6}(\overline{c}_p+\underline{c}_p+4c_p)(NP+1) \\ v_{55} &= \frac{P}{6}(\overline{h}+\underline{h}+4h) \\ v_{56} &= \frac{P(\overline{A}+\underline{A}+4A)}{6(a-be^{-cN})}+\frac{P(\overline{h}+\underline{h}+4h)}{6(a-be^{-cN})}\{\frac{b}{c^2}(e^{-cN}-1) \\ &+\frac{bN}{c}-\frac{1}{2}bN^2e^{-cN}\}+\frac{1}{12}I_kN^2(\overline{c}_p+\underline{c}_p+4c_p), \\ v_{57} &= \frac{P^2(\overline{h}+\underline{h}+4h)}{12(a-be^{-cN})} \\ v_{58} &= \frac{P^2N(\overline{h}+\underline{h}+4h)}{6(a-be^{-cN})}+\frac{I_kP^2(N-M)(\overline{c}_p+\underline{c}_p+4c_p)}{6(a-be^{-cN})}\end{aligned}$$

Subcase-2.5: $t_1 \le M \le T \le N$ In this subcase possibilistic mean value of the fuzzy profit function $\widetilde{P}_{25}$ is

$$\begin{aligned}\overline{M}(\widetilde{P}_{25}) &= \frac{(m-1)Pt_1(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(\overline{A}+\underline{A}+4A)}{6T}-\frac{(\overline{h}+\underline{h}+4h)(I_8+I_9)}{6T} \\ &-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_kP(N-M)t_1}{6T}\end{aligned} \tag{5.39}$$

$$\begin{aligned}\frac{d(M(\widetilde{P}_{25}))}{dt_1} &= \frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)t_1}{6T^2(a-be^{-cN})}\\ &+\frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cT})}+\frac{(\overline{h}+\underline{h}+4h)P(I_8+I_9)}{6T^2(a-be^{-cT})}+\frac{(\overline{h}+\underline{h}+4h)}{6T}\\ &\{(a-P)T+\frac{b}{c}(e^{-cT}-1)\}\\ &+\frac{b(\overline{c}_p+\underline{c}_p+4c_p)I_kP(N-M)(e^{-cT}-1)}{6cT^2(a-be^{-cT})}\\ &+\frac{b(\overline{c}_p+\underline{c}_p+4c_p)I_kP(N-M)e^{-cT}}{6T(a-be^{-cT})}\end{aligned} \tag{5.40}$$

Substituting the expressions of T, I_8 and I_9 in the equation $\frac{d(M(\widetilde{P}_{25}))}{dt_1}=0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{25})$(since for t=t_1^*, $\frac{d^2(M(\widetilde{P}_{25}))}{dt_1^2}$<0), as

$$\begin{aligned}t_1^* =& \big((P-1)v_1v_1^{'}-12(a-b)v_1v_2v_{59}-12bcv_1v_2^2v_{59}-12v_1v_2v_{60}+6v_1v_{61}\\ &-6v_{63}-6v_{64}+6cv_2v_{64}\\ &+\sqrt{(((P-1)v_1v_1^{'}-12(a-b)v_1v_2v_{59}-12bcv_1v_2^2v_{59}-12v_1v_2v_{60}+6v_1v_{61}}\\ &-6v_{63}-6v_{64}+6cv_2v_{64})^2-4(v_1^2v_2+6av_2^2v_{59}-6bv_2^2v_{59}+6bcv_2^2v_{59}+6v_2^2v_{60}\\ &-6v_2v_{61}+6v_{62})(6av_1^2v_{59}-6bv_1^2v_{59}+6bcv_1^2v_2v_{59}+6v_1^2v_{60}-6cv_1v_{64}+6v_{65})\big)\\ &/(2(av_1^2v_{59}-bv_1^2v_{59}+bcv_1^2v_2v_{59}+v_1^2v_{60}-cv_1v_{64}+v_{65})),\end{aligned} \tag{5.41}$$

$$\begin{aligned}\text{where } v_{59} &= \frac{1}{6}(\overline{h}+\underline{h}+4h)(a-P-b)\\ v_{60} &= \frac{1}{12}c^2P(\overline{h}+\underline{h}+4h)-\frac{1}{12}bcP(N-M)(\overline{c}_p+\underline{c}_p+4c_p)I_k\\ v_{61} &= \frac{1}{6}cPv_{26}(\overline{h}+\underline{h}+4h),\\ v_{62} &= \frac{P}{6}\{(\overline{A}+\underline{A}+4A)+(\overline{h}+\underline{h}+4h)(v_{25}+v_{26})\}\\ v_{63} &= \frac{P}{6}v_{24}(\overline{h}+\underline{h}+4h),\\ v_{64} &= \frac{Pb}{6c}(v_1-1)(\overline{h}+\underline{h}+4h),\ \ v_{65}=\frac{1}{6}Pv_{23}(\overline{h}+\underline{h}+4h)\end{aligned}$$

Subcase-2.6: $T \le M \le N$ In this subcase possibilistic mean value of the fuzzy profit function $\widetilde{P}_{26}$ is

$$\begin{aligned}\overline{M}(\widetilde{P}_{26}) =& \frac{(m-1)Pt_1(\overline{c}_p+\underline{c}_p+4c_p)}{6T}-\frac{(\overline{A}+\underline{A}+4A)}{6T}-\frac{(\overline{h}+\underline{h}+4h)(I_8+I_9)}{6T}\\ &-\frac{(\overline{c}_p+\underline{c}_p+4c_p)I_kP(N-M)t_1}{6T}\end{aligned} \tag{5.42}$$

$$\frac{d(M(\widetilde{P}_{26}))}{dt_1} = \frac{(m-1)P(\overline{c}_p+\underline{c}_p+4c_p)}{6T} - \frac{(m-1)P^2(\overline{c}_p+\underline{c}_p+4c_p)t_1}{6T^2(a-be^{-cN})} + \frac{(\overline{A}+\underline{A}+4A)P}{6T^2(a-be^{-cT})} + \frac{(\overline{h}+\underline{h}+4h)P(I_8+I_9)}{6T^2(a-be^{-cT})} + \frac{(\overline{h}+\underline{h}+4h)}{6T} \{(a-P)T+\frac{b}{c}(e^{-cT}-1)\} + \frac{b(\overline{c}_p+\underline{c}_p+4c_p)I_kP(N-M)(e^{-cT}-1)}{6cT^2(a-be^{-cT})} + \frac{b(\overline{c}_p+\underline{c}_p+4c_p)I_kP(N-M)e^{-cT}}{6T(a-be^{-cT})} \tag{5.43}$$

Substituting the expressions of T, I_8 and I_9 in the equation $\frac{d(M(\widetilde{P}_{26}))}{dt_1}=0$, we get the optimal t_1 (say t_1^*), which maximizes $M(\widetilde{P}_{26})$(since for t=t_1^*, $\frac{d^2(M(\widetilde{P}_{26}))}{dt_1^2}$<0), as

$$t_1^* = \big((P-1)v_1v_1^{'} - 12(a-b)v_1v_2v_{59} - 12bcv_1v_2^2v_{59} - 12v_1v_2v_{60} + 6v_1v_{61} - 6v_{63} - 6v_{64} + 6cv_2v_{64} + \sqrt{(((P-1)v_1v_1^{'} - 12(a-b)v_1v_2v_{59} - 12bcv_1v_2^2v_{59} - 12v_1v_2v_{60} + 6v_1v_{61} - 6v_{63} - 6v_{64} + 6cv_2v_{64})^2 - 4(v_1^2v_2 + 6av_2^2v_{59} - 6bv_2^2v_{59} + 6bcv_2^2v_{59} + 6v_2^2v_{60} - 6v_2v_{61} + 6v_{62})(6av_1^2v_{59} - 6bv_1^2v_{59} + 6bcv_1^2v_2v_{59} + 6v_1^2v_{60} - 6cv_1v_{64} + 6v_{65})}\big) / (2(av_1^2v_{59} - bv_1^2v_{59} + bcv_1^2v_2v_{59} + v_1^2v_{60} - cv_1v_{64} + v_{65})) \tag{5.44}$$

6. Algorithm

Depending upon the values of M and N six subcases may occur for each of two cases (i) N<M and (ii) $N \geq M$.

(6.1)

Find optimal t_1(say t_1^*) to maximize $\overline{M}(\widetilde{P}_{ij}(t_1)), i=1,2; j=1,2,...,6$

If N<M

If 0<N<$M \leq t_1$. Perform (6.1)(Profit and t_1^* are given by (5.8) and (5.11))
If 0<$N \leq t_1$<M. Perform (6.1)(Profit and t_1^* are given by (5.12) and (5.14))
If 0<$N \leq t_1$, M>T. Perform (6.1)(Profit and t_1^* are given by (5.15) and (5.17))
If $t_1 \leq N$<$M \leq T$. Perform (6.1)(Profit and t_1^* are given by (5.18) and (5.20))
If $t_1 \leq N \leq T$, M>T. Perform (6.1)(Profit and t_1^* are given by (5.21) and (5.23))
If $T \leq N$<M. Perform (6.1)(Profit and t_1^* are given by (5.24) and (5.26))

Else

If $M \leq N \leq t_1 \leq T$. Perform (6.1)(Profit and t_1^* are given by (5.27) and (5.29))
If 0<$M \leq t_1 \leq N \leq T$. Perform (6.1)(Profit and t_1^* are given by (5.30) and (5.32))
If 0<$M \leq t_1$,$N \geq T$. Perform (6.1)(Profit and t_1^* are given by (5.33) and (5.35))
If $t_1 \leq M \leq N \leq T$. Perform (6.1)(Profit and t_1^* are given by (5.36) and (5.38))
If $t_1 \leq M \leq T \leq N$. Perform (6.1)(Profit and t_1^* are given by (5.39) and (5.41))

If $T \leq M \leq N$. Perform (6.1)(Profit and t_1^* are given by (5.42) and (5.44))

7. Numerical illustration

For twelve different subcases twelve different examples are used. In Table 7.1, different crisp data for different examples are given and in Table 7.2 different fuzzy data for different examples are given and in Table 7.3 optimal cycle length, optimal replenishment time and maximum profits are given for twelve different examples.

Table 7.1

Crisp data for different examples

Subcase	Example	P	a	b	c	N	M	I_k	I_e	m
1.1	1.1	1300	1000	200	2.5	.25	.4	.19	.14	1.5
1.2	1.2	1300	1000	200	2.5	.2	.5	.19	.14	1.5
1.3	1.3	1300	1000	200	2.5	.2	.9	.19	.14	1.5
1.4	1.4	1900	1000	200	2.5	.3	.6	.19	.14	1.5
1.5	1.5	2200	1000	200	2.5	.19	.47	.19	.14	1.5
1.6	1.6	3000	1000	200	2.5	.70	.90	—	.14	1.5
2.1	2.1	1100	1000	200	2.5	.50	.30	.19	—	1.5
2.2	2.2	1400	1000	200	2.5	.35	.20	.19	—	1.5
2.3	2.3	2000	1000	200	2.5	1.2	.2	.19	—	1.5
2.4	2.4	1500	1000	200	2.5	.22	.2	.19	—	1.5
2.5	2.5	1400	1000	200	2.5	1.3	.9	.19	—	1.5
2.6	2.6	1500	1000	200	2.5	1.3	1.2	—	—	1.5

Table 7.2

Fuzzy data for different examples

Subcase	Example	$\widetilde{h}$	$\widetilde{c_p}$	$\widetilde{A}$
1.1	1.1	(4.2,4.5,4.8)	(.29,30,.37)	(1045,1050,1055)
1.2	1.2	(4.2,4.5,4.8)	(.29,30,.37)	(1045,1050,1055)
1.3	1.3	(4.2,4.5,4.8)	(.29,30,.37)	(1045,1050,1055)
1.4	1.4	(4.2,4.5,4.8)	(.29,30,.37)	(645,650,655)
1.5	1.5	(4.2,4.5,4.8)	(.29,30,.37)	(645,650,655)
1.6	1.6	(7.2,7.5,7.8)	(.29,30,.37)	(445,450,455)
2.1	2.1	(4.2,4.5,4.8)	(.29,30,.37)	(1045,1050,1055)
2.2	2.2	(4.2,4.5,4.8)	(.29,30,.37)	(645,650,655)
2.3	2.3	(7.2,7.5,7.8)	(.29,30,.37)	(445,450,455)
2.4	2.4	(4.2,4.5,4.8)	(.29,30,.37)	(645,650,655)
2.5	2.5	(7.2,7.5,7.8)	(.29,30,.37)	(445,450,455)
2.6	2.6	(7.2,7.5,7.8)	(.29,30,.37)	(445,450,455)

Table 7.3

Results of different examples

Subcase	Example	T year	t_1^* year	$\frac{d^2(M(\widetilde{P_{12}}))}{dt_1^2}$	Profit (units)
1.1	1.1	.7147	.4829	-6508.238	11283.01
1.2	1.2	.6214	.4145	-20180.30	11753.91
1.3	1.3	.6472	.4117	-15456.87	13932.80
1.4	1.4	.6378	.2997	-28346.55	11997.99
1.5	1.5	.4660	.1800	$-.54\times 10^8$	12034.60
1.6	1.6	.6729	.2026	-24201.77	12073.90
2.1	2.1	1.60	1.35	-886.41	11690.55
2.2	2.2	.55	.35	-36715.11	10882.23
2.3	2.3	1.09	.51	-405779.20	6596.14
2.4	2.4	.229	.124	-104208.30	9451.265
2.5	2.5	.98	.645	-22803.70	10057.07
2.6	2.6	.94	.58	-25187.52	11491.14

From Table 7.3, it reveals that the Subcase 1.3 is the most profitable scenario, since in this case (M-N) is largest, so retailer can earn more interest than the other subcases and Subcase 2.3 is worst subcase for the opposite reason.

8. Sensitivity analysis

Results are obtained due to different values of a, for all the examples(i.e., for the subcases 1.1 to 1.6 and the subcases 2.1 to 2.6) and it is observed that(see table-8.1 and table-8.2) profit increases with a i.e., profit increases with the increase of demand, which agrees with reality.

Table-8.1

Profits with various values of 'a' for case-1 (N$\leq$ M)

a		1000	1100	1200	1300	1400
P	Example-1.1	11283.01	12927.35	14664.10	16552.07	18952.78
R	Example-1.2	11753.91	13412.52	15094.36	16793.53	18492.73
O	Example-1.3	13932.80	15843.51	17777.37	19732.98	21709.39
F	Example-1.4	11997.99	13533.39	15045.57	16534.26	17999.47
I	Example-1.5	12034.60	13641.69	15246.16	16845.28	18439.03
T	Example-1.6	12073.90	13585.41	15115.96	16664.45	18230.16

Table-8.2

Profits with various values of 'a' for case-2 (N$\geq$ M)

a		1000	1050	1100	1150	1200
P	Example-2.1	11690.55	12744.24	13940.27	15576.56	14718.33
R	Example-2.2	10882.23	11583.76	12281.85	12976.49	13667.7
O	Example-2.3	6596.14	7826.68	10024.20	12374.07	14723.95
F	Example-2.4	9451.265	11186.99	11847.62	12506.01	13162.17
I	Example-2.5	10057.07	11482.92	13001.79	14683.92	17208.00
T	Example-2.6	11491.14	13044.21	14665.08	16382.67	18271.20

Now, results are obtained due to different values of M, for all the examples(i.e., for the subcases-1.1 to 1.6 and the subcases-2.1 to 2.6) and it is observed that(see tables-8.3 to 8.14) profit increases with the increase of retailer's trade credit period (M) , which agrees with reality.

Table-8.3

Profits with various values of 'M' for subcase-1.1

M		.40	.42	.44	.46	.48
P R O F I T	Example-1.1	11283.01	11376.13	11478.77	11581.01	11675.94

Table-8.4

Profits with various values of 'M' for subcase-1.2

M		.50	.52	.54	.56	.58
P R O F I T	Example-1.2	11753.91	11861.28	11696.00	12058.29	12185.52

Table-8.5

Profits with various values of 'M' for subcase-1.3

M		.90	.92	.94	.96	.98
P R O F I T	Example-1.3	13932.80	14042.04	14151.29	14260.53	14369.77

Table-8.6

Profits with various values of 'M' for subcase-1.4

M		.60	.61	.62	.63	.637
P R O F I T	Example-1.4	11997.99	12029.91	12062.18	12094.79	12117.81

Table-8.7

Profits with various values of 'M' for subcase-1.5

M		.47	.48	.49	.50	.51
P R O F I T	Example-1.5	12034.60	12088.87	12143.14	12197.41	12251.69

Table-8.8

Profits with various values of 'M' for subcase-1.6

M		.90	.92	.94	.96	.98
P r o f i t	Example-1.6	12073.9	12187.6	12301.3	12415.0	12528.9

Table-8.9

Profits with various values of 'M' for subcase-2.1

M		.30	.32	.34	.36	.38	.40
P r o f i t	Ex-2.1	11690.5	11715.0	11741.3	11769.5	11799.8	11832.2

Table-8.10

Profits with various values of 'M' for subcase-2.2

M		.20	.21	.22	.23	.24	.25
P r o f i t	Ex-2.2	10882.2	10911.8	10942.8	10975.2	11009.1	11044.4

Table-8.11

Profits with various values of 'M' for subcase-2.3

M		.20	.22	.24	.26	.28	.30
P r o f i t	Ex-2.3	6596.10	6641.20	6693.90	6756.50	6833.70	7301.00

Table-8.12

Profits with various values of 'M' for subcase-2.4

M		.20	.21	.22	.23	.24	.25
P r o f i t	Ex-2.4	9451.30	10635.7	10736.8	10786.0	10835.1	10884.3

Table-8.13

Profits with various values of 'M' for subcase-2.5

M		.90	.91	.92	.93	.94	.95
P r o f i t	Ex-2.5	10057.1	10109.8	10162.6	10215.3	10268.1	10320.9

Table-8.14

Profits with various values of 'M' for subcase-2.6

M		1.20	1.22	1.24	1.26	1.28	1.30
P r o f i t	Ex-2.6	11491.1	11596.4	11701.7	11807.0	11912.3	12017.6

Now, results are obtained due to different values of N, for all the examples and it is observed that(see tables-8.15-8.19 and tables-8.21, 8.22, 8.24) profit increases with the increase of customer's trade credit period (N) up to certain period after that profit decreases with the increase of N. It happens because initially increase of N increases the demand of the item which in turn increases profit. Again increase of N decreases profit due to bank interest. But profit due to increase of demand dominates loss of bank interest. As a result increase of N initially increases the resultant profit. As demand increases with time at a decreasing rate so after certain level of N, increase of profit due to increase of demand is less than the loss of bank interest due to increase of N. As a result resultant profit decreases after certain level of N. But this situation does not occur for Models-1.6, 2.3, 2.5, 2.6 (see tables-8.20, 8.23, 8.25, 8.26), as in that case $N > T$, so increase of N does not effect the demand during [0,T]. So profit decreases with increase of N in this case.

Table-8.15

Profits with various values of 'N' for subcase-1.1

N		.25	.27	.29	.31	.33	.35	.37
P r o f i t	Ex-1.1	11283	11687	11690	11684	11671	11651	11627

Table-8.16

Profits with various values of 'N' for subcase-1.2

N		.20	.24	.28	.30	.32	.36	.40
P r o f i t	Ex-1.2	11754	11796	11806	11801	11789	11748	11685

Table-8.17

Profits with various values of 'N' for subcase-1.3

N		.20	.24	.28	.31	.33	.35	.39
P r o f i t	Ex-1.3	13933	13993	14018	14016	14007	13991	13943

Table-8.18

Profits with various values of 'N' for subcase-1.4

N		.30	.34	.38	.40	.44	.48	.52
P r o f i t	Ex-1.4	11998	12020	12026	12022	12006	11977	11937

Table-8.19

Profits with various values of 'N' for subcase-1.5

N		.20	.21	.22	.23	.24	.25	.26
P r o f i t	Ex-1.5	12040	12043	12043	12041	12037	12031	12022

Table-8.20

Profits with various values of 'N' for subcase-1.6

N		.70	.74	.78	.82	.84	.86	.88
P r o f i t	Ex-1.6	12074	11845	11617	11391	11278	11165	11053

Table-8.21

Profits with various values of 'N' for subcase-2.1

N		.50	.51	.52	.53	.54	.55	.56
P r o f i t	Ex-2.1	11691	11692	11693.4	11694	11693.2	11692.6	11692.2

Table-8.22

Profits with various values of 'N' for subcase-2.2

N		.35	.37	.39	.41	.43	.45	.47
P r o f i t	Ex-2.2	10882	10889	10890.3	10885	10875	10861	10843.1

Table-8.23

Profits with various values of 'N' for subcase-2.3

N		1.20	1.22	1.24	1.26	1.28	1.30	1.32
P r o f i t	Ex-2.3	6596	6490	6384	6277.8	6171.8	6065.8	5959.9

Table-8.24

Profits with various values of 'N' for subcase-2.4

N		.22	.23	.24	.25	.26	.27	.28
P r o f i t	Ex-2.4	9451.3	10506	10484	10460	10433	10403	10371

Table-8.25

Profits with various values of 'N' for subcase-2.5

N		1.30	1.32	1.34	1.36	1.38	1.40	1.42
P r o f i t	Ex-2.5	10057	9951.6	9846.2	9740.8	9635.4	9530.0	9424.7

Table-8.26

Profits with various values of 'N' for subcase-2.6

N		1.30	1.32	1.34	1.36	1.38	1.40	1.42
P r o f i t	Ex-2.6	11491	11386	11281	11176	11070	10965	10860

9. Conclusions

This research addresses a production inventory model with dynamic demand under bi-level trade credit policy in imprecise environment. An easy-to-use algorithm is proposed which gives the optimal values of both profit and replenishment time. Here we have determined the optimal replenishment time in maximizing the expected resultant profit using possibilistic mean value approach and finally, numerical examples are used to illustrate all results obtained in this paper. In addition, we obtain a lot of managerial insights from numerical examples. This retailer's model has wide range of applications in wholesale-retail-customer business where the competition is stiff, especially in grocery, stationary goods shop, building materials shop etc.

A future study will further incorporate the proposed model into more realistic assumptions, such as fuzzy demand, deteriorating items, allowable shortages, multi-supplier, multi-retailer, multi-customer etc

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