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Topological structure formed by soft multi sets and soft multi compact space

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Abstract. The purpose of this paper is to study the concept of topological structure formed by soft multi sets. The notion of relative complement of soft multi set, soft multi point, soft multi open set, soft multi closed set, soft multi basis, soft multi sub basis, neighbourhoods and neighbourhood system, interior and closure of a soft multi set etc are to be introduced and their basic properties are also to be investigated . It is seen that a soft multi topological space gives a parameterized family of topological spaces. Lastly the concept of soft multi compact space is also introduced.

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Keywords: Soft multi set, Soft multi topology, Soft multi point, Soft multi open set, Soft multi closed set, Neighbourhood of a soft multi set, Interior of a soft multi set, Closure of a soft multi set, Soft multi basis, Soft multi compact space.

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1. Introduction

In 1999, Molodtsov initiated soft set theory as a completely generic mathematical tool for modelling vague concepts. In soft set theory there is no limited condition to the description of objects; so researchers can choose the form of parameters as they need, which greatly simplifies the decision making process and make the process more efficient in the absence of partial information. Although many mathematical tools are available for modelling uncertainties such as probability theory, fuzzy set theory, rough set theory, interval valued mathematics etc, but there are inherent difficulties associated with each of these techniques. Moreover all these techniques lack in the parameterization of the tools and hence they could not be applied successfully in tackling problems especially in areas like economic, environmental and social problems domains. Soft set theory is standing in a unique way in the sense that it is free from the above difficulties.

Soft set theory has a rich potential for application in many directions, some of which are reported by Molodtsov [6] in his work. Later on Maji et al.[5] presented some new definitions on soft sets such as subset, union, intersection and complements of soft sets and discussed in detail the application of soft set in decision making problem. Based on the analysis of several operations on soft sets introduced in [5], Ali et al. [1] presented some new algebraic operations for soft sets and proved that certain De Morgan's law holds in soft set theory with respect to these new definitions. Alkhazaleh et al. [4] as a generalization of Molodtsov's soft set, presented the definition of a soft multi set and its basic operations such as complement, union, and intersection etc. Recently, D.Tokat and I.Osmanoglu ([7],[9]) introduced soft multi topology . In this paper we study the concept of soft multi topological spaces in details .

Then the notion of relative complement of soft multi set, soft multi point, soft multi open set, soft multi closed set, soft multi basis, soft multi sub basis, neighbourhood and neighbourhood system, interior and closure of a soft multi set etc are to be introduced and their basic properties are investigated. It is seen that a soft multi topological space gives a parameterized family of topological spaces (see [3], [8]). Lastly we introduce the soft multi compact space.

2. Preliminaries

Definition 2.1 ([6]). Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A$\subseteq$ E. Then the pair (F, A) is called a soft set over U, where F is a mapping given by F: A$\rightarrow$ P(U).

Definition 2.2 ([2]). Let $\{U_i\text{: } i\in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i=\phi$ and let $\{E_{U_i} : i\in I\}$ be a collection of sets of parameters. Let $U=\Pi_{i\in I}P(U_i)$ where $P(U_i)$ denotes the power set of U_i, $E=\Pi_{i\in I}E_{U_i}$ and A$\subseteq$E . A pair (F, A) is called a soft multiset over U, where F is a mapping given by F: A$\rightarrow$U.

Definition 2.3 ([2]). A soft multiset (F, A) over U is called a null soft multi set denoted by $\widetilde{\phi}$, if for all a$\in$A, F(a)=ϕ.

Definition 2.4 ([2]). A soft multiset (F, A) over U is called an absolute soft multi set denoted by $\widetilde{A}$, if for all a$\in$A, F(a)=U.

Definition 2.5 ([2]). For any soft multi set (F, A), a pair $(e_{U_i,j},F_{e_{U_i},j})$ is called a U_i- soft multi part , $\forall e_{U_i,j}\in a_k$, and $F_{e_{U_i},j}\subseteq F(A)$ is an approximate value set, where $F(A)\subseteq U=\Pi_{i\in I}P(U_i)$,$a_k\in A$, k={1,2,3,..,n}, i$\in${1,2,3,..,m} and j $\in${1,2,3,..,r}.

Definition 2.6 ([2]). For two soft multi sets (F, A) and (G, B) over U, (F, A) is called a soft multi subset of (G, B) if

1. A$\subseteq$B and
2. $\forall e_{U_i,j}\in a_k$, $(e_{U_i,j},F_{e_{U_i},j})\subseteq(e_{U_i,j},G_{e_{U_i},j})$

where $a_k\in A$, k={1,2,3,..,n}, i$\in${1,2,3,..,m} and j $\in${1,2,3,..,r}. This relation is denoted by (F, A)$\widetilde{\subseteq}$(G, B). In this case (G, B) is called a soft multi superset of (F, A).

Definition 2.7 ([2]). Two soft multi sets (F, A) and (G, B) over U are said to be equal if (F , A) is a soft multi subset of (G, B) and (G, B) is a soft multi subset of (F , A).

Definition 2.8 ([2]). Union of two soft multi sets (F, A) and (G, B) over U denoted by (F,A)$\widetilde{\cup}$(G, B) is the soft multi set (H, C), where C= A$\cup$B and $\forall$e$\in$C,
H(e)= F(e), if e$\in$A-B
= G(e), if e$\in$B-A
=F(e)$\bigcup$G(e), if e$\in$A$\cap$B

Definition 2.9 ([2]). Intersection of two soft multi sets (F, A) and (G, B) over U denoted by (F,A)$\widetilde{\cap}$(G, B) is the soft multi set (H, C), where C= A$\cup$B and $\forall$e$\in$C,
H(e)= F(e), if e$\in$A-B
= G(e), if e$\in$B-A
=F(e)$\bigcap$G(e), if e$\in$A$\cap$B

3. Soft multi sets and their basic properties

In this section, for the sake of simplicity, we restate a few basic definitions e.g., Definitions 2.3 and 2.4 etc. in the following form to study a few results of soft multi topological spaces properly. Let {U$_i$: i$\in$I} be a collection of universes such that $\bigcap_{i\in I}$U$_i$=ϕ and let {E$_{U_i}$: i$\in$I} be a collection of sets of parameters. Let U=$\Pi_{i\in I}$P(U$_i$) where P(U$_i$) denotes the power set of U$_i$, E=$\Pi_{i\in I}$E$_{U_i}$ .The set of all soft multi set over (U,E) is denoted by SMS(U,E).

Definition 3.1. A soft multiset (F, E)$\in$SMS(U,E) is called a null soft multi set denoted by $\widetilde{\phi}$, if for all e$\in$E, F(e)=ϕ.

Definition 3.2. A soft multiset (F, E)$\in$SMS(U,E) is called an absolute soft multi set denoted by $\widetilde{E}$, if for all e$\in$E, F(e)=U.

Definition 3.3. The relative complement of a soft multi set (F, A) over (U,E) is denoted by (F, A)$'$ and is defined by (F, A)$'$=(F$'$,A) , where F$'$:A$\to$U is a mapping given by F$'$(e)=U-F(e), $\forall$e$\in$E.

Example 3.4. Let us consider there are three universes U$_1$, U$_2$ and U$_3$.Let us consider a soft multiset (F, A) which describes the "attractiveness of house", "cars" and "hotels" that Mr. Sen is considering for accommodation purchase, transportation purchase and venue to hold a wedding celebration respectively. Let U$_1$={h$_1$, h$_2$, h$_3$, h$_4$}, U$_2$={c$_1$, c$_2$, c$_3$} and U$_3$={v$_1$, v$_2$}. Let {E$_{U_1}$, E$_{U_2}$, E$_{U_3}$} be a collection of sets of parameters related to the above universes, where
E$_{U_1}$={e$_{U_1,1}$=expensive, e$_{U_1,2}$=cheap, e$_{U_1,3}$=wooden},
E$_{U_2}$={e$_{U_2,1}$=expensive, e$_{U_2,2}$=cheap, e$_{U_2,3}$=sporty},
E$_{U_3}$={e$_{U_3,1}$=expensive, e$_{U_3,2}$=cheap, e$_{U_3,3}$=in Kuala Lumpur},
U=$\Pi_{i=1}^3$P(U$_i$), E=$\Pi_{i=1}^3$E$_{U_i}$ and
A={e$_1$=(e$_{U_1,1}$, e$_{U_2,1}$, e$_{U_3,1}$), e$_2$=(e$_{U_1,1}$, e$_{U_2,2}$, e$_{U_3,1}$)}
Then the relative complement of the soft multiset
(F,A)= {(e$_1$,({h$_1$, h$_2$}, {c$_1$, c$_2$}, {v$_1$})), (e$_2$,({h$_3$, h$_4$}, {c$_1$, c$_3$}, {v$_2$}))} is
(F,A)$'$= {(e$_1$,({h$_3$, h$_4$}, {c$_3$}, {v$_2$})), (e$_2$,({h$_1$, h$_2$}, {c$_2$}, {v$_1$}))}
Clearly $\widetilde{\phi}'$=$\widetilde{E}$ and $\widetilde{E}'$=$\widetilde{\phi}$.

Proposition 3.5. *If (F, A) and (G, B) are two soft multi sets over (U, E), then we have the following*

(i) $((F, A)\widetilde{\cup}(G, B))' = (F, A)'\widetilde{\cap}(G, B)'$

(ii) $((F, A)\widetilde{\cap}(G, B))' = (F, A)'\widetilde{\cup}(G, B)'$

Proof. Straight forward. □

Definition 3.6. A soft multi set(F,A)∈SMS(U, E) is called a soft multi point in (U, E), denoted by $e_{(F,A)}$, if for the element e∈A, F(e)≠φ and ∀e'∈A-{e}, F(e')=φ.

Example 3.7. Let us consider there are three universes U_1, U_2 and U_3. Let us consider a soft multiset (F, A) which describes the "attractiveness of house", "cars" and "hotels" that Mr. Sen is considering for accommodation purchase, transportation purchase and venue to hold a wedding celebration respectively. Let U_1={h_1, h_2, h_3, h_4}, U_2={c_1, c_2, c_3} and U_3={v_1, v_2}. Let {E_{U_1}, E_{U_2}, E_{U_3}} be a collection of sets of parameters related to the above universes, where

E_{U_1}={$e_{U_1,1}$=expensive, $e_{U_1,2}$=cheap, $e_{U_1,3}$=wooden},

E_{U_2}={$e_{U_2,1}$=expensive, $e_{U_2,2}$=cheap, $e_{U_2,3}$=sporty},

E_{U_3}={$e_{U_3,1}$=expensive, $e_{U_3,2}$=cheap, $e_{U_3,3}$=in Kuala Lumpur},

$U=\Pi_{i=1}^{3}P(U_i)$, $E=\Pi_{i=1}^{3}E_{U_i}$ and

A={e_1=($e_{U_1,1}$, $e_{U_2,1}$, $e_{U_3,1}$), e_2=($e_{U_1,1}$, $e_{U_2,2}$, $e_{U_3,1}$)}

Then the soft multi set (F,A)= {(e_1,({h_1, h_2}, {c_1, c_2}, ϕ))} is the soft multi point and it is denoted by $e1_{(F,A)}$.

Definition 3.8. A soft multi point $e_{(F,A)}$ e is said to be in the soft multi set (G, B), denoted by $e_{(F,A)}\widetilde{\in}$(G, B), if (F, A)$\widetilde{\subseteq}$(G, B).

Example 3.9. The soft multi point $e1_{(F,A)}$ as in the example 3.7, in the soft multi set (G,B)= {(e_1,({h_1, h_2}, {c_1, c_2}, {v_1})), (e_2,({h_3, h_4}, {c_1, c_3}, {v_2})), (e_3,({h_1, h_3, h_4}, {c_1, c_3}, {v_1, v_2}))}, i.e. $e_{(F,A)}$ $\widetilde{\in}$(G, B).

Proposition 3.10. *Let $e_{(F,A)}$ be a soft multi point and (G, B) be the soft multiset in SMS(U, E). If $e_{(F,A)}$ $\widetilde{\in}$(G, B), then $e_{(F,A)}$ $\widetilde{\notin}$(G, B)$'$.*

Proof. If $e_{(F,A)}$ $\widetilde{\in}$(G, B), then (F , A)$\widetilde{\subseteq}$(G, B), i.e. for the element e∈A, F(e)⊆G(e). This implies F(e)⊈U-G(e)=G'(e), i.e. (F, A)$\widetilde{\nsubseteq}$(G, B)$'$. Therefore, we have $e_{(F,A)}$ $\widetilde{\notin}$(G, B)$'$. □

Remark 3.11. The converse of the above proposition is not true in general.

Example 3.12. If we consider the soft multi point

$e1_{(F,A)}$={(e_1,({h_1, h_2}, {c_1, c_2}, ϕ))} as in example 3.7 and a soft multi set (G,B)= {(e_1,({h_1, h_3}, {c_2, c_3}, {v_1})), (e_2,({h_2, h_4}, {c_1, c_3}, {v_2})), (e_3,({h_4}, {c_1}, {v_1}))},

Then $e_{(F,A)}$ $\widetilde{\notin}$(G, B) and also $e_{(F,A)}$ $\widetilde{\notin}$(G, B)$'$={(e_1,({h_2, h_4}, {c_1}, {v_2})), (e_2,({h_1, h_3}, {c_2}, {v_1})), (e_3,({h_1, h_2, h_3}, {c_2, c_3}, {v_1}))}.

Definition 3.13. Let (F, A)∈SMS(U, E) and x∈U_i , for some i. Then we say that x ∈(F, A) and read as x belong to the soft multiset (F, A) if x∈$F_{e_{U_i},j}$, ∀j.

Example 3.14. Let us consider the soft multi set (F,A)= $\{(e_1,(\{h_1, h_2\}, \{c_1, c_2\}, \{v_1\})), (e_2,(\{h_3, h_4\}, \{c_1, c_3\}, \{v_2\}))\}$, as in example 3.4 then for the element $c_1 \in U_2$ we say that $c_1 \in (F, A)$, since $c_1 \in F_{e_{U_2},1}=\{c_1, c_2\}$ and $c_1 \in F_{e_{U_2},2} =\{c_1, c_3\}$ but h_1, $h_2 \notin (F,A)$, since $h_1, h_2 \in F_{e_{U_i},j}=\{h_1, h_2\}$ but $h_1, h_2 \notin F_{e_{U_1},2}=\{h_3, h_4\}$.

Remark 3.15. For any $x \in U_i$, we say that $x \notin (F, A)$ if $x \notin F(e_{U_i,j})$ for some $e_{U_i,j} \in a_k$, $a_k \in A$.

4. Soft multi topological spaces

Recently, D.Tokat and I.Osmanoglu ([7],[9]) introduced soft multi topology . In this section we study the notion of relative complement of soft multi set, soft multi point, soft multi set topology, soft multi closed set, soft multi basis, soft multi sub basis, neighbourhood and neighbourhood system, interior and closure of a soft multi set etc are to be introduced and their basic properties are investigated. It is seen that a soft multi topological space gives a parameterized family of topological spaces.

Definition 4.1. A sub family τ of SMS(U, E), is called soft multi set topology on (U, E), if the following axioms are satisfied:

$[O_1]$. $\widetilde{\phi}$, $\widetilde{E} \in \tau$,

$[O_2]$. the union of any number of soft multi sets in τ belongs to τ, i.e. for any $\{(F^k,A^k)|\ k \in K\} \subseteq \tau \Rightarrow \widetilde{\cup}_{k\in K}(F^k,A^k) \in \tau$,

$[O_3]$. If (F, A), (G, B)$\in\tau$, then (F, A)$\widetilde{\cap}$(G, B)$\in\tau$.

Then the pair ((U, E), τ) is called soft multi topological space. The members of τ are called soft multi open sets (or τ-open soft multi sets or simply open sets) and the conditions $[O_1]$, $[O_2]$ and $[O_3]$ are called the axioms for soft multi open sets.

Example 4.2. Let us consider there are three universes U_1, U_2 and U_3. Let $U_1=\{h_1, h_2, h_3, h_4\}$, $U_2=\{c_1, c_2, c_3\}$ and $U_3=\{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$E_{U_1}=\{e_{U_1,1}$=expensive, $e_{U_1,2}$=cheap, $e_{U_1,3}$=wooden, $e_{U_1,4}$= in green surroundings$\}$,

$E_{U_2}=\{e_{U_2,1}$=expensive, $e_{U_2,2}$=cheap, $e_{U_2,3}$=sporty$\}$,

$E_{U_3}=\{e_{U_3,1}$=expensive, $e_{U_3,2}$=cheap, $e_{U_3,3}$=in Kuala Lumpur, $e_{U_3,4}$= majestic$\}$,

Let $U=\Pi_{i=1}^3 P(U_i)$, $E=\Pi_{i=1}^3 E_{U_i}$ and

$A^1=\{e_1=(e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), e_2=(e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$,

$A^2=\{e_1=(e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), e_3=(e_{U_1,2}, e_{U_2,3}, e_{U_3,1})\}$

Suppose that

$(F^1,A^1)= \{(e_1,(\{h_1, h_2\}, \{c_1, c_2\}, \{v_1\})), (e_2,(\{h_3, h_4\}, \{c_1, c_3\}, \{v_2\}))\}$,

$(F^2,A^2)= \{(e_1,(\{h_1, h_3\}, \{c_2, c_3\}, \{v_1,v_2\})), (e_3,(\{h_2, h_4\}, \{c_1, c_2\}, \{v_2\}))\}$,

$(F^3,A^3)=(F^1,A^1)\widetilde{\cup}(F^2,A^2)$

$=\{(e_1,(\{h_1, h_2, h_3\}, \{c_1, c_2, c_3\}, \{v_1, v_2\})), (e_2,(\{h_3, h_4\}, \{c_1, c_3\}, \{v_2\})), (e_3,(\{h_2, h_4\}, \{c_1, c_2\}, \{v_2\}))\}$,

$(F^4,A^4)=(F^1,A^1)\widetilde{\cap}(F^2,A^2)$

$=\{(e_1,(\{h_1\}, \{c_2\}, \{v_1\})), (e_2,(\{h_3, h_4\}, \{c_1, c_3\}, \{v_2\})), (e_3,(\{h_2, h_4\}, \{c_1, c_2\}, \{v_2\}))\}$,

where $A^3=A^4=A^1\cup A^2=\{e_1=(e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), e_2=(e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), e_3=(e_{U_1,2}, e_{U_2,3}, e_{U_3,1})\}$

Then we observe that the sub family τ_1={$\widetilde{\phi}$, $\widetilde{E}$, (F^1,A^1), (F^2,A^2), (F^3,A^3), (F^4,A^4)}
of SMS(U, E) is a soft multi topology on (U, E), since it satisfies the necessary three axioms [O_1], [O_2] and [O_3] and ((U, E), τ_1) is a soft multi topological space. But the sub family τ_2={$\widetilde{\phi}$, $\widetilde{E}$, (F^1,A^1), (F^2,A^2)} of SMS(U, E) is not a soft multi topology on of (U, E), since the union (F^1,A^1)$\widetilde{\cup}$(F^2,A^2) and the intersection (F^1,A^1)$\widetilde{\cap}$(F^2,A^2) does not belongs to τ_2.

Definition 4.3. As every soft multi topology on (U, E) must contain the sets $\widetilde{\phi}$ and $\widetilde{E}$, so the family $\mathbb{I}$={$\widetilde{\phi}$, $\widetilde{E}$}, forms a soft multi topology on (U, E). This topology is called indiscrete soft multi set topology and the pair ((U, E), $\mathbb{I}$) is called an indiscrete soft multi topological space.

Definition 4.4. Let $\mathbb{D}$ denote family of all the soft multi subsets of (U, E). Then we observe that $\mathbb{D}$ satisfies all the axioms for topology on (U, E). This topology is called discrete soft multi topology and the pair ((U, E),$\mathbb{D}$) is called a discrete soft multi topological space.

Proposition 4.5. *Let ((U, E), τ) be a soft multi topological space over (U, E). Then the collection τ_e={F(e):(F, A)$\in$ τ)} for each e$\in$E, defines a topology on U .*

Proof. [O_1]. Since $\widetilde{\phi}$, $\widetilde{E}\in\tau$ implies that φ, U$\in\tau_e$.
[O_2]. Let {F^k(e): k$\in$K}$\subseteq\tau_e$,for some {(F^k,A^k)| k$\in$K}$\subseteq\tau$.Since
$\widetilde{\cup}_{k\in K}$($F^k$,$A^k$)$\in\tau$, so $\widetilde{\cup}_{k\in K}F^k(e)\in\tau_e$, for each e$\in$E.
[O_3]. Let F(e), G(e)$\in\tau_e$, for some (F, A), (G,B)$\in\tau$.Since (F, A)$\widetilde{\cap}$(G,B)$\in\tau$, so F(e)$\widetilde{\cap}$G(e)$\in\tau_e$,for each e$\in$E. □

Example 4.6. Let us consider the soft multi topology τ_1={$\widetilde{\phi}$, $\widetilde{E}$, (F^1,A^1), (F^2,A^2), (F^3,A^3), (F^4,A^4)} as in the example: 4.2, where
$F^1(e_1)$=({h_1, h_2}, {c_1, c_2}, {v_1}),
$F^1(e_2)$=({h_3, h_4}, {c_1, c_3}, {v_2}),
$F^2(e_1)$=({h_1, h_3}, {c_2, c_3}, {v_1,v_2}),
$F^2(e_3)$=({h_2, h_4}, {c_1, c_2}, {v_2}),
$F^3(e_1)$=({h_1, h_2, h_3}, {c_1, c_2, c_3}, {v_1, v_2}),
$F^3(e_2)$=({h_3, h_4}, {c_1, c_3}, {v_2}),
$F^3(e_3)$=({h_2, h_4}, {c_1, c_2}, {v_2}),
$F^4(e_1)$=({h_1}, {c_2}, {v_1}),
$F^4(e_2)$= ({h_3, h_4}, {c_1, c_3}, {v_2})),
$F^4(e_3)$=({h_2, h_4}, {c_1, c_2}, {v_2}).
It can be easily seen that
τ_{e_1}={φ, U, ({h_1, h_2}, {c_1, c_2}, {v_1}),({h_1, h_3}, {c_2, c_3}, {v_1,v_2}), ({h_1}, {c_2}, {v_1}), ({h_1, h_2, h_3}, {c_1, c_2, c_3}, {v_1, v_2})},
τ_{e_2}={φ, U, ({h_3, h_4}, {c_1, c_3}, {v_2})} and
τ_{e_3}={φ, U, ({h_2, h_4}, {c_1, c_2}, {v_2})} are topologies on U.

Definition 4.7. Let ((U, E), τ_1) and ((U, E), τ_2) be two soft multi topological spaces. If each (F, A)$\in\tau_1\Rightarrow$(F, A)$\in\tau_2$, then τ_2 is called soft multi finer (stronger) topology than τ_1 and τ_1 is called soft multi coarser (or weaker) topology than τ_2 and denoted by $\tau_1\overrightarrow{\subseteq}\tau_2$.

Two soft multi topologies, one of which is finer than other, are said to be comparable.

Example 4.8. The indiscrete soft multi topology on (U, E) is the soft multi coarsest (weakest) and discrete soft multi topology on (U, E) is the soft multi finest (strongest) of all topologies of (U, E). Any other soft multi topology on (U, E) will be in between these two soft multi set topologies.

Example 4.9. If we consider the topologies τ_1 as in the example 4.2 and τ_3={$\widetilde{\phi}$, $\widetilde{E}$, (F^1,A^1)} on (U, E) . Then τ_1 is soft multi finer topology than τ_3 and τ_3 is soft multi coarser topology than τ_1 i.e. $\tau_3 \overrightarrow{\subseteq} \tau_1$.

Theorem 4.10. *Let {τ_i: i∈I} be arbitrary collection of soft multi topologies on (U,E). Then their intersection $\bigcap_{i\in I}\tau_i$ is also a soft multi topology on(U,E).*

Proof. [O_1]. Since $\widetilde{\phi}$, $\widetilde{E}\in\tau_i$, for each i∈I, hence $\widetilde{\phi}$, $\widetilde{E}\in\bigcap_{i\in I}\tau_i$.

[O_2]. Let {(F^k,A^k)| k∈K} be an arbitrary family of soft multi sets where

(F^k,A^k)$\in\bigcap_{i\in I}\tau_i$ for each k∈K. Then for each i∈I, (F^k,A^k)$\in\tau_i$ for k∈K and since for each i∈I, τ_i is an topology, therefore $\widetilde{\cup}_{k\in K}$($F^k$,$A^k$)$\in\tau_i$, for each i∈I. Hence $\widetilde{\cup}_{k\in K}$($F^k$,$A^k$)$\in\bigcap_{i\in I}\tau_i$.

[O_3]. Let (F,A) and (G,B)$\in\bigcap_{i\in I}\tau_i$, then (F,A),G,B)$\in\tau_i$, for each i∈I and since τ_i is a soft multi topology for each i∈I, therefore (F,A)$\widetilde{\cap}$(G,B)$\in\tau_i$ for each i∈I. Hence (F,A)$\widetilde{\cap}$(G,B)$\in\bigcap_{i\in I}\tau_i$.

Thus $\bigcap_{i\in I}\tau_i$ satisfies all the axioms of topology. Hence $\bigcap_{i\in I}\tau_i$ forms a topology. But union of topologies need not be a topology; we can show this with following example. □

Example 4.11. The union of two soft multi topologies may not be a soft multi topology. If we consider the example 4.2, then the sub families τ_3={$\widetilde{\phi}$, $\widetilde{E}$, (F^1,A^1)} and τ_4={$\widetilde{\phi}$, $\widetilde{E}$, (F^2,A^2)}are the soft multi topologies set on (U,E). But their union $\tau_3\cup\tau_4$={$\widetilde{\phi}$, $\widetilde{E}$, (F^1,A^1), (F^2,A^2)} =τ_2 which is not a soft multi topology on (U,E).

Definition 4.12. Let ((U, E), τ) be a soft multi topological space over (U, E). A soft multi subset (F, A) of (U, E) is called soft multi closed if its relative complement $(F, A)'$ is a member of τ.

Example 4.13. Let us consider example: 4.2, then the soft multi closed sets in ((U, E), τ_1) are $(\widetilde{\phi})'=\widetilde{E}$,$(\widetilde{E})'=\widetilde{\phi}$,

$(F^1,A^1)'$= {(e_1,({h_3, h_4}, {c_3}, {v_2})), (e_2,({h_1, h_2}, {c_2}, {v_1}))},

$(F^2,A^2)'$= {(e_1,({h_2, h_4}, {c_1}, ϕ)), (e_3,({h_1, h_3}, {c_3}, {v_1}))},

$(F^3,A^3)'$= {(e_1,({h_4}, ϕ, ϕ)), (e_2,({h_1, h_2}, {c_2}, {v_1})), (e_3,({h_1, h_3}, {c_3}, {v_1}))},

$(F^4,A^4)'$= {(e_1,({h_2, h_3, h_4}, {c_1, c_3}, {v_2})), (e_2,({h_1, h_2}, {c_2}, {v_1})), (e_3,({h_1, h_2}, {c_3}, {v_1}))},

Proposition 4.14. *Let ((U, E), τ) be a soft multi set topological space over (U, E). Then*

(1) *($\widetilde{\phi}$) and $\widetilde{E}$ are soft multi closed sets over (U, E).*

(2) *The intersection of arbitrary collection of soft multi closed sets is a soft multi closed set over (U, E).*

(3) *The union of any two soft multi closed sets is a soft multi closed set over (U, E).*

Proof. Straight forward □

5. Soft multi basis and soft multi sub basis

In this section soft multi basis and soft multi sub basis are to be introduced.

Definition 5.1. Let ((U, E), τ) be a soft multi set topological space on (U, E) and $\mathbb{B}$ be a subfamily of τ. If every element of τ can be express as the arbitrary soft multi set union of some element of $\mathbb{B}$, then $\mathbb{B}$ is called soft multi basis(in short base) for the soft multi topology τ.

Definition 5.2. A collection S$\subseteq\tau$ is called a multi soft sub basis (in short sub base)for the topology τ if the set $\mathbb{B}$(S) consisting of finite intersections of elements of S forms a multi soft basis for τ.

Example 5.3. In the example: 4.2 for the topology τ_1 the sub family $\mathbb{B}$=\{$\widetilde{\phi}$, $\widetilde{E}$, (F^1,A^1), (F^2,A^2), (F^4,A^4)\} of SMS(U, E) is a multi soft basis for the topology τ_1 and S=\{$\widetilde{\phi}$, $\widetilde{E}$, (F^1,A^1), (F^2,A^2)\} is a multi soft sub basis for the topology τ_1, since $\mathbb{B}$(S)=\{$\widetilde{\phi}$, $\widetilde{E}$, (F^1,A^1), (F^2,A^2), (F^4,A^4)\} is a multi soft basis for the topology τ_1.

Theorem 5.4. *Let ((U, E), τ) be soft multi topological space on (U, E). A subfamily $\mathbb{B}$ of τ forms a base for a topology τ if and only if*
(1) $\widetilde{E}$ $=\widetilde{\cup}$\{*(G, B): (G, B)*$\in\mathbb{B}$\}
(2) *For every (G_1, B_1), (G_2, B_2)$\in\mathbb{B}$, (G_1, B_1)$\widetilde{\cap}$(G_2, B_2) is the union of members of* $\mathbb{B}$.

Proof. Necessity: Let $\mathbb{B}$ be a base for a topology τ on (U, E).
(1) Since $\widetilde{E}\in\tau$, we have $\widetilde{E}$ $=\widetilde{\cup}$\{(G, B): (G, B)$\in\mathbb{B}$\}.
(2) If (G_1, B_1), (G_2, B_2)$\in\mathbb{B}$, then (G_1, B_1), (G_2, B_2)$\in\tau$, since $\mathbb{B}$ subfamily of τ and since τ is a topology on (U, E), therefore (G_1, B_1)$\widetilde{\cap}$(G_2, B_2)$\in\tau$ and thus (G_1, B_1)$\widetilde{\cap}$(G_2, B_2) is the union of members of $\mathbb{B}$.

Sufficiency: Let $\mathbb{B}$ be a family with the given properties and let τ be the family of all unions of members of $\mathbb{B}$. Now if we can prove that τ is a topology on (U, E), then it is obvious that $\mathbb{B}$ is a base for this topology.

[O_1]. $\widetilde{\phi}$, $\widetilde{E}\in\tau$, since $\widetilde{\phi}\in\tau$ is the union of empty sub collection from $\mathbb{B}$ (i.e. $\widetilde{\phi}$ $=\widetilde{\cup}$\{(G, B): (G, B)$\in\widetilde{\phi}\widetilde{\subseteq}\mathbb{B}$\}) and $\widetilde{E}\in\tau$, by condition (1), $\widetilde{E}$ $=\widetilde{\cup}$\{(G, B): (G, B)$\in\mathbb{B}$\}.

[O_2]. Let (F^k,A^k)$\in\tau$ for all k. By definition of τ, each (F^k,A^k)= $\widetilde{\cup}$\{(G, B): (G, B)$\in\mathbb{B}$\}, hence $\widetilde{\cup}_k$(F^k,A^k)= $\widetilde{\cup}_k$($\widetilde{\cup}$\{(G, B): (G, B)$\in\mathbb{B}$\}) is also the union of members of $\mathbb{B}$ and so belongs to τ. Thus τ satisfies [O_2].

[O_3]. Let (F^1,A^1), (F^2,A^2)$\in\tau$. By definition of τ, (F^1,A^1)=$\widetilde{\cup}$\{(G, B): (G, B)$\in\mathbb{B}$\} and (F^2,A^2)= $\widetilde{\cup}$\{(H, C): (H, C)$\in\mathbb{B}$\},
hence (F^1,A^1)$\widetilde{\cap}$(F^2,A^2)= ($\widetilde{\cup}$\{(G, B): (G, B)$\in\mathbb{B}$\})$\widetilde{\cap}$($\widetilde{\cup}$\{(H, C): (H, C)$\in\mathbb{B}$\}) = $\widetilde{\cup}$\{(G, B)$\widetilde{\cap}$(H, C): (G, B), (H, C)$\in\mathbb{B}$\}).

Condition (2) implies that (F^1,A^1)$\widetilde{\cap}$(F^2,A^2) is the expressible as the union of the member of $\mathbb{B}$ and hence is a member of τ.

The topology τ obtained as above, forms a base is called the topology generated by the base $\mathbb{B}$. Since the base, defined as above is a subfamily of τ, i.e. members of base are open, it is called an open base. □

6. Neighbourhoods and neighbourhood systems

We introduce neighbourhoods and neighbourhood systems in a soft multi topological space.

Definition 6.1. Let τ be the soft multi topology on (U , E) . A soft multi set (F , A) in SMS(U , E) is a neighbourhood of a soft multi set (G, B) if and only if there exists an τ-open soft multi set (H ,C)i.e. (H,C)$\in\tau$ such that (G,B)$\widetilde{\subseteq}$(H,C)$\widetilde{\subseteq}$(F,A).

Example 6.2. Let us consider there are three universes U_1, U_2 and U_3. Let U_1={h_1, h_2, h_3, h_4}, U_2={c_1, c_2, c_3} and U_3={v_1, v_2}. Let {E_{U_1}, E_{U_2}, E_{U_3}} be a collection of sets of decision parameters related to the above universes, where

E_{U_1}={$e_{U_1,1}$=expensive, $e_{U_1,2}$=cheap, $e_{U_1,3}$=wooden, $e_{U_1,4}$= in green surroundings},

E_{U_2}={$e_{U_2,1}$=expensive, $e_{U_2,2}$=cheap, $e_{U_2,3}$=sporty},

E_{U_3}={$e_{U_3,1}$=expensive, $e_{U_3,2}$=cheap, $e_{U_3,3}$=in Kuala Lumpur, $e_{U_3,4}$= majestic},

Let U=$\Pi_{i=1}^{3}$P(U_i), E=$\Pi_{i=1}^{3}E_{U_i}$ and let

A={e_1=($e_{U_1,1}$, $e_{U_2,1}$, $e_{U_3,1}$), e_2=($e_{U_1,1}$, $e_{U_2,2}$, $e_{U_3,1}$), e_3=($e_{U_1,2}$, $e_{U_2,3}$, $e_{U_3,1}$), e_4=($e_{U_1,2}$, $e_{U_2,3}$, $e_{U_3,2}$)},

B={e_1=($e_{U_1,1}$, $e_{U_2,1}$, $e_{U_3,1}$),
e_2=($e_{U_1,1}$, $e_{U_2,2}$, $e_{U_3,1}$)},

C={e_1=($e_{U_1,1}$, $e_{U_2,1}$, $e_{U_3,1}$),
e_2=($e_{U_1,1}$, $e_{U_2,2}$, $e_{U_3,1}$),
e_3=($e_{U_1,2}$, $e_{U_2,3}$, $e_{U_3,1}$)},

In a soft multi topology

τ={$\widetilde{\phi}$, $\widetilde{E}$, {(e_1,({h_1, h_2}, {c_1, c_2}, {v_1})), (e_2,({h_3, h_4}, {c_1, c_3}, {v_2})), (e_3,({h_1, h_2, h_3}, {c_2, c_3}, {v_2}))}}

the soft multi set

(F,A)={(e_1,({h_1, h_2, h_3}, {c_1, c_2}, {v_1})), (e_2,({h_2, h_3, h_4}, {c_1, c_3}, {v_1, v_2})), (e_3,({h_1, h_2, h_3}, {c_2, c_3}, {v_2})), (e_4,({h_1}, {c_2}, {v_2}))}

is a neighbourhood of the soft multi set

(G,B)= {(e_1,({h_1}, {c_2}, {v_1})), (e_2,({h_4}, {c_1, c_3}, {v_2}))},

because there exists an τ - open soft multi set

(H,C)={(e_1,({h_1, h_2}, {c_1, c_2}, {v_1})), (e_2,({h_3, h_4}, {c_1, c_3}, {v_2})), (e_3,({h_1, h_2, h_3}, {c_2, c_3}, {v_2}))}$\in\tau$ such that (G,B)$\widetilde{\subseteq}$(H,C)$\widetilde{\subseteq}$(F,A)

Theorem 6.3. *A soft multi set (F, A) in SMS(U, E) is a soft multi open set if and only if (F, A) is a neighbourhood of each soft multi set (G, B) contained in (F, A).*

Proof. Straight forward. □

Definition 6.4. Let ((U,E),τ) be a soft multi topological space on (U, E) and (F, A) be a soft multi set in SMS(U, E). The family of all neighbourhoods of (F, A) is called the neighbourhood system of (F, A) up to topology and is denoted by $N_{(F,A)}$.

Theorem 6.5. *Let ((U,E),τ) be a soft multi set topological space. If $N_{(F,A)}$ is the neighbourhood system of a soft multi set (F,A). Then,*

1 *$N_{(F,A)}$ is non empty and (F,A) is soft multi subset of the each member of $N_{(F,A)}$.*

2 *The intersection of any two members of $N_{(F,A)}$ belong to $N_{(F,A)}$.*

3 *Each soft multi set which contains a member of $N_{(F,A)}$ belong to $N_{(F,A)}$.*

Proof. Straight forward. □

Definition 6.6. Let ((U,E),τ) be a soft multi topological space on (U , E) and (F, A) be a soft multi set in SMS(U, E) . A collection $B_{(F,A)}\subseteq$SMS(U, E) subsets of all containing the neighbourhood of (F, A) is called a neighbourhood basis of (F, A) if

(1) Every element of $B_{(F,A)}$ is a neighbourhood of (F, A);

(2) Every neighbourhood of (F, A) contains an element of $B_{(F,A)}$ as a subset.

7. Interior and closure

Definition 7.1. Let ((U,E),τ) be a soft multi topological space on (U , E) and (F, A) be a soft multi set in SMS(U, E). Then the union of all soft multi open sets contained in (F, A) is called the interior of (F, A) and is denoted by int(F, A) and defined by

int(F, A)=$\widetilde{\cup}${(G,B)| (G,B) is a soft multi open set contained in (F,A)}

Example 7.2. Let us consider the soft multi topology τ_1 as in the example 4.2 and let

(F,A)={(e_1,({h_1, h_2, h_3}, {c_1, c_2}, {v_1})), (e_2,({h_1, h_3, h_4}, {c_1, c_3}, {v_2}))} be a soft multi set, then int(F,A)=$\widetilde{\cup}${(G,B)| (G,B) is a soft multi open set contained in (F,A)}

=(F^1,A^1)$\widetilde{\cup}$(F^4,A^4) ={(e_1,({h_1, h_2}, {c_1, c_2}, {v_1})), (e_2,({h_3, h_4}, {c_1, c_3}, {v_2})),(e_3,({h_2, h_4}, {c_1, c_2}, {v_2}))}, Since (F^1,A^1) and (F^4,A^4) are two soft multi open sets contained in (F,A).

Theorem 7.3. *Let ((U,E),τ) be a soft multi set topological space on (U,E) and (F, A) be a soft multi set in SMS(U,E). Then*

1 *int(F, A) is an open and int(F, A) is the largest open soft multi set contained in (F, A).*

2 *The soft multi set (F, A) is open if and only if (F, A) = int(F, A).*

Proof. Straight forward. □

Proposition 7.4. *For any two soft multi sets (F, A) and (G, B) in a soft multi topological space ((U,E),τ) on (U,E),*

(i) *(G, B)$\widetilde{\subseteq}$(F, A)$\Rightarrow$ int(G, B) $\widetilde{\subseteq}$ int(F, A)*

(ii) *int$\widetilde{\phi}$=$\widetilde{\phi}$ and int$\widetilde{E}$=$\widetilde{E}$*

(iii) *int(int(F, A))= int(F, A)*

(iv) *int(F, A)$\widetilde{\cap}$int(G, B)$\widetilde{\subseteq}$ int((F, A)$\widetilde{\cap}$(G, B))*

(v) *int(F, A)$\widetilde{\cup}$int(G, B)$\widetilde{\subseteq}$ int((F, A)$\widetilde{\cup}$(G, B))*

Proof. Straight forward. □

Definition 7.5. Let ((U,E),τ) be a soft multi topological space on (U, E) and (F, A) be a soft multi set in SMS(U, E). Then the intersection of all soft multi closed set containing (F, A) is called the closure of (F, A) and is denoted by cl(F, A) and defined by

cl(F, A) =$\widetilde{\cap}$\{(G,B)|(G,B) is a soft multi closed set containing (F, A)\}

Observe first that cl(F, A) is a soft multi closed set, since it is the intersection of soft multi closed sets. Furthermore, cl(F, A) is the smallest soft multi closed set containing (F, A).

Example 7.6. Let us consider the soft multi topology τ_1 as in the example 4.2 and let

(F,A)=\{(e_1,(\{h_4\}, \{c_3\}, \{v_2\})), (e_2,(\{h_2\}, \{c_2\}, \{v_1\}))\} be a soft multi set, then cl(F,A)=$\widetilde{\cap}$\{(G,B)| (G,B) is a soft multi closed set containing (F,A)\}

=$(F^1,A^1)'\widetilde{\cap}(F^4,A^4)'$ =\{(e_1,(\{h_3, h_4\}, \{c_3\}, \{v_2\})), (e_2,(\{h_1, h_2\}, \{c_2\}, \{v_1\})),(e_3, (\{h_1, h_2\}, \{c_3\}, \{v_1\}))\}, Since $(F^1,A^1)'$ and $(F^4,A^4)'$ are two soft multi closed sets contained in (F,A).

Proposition 7.7. *For any two soft multi sets (F, A) and (G, B) in a soft multi set topological space ((U,E),τ) on (U,E),*

(i) *cl$\widetilde{\phi}$=$\widetilde{\phi}$ and cl$\widetilde{E}$= $\widetilde{E}$*
(ii) *(F, A))$\widetilde{\subseteq}$ cl(F, A)*
(iii) *(F, A) is a soft multi closed set if and only if (F, A)=cl(F, A).*
(iv) *cl(cl(F, A))=cl(F, A)*
(v) *(G, B)$\subseteq$(F, A)$\Rightarrow$cl(G, B)$\subseteq$cl(F, A)*
(vi) *cl((F, A)$\widetilde{\cap}$(G, B))$\widetilde{\subseteq}$ cl(F, A)$\widetilde{\cap}$cl(G, B)*
(vii) *cl((F, A)$\widetilde{\cup}$(G, B))= cl(F, A)$\widetilde{\cup}$cl(G, B)*

Proof. Straight forward. □

Theorem 7.8. *Let ((U,E),τ) be a soft multi set topological space on (U,E) and let (F , A) be a soft multi set in SMS(U,E). Then*

(i) *$(cl(F, A))' = int((F, A)')$*
(ii) *$(int(F, A))' = cl((F, A)')$*

Proof. Straight forward. □

Definition 7.9. Let ((U,E),τ) be a soft multi topological space on (U, E) and (G, B) be a soft multi set in SMS(U, E). The soft multi point $e_{(F,A)}$ in SMS(U, E) is called a soft multi interior point of a soft multi set (G, B) if there exists a soft multi open set (H ,C), such that $e_{(F,A)}$ $\widetilde{\in}$ (H, C)$\widetilde{\subseteq}$(G, B).

Example 7.10. Let us consider the soft multi topology τ_1 as in the example: 4.2 and let

(F,A)=\{(e_1,(\{h_1, h_2, h_3\}, \{c_1, c_2\}, \{v_1\})), (e_2,(\{h_1, h_3, h_4\}, \{c_1, c_3\}, \{v_2\}))\}

be a soft multi set, then $e_{1_{(F,A)}}$=\{(e_1,(\{h_1\}, \{c_1\}, \{v_1\}))\} is a soft multi interior point of the soft multi set (F , A), since there exist a soft multi open set $(F^1,A^1)\in\tau_1$, such that $e_{1_{(F,A)}}\widetilde{\in}(F^1,A^1)\widetilde{\subseteq}$(F,A).

But $e_{2_{(F,A)}}$={(e_2,({h_1,h_2, h_3} ϕ, ϕ))} is not a soft multi interior point of the soft multi set (F, A), since there does not exists an soft multi open set (H, C), such that $e_{2_{(F,A)}} \widetilde{\in}$ (H, C)$\widetilde{\subseteq}$(F, A).

Proposition 7.11. *Let ((U,E),τ) be a soft multi topological space on (U, E) and (G, B) be a soft multi open set in SMS(U, E). Then every soft multi point $e_{(F,A)} \widetilde{\in}$ (G, B) is a soft multi interior point.*

Proof. The proof is straight forward. □

Definition 7.12. Let ((U,E),τ) be a soft multi topological space on (U, E) and (F, A) be a soft multi set in SMS(U, E). Then we defined a soft multi set associate with (F, A) over (U, E) is denoted by (cl(F),A) and defined by cl(F)(e)=cl(F(e)), where cl(F(e)) is the closer of F(e) in τ_e for each e∈A.

Proposition 7.13. *Let ((U,E),τ) be a soft multi topological space on (U, E) and (F, A) be a soft multi set in SMS(U, E). Then (cl(F),A)$\widetilde{\subseteq}$cl(F, A).*

Proof. For any e∈A, cl(F(e)) is the smallest closed set in (U, τ_e), which contains F(e). Moreover if cl(F ,A)=(G, B), then G(e) is also closed set in (U, τ_e) containing F(e). This implies that cl(F)(e)=cl(F(e))⊆ G(e). Thus (cl(F),A) $\widetilde{\subseteq}$cl(F, A). □

Corollary 7.14. *Let ((U,E),τ) be a soft multi topological space on (U, E) and (F, A) be a soft multi set in SMS(U, E). Then (cl(F),A)= cl(F ,A)if and only if (cl(F),A)$'$∈τ.*

Proof. If (cl(F),A)= cl(F ,A), then (cl(F),A)= cl(F ,A) is a soft multi closed set and so (cl(F),A)$'$∈τ.

Conversely if (cl(F),A)$'$∈τ then (cl(F),A) is a soft multi closed set containing (F, A). By proposition 7.13 (cl(F),A)$\widetilde{\subseteq}$cl(F ,A) and by the definition of soft multi closure of (F, A), any soft multi closed set over which contains (F, A) will contain cl(F, A). This implies that cl(F ,A)$\widetilde{\subseteq}$(cl(F),A). Thus (cl(F),A)=cl(F ,A). □

8. Soft multi subspace topology

In this section we introduce soft multi subspace topology.

Theorem 8.1. *Let ((U,E),τ) be a soft multi topological space on (U, E) and (F , A) be a soft multi set in SMS(U, E) . Then the collection $\tau_{(F,A)}$= {(F, A)$\widetilde{\cap}$(G, B)|(G, B)∈τ} is a soft multi topology on the soft multi set (F , A).*

Proof. [O_1]. Since $\widetilde{\phi}$, $\widetilde{E}$∈τ, therefore (F, A)=(F, A)$\widetilde{\cap}\widetilde{E}$
and $\widetilde{\phi}_{(F,A)}$=(F, A)$\widetilde{\cap}\widetilde{\phi}$, therefore $\widetilde{\phi}_{(F,A)}$, (F, A)∈$\tau_{(F,A)}$.

[O_2]. Let {(F^k,A^k)| k∈K} be an arbitrary family of soft multi open sets in $\tau_{(F,A)}$, then for each k∈K, there exist (G^k,B^k)∈τ such that (F^k,A^k)=(F,A)$\widetilde{\cap}$(G^k,B^k)

Now $\widetilde{\cup}_{k\in K}$($F^k$,$A^k$)=$\widetilde{\cup}_{k\in K}$((F,A)$\widetilde{\cap}$($G^k$,$B^k$)) =(F,A)$\widetilde{\cap}$($\widetilde{\cup}_{k\in K}$($G^k$,$B^k$)) and since $\widetilde{\cup}_{k\in K}$($G^k$,$B^k$)∈$\tau$⇒$\widetilde{\cup}_{k\in K}$($G^k$,$B^k$)∈$\tau_{(F,A)}$.

[O_3]. Let (F^1,A^1)and(F^2,A^2)are the two soft multi open sets in $\tau_{(F,A)}$, then for each i=1, 2, there exist(G^i,B^i))∈τ such that(F^i,A^i)=(F,A)$\widetilde{\cap}$(G^i,B^i)).

Now(F^1,A^1)$\widetilde{\cap}$(F^2,A^2)=((F,A)$\widetilde{\cap}$(G^1,B^1))$\widetilde{\cap}$((F,A)$\widetilde{\cap}$(G^2,B^2))

=(F,A)$\widetilde{\cap}$((G^1,B^1)$\widetilde{\cap}$(G^2,B^2))and since (G^1,B^1)$\widetilde{\cap}$(G^2,B^2)$\in\tau$,hence
(F^1,A^1)$\widetilde{\cap}$(F^2,A^2)$\in\tau_{(F,A)}$
Thus $\tau_{(F,A)}$ is a soft multi topology on the soft multi set (F,A). □

Definition 8.2. Let ((U,E),τ) be an soft multi topological space on (U, E) and (F,A) be an soft multi set in SMS(U, E). Then the soft multi topology $\tau_{(F,A)}$= {(F,A)$\widetilde{\cap}$(G,B)|(G,B)$\in\tau$} is called soft multi subspace topology and ((F,A),$\tau_{(F,A)}$) is called soft multi topological subspace of ((U,E),τ).

Example 8.3. Let us consider the soft multi topological space((U,E),τ_1) given in the example 4.2 and let a soft multi set be

(F,A)={(e$_1$,({h$_1$, h$_4$}, {c$_1$, c$_3$}, {v$_1$})), (e$_4$,({h$_2$, h$_3$, h$_4$}, {c$_1$, c$_3$}, {v$_1$, v$_2$}))}

where A={e$_1$=($e_{U_1,1}$, $e_{U_2,1}$, $e_{U_3,1}$), e$_4$=($e_{U_1,1}$, $e_{U_2,2}$, $e_{U_3,3}$)},then

(G^1,B^1)=(F,A)$\widetilde{\cap}$(F^1,A^1)= {(e$_1$,({h$_1$}, {c$_1$}, {v$_1$})), (e$_2$,({h$_3$, h$_4$}, {c$_1$, c$_3$}, {v$_2$})), (e$_4$,({h$_2$, h$_3$, h$_4$}, {c$_1$, c$_3$}, {v$_1$, v$_2$}))},

(G^2,B^2)=(F,A)$\widetilde{\cap}$(F^2,A^2)= {(e$_1$,({h$_1$}, {c$_3$}, {v$_1$})), (e$_3$,({h$_2$, h$_4$}, {c$_1$, c$_2$}, {v$_2$})), (e$_4$,({h$_2$, h$_3$, h$_4$}, {c$_1$, c$_3$}, {v$_1$, v$_2$}))},

(G^3,B^3)=(F,A)$\widetilde{\cap}$(F^3,A^3)= {(e$_1$,({h$_1$}, {c$_1$, c$_3$}, {v$_1$})), (e$_2$,({h$_3$, h$_4$}, {c$_1$, c$_3$}, {v$_2$})), (e$_3$,({h$_2$, h$_4$}, {c$_1$, c$_2$}, {v$_2$})), (e$_4$,({h$_2$, h$_3$, h$_4$}, {c$_1$, c$_3$}, {v$_1$, v$_2$}))},

(G^4,B^4)=(F,A)$\widetilde{\cap}$(F^4,A^4)= {(e$_1$,({h$_1$}, ϕ, {v$_1$})), (e$_2$,({h$_3$, h$_4$}, {c$_1$, c$_3$}, {v$_2$})), (e$_3$,({h$_2$, h$_4$}, {c$_1$, c$_2$}, {v$_2$})), (e$_4$,({h$_2$, h$_3$, h$_4$}, {c$_1$, c$_3$}, {v$_1$, v$_2$}))},

Then $\tau_{1_{(F,A)}}$= {$\widetilde{\phi}_{(F,A)}$, (F, A), (G^1,B^1), (G^2,B^2), (G^3,B^3), (G^4,B^4)},

where

B^1=A∪A^1={e$_1$=($e_{U_1,1}$, $e_{U_2,1}$, $e_{U_3,1}$), e$_2$=($e_{U_1,1}$, $e_{U_2,2}$, $e_{U_3,1}$), e$_4$=($e_{U_1,1}$, $e_{U_2,2}$, $e_{U_3,3}$)},

B^2=A∪A^2={e$_1$=($e_{U_1,1}$, $e_{U_2,1}$, $e_{U_3,1}$), e$_3$=($e_{U_1,2}$, $e_{U_2,3}$, $e_{U_3,1}$), e$_4$=($e_{U_1,1}$, $e_{U_2,2}$, $e_{U_3,3}$)}

B^3=A∪A^3=B^4=A∪A^4={e$_1$=($e_{U_1,1}$, $e_{U_2,1}$, $e_{U_3,1}$), e$_2$=($e_{U_1,1}$, $e_{U_2,2}$, $e_{U_3,1}$), e$_3$= ($e_{U_1,2}$, $e_{U_2,3}$, $e_{U_3,1}$), e$_4$=($e_{U_1,1}$, $e_{U_2,2}$, $e_{U_3,3}$)}

is a soft multi subspace topology for τ_1 and ((F,A),$\tau_{1_{(F,A)}}$) is called a soft multi subspace of ((U,E),τ_1).

Theorem 8.4. *Let ((U,E),τ) be a soft multi topological space on (U , E) and let B be a soft multi basis for τ and (F, A) be a soft multi set in SMS(U, E) . Then the family* $B_{(F,A)}$*={(F, A)$\widetilde{\cap}$(G, B)|(G, B)$\in$B} is a soft multi basis for soft multi subspace topology* $\tau_{(F,A)}$.

Proof. Let (H, D)$\in\tau_{(F,A)}$, then there exists a soft multi set (G, B)$\in\tau$, such that (H, D)=(F, A)$\widetilde{\cap}$(G, B). Since B is a base for τ, so there exists sub collection {(G^i,B^i)| i∈I} of B such that

(G, B)=$\widetilde{\cup}_{i\in I}$(G^i,B^i).Therefore (H, D)=(F, A)$\widetilde{\cap}$(G, B)= (F,A)$\widetilde{\cap}$($\widetilde{\cup}_{i\in I}$(G^i,B^i))= $\widetilde{\cup}_{i\in I}$((F,A)$\widetilde{\cap}$(G^i,B^i)). Since (F,A)$\widetilde{\cap}(G^i,B^i)\in B_{(F,A)}$, which implies that $B_{(F,A)}$ is a soft multi basis for the soft multi subspace topology $\tau_{(F,A)}$. □

Theorem 8.5. *Let ((U,E),τ) be a soft multi topological space on (U, E) and ((F, A),τ^*) be a soft multi topological subspace of ((U,E),τ) and ((G, B),τ^{**}) be a soft multi topological subspace of ((F, A),τ^*). Then ((G, B),τ^{**}) is also a soft multi topological subspace of ((U,E),τ).*

Proof. Straight forward. □

9. Soft multi compact spaces

In this section we define soft multi cover and soft multi compact space.

Definition 9.1. Let ((U,E),τ) be a soft multi topological space on (U, E) and let (F, A) be any soft multi set in SMS(U, E). Then a sub family Ω of SMS(U, E) is called a soft multi cover for (F, A) if and only if (F , A)$\widetilde{\subseteq}$ { (G, B): (G, B)$\in\Omega$} and we say that Ω covers (F, A).

Definition 9.2. If a sub collection of soft multi cover Ω also covers âv F , A âw , then it is called an soft multi sub cover of Ω for âv F , A âw .

Definition 9.3. If the members of soft multi cover Ω are open, then $\Omega$is called soft multi open cover.

Definition 9.4. If the members of soft multi cover Ω are finite in number, then it is called finite soft multi cover.

Definition 9.5. Let ((U,E),τ) be a soft multi topological space on (U , E) . A soft multi set âv F , A âw in SMS (U , E) is called soft multi compact set if and only if every soft multi open covers of âv F , A âw has a finite soft multi sub cover.

Definition 9.6. A soft multi topological space ((U,E),τ) is called soft multi compact space if and only if E is soft multi compact.

Definition 9.7. Let ((U,E),τ) be a soft multi topological space on (U , E) . A sub family Ω of SMS (U , E) has the finite intersection property if and only if the interaction of any finite sub collection of Ω is not null soft multi set.

Theorem 9.8. *A soft multi topological space ((U,E),τ) is soft multi compact space if and only if for every family of soft multi closed subsets with finite intersection property has non null intersection.*

Proof. Let ((U,E),τ) be a soft multi compact space and let {(F^k,A^k)| k$\in$K} be an arbitrary family of soft multi closed sets in τ with finite intersection property.

If possible let $\widetilde{\cap}_{k\in K}$($F^k$,$A^k$)=$\widetilde{\phi}$, then by taking complements, $(\widetilde{\cap}_{k\in K}(F^k,A^k))'=(\widetilde{\phi})'$ i.e. $\widetilde{\cup}_{k\in K}(F^k,A^k)'=\widetilde{E}$. So that $\{(F^k,A^k)'|$ k$\in$K} forms a soft multi open cover for $\widetilde{E}$. Since $\widetilde{E}$ is compact, there is a finite soft multi sub cover $\{(F^i,A^i)'|$ i=1,2,3,....,n}, such that $\widetilde{E}=\widetilde{\cup}_{i=1}^{n}(F^i,A^i)'$. Then by taking complements,

$(\widetilde{E})'=(\widetilde{\cup}_{i=1}^{n}(F^i,A^i)')'$, i.e. $\widetilde{\phi}=\widetilde{\cap}_{i=1}^{n}(F^i,A^i)$.

Thus {(F^k,A^k)| k$\in$K} does not have the finite intersection property, which contrary to our assumption. Hence $\widetilde{\cap}_{k\in K}(F^k,A^k)\neq\widetilde{\phi}$.

Conversely let every family of soft multi closed subsets in ((U,E),τ) with finite intersection property has non null intersection in ((U,E),τ). Now suppose that ((U,E),τ) is not soft multi compact space. Then there is a soft multi open cover $\{(G^k,B^k)'|$ k$\in$K} of $\widetilde{E}$ that has no finite soft multi sub cover, i.e. $\widetilde{E}\neq\widetilde{\cup}_{i=1}^{n}(G^i,B^i)$, then by taking complements, $(\widetilde{E})'\neq(\widetilde{\cup}_{i=1}^{n}(G^i,B^i))'$, i.e. $\widetilde{\phi}\neq\widetilde{\cap}_{i=1}^{n}(G^i,B^i)'$ which implies $\{(G^k,B^k)'|$ k$\in$K} has the finite intersection property. But by soft multi cover property $\widetilde{E}=\widetilde{\cup}_{k\in K}(G^k,B^k)$, then by taking complements, $\widetilde{\cap}_{k\in K}(G^k,B^k)'=\widetilde{\phi}$, i.e. the intersection of all members of the family of soft multi closed sets is null soft multi set, which contradicting the given condition. Hence ((U,E),τ) is soft multi compact space. □

Theorem 9.9. *Let ((U,E),τ) be a soft multi compact space and let (F , A) be a soft multi closed sets in τ. Then the soft multi closed subspace ((F, A),$\tau_{(F,A)}$) of ((U,E),τ) is soft multi compact space.*

Proof. Let ((U,E),τ) be a soft multi compact space and let (F, A) be a soft multi closed sets in τ. Let {(F^k,A^k)| k∈K} be an arbitrary family of soft multi closed sets in ((F, A),$\tau_{(F,A)}$) with finite intersection property. Then soft multi sets (F^k,A^k) for each k∈K are soft multi closed sets in ((U,E),τ); since (F, A) is a soft multi closed sets in τ. Thus {(F^k,A^k)| k∈K} is a family of soft multi closed sets in ((U,E),τ), possessing finite intersection property and as ((U,E),τ) is soft multi set compact, it follows that $\widetilde{\cap}_{k\in K}(F^k,A^k)\neq\widetilde{\phi}$ (by theorem 9.8). This implies that the soft multi closed subspace ((F, A),$\tau_{(F,A)}$) of ((U,E),τ) is soft multi compact space. □

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