Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 7, No. 6, (June 2014), pp. 867–877
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Soft proximity

H. Hazra, P. Majumdar, S. K. Samanta

Received 17 January 2013; Revised 6 April 2013; Accepted 5 May 2013

Abstract. In this paper the notion of soft proximity is introduced and its properties are studied. The notion of Lodato soft proximities is also defined. Topologies of soft sets are obtained using Kuratowski closure operators of soft sets. The notion of soft proximal continuity and soft proximal neighbourhood are also introduced.

2010 AMS Classification: 54A40, 03E72

Keywords: Soft set, Basic soft proximity, Lodato soft proximity, Čech closure operator of soft sets, Kuratowski closure operator of soft sets, Topology of soft sets, Soft proximal continuity, Soft proximal neighbourhood.

Corresponding Author: S. K. Samanta (syamal_123@yahoo.co.in)

1. Introduction

After the publication of the research paper on 'Fuzzy Sets' in 1965 by Lotfi A. Zadeh, a paradigmatic change has occurred in understanding the concept of uncertainty. Before the invention of fuzzy sets, probability theory was the only tool for modeling uncertainty. But this traditional tool of mathematics can not handle all different types of uncertainties present in our daily existence, especially those which results from imprecise natural languages. Whenever we say that 'today is very cold and you should wear adequate warm clothing', a person understands well and can take necessary measures but a machine can not. Here the words 'very cold' are not precise or in other words 'vague'. Vagueness is another kind of uncertainty which is present in our daily language but is different from 'randomness' and hence can not be modeled using probability. Hence the need for a fundamentally different approach to study uncertainty present in physical process motivated the development in this area of Mathematics. But Fuzzy sets are not the only tool available for modeling uncertainty. There are also other theories namely, intuitionistic fuzzy sets, rough sets, vague sets etc, which are available and are quite useful in their domain of applications. But still these theories have certain limitations. Therefore research is still going for finding better theories which can model the natural phenomena more realistically. The problem with the fuzzy set is that it lacks parameterization of tools. In 1999 Molodtsov [27] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which traditional mathematical tools can not handle. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Later other authors like Maji, Roy and Biswas [23] have further studied the theory of soft sets and used this theory to solve some decision making problems. They have also introduced the concept of fuzzy soft set and intuitionistic fuzzy soft set[24, 21, 20, 22] a more generalized concept, which is a combination of fuzzy set and soft set and studied its properties. In 2009, Ali et al [2] has defined some new operations on soft sets. It also is interesting to see that soft sets are closely related to many other soft computing models such as rough sets and fuzzy sets. Feng et al. [11] first considered the combination of soft sets, fuzzy sets and rough sets. Using soft sets as the granulation structures, Feng et al. [12] initiated soft approximation spaces and soft rough sets, which extended Pawlak's rough set model using soft sets. In some cases Feng's soft rough set model could provide better approximations than classical rough sets. Research in soft set theory (SST) has been done in many areas like algebra, topology, applications etc. In 2007, Aktas & Cagman [1] have introduced a notion of soft group. The idea of soft semirings has been introduced by Feng et al [10]. Jun [16, 15] investigated soft BCK/BCI-Algebras and its applications. Ali et al.[2] and in 2009, Shabir & Irfan Ali [32] studied soft semigroups and soft ideals and idealistic soft semirings. Das and Samanta [8, 9] introduced the notions of soft real and complex numbers and opened up the scope of studying soft real and complex analysis. Authors like Kharal & Ahmed [17]and Majumdar & Samanta [25] has introduced different notions of mappings on soft sets. Several authors like Shabir & Naz [31], Hazra, Majumdar & Samanta [14] have studied the notion of soft topological spaces. Also Aygunoglu & Aygun [3] have studied soft product topologies and soft compactness. The notion of fuzzy soft topologies has been also studied by few authors [29, 33, 30]. Kong et al [18, 19] have applied the soft set theoretic approach in decision making problems. Recently, Feng et al. [13] ascertained the relationships among five different types of soft subsets and considered the free soft algebras associated with soft product operations. It has been shown that soft sets have some non-classical algebraic properties which are distinct from those of crisp sets or fuzzy sets. On the other hand proximities have been studied by several authors in crisp sense as well as in fuzzy sense. In [7], Chattopadhyay, Hazra and Samanta introduced basic fuzzy proximities and Lodato fuzzy proximities and investigated some properties concerning them and eventually proved that there is a bijection between a class of Lodato fuzzy proximities compatible with a given strongly T_1- topological space of fuzzy sets (X, c) and the class of strongly T_1 principal Type-II fuzzy linkage compactifications of (X, c).

Here we have introduced the notion of proximity on soft sets. The rest of the paper is constructed as follows: In Section 2, some preliminary definitions and results regarding soft sets, soft topology and crisp proximity are given which will be used in the rest of the paper. In Section 3 the notion of proximities of soft sets is introduced and some of their important properties are studied. Section 4 concludes the paper.

2. Preliminaries

In this section some definitions, results and examples regarding soft sets are given which will be used in the rest of this paper. The idea of soft sets was first given by Molodtsov. Later Maji & Roy have defined operations on soft sets and studied their properties.

Definition 2.1 ([27]). Let U be an initial universal set and let E be a set of parameters. Let $P(U)$ denote the power set of U. Let A be a subset of E. A pair (F, A) is called a *soft set* over U if F is a mapping given by $F : A \to P(U)$.

Definition 2.2 ([24]). For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a *soft subset* of (G, B) if (i) $A \subset B$, (ii) $\forall \epsilon \in A, F(\epsilon) \subset G(\epsilon)$.

Definition 2.3 ([24]). Two soft sets (F, A) and (G, B) over a common universe U are said to be *soft equal* if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) .

In 2008, Majumdar & Samanta have given a new definition of complement of soft sets as follows:

Definition 2.4 ([26]). The *complement of a soft set* (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\alpha), \forall \alpha \in A$.

Definition 2.5 ([24]). A soft set (F, A) over U is said to be *null soft set* denoted by Φ if $\forall \epsilon \in A, F(\epsilon) = \phi$.

Definition 2.6 ([24]). A soft set (F, A) over U is said to be *absolute soft set* denoted by $\tilde{A}$ if , if $\forall \epsilon \in A, F(\epsilon) = U$.

Definition 2.7 ([24]). The *union of two soft sets* (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$ and $\forall e \in C$,

$$
\begin{aligned}
H(e) &= F(e) \text{ if } e \in A - B \\
&= G(e) \text{ if } e \in B - A \\
&= F(e) \cup G(e) \text{ if } e \in A \cap B.
\end{aligned}
$$

We write $(F, A)\tilde{\cup}(G, B)$.

Definition 2.8 ([24]). The *intersection of two soft sets* (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cap B$ and

$$\forall e \in C,\ H(e) = F(e) \cap G(e).$$

We write $(F, A)\tilde{\cap}(G, B)$.

Definition 2.9 ([17]). Let $\tilde{f} : U_1 \to U_2$ and $\hat{f} : E_1 \to E_2$ be two mappings. Then the pair $f = (\tilde{f}, \hat{f})$ is said to be a *soft mapping* from $P(U_1)^{E_1}$ to $P(U_2)^{E_2}$ and the *image* $f(F)$ of any $F \in P(U_1)^{E_1}$ is defined as:

$$
\begin{aligned}
f(F)(e') &= \tilde{f}\Big(\bigcup_{e \in \hat{f}^{-1}(e')} F(e)\Big) \text{ if } \hat{f}^{-1}(e') \neq \phi \\
&= \phi \text{ if } \hat{f}^{-1}(e') = \phi, \forall e' \in E_2.
\end{aligned}
$$

Definition 2.10 ([14]). Let τ be a family of soft sets over (U, E).
Define $\tau(e) = \{F(e) : F \in \tau\}$ for $e \in E$.

Then τ is said to be a *topology of soft subsets* over (U, E) if $\tau(e)$ is a crisp topology on $U, \forall e \in E$.
In this case $((U, E), \tau)$ is said to be a *topological space of soft subsets.*
If τ is a topology of soft subsets over (U, E) , then the members of τ are called *open soft sets* and a soft set F over (U, E) is said to be *closed* if $F^c \in \tau$.

Theorem 2.11 ([14])**.** *Let Ω be the family of all closed soft sets over (U, E) , then (i)$\tilde{\Phi}, \tilde{A} \in \Omega$ (ii) $F_i \in \Omega \Rightarrow \tilde{\cap}_i F_i \in \Omega$ and (iii) $F_1, F_2 \in \Omega \Rightarrow F_1 \tilde{\cup} F_2 \in \Omega$.*

Note 2.12. The family of all open soft sets over (U, E) will form a soft topology in the sense of Shabir & Naz [31] .

Example 2.13 ([14])**.** Let $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$.
Also let $F_1 \in P(U)^E$ be defined as follows:
$F_1 = \{F_1(e_1) = \{x_1\}, F_1(e_2) = \{x_2, x_3\}\}$.
Here $\tau(e_1) = \{\phi, U, \{x_1\}\}$ and $\tau(e_2) = \{\phi, U, \{x_2, x_3\}\}$ are crisp topologies on U.
Thus $\tau = \{\tilde{\Phi}, \tilde{A}, F_1\} \subset P(P(U)^E)$ is a topology of soft subsets over (U, E).

Definition 2.14 ([14])**.** Let $\mathfrak{T}_1$ and $\mathfrak{T}_2$ be two soft topologies over (U_1, E_1) and (U_2, E_2) respectively. A soft mapping $f = (\tilde{f}, \hat{f})$ from $P(U_1)^{E_1}$ to $P(U_2)^{E_2}$ is said to be *soft continuous* if the inverse image of every $e-$open set of $\mathfrak{T}_2$ under f is $\hat{f}^{-1}(e)$-open in $\mathfrak{T}_1$ $\forall e \in E_2$.

Theorem 2.15 ([14])**.** *$f = (\tilde{f}, \hat{f})$ is soft continuous if and only if inverse of each e-closed set in $\mathfrak{T}_2$ under f is $\hat{f}^{-1}(e)$-closed set in $\mathfrak{T}_1$, $\forall e \in E_2$.*

Next we give some basic definitions regarding proximity of ordinary sets, i.e. crisp sets. One may read [28, 5] for further details.

Definition 2.16. A *basic proximity* Π on X is a binary relation on $P(X)$ satisfying the following conditions:
(i) $\Pi = \Pi^{-1}$
(ii) $\forall A, B, C \subset X, (A \cup B, C) \in \Pi \Leftrightarrow (A, C) \in \Pi$ or $(B, C) \in \Pi$
(iii) $\forall A, B \subset X, A \cap B \neq \phi \Rightarrow (A, B) \in \Pi$
(iv) $(A, \phi) \notin \Pi \; \forall A \subset X$.
If Π is a basic proximity on X then the pair (X, Π) is called a *basic proximity space.*

Definition 2.17. A basic proximity Π on X is called *separated* if for every $x, y \in X$, $(\{x\}, \{y\}) \in \Pi \Rightarrow x = y$.

Definition 2.18. A basic proximity Π on X is called *Lodato proximity* if for all $\forall A, B, C \subset X, (A, B) \in \Pi$ and $(b, C) \in \Pi \; \forall b \in B \Rightarrow (A, C) \in \Pi$.

Definition 2.19. Let X be a set and $P(X)$ be the power set of X. A mapping $c : P(X) \to P(X)$ is called a *Čech closure operator* on X if it satisfies the following conditions:
(i) $c(\phi) = \phi$
(ii) $c(A) \supset A \; \forall A \subset X$
(iii) $c(A \cup B) = c(A) \cup c(B) \; \forall A, B \subset X$.
The pair (X, c) is called a *closure space* whenever c is a Čech closure operator on X.

Note 2.20. A Čech closure operator c on X satisfying the additional condition: $c(c(A)) = c(A)\ \forall A \subset X$ is called *Kuratowski closure operator* on X. The pair (X, c), where c is a Kuratowski closure operator on X, is called a *topological space*.

Definition 2.21. A closure space (X, c) is called R_0 if for any two points x, y in X, $x \in c(\{y\})$ implies $y \in c(\{x\})$.

We now give the definitions of filters and grills. Filters were introduced by Carton [4] and grills were introduced by Choquet [6].

Definition 2.22 ([4])**.** A *filter* $\mathcal{F}$ on X is a non-empty family of subsets of X satisfying

(i) $\forall A, B \subset X$, $B \in \mathcal{F}$ and $B \subset A \Rightarrow A \in \mathcal{F}$.

(ii) $A, B \in \mathcal{F}$ implies $A \cap B \in \mathcal{F}$.

A filter $\mathcal{F}$ is called a *proper filter* if $\phi \notin \mathcal{F}$. A proper filter which is not contained in any other filter is called an *ultrafilter*.

Definition 2.23 ([6])**.** A *grill* $\mathcal{G}$ on X is a collection of subsets of X satisfying

(i) $\phi \notin \mathcal{G}$.

(ii) $\forall A, B \subset X$, $B \in \mathcal{G}$ and $B \subset A \Rightarrow A \in \mathcal{G}$.

(iii) $\forall A, B \subset X$, $A \cup B \in \mathcal{G}$ implies $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

A grill $\mathcal{G}$ is called a *proper grill* if $\mathcal{G} \neq \phi$

Definition 2.24. Let Π be a basic proximity on X. Then for each $A \subset X$ define $\Pi(A) = \{B \subset X : (A, B) \in \Pi\}$.

Theorem 2.25. *Let Π be a binary relation on $P(X)$. Then Π is a basic proximity on X if and only if*

(i) $\Pi = \Pi^{-1}$

(ii) *for each $A(\neq \phi) \subset X, \Pi(A)$ is a grill on X such that $\Pi(A) \supset \bigcup\{\omega \in \Omega(X) : A \in \omega\}$, where $\Omega(X)$ is the set of all ultrafilters on X.*

3. Soft proximity

In this section we introduce the notion of soft proximity and study its properties.

Definition 3.1. Let E be a set of parameters and X be a nonempty set and $\mathcal{A}$ be a set of basic proximities on X. Then the pair (π, E) is called a *basic soft proximity* on (X, E) if π is a mapping given by $\pi : E \to \mathcal{A}$.
The set of all basic soft proximities on (X, E) will be denoted by $M_S(X, E)$

If $(\pi, E) \in M_S(X, E)$, then $((X, E), \pi)$ is called a *basic proximity space.*

Example 3.2. Let $X = \{x_1, x_2, x_3\}, E = \{e_1, e_2\}$.
Define $\pi(e_1) = \{(F(e_1), G(e_1)) : F, G \in P(X)^E\} - \{(F_1(e_1), G_1(e_1)), (F_2(e_1), G_2(e_1))\}$, where $F_1(e_1) = \{x_1\}, G_1(e_1) = \{x_2\}, F_2(e_1) = \{x_2\}, G_2(e_1) = \{x_1\}$
and $\pi(e_2) = \{(F(e_2), G(e_2)) : F, G \in P(X)^E\} - \{(F_3(e_2), G_3(e_2)), (F_4(e_2), G_4(e_2))\}$, where $F_3(e_2) = \{x_1\}, G_3(e_2) = \{x_3\}, F_4(e_2) = \{x_3\}, G_4(e_2) = \{x_1\}$.
Clearly $\pi(e_1), \pi(e_2)$ are proximities on X (in the crisp sense). Thus (π, E) is a basic soft proximity on (X, E).

Definition 3.3. The pair $(\mathcal{G}, E)$ is said to be a *soft grill* on X if $\mathcal{G}$ is a mapping given by $\mathcal{G} : E \to \mathcal{B}$, where $\mathcal{B}$ is the set of all grills on X.

Example 3.4. Let $X = \{x_1, x_2, x_3\}, E = \{e_1, e_2\}$.
Define $\mathcal{G}(e_1) = \{\{x_1\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}, X\}$
and $\mathcal{G}(e_2) = \{\{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}, X\}$.
Clearly $\mathcal{G}(e_1), \mathcal{G}(e_2)$ are grills on X (in the crisp sense). Thus $(\mathcal{G}, E)$ is a soft grill on (X, E).

Definition 3.5. Let (π, E) be a basic soft proximity on X and (F, E) be a soft set. Define $\pi(F) : E \to \mathcal{B}$ by
$\pi(F)(e) = \pi(e)(F(e))$.
Clearly $(\pi(F), E)$ is a soft grill on X.

Definition 3.6. A mapping $c : P(X)^E \to P(X)^E$ is said to be a *Čech closure operator of soft sets* on (X, E) if
(i) $c(\tilde{\Phi}) = \tilde{\Phi}$,
(ii) $c(F) \supset F, \ \forall F \in P(X)^E$,
(iii) $c(F \tilde{\cup} G) = c(F) \tilde{\cup} c(G), \ \forall F, G \in P(X)^E$.

Moreover if c satisfies the additional condition $c(c(F)) = c(F), \ \forall F \in P(X)^E$, then c said to be a *Kuratowski closure operator of soft sets* on (X, E). If c is a Čech closure operator of soft sets, then $((X, E), c)$ is called *closure space of soft sets.*

Theorem 3.7. *Let c be a Kuratowski closure operator of soft sets on (X, E). Let us define $\tau_c = \{F \in P(X)^E : c(F') = F'\}$, where F' is the complement of the soft set F.*
Then τ_c forms a topology of soft sets (in the sense of [31]*) and the closure operator induced by τ_c coincides with c.*

Proof. It can be easily checked that τ_c is a topology of soft sets.
Let $F \in P(X)^E$. Then
$cl_{\tau_c} F = \tilde{\bigcap}\{G : G$ is a closed soft set in $((X, E), \tau_c)$ such that $G \supset F\}$
$= \tilde{\bigcap}\{G : c((G')') = (G')'$ such that $G \supset F\}$
$= \tilde{\bigcap}\{G : c(G) = G$ such that $G \supset F\}$
$= \tilde{\bigcap}\{c(G) : G \supset F\}$
$= c(F)$.
This completes the proof. □

Hence without any loss of generality, where c is a Kuratowski closure operator of soft sets on (X, E), the triple $((X, E), c)$ will be called a *topological space of soft sets.*

Theorem 3.8. *A soft set F is closed if and only if $c(F) = F$.*

Definition 3.9. Let $(\pi, E) \in M_S(X, E)$. For $F \in P(X)^E$, and $e \in E$ define
$c_\pi(F)(e) = \{x \in X : \{x\} \in \pi(e)(F(e))\}$ i.e., $c_\pi(F)(e) = c_{\pi(e)}(F(e))$.

Definition 3.10. Let $x \in X$ and $e \in E$. A *soft point* $\{x\}_e$ is a soft set on E such that $\{x\}_e(e_1) = \{x\}$ if $e_1 = e$ and $= \phi$ if $e_1 \neq e$.

Definition 3.11. Let c be a Čech closure operator of soft sets on (X, E). Then c is said to be R_0 if for any $x, y \in X$ and $\forall P \in P(X)^E$ with $P(e) = \{x\}, y \in c(P)(e) \Rightarrow x \in c(Q)(e) \ \forall Q \in P(X)^E$ with $Q(e) = \{y\}$.

Theorem 3.12. *Let $(\pi, E) \in M_S(X, E)$. Then c_π is a $R_0 - \check{C}ech$ closure operator of soft sets.*

Proof. Clearly $c_\pi(\tilde{\Phi})(e) = \phi\ \forall e \in E$. Therefore $c_\pi(\tilde{\Phi}) = \tilde{\Phi}$.
Let $F \in P(X)^E$ and $e \in E$.
Therefore $x \in F(e) \Rightarrow \{x\} \cap F(e) \neq \phi \Rightarrow \{x\} \in \pi(e)(F(e)) \Rightarrow x \in c_\pi(F)(e)$.
Therefore $F(e) \subset c_\pi(F)(e)$. This is true for all $e \in E$. Therefore $F \subset c_\pi(F)$.
Let $F, G \in P(X)^E$ and $e \in E$.
Therefore $c_\pi(F \tilde{\cup} G)(e) = \{x \in X : \{x\} \in \pi(e)(F\tilde{\cup}G(e))\} = \{x \in X : \{x\} \in \pi(e)(F(e) \cup G(e))\} = \{x \in X : (\{x\}, F(e) \cup G(e)) \in \pi(e)\} = \{x \in X : (\{x\}, F(e)) \in \pi(e)$ or $(\{x\}, G(e)) \in \pi(e)\} = \{x \in X : (\{x\}, F(e)) \in \pi(e)\} \cup \{x \in X : (\{x\}, G(e)) \in \pi(e)\} = \{x \in X : \{x\} \in \pi(e)(F(e))\} \cup \{x \in X : \{x\} \in \pi(e)(G(e))\} = c_\pi(F)(e) \cup c_\pi(G)(e) = (c_\pi(F)\tilde{\cup}c_\pi(G))(e)$.
Therefore $c_\pi(F\tilde{\cup}G) = c_\pi(F)\tilde{\cup}c_\pi(G)$.
Thus c_π is a $\check{C}$ech closure operator of soft sets on (X, E).
Let $x, y \in X$. Let $P \in P(X)^E$ such that $P(e) = \{x\}$. Then
$y \in c_\pi(P)(e) \Leftrightarrow \{y\} \in \pi(e)(P(e)) \Leftrightarrow (\{y\}, P(e)) \in \pi(e) \Leftrightarrow (\{y\}, \{x\}) \in \pi(e) \Leftrightarrow (\{x\}, \{y\}) \in \pi(e) \Leftrightarrow \{x\} \in \pi(e)(\{y\}) \Leftrightarrow \{x\} \in \pi(e)(Q(e))\ \forall Q \in P(X)^E$ with $Q(e) = \{y\} \Leftrightarrow x \in c_\pi(Q)(e)\ \forall Q \in P(X)^E$ with $Q(e) = \{y\}$.
Therefore c_π is a $R_0 - \check{C}$ech closure operator of soft sets on (X, E). □

Theorem 3.13. *If c be a $R_0 - \check{C}ech$ closure operator of soft sets on (X, E). Then there is a basic soft proximity π on (X, E) such that $c_\pi = c$.*

Proof. Let c be a $R_0 - \check{C}$ech closure operator of soft sets on (X, E).
Define $\forall e \in E$,
$\pi(e) = \{(F(e), G(e)) : F, G \in P(X)^E, (c(F)(e) \cap G(e)) \cup (F(e) \cap c(G)(e)) \neq \phi\}$.
Clearly $\pi(e) = \{\pi(e)\}^{-1}\ \forall e \in E$.
Let $e \in E$ and $F, G, H \in P(X)^E$. Then,
$(F(e), G(e) \cup H(e)) \in \pi(e) \Leftrightarrow \{c(F)(e) \cap (G\tilde{\cup}H)(e)\} \cup \{F(e) \cap c(G\tilde{\cup}H)(e)\} \neq \phi \Leftrightarrow \{(c(F)(e) \cap (G(e) \cup H(e))\} \cup \{(F(e) \cap (c(G)(e) \cup c(H)(e))\} \neq \phi \Leftrightarrow \{(c(F)(e) \cap G(e)\} \cup \{(c(F)(e) \cap H(e)\} \cup \{(F(e) \cap c(G)(e)\} \cup \{(F(e) \cap c(H)(e)\} \neq \phi \Leftrightarrow \{(c(F)(e) \cap G(e)\} \cup \{(F(e) \cap c(G)(e)\} \neq \phi$ or $\{(c(F)(e) \cap H(e)\} \cup \{(F(e) \cap c(H)(e)\} \neq \phi \Leftrightarrow (F(e), G(e)) \in \pi(e)$ or $(F(e), H(e)) \in \pi(e)$.
Let $F, G \in P(X)^E$ such that $F(e) \cap G(e) \neq \phi$. Therefore $\{(c(F)(e) \cap G(e)\} \cup \{(F(e) \cap c(G)(e)\} \neq \phi$. Thus $(F(e), G(e)) \in \pi(e)$.
Let $F \in P(X)^E$. Then $\{(c(F)(e) \cap \tilde{\Phi}(e)\} \cup \{(F(e) \cap c(\tilde{\Phi})(e)\} = \phi$. Therefore $(F(e), \tilde{\Phi}(e)) \notin \pi(e)$. i.e., $(F(e), \phi) \notin \pi(e)$.
Hence $\pi(e)$ is a basic proximity on X. Therefore $\pi : E \to \mathcal{A}$ is a mapping.
Therefore (π, E) is a basic soft proximity on (X, E).
Let $F \in P(X)^E$ and $e \in E$.
Then $c_\pi(F)(e) = \{x \in X : \{x\} \in \pi(e)(F(e))\} = \{x \in X : (\{x\}, F(e)) \in \pi(e)\}$.
Therefore $x \in c_\pi(F)(e) \Rightarrow \exists P \in P(X)^E$ such that $P(e) = \{x\}$ and $(c(P)(e) \cap F(e)) \cup (P(e) \cap c(F)(e)) \neq \phi$.
Now $P(e) \cap c(F)(e) \neq \phi \Rightarrow \{x\} \cap c(F)(e) \neq \phi \Rightarrow x \in c(F)(e)$.
Again $c(P)(e) \cap F(e) \neq \phi \Rightarrow \exists y \in c(P)(e) \cap F(e) \Rightarrow y \in c(P)(e)$ and $y \in F(e) \Rightarrow x \in c(Q)(e) \forall Q \in P(X)^E$ with $Q(e) = \{y\} \Rightarrow x \in c(\{y\}_e)(e)$ and $y \in F(e) \Rightarrow x \in$

$c(\{y\}_e)(e)$ and $\{y\}_e \subset F \Rightarrow x \in c(F)(e)$.
Also $x \in c(F)(e) \Rightarrow \{x\} \cap c(F)(e) \neq \phi \Rightarrow (\{x\}, F(e)) \in \pi(e) \Rightarrow x \in c_\pi(F)(e)$.
Thus $x \in c_\pi(F)(e) \Leftrightarrow x \in c(F)(e)$. Therefore $c_\pi(F)(e) = c(F)(e)$. Thus $c_\pi(F) = c(F)$ and hence $c_\pi = c$. □

Definition 3.14. Let E be a set of parameters and X be a nonempty set and $\mathcal{L}$ be the set of all Lodato proximities on X. Then the pair (π, E) is called a *Lodato soft proximities* on (X, E) if π is a mapping given by $\pi : E \to \mathcal{L}$.
The set of all Lodato soft proximities will be denoted by $M_S^{LO}(X, E)$.

Theorem 3.15. *If $(\pi, E) \in M_S^{LO}(X, E)$, then c_π is a Kuratowski closure operator of soft sets on (X, E) i.e., $((X, E), c_\pi)$ is a topological space of soft sets .*

Proof. Let $(\pi, E) \in M_S^{LO}(X, E)$. Then c_π is a Čech closure operator of soft sets on (X, E).
Let $F \in P(X)^E$. Then $c_\pi(c_\pi(F)) \supset c_\pi(F)$.
Let $e \in E$. Then $c_\pi(F)(e) = \{x \in X : \{x\} \in \pi(e)(F(e))\}$.
Since $\pi(e)$ is a Lodato proximity on X, for $x \in X$,
$\{x\} \in \pi(e)(c_\pi(F)(e)) \Rightarrow \{x\} \in \pi(e)(F(e))$.
Therefore $x \in c_\pi(c_\pi(F))(e) \Rightarrow x \in c_\pi(F)(e)$. Thus $c_\pi(c_\pi(F))(e) \subset c_\pi(F)(e)$.
Therefore $c_\pi(c_\pi(F)) \subset c_\pi(F)$. Hence $c_\pi(c_\pi(F)) = c_\pi(F)$.
Therefore c_π is a Kuratowski closure operator of soft sets on (X, E). □

Theorem 3.16. *If c is a R_0-Kuratowski closure operator of soft sets on (X, E). Then there is a Lodato soft proximity π on (X, E) such that $c_\pi = c$.*

Proof. Let c be a R_0-Kuratowski closure operator of soft sets on (X, E).
Define for all $e \in E$,
$\pi(e) = \{(F(e), G(e)) : F, G \in P(X)^E, c(F)(e) \cap c(G)(e) \neq \phi\}$.
Clearly $\pi(e) = (\pi(e))^{-1}$ $\forall e \in E$.
Let $e \in E$ and $F, G, H \in P(X)^E$. Then
$(F(e), (G\tilde{\cup}H)(e)) \in \pi(e) \Leftrightarrow c(F)(e) \cap c(G\tilde{\cup}H)(e) \neq \phi \Leftrightarrow c(F)(e) \cap (c(G)\tilde{\cup}c(H))(e) \neq \phi \Leftrightarrow (c(F)(e) \cap c(G)(e)) \cup (c(F)(e) \cap c(H)(e)) \neq \phi \Leftrightarrow c(F)(e) \cap c(G)(e) \neq \phi$ or $c(F)(e) \cap c(H)(e) \neq \phi \Leftrightarrow (F(e), G(e)) \in \pi(e)$ or $(F(e), H(e)) \in \pi(e)$.
Let $F, G \in P(X)^E$ such that $F(e) \cap G(e) \neq \phi$.
Therefore $c(F)(e) \cap c(G)(e) \neq \phi$ and hence $(F(e), G(e)) \in \pi(e)$.
Clearly $(F(e), \phi) \notin \pi(e)$, since $c(F)(e) \cap c(\tilde{\Phi})(e) = \phi$.
Thus $\pi(e)$ is a basic proximity on X.
Let $F \in P(X)^E$ and $e \in E$. Then
$c_\pi(F)(e) = \{x : \{x\} \in \pi(e)(F(e))\} = \{x : (\{x\}, F(e)) \in \pi(e)\}$.
Therefore $x \in c_\pi(F)(e) \Rightarrow \exists P \in P(X)^E$ such that $P(e) = \{x\}$ and $c(P)(e) \cap c(F)(e) \neq \phi \Rightarrow \exists P \in P(X)^E$ such that $P(e) = \{x\}$ and $\exists y \in c(P)(e) \cap c(F)(e) \Rightarrow \exists P \in P(X)^E$ such that $P(e) = \{x\}$ and $\exists y \in c(P)(e), y \in c(F)(e) \Rightarrow x \in c(Q)(e) \forall Q \in P(X)^E$ with $Q(e) = \{y\}$ and $y \in c(F)(e)$ (since c is R_0) $\Rightarrow x \in c(\{y\}_e)(e)$ and $\{y\}_e \subset c(F) \Rightarrow x \in c(c(F))(e) \Rightarrow x \in c(F)(e)$.
Again $x \in c(F)(e) \Rightarrow \{x\}_e(e) \cap c(F)(e) \neq \phi \Rightarrow c(\{x\}_e)(e) \cap c(F)(e) \neq \phi \Rightarrow (\{x\}_e(e), F(e)) \in \pi(e) \Rightarrow (\{x\}, F(e)) \in \pi(e) \Rightarrow x \in c_\pi(F)(e)$.
Thus $c(F)(e) \subset c_\pi(F)(e)$. Hence $c_\pi(F)(e) = c(F)(e)$.
This is true $\forall e \in E$. Therefore $c_\pi(F) = c(F)$. This is true $\forall F \in P(X)^E$. Thus

$c_\pi = c$.
It is clear that $\forall F, G \in P(X)^E, \forall e \in E$,
$(F(e), G(e)) \in \pi(e) \Rightarrow (c_\pi(F)(e), c_\pi(G)(e)) \in \pi(e)$.
Let $F, G \in P(X)^E, e \in E$ such that $(c_\pi(F)(e), c_\pi(G)(e)) \in \pi(e)$.
Therefore $(c(F)(e), c(G)(e)) \in \pi(e)$. Therefore $c(c(F))(e) \cap c(c(G))(e)) \neq \phi$.
Therefore $c(F)(e) \cap c(G)(e)) \neq \phi$. Thus $(F(e), G(e)) \in \pi(e)$.
Therefore $\pi(e)$ is a Lodato proximity on X $\forall e \in E$.
Hence (π, E) is a Lodato soft proximity on (X, E) and $c_\pi = c$. □

Definition 3.17. A soft mapping $f = (\tilde{f}, \hat{f}) : ((X_1, E_1), \pi_1) \to ((X_2, E_2), \pi_2)$ is said to be *soft proximally continuous* if for each $F, G \in P(X_1)^{E_1}$,
$(F(e), G(e)) \in \pi_1(e) \Rightarrow (\tilde{f}(F(e)), \tilde{f}(G(e))) \in \pi_2(\hat{f}(e)), \forall e \in E_1$.

Theorem 3.18. *Every soft proximally continuous mapping is soft continuous.*

Proof. Let $f = (\tilde{f}, \hat{f}) : ((X_1, E_1), \pi_1) \to ((X_2, E_2), \pi_2)$ be soft proximally continuous.
Let $F \in P(X_1)^{E_1}$. Let $e' \in E_2$ such that $\hat{f}^{-1}(e') \neq \phi$.
Therefore $f(c_{\pi_1}(F))(e') = \tilde{f}(\bigcup_{e \in \hat{f}^{-1}(e')} c_{\pi_1}(F)(e)) = \tilde{f}(\bigcup_{e \in \hat{f}^{-1}(e')} c_{\pi_1(e)}(F(e)))$.
Let $x \in \bigcup_{e \in \hat{f}^{-1}(e')} c_{\pi_1(e)}(F(e))$.
Therefore $x \in c_{\pi_1(e_0)}(F(e_0))$ for some $e_0 \in \hat{f}^{-1}(e')$. Therefore $(\{x\}, F(e_0)) \in \pi_1(e_0)$.
Thus $(\tilde{f}(\{x\}), \tilde{f}(F(e_0))) \in \pi_2(\hat{f}(e_0))$, by soft proximal continuity.
Therefore $(\{\tilde{f}(x)\}, \tilde{f}(F(e_0))) \in \pi_2(e')$.
Thus $\tilde{f}(x) \in c_{\pi_2(e')}\tilde{f}(\bigcup_{e \in \hat{f}^{-1}(e')} F(e))$, since $\tilde{f}(\bigcup_{e \in \hat{f}^{-1}(e')} F(e)) \supset \tilde{f}(F(e_0))$.
Therefore $\tilde{f}(\bigcup_{e \in \hat{f}^{-1}(e')} c_{\pi_1(e)}(F(e))) \subset c_{\pi_2(e')}\tilde{f}(\bigcup_{e \in \hat{f}^{-1}(e')} F(e))$.
Thus $f(c_{\pi_1}(F))(e') \subset c_{\pi_2(e')} f(F)(e') = (c_{\pi_2} f(F))(e')$.
Therefore $f(c_{\pi_1}(F)) \subset c_{\pi_2} f(F)$.
Hence f is soft continuous. □

Definition 3.19. Let $(\pi, E) \in M_S(X, E)$ and $F, G \in P(X)^E$. Then G is said to be a *soft proximal neighbourhood* of F, denoted by $G \gg F$ if $X - G(e) \notin \pi(e)(F(e))$ $\forall e \in E$.
The set of all soft proximal neighbourhoods of F w.r.t π will be denoted by $\mathcal{N}(\pi, F)$.

Remark 3.20. Let $(\pi, E) \in M_S(X, E)$, $F \in P(X)^E, x \in X$ and $e \in E$.
Then $F \gg \{x\}_e$ if and only if $(X - F(e'), \{x\}_e(e')) \notin \pi(e')$ $\forall e' \in E$ if and only if $(X - F(e), \{x\}) \notin \pi(e)$ if and only if $\{x\} \notin \pi(e)(X - F(e))$ if and only if $x \notin c_{\pi(e)}(X - F(e))$ if and only if $\{x\} \cap c_{\pi(e)}(X - F(e)) = \phi$ if and only if $F(e)$ is a neighbourhood of x in the closure space $(X, c_{\pi(e)})$.

Theorem 3.21. *The following results hold:*
(i) *For each* $F, G \in P(X)^E, G \in \mathcal{N}(\pi, F) \Rightarrow F' \in \mathcal{N}(\pi, G')$.
(ii) *For each* $F \in P(X)^E, c_\pi(F) = \tilde{\cap}\{G : G \in \mathcal{N}(\pi, F)\}$.

Proof. (i) Let $F, G \in P(X)^E$. Then
$G \in \mathcal{N}(\pi, F) \Rightarrow X - G(e) \notin \pi(e)(F(e))\ \forall e \in E \Rightarrow (F(e), X - G(e)) \notin \pi(e)\ \forall e \in E \Rightarrow (X - G(e), F(e)) \notin \pi(e)\ \forall e \in E \Rightarrow F(e) \notin \pi(e)(G'(e))\ \forall e \in E \Rightarrow X - F'(e) \notin \pi(e)(G'(e))\ \forall e \in E \Rightarrow F' \in \mathcal{N}(\pi, G')$.

(ii) Let $e \in E$. Then $c_\pi(F)(e) = c_{\pi(e)}F(e) = \{x \in X : (\{x\}, F(e)) \in \pi(e)\}$.
Therefore $x \in c_\pi(F)(e) \Leftrightarrow (\{x\}, F(e)) \in \pi(e) \Leftrightarrow X - (\{x\}_e)'(e) \in \pi(e)(F(e)) \Leftrightarrow (\{x\}_e)' \notin \mathcal{N}(\pi, F) \Leftrightarrow \forall G \in \mathcal{N}(\pi, F), G \not\subset (\{x\}_e)' \Leftrightarrow \forall G \in \mathcal{N}(\pi, F), x \in G(e)$, since $G \not\subset (\{x\}_e)' \Leftrightarrow \exists e' \in E$ such that $G(e') \not\subset (\{x\}_e)'(e') \Leftrightarrow \exists e' \in E$ such that $G(e') \not\subset X - \{x\}_e(e') \Leftrightarrow G(e) \not\subset X - \{x\} \Leftrightarrow x \in G(e)$.
Thus $c_\pi(F)(e) = \tilde{\cap}\{G : G \in \mathcal{N}(\pi, F)\}(e)$.
Since $e \in E$ is arbitrary, $c_\pi(F) = \tilde{\cap}\{G : G \in \mathcal{N}(\pi, F)\}$. □

4. Conclusions

Proximity is a structure which plays a crucial role in the compactification /extension problems of topological spaces. The fuzzyfication of this structure has been studied by many authors. In this paper an attempt has been made, for the first time, to introduce this structure in soft set setting. In the field of soft topology, studies on soft proximities will be a potential area of research.

Acknowledgements. The authors are thankful to the editors-in-chief and managing editors for their important comments which helped to improve the presentation of the paper. This present research work has been financially supported by the University Grants Commission, in forms of UGC-DRS (Phase-II) scheme.

References

[1] H. Aktas and N. Cagman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2726–2735.
[2] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57(9) (2009) 1547–1553.
[3] A. Aygunoglu and H. Aygun, Introduction to fuzzy soft groups, Comput. Math. Anal. 58 (2009) 1279–1286.
[4] H. Cartan, Theorie des filtres, Comptes Rendus Acad Sci. Paris 205 (1937) 595–598.
[5] E. Čech, Topological Spaces, rev. ed. Publ. House Czech. Acad. Sc. Prague, English transl. Wiley, New York (1966).
[6] G. Choquet, Sur les notions de filtre et de grille, C. R. Acad. Sci. Paris 224 (1947) 171–173.
[7] K. C. Chattopadhyay, H. Hazra and S. K. Samanta, A correspondence between Lodato fuzzy proximities and a class of principal type-II fuzzy extensions, J Fuzzy Math. 20(1) (2012) 29–46.
[8] Sujoy Das and S. K. Samanta, Soft real set, soft real number and their properties, J Fuzzy Math. 20(3) (2012) 551–576.
[9] Sujoy Das and S. K. Samanta, On soft complex sets and soft complex numbers, J. Fuzzy Math. 21(1) (2013) 195–216.
[10] F. Feng, Y. B. Jun and X. Z. Zhao, Soft semirings, Comput. Math. Appl. 56(10) (2008) 2621–2628.
[11] F. Feng, C. X. Li, B. Davvaz and M. Irfan Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Computing 14 (2010) 899–911.
[12] F. Feng, X. Y. Liu, V. Leoreanu-Fotea and Y. B. Jun, Soft sets and soft rough sets, Inform. Sci. 181 (2011) 1125–1137.
[13] F. Feng and Y. M. Li, Soft subsets and soft product operations, Inform. Sci. 232 (2013) 44–57.
[14] H. Hazra, P. Majumdar and S. K. Samanta, Soft topology, Fuzzy Inf. Eng. 4(1) (2012) 105–115
[15] Y. B. Jun, Soft BCK/BCI-algebras, Comput. Math. Appl. 56 (2008) 1408–1413

[16] Y. B. Jun and C. H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Inform. Sci. 178 (2008) 2466–2475.
[17] A. Kharal and B. Ahmad, Mapping on soft classes, New Math. Nat. Comput. 7(3) (2011) 471–481.
[18] Z. Kong, L. Gao, L. Wang and S. Li, The normal parameter reduction of soft sets and its algorithm, Comput. Math. Appl. 56(12) (2008) 3029–3037.
[19] Z. Kong, L. Gao and L. Wang, Comment on 'A fuzzy soft set theoretic approach to decision making problems', J. Comput. Appl. Math. 223 (2009) 540–542.
[20] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9(3) (2001) 589–602.
[21] P. K. Maji, R. Biswas and A. R. Roy, Intuitionistic fuzzy soft sets, J. Fuzzy Math. 9(3) (2001) 677–692.
[22] P. K. Maji, A. R. Roy and R. Biswas, On intuitionistic fuzzy soft sets, J. Fuzzy Math. 12(3) (2004) 669–683.
[23] P. K. Maji, A. R. Roy and R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (2002) 1077–1083.
[24] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[25] P. Majumdar and S. K. Samanta, On soft mappings, Comput. Math. Appl. 60(9) (2010) 2666–2672.
[26] P. Majumdar and S. K. Samanta, Similarity measure of soft sets, New Math. Nat. Comput. 4(1) (2008) 1–12.
[27] D. Molodtsov, Soft set theory - first results, Comput. Math. Appl. 37 (1999) 19–31.
[28] S. A. Naimpally and B. D. Warrack, The proximities spaces, Cambridge University Press, 1970.
[29] S. Roy and T. K. Samanta, A note on fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 3(2) (2012) 305–311.
[30] Tugbahan Simsekler and Saziye Yuksel, Fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 5(1) (2013) 87–96.
[31] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61(7) (2011) 1786–1799.
[32] M. Shabir and M. I. Ali, Soft ideals and generalized fuzzy ideals in semigroups, New Math. Nat. Comput. 5(3) (2009) 599–615.
[33] B. Tanay and M. B. Kandemir, Topological structure of fuzzy soft sets, Comput. Math. Appl. 61 (2011) 2952–2957.

H. Hazra (h.hazra2010@gmail.com)
Department of Mathematics, Bolpur College, Bolpur-731204, West Bengal, India

P. Majumdar (pmajumdar2@rediffmail.com)
Department of Mathematics, M.U.C Women's College, Burdwan, West Bengal, India

S. K. Samanta (syamal_123@yahoo.co.in)
Department of Mathematics, Visva Bharati, Santiniketan-731235, W.Bengal, India