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On $\alpha\delta$-neighbourhoods in topological spaces

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Abstract. In this paper, we introduce $\alpha\delta$-derived set, $\alpha\delta$-border, $\alpha\delta$-frontier, $\alpha\delta$-exterior and $\alpha\delta$-saturated and further the relationships between them are derived. Also we introduce a new function called $\alpha\delta$-Totally-Continuous Functions. Furthermore, basic properties of these functions and preservation theorems of $\alpha\delta$-totally continuous functions are investigated. Also $\alpha\delta$-Totally open functions in topological spaces are introduced and studied.

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Keywords: $\alpha\delta$-derived set, $\alpha\delta$-border, $\alpha\delta$-frontier, $\alpha\delta$-exterior, $\alpha\delta$-saturated and $\alpha\delta$-totally-continuous functions.

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1. Introduction

The importance of general topological spaces rapidly increases in many fields of applications such as data mining [4]. Information systems are basic tools for producing knowledge from data in any real-life field. Topological structures on the collection of data are suitable mathematical models for mathematizing not only quantitative data but also qualitative ones. Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets. One of the most well known notions and also an inspiration source is the notion of $\alpha\delta$-open [3] sets introduced by R.Devi, V.Kokilavani and P.Basker. In this paper, we will continue the study of related functions with $\alpha\delta$-open and $\alpha\delta$-closed sets. We introduce and characterize the concept of $\alpha\delta$-derived set, $\alpha\delta$-border, $\alpha\delta$-frontier, $\alpha\delta$-exterior and $\alpha\delta$-saturated and further the relationship between them are derived. Also we introduce a new function called $\alpha\delta$-Totally-Continuous Functions. Furthermore, basic properties of these functions and preservation theorems of

$\alpha\delta$-totally continuous functions are investigated. Also $\alpha\delta$-Totally open functions in topological spaces are introduced and studied.

2. Preliminaries

Throughout the present paper, spaces X and Y always mean topological spaces. Let X be a topological space and A, a subset of X. The closure of A and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively.

Definition 2.1. A subset A of a space (X, τ) is called
(1) *regularopen* [5] if $A = int(cl(A))$.
(2) *semiopen* [5] if $A \subset cl(int(A))$.
(3) *α-open* [3] if $A \subset int(cl(int(A)))$.
(4) *δ-semiopen* [3] $A \subset cl(Int_\delta(A))$.
The δ-interior [2] of a subset A of X is the union of all *regularopen* sets of X contained in A and is denoted by $Int_\delta(A)$. The subset A is called δ-open [2] if $A = Int_\delta(A)$, *i.e.*, a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed. Alternatively, a set $A \subset (X, \tau)$ is called δ-closed [2] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x/x \in U \in \tau \Rightarrow int(cl(U)) \cap A \neq \phi\}$. The family of all δ-open (resp. δ-closed) sets in X is denoted by $\delta O(X)$ (resp. $\delta C(X)$).
The intersection of all *semiclosed* (resp. *α-closed*, *δ-semiclosed*) sets containing A is called the *semi-closure* (resp. *α-closure*, *δ-semiclosure*) of A and is denoted by $scl(A)$ (resp. $\alpha cl(A)$, $\delta scl(A)$). Dually, *semi-interior* (resp. *α-interior*, *δ-semi-interior*) of A is defined to be the union of all *semiopen* (resp. *α-open*, *δ-semiopen*) sets contained in A and is denoted by $sint(A)$ (resp. $\alpha\ int(A)$, δ-$sint(A)$). Note that δ-$scl(A)$=$A \cup int(cl_\delta(A))$ and δ-$sint(A)$=$A \cup cl(int_\delta(A))$.
We recall the following definition used in sequel.

Definition 2.2. A subset A of a space (X, τ) is called
• an α-generalized closed [1] (αg-closed) set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).
• an $\alpha\delta$-closed set [1] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in (X, τ).

Definition 2.3. A function $f : (X, \tau) \to (Y, \sigma)$ is called
• *$\alpha\delta$-continuous* [1] if $f^{-1}(V)$ is *$\alpha\delta$-closed* in (X, τ) for every *closedset* V of (Y, σ).
• *$\alpha\delta$-irresolute* [1] if $f^{-1}(V)$ is *$\alpha\delta$-closed* in (X, τ) for every $\alpha\delta$-closed set V of (Y, σ).

3. $\alpha\delta$-deroved and $\alpha\delta$-border

Definition 3.1. Let A be a subset of a space X.
• A point $x \in X$ is said to be $\alpha\delta$-limit point of A if for each $\alpha\delta$-open set U containing x, $U \cap (A - \{x\}) \neq \phi$. The set of all $\alpha\delta$-limit point of A is called $\alpha\delta$-derived (briefly.$D_{\lfloor\alpha\delta\rfloor}$) set of A and is denoted by $D_{\lfloor\alpha\delta\rfloor}(A)$.
• A point $x \in X$ is said to be δ-limit point of A if for each δ-open set U containing x, $U \cap (A - \{x\}) \neq \phi$. The set of all δ-limit point of A is called δ-derived set of A and is denoted by $d_\delta(A)$.

Theorem 3.2. *For subsets A, B of a space X, the following statements hold:*
(1) $D_{\lfloor\alpha\delta\rfloor}(A) \subset d_\delta(A)$ *where* $d_\delta(A)$ *is the* δ*-derived set of* A.
(2) *If* $A \subset B$, *then* $D_{\lfloor\alpha\delta\rfloor}(A) \subset D_{\lfloor\alpha\delta\rfloor}(B)$.
(3) $D_{\lfloor\alpha\delta\rfloor}(A) \cup D_{\lfloor\alpha\delta\rfloor}(B) \subset D_{\lfloor\alpha\delta\rfloor}(A \cup B)$.
(4) $D_{\lfloor\alpha\delta\rfloor}(D_{\lfloor\alpha\delta\rfloor}(A)) - A \subset D_{\lfloor\alpha\delta\rfloor}(A)$.
(5) $D_{\lfloor\alpha\delta\rfloor}(A \cup D_{\lfloor\alpha\delta\rfloor}(A)) \subset A \cup D_{\lfloor\alpha\delta\rfloor}(A)$.

Proof. (1) It suffices to observe that every δ-open set is $\alpha\delta$-open.
(2) It is obvious.
(3) It is an immediate consequence of (2).
(4) If $x \in D_{\lfloor\alpha\delta\rfloor}(D_{\lfloor\alpha\delta\rfloor}(A)) - A$ and U is an $\alpha\delta$-open set containing x, then $U \cap (D_{\lfloor\alpha\delta\rfloor}(A) - \{x\}) \neq \phi$. Let $y \in U \cap (D_{\lfloor\alpha\delta\rfloor}(A) - \{x\})$. Then since $y \in D_{\lfloor\alpha\delta\rfloor}(A)$ and $y \in U$, $U \cap (A - \{y\}) \neq \phi$. Let $Z \in U \cap (A - \{y\}))$. Then $Z \neq x$ for $Z \in A$ and $x \notin A$. Hence $U \cap (A - \{x\}) \neq \phi$. Therefore $x \in D_{\lfloor\alpha\delta\rfloor}(A)$.
(5) Let $x \in D_{\lfloor\alpha\delta\rfloor}(A \cup D_{\lfloor\alpha\delta\rfloor}(A))$. If $x \in A$, the result is obvious. So let $x \in D_{\lfloor\alpha\delta\rfloor}(A \cup D_{\lfloor\alpha\delta\rfloor}(A)) - A$, then for $\alpha\delta$-open set U containing x, $U \cap (A \cup D_{\lfloor\alpha\delta\rfloor}(A) - \{x\}) \neq \phi$. Thus $U \cap (A - \{x\}) \neq \phi$ or $U \cap (D_{\lfloor\alpha\delta\rfloor}(A) - \{x\}) \neq \phi$. Now it follows (4) that $U \cap (A - \{x\}) \neq \phi$. Hence $x \in D_{\lfloor\alpha\delta\rfloor}(A)$. Therefore, in any case $D_{\lfloor\alpha\delta\rfloor}(A \cup D_{\lfloor\alpha\delta\rfloor}(A)) \subset A \cup D_{\lfloor\alpha\delta\rfloor}(A)$. □

Theorem 3.3. *For any subset A of a space X,* $\alpha\delta_{Cl}(A) = A \cup D_{\lfloor\alpha\delta\rfloor}(A)$.

Proof. Since $D_{\lfloor\alpha\delta\rfloor}(A) \subset \alpha\delta_{Cl}(A)$, $A \cup D_{\lfloor\alpha\delta\rfloor}(A) \subset \alpha\delta_{Cl}(A)$. On the other hand, let $x \in \alpha\delta_{Cl}(A)$. If $x \in A$, then the proof is complete. If $x \notin A$, then each $\alpha\delta$-open set U containing x intersects A at a point distinct from x. Therefore $x \in D_{\lfloor\alpha\delta\rfloor}(A)$. Thus $\alpha\delta_{Cl}(A) \subset A \cup D_{\lfloor\alpha\delta\rfloor}(A)$ which implies that $\alpha\delta_{Cl}(A) = A \cup D_{\lfloor\alpha\delta\rfloor}(A)$. This completes the proof. □

Theorem 3.4. *For subsets A, B of a space X, the following statements hold:*
(1) $\alpha\delta_{Int}(A)$ *is the largest* $\alpha\delta$*-open set contained in* A.
(2) A *is an* $\alpha\delta$*-open if and only if* $A = \alpha\delta_{Int}(A)$.
(3) $\alpha\delta_{Int}(\alpha\delta_{Int}(A)) = \alpha\delta_{Int}(A)$.
(4) $\alpha\delta_{Int}(A) = A - D_{\lfloor\alpha\delta\rfloor}(X - A)$.
(5) $X - \alpha\delta_{Int}(A) = \alpha\delta_{Cl}(X - A)$.
(6) $X - \alpha\delta_{Cl}(A) = \alpha\delta_{Int}(X - A)$.
(7) $A \subset B$, *then* $\alpha\delta_{Int}(A) \subset \alpha\delta_{Int}(B)$.
(8) $\alpha\delta_{Int}(A) \cup \alpha\delta_{Int}(B) \subset \alpha\delta_{Int}(A \cup B)$.

Proof. (1), (2), (3) are obvious.
(4) If $x \in A - D_{\lfloor\alpha\delta\rfloor}(X - A)$, then $x \notin D_{\lfloor\alpha\delta\rfloor}(X - A)$ and so there exists an $\alpha\delta$-open set U containing x such that $U \cap (X - A) = \phi$. Then $x \in U \subset A$ and hence $x \in \alpha\delta_{Int}(A)$, *i.e.*, $A - D_{\lfloor\alpha\delta\rfloor}(X - A) \subset \alpha\delta_{Int}(A)$. On the other hand, if $x \in \alpha\delta_{Int}(A)$, then $x \notin D_{\lfloor\alpha\delta\rfloor}(X - A)$. Since $\alpha\delta_{Int}(A)$ is an $\alpha\delta$-open and $\alpha\delta_{Int}(A) \cap (X - A) = \phi$. Hence $\alpha\delta_{Int}(A) = A - D_{\lfloor\alpha\delta\rfloor}(X - A)$.
(5) $X - \alpha\delta_{Int}(A) = X - (A - D_{\lfloor\alpha\delta\rfloor}(X - A)) = (X - A) \cup D_{\lfloor\alpha\delta\rfloor}(X - A) = \alpha\delta_{Cl}(X - A)$.
(6), (7) and (8) are obvious. □

Definition 3.5. For a subset A of a space X,

- $Bd^{\prec\alpha\delta\succ}(A) = A - \alpha\delta_{Int}(A)$ is said to be $\alpha\delta$-border of A.
- $b^{\delta}(A) = A - \delta\text{-}int(A)$ is said to be δ-border of A.

Theorem 3.6. *For a subset A of a space X, the following statements hold:*
(1) $Bd^{\prec\alpha\delta\succ}(A) \subset b^{\delta}(A)$,*where* $b^{\delta}(A)$ *denotes the* δ*-border of* A.
(2) $A = \alpha\delta_{Int}(A) \cup Bd^{\prec\alpha\delta\succ}(A)$.
(3) $\alpha\delta_{Int}(A) \cap Bd^{\prec\alpha\delta\succ}(A) = \phi$.
(4) *A is an $\alpha\delta$-open iff* $Bd^{\prec\alpha\delta\succ}(A) = \phi$.
(5) $Bd^{\prec\alpha\delta\succ}(\alpha\delta_{Int}(A)) = \phi$.
(6) $\alpha\delta_{Int}(Bd^{\prec\alpha\delta\succ}(A)) = \phi$.
(7) $Bd^{\prec\alpha\delta\succ}(Bd^{\prec\alpha\delta\succ}(A)) = Bd^{\prec\alpha\delta\succ}(A)$.
(8) $Bd^{\prec\alpha\delta\succ}(A) = A \cap \alpha\delta_{Cl}(X - A)$
(9) $Bd^{\prec\alpha\delta\succ}(A) = D_{\lfloor\alpha\delta\rfloor}(X - A)$

Proof. (1), (2), (3), (4), (5) and (7) are obvious.
(6) If $x \in \alpha\delta_{Int}(Bd^{\prec\alpha\delta\succ}(A))$, then $x \in Bd^{\prec\alpha\delta\succ}(A)$. On the other hand, since $Bd^{\prec\alpha\delta\succ}(A) \subset A$, $x \in \alpha\delta_{Int}(Bd^{\prec\alpha\delta\succ}(A)) \subset \alpha\delta_{Int}(A)$. Hence $x \in \alpha\delta_{Int}(A) \cap Bd^{\prec\alpha\delta\succ}(A)$, which contradicts (3). Thus $\alpha\delta_{Int}(A) \cap Bd^{\prec\alpha\delta\succ}(A) = \phi$.
(8) $Bd^{\prec\alpha\delta\succ}(A) = A - \alpha\delta_{Int}(A) = A - (X - \alpha\delta_{Cl}(X - A)) = A \cap \alpha\delta_{Cl}(X - A))$.
(9) $Bd^{\prec\alpha\delta\succ}(A) = A - \alpha\delta_{Int}(A) = A - (A - D_{\lfloor\alpha\delta\rfloor}(X - A)) = D_{\lfloor\alpha\delta\rfloor}(X - A))$.
In general the converse of (1) may not be true by the following example. □

Example 3.7. Let $X = \{a, b, c\}$ with topology $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, δ-open= $\{\varphi,\ X,\ \{b\},\ \{a,\ c\}\}$ and $\alpha\delta$-open= $\{\varphi,\ X,\ \{a\},\ \{b\},\ \{a,\ b\},\ \{a,\ c\}\}$. If $A = \{a,\ b\}$ Then $Bd^{\prec\alpha\delta\succ}(\{a,\ b\}) = \varphi$ and $b^{\delta}(\{a,\ b\}) = \{a\}$ which implies that $b^{\delta}(A)$ doesnot contained in $Bd^{\prec\alpha\delta\succ}(A)$.

4. $\alpha\delta$-frontier and $\alpha\delta$-exterior

Definition 4.1. For a subset A of a space X,
- $F_{\approx\alpha\delta}(A) = \alpha\delta_{Cl}(A) \cap \alpha\delta_{Cl}(X/A)$ is said to be $\alpha\delta$-frontier of A.
- $Fr_{\delta}(A) = \delta\text{-}cl(A) \cap \delta\text{-}cl(X/A)$ is said to be δ-frontier of A.

Theorem 4.2. *For a subset A of a space X, the following statements hold:*
(1) $F_{\approx\alpha\delta}(A) \subset Fr_{\delta}(A)$, *where* $Fr_{\delta}(A)$ *denotes the* δ*-frontier of* A.
(2) $\alpha\delta_{Cl}(A) = \alpha\delta_{Int}(A) \cup F_{\approx\alpha\delta}(A)$.
(3) $\alpha\delta_{Int}(A) \cap F_{\approx\alpha\delta}(A) = \varphi$.
(4) $Bd^{\prec\alpha\delta\succ}(A) \subset F_{\approx\alpha\delta}(A)$.
(5) $F_{\approx\alpha\delta}(A) = Bd^{\prec\alpha\delta\succ}(A) \cup D_{\lfloor\alpha\delta\rfloor}(A)$.
(6) *A is $\alpha\delta$-open set iff* $F_{\approx\alpha\delta}(A) = D_{\lfloor\alpha\delta\rfloor}(A)$.
(7) $F_{\approx\alpha\delta}(A) = \alpha\delta_{Cl}(A) \cap \alpha\delta_{Cl}(X/A)$.
(8) $F_{\approx\alpha\delta}(A) = F_{\approx\alpha\delta}(X/A)$.
(9) $F_{\approx\alpha\delta}(A)$ *is $\alpha\delta$-closed.*
(10) $F_{\approx\alpha\delta}(F_{\approx\alpha\delta}(A)) \subset F_{\approx\alpha\delta}(A)$.
(11) $F_{\approx\alpha\delta}(\alpha\delta_{Int}(A)) \subset F_{\approx\alpha\delta}(A)$.
(12) $F_{\approx\alpha\delta}(\alpha\delta_{Cl}(A)) \subset F_{\approx\alpha\delta}(A)$.
(13) $\alpha\delta_{Int}(A) = A - F_{\approx\alpha\delta}(A)$.

Proof. (1), (4), (6), (8) and (11) are obvious.

(2) $\alpha\delta_{Int}(A) \cup F_{\approx\alpha\delta}(A) = \alpha\delta_{Int}(A) \cup (\alpha\delta_{Cl}(A) - \alpha\delta_{Int}(A)) = \alpha\delta_{Cl}(A)$

(3) $\alpha\delta_{Int}(A) \cap F_{\approx\alpha\delta}(A) = \alpha\delta_{Int}(A) \cap (\alpha\delta_{Cl}(A) - \alpha\delta_{Int}(A)) = \varphi$.

(5) Since $\alpha\delta_{Int}(A) \cup F_{\approx\alpha\delta}(A) = \alpha\delta_{Int}(A) \cup Bd^{\prec\alpha\delta\succ}(A) \cup D_{\lfloor\alpha\delta\rfloor}(A)$, $F_{\approx\alpha\delta}(A) = Bd^{\prec\alpha\delta\succ}(A) \cup D_{\lfloor\alpha\delta\rfloor}(A)$.

(7) $F_{\approx\alpha\delta}(A) = \alpha\delta_{Cl}(A) - \alpha\delta_{Int}(A) = \alpha\delta_{Cl}(A) \cap \alpha\delta_{Cl}(X/A)$.

(9) $\alpha\delta_{Cl}(F_{\approx\alpha\delta}(A)) = \alpha\delta_{Cl}(\alpha\delta_{Cl}(A) \cap \alpha\delta_{Cl}(X/A)) \subset \alpha\delta_{Cl}(\alpha\delta_{Cl}(A)) \cap \alpha\delta_{Cl}(\alpha\delta_{Cl}((X/A))) = F_{\approx\alpha\delta}(A)$. Hence $F_{\approx\alpha\delta}(A)$ is $\alpha\delta$-closed.

(10) $F_{\approx\alpha\delta}(F_{\approx\alpha\delta}(A)) = \alpha\delta_{Cl}(F_{\approx\alpha\delta}(A)) \cap \alpha\delta_{Cl}(X - F_{\approx\alpha\delta}(A)) \subset \alpha\delta_{Cl}(F_{\approx\alpha\delta}(A)) = F_{\approx\alpha\delta}(A)$.

(12) $F_{\approx\alpha\delta}(\alpha\delta_{Cl}(A)) = \alpha\delta_{Cl}(\alpha\delta_{Cl}(A)) - \alpha\delta_{Int}(\alpha\delta_{Cl}(A)) = \alpha\delta_{Cl}(A) - \alpha\delta_{Int}(\alpha\delta_{Cl}(A)) = \alpha\delta_{Cl}(A) - \alpha\delta_{Int}(A) = F_{\approx\alpha\delta}(A)$.

(13) $A - F_{\approx\alpha\delta}(A) = A - (\alpha\delta_{Cl}(A) - \alpha\delta_{Int}(A)) = \alpha\delta_{Int}(A)$.

In general the converse of (1) and (4) may not be true by the following example. □

Example 4.3. Let $X = \{a, b, c\}$ with topology $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, δ-closed set= $\{\varphi, X, \{b\}, \{a, c\}\}$ and $\alpha\delta$-closed set= $\{\varphi, X, \{b\}, \{c\}, \{b, c\}, \{c, a\}\}$.

(1) If $A = \{a,\ b\}$ Then $F_{\approx\alpha\delta}(\{a,\ b\}) = \{c\}$ and $Fr_\delta(\{a,\ b\}) = \{a,\ c\}$ which implies that $Fr_\delta(A)$ doesnot contained in $F_{\approx\alpha\delta}(A)$.

(4) If $A = \{a,\ b\}$ Then $F_{\approx\alpha\delta}(\{a,\ b\}) = \{c\}$ and $Bd^{\prec\alpha\delta\succ}(\{a,\ b\}) = \varphi$ which implies that $F_{\approx\alpha\delta}(A)$ doesnot contained in $Bd^{\prec\alpha\delta\succ}(A)$.

In the following theorem $\alpha\delta^{\succ\succ}C$ denote the set of points x of X for which a function $f : (X,\ \tau) \to (Y,\ \sigma)$ is not $\alpha\delta$-continuous.

Theorem 4.4. *$\alpha\delta^{\succ\succ}C$ is identical with the union of the $\alpha\delta$-frontiers of the inverse images of $\alpha\delta$-open sets containing $f(x)$.*

Proof. Suppose that f is not $\alpha\delta$-continuous at a point x of X. Then there exists an open set $V \subset Y$ containing $f(x)$ such that $f(U)$ is not a subset of V for every $U \in \alpha\delta O(X)$ containing x. Hence we have $U \cap (X - f^{-1}(V)) \neq \varphi$ for every $U \in \alpha\delta O(X)$ containing x. It follows that $x \in \alpha\delta_{Cl}(X - f^{-1}(A))$. We also have $x \in f^{-1}(V) \subset \alpha\delta_{Cl}(f^{-1}(A))$. This means that $x \in F_{\approx\alpha\delta}(f^{-1}(V))$. Now, let f be $\alpha\delta$-continuous at $x \in X$ and $V \subset Y$ be any open set containing $f(x)$. Then $x \in f^{-1}(V)$ is a $\alpha\delta$-open set of X. Thus $x \in \alpha\delta_{Int}(f^{-1}(V))$ and therefore $x \notin F_{\approx\alpha\delta}(f^{-1}(V))$ for every open set V containing $f(x)$. □

Definition 4.5. For a subset A of a space X,

- $\xi xt^{\langle\alpha\delta\rangle}(A) = \alpha\delta_{Int}(X - A)$ is said to be $\alpha\delta$-exterior of A.
- $Ext_\delta(A) = \delta\text{-}int(X - A)$ is said to be δ-exterior of A.

Theorem 4.6. *For a subset A of a space X, the following statements hold:*

(1) *$Ext_\delta(A) \subset \xi xt^{\langle\alpha\delta\rangle}(A)$, where $Ext_\delta(A)$ denotes the δ-exterior of A.*

(2) *$\xi xt^{\langle\alpha\delta\rangle}(A)$ is $\alpha\delta$-open.*

(3) *$\xi xt^{\langle\alpha\delta\rangle}(A) = \alpha\delta_{Int}(X - A) = X - \alpha\delta_{Cl}(A)$.*

(4) *$\xi xt^{\langle\alpha\delta\rangle}(\xi xt^{\langle\alpha\delta\rangle}(A)) = \alpha\delta_{Int}(\alpha\delta_{Cl}(A))$.*

(5) *If $A \subset B$, then $\xi xt^{\langle\alpha\delta\rangle}(A) \supset \xi xt^{\langle\alpha\delta\rangle}(B)$.*

(6) *$\xi xt^{\langle\alpha\delta\rangle}(A \cup B) \subset \xi xt^{\langle\alpha\delta\rangle}(A) \cup \xi xt^{\langle\alpha\delta\rangle}(B)$.*

(7) *$\xi xt^{\langle\alpha\delta\rangle}(X) = \varphi$.*

(8) $\xi xt^{\langle\alpha\delta\rangle}(\varphi) = X$.
(9) $\xi xt^{\langle\alpha\delta\rangle}(A) = \xi xt^{\langle\alpha\delta\rangle}(X - \xi xt^{\langle\alpha\delta\rangle}(A))$.
(10) $\alpha\delta_{Int}(A) \subset \xi xt^{\langle\alpha\delta\rangle}(\xi xt^{\langle\alpha\delta\rangle}(A))$.
(11) $X = \alpha\delta_{Int}(A) \cup \xi xt^{\langle\alpha\delta\rangle}(A) \cup F_{\approx\alpha\delta}(A)$.

Proof. (1), (2), (3), (5), (6), (7), (8) and (11) are obvious.

(4) $\xi xt^{\langle\alpha\delta\rangle}(\xi xt^{\langle\alpha\delta\rangle}(A)) = \xi xt^{\langle\alpha\delta\rangle}(X - \alpha\delta_{Cl}(A)) = \alpha\delta_{Int}(X - (X - \alpha\delta_{Cl}(A))) = \alpha\delta_{Int}(\alpha\delta_{Cl}(A))$.

(9) $\xi xt^{\langle\alpha\delta\rangle}(X - \xi xt^{\langle\alpha\delta\rangle}(A)) = \xi xt^{\langle\alpha\delta\rangle}(X - \alpha\delta_{Int}(X - A)) = \alpha\delta_{Int}(X - (X - \alpha\delta_{Int}(X - A))) = \alpha\delta_{Int}(\alpha\delta_{Int}(X - A)) = \alpha\delta_{Int}(X - A) = \xi xt^{\langle\alpha\delta\rangle}(A)$.

(10) $\alpha\delta_{Int}(A) \subset \alpha\delta_{Int}(\alpha\delta_{Cl}(A)) = \alpha\delta_{Int}(X - \alpha\delta_{Int}(X - A)) = \alpha\delta_{Int}(X - \xi xt^{\langle\alpha\delta\rangle}(A)) = \xi xt^{\langle\alpha\delta\rangle}(\xi xt^{\langle\alpha\delta\rangle}(A))$.

In general the converse of (1) and (6) may not be true by the following example. □

Example 4.7. (1) Let $X = \{a, b, c\}$ with topology $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, δ-closed set= $\{\varphi, X, \{b\}, \{a, c\}\}$ and $\alpha\delta$-closed set= $\{\varphi, X, \{b\}, \{c\}, \{b, c\}, \{c, a\}\}$. If $A = \{c\}$ Then $\xi xt^{\langle\alpha\delta\rangle}(\{c\}) = \{a, b\}$ and $Ext_\delta(\{c\}) = \{b\}$ which implies that $\xi xt^{\langle\alpha\delta\rangle}(A)$ doesnot contained in $Ext_\delta(A)$.

(6) Let $X = \{a,\ b,\ c,\ d\}$ with topology $\tau = \{\varphi,\ X,\ \{a\},\ \{c\},\ \{a,\ c\},\ \{a,\ c,\ d\}\}$ and $\alpha\delta$-closed set= $\{\varphi,\ X,\ \{b,\ d\},\ \{a,\ b,\ d\},\ \{b,\ c,\ d\}\}$. If $A = \{a\}$ and $B = \{b\}$, then $\xi xt^{\langle\alpha\delta\rangle}(A) = \{c\}$, $\xi xt^{\langle\alpha\delta\rangle}(B) = \{a,\ c\}$ and $\xi xt^{\langle\alpha\delta\rangle}(A \cup B) = \{c\}$ which implies that $\xi xt^{\langle\alpha\delta\rangle}(A) \cup \xi xt^{\langle\alpha\delta\rangle}(B)$ doesnot contained in $\xi xt^{\langle\alpha\delta\rangle}(A \cup B)$.

Definition 4.8. Let X be a topological space. A set $A \subset X$ is said to be $\alpha\delta$-saturated if for every $x \in A$ it follows $\alpha\delta_{Cl}(\{x\}) \subset A$. The set of all $\alpha\delta$-saturated sets in X we denote by $Sat^{\star\star}\alpha\delta(X)$.

Theorem 4.9. *Let X be a topological space. Then $Sat^{\star\star}\alpha\delta(X)$ is a complete Boolean set algebra.*

Proof. We will prove that all the unions and complements of elements of $Sat^{\star\star}\alpha\delta(X)$ are members of $Sat^{\star\star}\alpha\delta(X)$. Obviously, only the proof regarding the complements is not trivial. Let $A \in Sat^{\star\star}\alpha\delta(X)$ and suppose that $\alpha\delta_{Cl}(\{x\})$ does not contained in $X - A$ for some $x \in X - A$. Then there exists $y \in A$ such that $y \in \alpha\delta_{Cl}(\{x\})$. It follows that x, y have no disjoint neighbourhoods. Then $x \in \alpha\delta_{Cl}(\{y\})$. But this is a contradiction, because by the definition of $Sat^{\star\star}\alpha\delta(X)$ we have $\alpha\delta_{Cl}(\{y\}) \subset A$. Hence, $\alpha\delta_{Cl}(\{x\}) \subset X - A$ for every $x \in X - A$, which implies $X - A \in Sat^{\star\star}\alpha\delta(X)$. □

Corollary 4.10. *$Sat^{\star\star}\alpha\delta(X)$ contains every union and every intersection of $\alpha\delta$-closed and $\alpha\delta$-open sets in X.*

5. $\alpha\delta$-totally continuous functions

Definition 5.1. A function $f : (X,\ \tau) \to (Y,\ \sigma)$ is called

(1) $\alpha\delta$-Totally-Continuous at a point $x \in X$ if for each open subset V in Y containing $f(x)$, there exists a $\alpha\delta$-clopen subset U in X containing x such that $f(U) \subset V$.

(2) $\alpha\delta$-Totally-Continuous, if it has this property at each point of X.

Theorem 5.2. *The following statements are equivalent for a function* $f : (X, \tau) \to (Y, \sigma)$*:*

(1) f *is* $\alpha\delta$*-Totally-Continuous.*
(2) *for every open set* V *of* Y*,* $f^{-1}(V)$ *is* $\alpha\delta$*-clopen in* X*.*

Proof. (1) $\Rightarrow$ (2) Let V be an open subset of Y and let $x \in f^{-1}(V)$. Since $f(x) \in V$, by (1), there exists a $\alpha\delta$-clopen set U_x in X containing x such that $U_x \subset f^{-1}(V)$. We obtain $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. Thus, $f^{-1}(V)$ is $\alpha\delta$-clopen in X.

(2) $\Rightarrow$ (1) Clear. □

Remark 5.3. It is clear that every $\alpha\delta$-Totally-Continuous function is $\alpha\delta$-continuous. But the converse is false.

Example 5.4. The identity function on the real line with the usual topology is continuous and hence $\alpha\delta$-continuous. The inverse image of (0, 1) is not $\alpha\delta$-closed and the function is not $\alpha\delta$-Totally-Continuous.

Definition 5.5. A space (X, τ) is said to be $\alpha\delta$-space if every $\alpha\delta$-open set of X is open in X.

Theorem 5.6. *If a function* $f : (X, \tau) \to (Y, \sigma)$ *is totally continuous and* X *is a* $\alpha\delta$*-space, then* f *is* $\alpha\delta$*-Totally-Continuous.*

Proof. Straightforward. □

Definition 5.7 ([1])**.** A topological space (X, τ) is said to be $\alpha\delta$-connected if it cannot be written as the union of two nonempty disjoint $\alpha\delta$-open sets.

Theorem 5.8. *If* f *is a* $\alpha\delta$*-Totally-Continuous function from a* $\alpha\delta$*-connected space* X *onto any space* Y*, then* Y *is an indiscrete space.*

Proof. If possible, suppose that Y is not indiscrete. Let A be a proper non-empty open subset of Y. Then $f^{-1}(A)$ is a proper non-empty $\alpha\delta$-clopen subset of (X, τ), which is a contradiction to the fact that X is $\alpha\delta$-connected. □

Theorem 5.9. *The set of all points* $x \in X$ *in which a function* $f : (X, \tau) \to (Y, \sigma)$ *is not an* $\alpha\delta$*-Totally-Continuous is the union of* $\alpha\delta$*-frontier of the inverse images of open sets containing* $f(x)$*.*

Proof. Suppose that f is not an $\alpha\delta$-Totally-Continuous function at $x \in X$. Then there exists an open set V of Y containing $f(x)$ such that $f(U)$ is not contained in V for each $U \in \alpha\delta O(X)$ containing x and hence $x \in \alpha\delta_{Cl}(X/f^{-1}(V))$. On the other hand, $X \in f^{-1}(V) \subset \alpha\delta_{Cl}(f^{-1}(V))$ and hence $X \in F_{\approx\alpha\delta}(f^{-1}(V))$. □

Conversely, suppose that f is an $\alpha\delta$-Totally-Continuous at $x \in X$ and let V be an open set of Y containing $f(x)$. Then there exists $U \in \alpha\delta O(X)$ containing x such that $U \subset f^{-1}(V)$. Hence $x \in \alpha\delta_{Int}(f^{-1}(V))$. Therefore, $X \in F_{\approx\alpha\delta}(f^{-1}(V))$ for each open set V of Y containing $f(x)$.

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