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On $(\in, \in \vee q_k)$-fuzzy filters of CI-algebras

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Abstract. In this paper, by considering the relationship between a fuzzy point and a fuzzy set we introduce the notion of $(\in, \in \vee q_k)$-fuzzy filter in CI-algebras. We state and prove some theorems in $(\in, \in \vee q_k)$-fuzzy filter of CI-algebras also we study relationship between $(\in, \in \vee q_k)$-fuzzy filters and fuzzy filters.

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1. Introduction

In 1966, Imai and Iseki defined a class of algebras of type (2,0) called BCK-algebras. There exist several generalizations of BCK-algebras such as BCH-algebras, BCC-algebras, BH-algebras, d-algebras, BE- algebras, etc.

In 2009, B. L. Meng introduced the notion of a CI-algebra as a generalization of BE-algebra and dual $BCK/BCI/BCH$-algebras[3]. For further study see [2, 4, 5].

The theory of fuzzy sets was first developed by Zadeh[7] and has been applied to many branches in mathematics. A new type of fuzzy subgroup, that is, the $(\in, \in \vee q)$-fuzzy subgroup, was introduced in an earlier paper of Bhakat and Das [1] by using the combined notions of "belongingness" and "quasicoincidence" of fuzzy points and fuzzy sets which was introduced by Pu and Liu [6].

In this paper, we introduce the concept of $(\in, \in \vee q_k)$-fuzzy filter of CI-algebras and deal with related properties. We investigate the relationship between $(\in, \in \vee q_k)$-fuzzy filter and fuzzy filters. We give some conditions for a fuzzy set μ to be an $(\in, \in \vee q_k)$-fuzzy filter. We establish characterizations of an $(\in, \in \vee q_k)$-fuzzy filter.

2. Preliminaries

Let $K(\tau)$ be the class of all algebras of type $\tau = (2, 0)$.

Definition 2.1. An element $X \in K(\tau)$ is called a CI-algebra if it satisfies the following axioms:
(a1) $x * x = 1$,
(a2) $1 * x = x$,
(a3) $x * (y * z) = y * (x * z)$,
for all $x, y, z \in X$. If a CI-algebra X satisfies:
(a4) $x * 1 = 1$ for all $x \in X$,
Then we say that X is a BE-algebra.

We can define a partial ordering $\leq$ on X by
$(\forall x, y \in X)(x \leq y \Leftrightarrow x * y = 1)$.
In a CI-algebra X, the following hold (see[3]);
(b1) $y * ((y * x) * x) = 1$
(b2) $(x * 1) * (y * 1) = (x * y) * 1$
(b3) $1 \leq x \Rightarrow x = 1$,
for all $x, y \in X$.
A CI-algebra X is said to be self-distributive if $x * (y * z) = (x * y) * (x * z)$, for all $x, y, z \in X$. A CI-algebra X is said to be transitive (see[3]) if it satisfies:
$(\forall x, y, z \in X)((y * z) * ((x * y) * (x * z)) = 1)$.
A subset F of a CI-algebra X is called a filter of X(see[3]) if it satisfies;
(F1) $1 \in F$
(F2) $(\forall x, y \in X)(x * y \in F, x \in F \Rightarrow y \in F)$.

Definition 2.2 ([4])**.** A Fuzzy set μ in a CI-algebra X is called a fuzzy filter of X if it satisfies;
(F3) $(\forall x \in X)(\mu(1) \geq \mu(x))$,
(F4) $(\forall x, y \in X)(\mu(y) \geq min\{\mu(x * y), \mu(x)\})$,
For any fuzzy set μ in X and $t \in (0, 1]$, the set

$$U(\mu; t) = \{x \in X \mid \mu(x) \geq t\}$$

is called a level subset of X. A fuzzy set μ in a set X of the form

$$\mu(y) := \begin{cases} t \in (0,1] \quad if & y = x, \\ 0 & y \neq x \end{cases} \qquad (1)$$

is said to be a fuzzy point with support x and value t and is denoted by (x, t).
For a fuzzy point (x,t) and a fuzzy set μ in a set X, Pu and Liu[6] introduced the symbol $(x, t)\alpha\mu$, where $\alpha \in \{\in, q, \in \vee q, \in \wedge q\}$. We say that $(x, t) \in \mu$(resp. $(x, t)q\mu$, it means $\mu(x) \geq t(resp.\mu(x)+t > 1)$, and in this case, (x, t) is said to belong to (resp. be quasi- coincident whit)a fuzzy set μ. To say that $(x, t \in \vee q\mu(resp.(x, t) \in \wedge q\mu))$, we mean $(x, t) \in \mu$ or $(x, t)q\mu$(resp.$(x, t)\mu$) and $(x, t)q\mu$.

3. Main results

In what follows, X is a CI-algebra and $\kappa \in [0, 1]$ unless otherwise specified.
Also we define for $\alpha \in \{\in, \in \vee q_k\}$

(i) $(x,t)q_k\mu$, we mean $\mu(x)+t+k>1$,
(ii) $(x,t)\in\vee q_k\mu$, means that $(x,t)\in\mu$ or $(x,t)q_k\mu$.
For say that $(x,t)\overline{\alpha}\mu$, we mean $(x,t)\alpha\mu$ dose not hold.

Definition 3.1. A fuzzy set μ in X is called an $(\in,\in\vee q_k)$-fuzzy filter of X if it satisfies
$(c_1)(x,t)\in\mu\Rightarrow(1,t)\in q_k\mu$,
$(c_2)(x,t)\in\mu$ and $(x*y,r)\in\mu\Rightarrow(y,min\{t,r\})\in\vee q_k\mu$ for all $x,y\in X$and $t,r\in(0,1]$.

An $(\in,\in\vee q_k)$-fuzzy filter of X with $k=0$ is called $(\in,\in\vee q)$-fuzzy filter of X.

Example 3.2. Let $X=\{1,a,b,c\}$ be a set with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	c	c	c	1

Then $(X,*,0)$ is a CI-algebra. Define a fuzzy set $\mu:X\to[0,1]$ on X, by $\mu(1)=0.8,\mu(a)=\mu(b)=0.4$ and $\mu(c)=0.3$. Then μ is a $(\in,\in\vee q_k)$-fuzzy filter of X.

Theorem 3.3. *A fuzzy Set μ in X is an $(\in,\in\vee q_k)$-fuzzy filter of X if and only if it satisfies two conditions:*
$(c_3)(\forall x\in X,\mu(1)\geq min\{\mu(x),\frac{1-k}{2}\})$,
$(c_4)(\forall x,y\in X,\mu(y)\geq min\{\mu(x),\mu(x*y),\frac{1-k}{2}\})$.

Proof. Assume that μ is an $(\in,\in\vee q_k)$-fuzzy filter of X, if (c_3) is not valid, then $\mu(1)<t_a\leq min\{\mu(a),\frac{1-k}{2}\}$, for some $a\in X$ and $t_a\in(0,\frac{1-k}{2}]$. Thus $(a,t_a)\in\mu$ but $(1,t_a)\overline{\in}\mu$. Also, $\mu(1)+t_a<2t_a\leq1-k$, i.e. $(1,t_a)\overline{q_k}\mu$. Therefore $(1,t_a)\in\overline{\vee q_k}\mu$, which is a contradiction. consequently, $\mu(1)\geq min\{\mu(x),\frac{1-k}{2}\}$ for all $x\in X$. Assume that (c_4) is not valid. Then there exist $a,b\in X$ and $t\in(0,\frac{1-k}{2}]$ such that $\mu(b)\leq t\leq min\{\mu(a),\mu(a*b),\frac{1-k}{2}\}$. If $min\{\mu(a),\mu(a*b)\}<\frac{1-k}{2}$, then $\mu(b)<t\leq min\{\mu(a),\mu(a*b)\}$, Hence $(a,t)\in\mu$ and $(a*b)\in\mu$ but $(y,t)\overline{\in}\mu$. Moreover, $\mu(b)+t<2t\leq1-k$, and so $(b,t)\overline{q_k}\mu$. Therefore $(b,t)\overline{\in\vee q_k}\mu$, which is a contradiction, if $min\{\mu(a),\mu(a+b)\geq\frac{1-k}{2},\}$ then $(a,\frac{1-k}{2})\in\mu$ and $(a+b,\frac{1-k}{2})\in\mu$ but $(b,\frac{1-k}{2})\overline{\in}\mu$. Also $\mu(b)+\frac{1-k}{2}<\frac{1-k}{2}+\frac{1-k}{2}=1-k$, i.e. $(b,\frac{1-k}{2})\overline{q_k}\mu$. Hence $(b,\frac{1-k}{2})\overline{\in\vee q_k}\mu$ which is a contradiction. Consequently, $\mu(b)\geq min\{\mu(a),\mu(a*b),\frac{1-k}{2}\}$ for all $x,y\in X$.

Conversely, let μ be a fuzzy set in X satisfying (c_3) and (c_4), Let $x\in X$ and $t\in(0,1]$ be such that $(x,t)\in\mu$. Then $\mu(x)\geq t$, and so $\mu(1)\geq min\{\mu(x),\frac{1-k}{2}\}\geq min\{t,\frac{1-k}{2}\}$. If $t\leq\frac{1-k}{2}$, then $\mu(1)\geq t$, i.e, $(1,t)\in\mu$. If $t>\frac{1-k}{2}$, then $\mu(1)\geq\frac{1-k}{2}$. Thus $\mu(1)+t>\frac{1-k}{2}+\frac{1-k}{2}=1-k$, i.e, $(1,t)q_k\mu$. Hence $(1,t)\in\vee q_k\mu$, which proves (c_1). Let $x,y\in X$ and $t,r\in(0,1]$ be such that $(x,t)\in\mu$ and $(x*y,r)\in\mu$. Then $\mu(x)\geq t$ and $\mu(x*y)\geq r$. It follows from (c_4) that $\mu(y)\geq min\{\mu(x),\mu(x,y),\frac{1-k}{2}\}$

$$\geq min\{t,r,\tfrac{1-k}{2}\}:=\begin{cases} min\{t,r\} & if \quad t\leq\frac{1-k}{2} \quad or \quad r\leq\frac{1-k}{2},\\ \frac{1-k}{2} & if \quad t>\frac{1-k}{2} \quad or \quad r>\frac{1-k}{2}\end{cases}$$

The case $\mu(y) \geq min\{t, r\}$ implies that $(y, min\{t, r\}) \in \mu$. From the $\mu(y) \geq \frac{1-k}{2}$ we have $\mu(y) + min\{t, r\} > \frac{1-k}{2} + \frac{1-k}{2} = 1 - k$,

i.e, $(y, min\{t, r\})q_k\mu$. Hence $(y, min\{t, r\}) \in \vee q_k\mu$. Therefore the condition (c_2) is valid . Consequently, μ is an $(\in, \in \vee q_k)$- fuzzy filter of L. □

Theorem 3.4. *A fuzzy set μ in L is an $(\in, \in \vee q_k)$-fuzzy filter of L if and only if it satisfies:*

$(*)\forall t \in (0, \frac{1-k}{2}]U(\mu; t) \neq \varnothing \Rightarrow U(\mu; t)$ *is a filter of L.*

Proof. Let μ be an $(\in, \in \vee q_k)$- fuzzy filter of L. Let $t \in (0, \frac{1-k}{2}]$ be such that $U(\mu; t) \neq \varnothing$. Obviously, $1 \in U(\mu; t)$ for all $t \in (0, \frac{1-k}{2}]$. Let $x, y \in X$ be Such that $x \in U(\mu; t)$ and $x * y \in U(\mu; t)$. Then $\mu(x) \geq t$ and $\mu(x * y) \geq t$. It follows from (c_4) that $\mu(x * y) \geq min\{\mu(x), \mu(x * y), \frac{1-k}{2}\} \geq min\{t, \frac{1-k}{2}\} = t$ so that $y \in U(\mu; t)$. Hence $U(\mu; t)$ is a filter of X.

Conversely, let μ be a fuzzy set in X in which $(*)$ is valid. If there exists a $a \in X$ such that $\mu(1) < min\{\mu(a), \frac{1-k}{2}\}$, then $\mu(1) < t_a \leq min\{\mu(a), \frac{1-k}{2}\}$ for some $t_a \in (0, \frac{1-k}{2}]$. Thus $(a, t_a) \in \mu$ but $(1, t_a)\overline{\in}\mu$. Also, $\mu(1) + t_a < 2t_a \leq 1 - k$, i.e, $(1, t_a)\overline{q_k}\mu$. Hence $(1, t_a)\overline{\in \vee q_k}\mu$, which is a contradiction. Therefore $\mu(1) \geq min\{\mu(x), \frac{1-k}{2}\}$ for all $x \in X$. Assume that exist $a, b \in X$ such that $\mu(b) < min\{\mu(a), \mu(a * b), \frac{1-k}{2}\}$. Then $\mu(b) < t \leq min\{\mu(a), \mu(a * b), \frac{1-k}{2}\}$ for some $t \in (0, \frac{1-k}{2}]$, and so $a \in U(\mu; t)$ and $a * b \in U(\mu; t)$, but $b \notin U(\mu; t)$. Since $U(\mu; t)$ is a filter of X, it is a contradiction. Therefore $\mu(y) \geq min\{\mu(x), \mu(x * y), \frac{1-k}{2}\}$ for all $x, y \in X$. Consequently, we conclude that μ is an $(\in, \in \vee q_k)$-fuzzy filter of X by Theorem 3.3. □

Corollary 3.5. *A fuzzy set μ in X is an $(\in, \in \vee q)$ fuzzy filter of X if and only if it satisfies:*

$\forall t \in (0, 0.5], U(\mu; t) \neq \varnothing \Rightarrow U(\mu; t)$ *is a filter of X.*

Theorem 3.6. *If A is a filter of X, then a fuzzy set μ in X defined by*

$$\mu : X \to [0, 1], x := \begin{cases} t_1 & if \quad x \in A, \\ t_2 & otherwise \end{cases}$$

where $t_1 \in [\frac{1-k}{2}, 1]$ and $t_2 \in (0, \frac{1-k}{2})$, is an $(\in, \in \vee q_k)$-fuzzy filter of X.

Proof. Note that

$$U(\mu; r) := \begin{cases} A & if \quad r \in (t_2, \frac{1-k}{2}], \\ X & r \in (0, t_2] \end{cases}$$

which is a filter of X. It follows from Theorem 3.4.

That μ is an $(\in, \in \vee q_k)$-fuzzy filter of X. □

Corollary 3.7. *If A is a filter of X, then a fuzzy set μ in X defined by*

$$\mu : L \to [0, 1], x := \begin{cases} t_1 & if \quad x \in A, \\ t_2 & otherwise \end{cases}$$

where $t_1 \in [0.5, 1]$ and $t_2 \in (0, 0.5)$, is an $(\in, \in \vee q_k)$-fuzzy filter of X.

Theorem 3.8. *Every fuzzy filter is an $(\in, \in \vee q_k)$-fuzzy filter.*

We show that the converse of above theorem is not true in general.

Example 3.9. Let $X = \{1, a, b, c\}$ be a set with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	1	a	b	1

Then $(X, *, 0)$ is a CI-algebra. Define a fuzzy set $\mu : X \to [0, 1]$ on X, by $\mu(1) = \mu(a) = \mu(b) = 0.6$ and $\mu(c) = 0.5$ and $k = 0.2$. Then μ is a $(\in, \in \vee q_k)$-fuzzy filter of X but is not a fuzzy filter because $\mu(c)$ is not $\geq min\{\mu(b), \mu(c * b)\}$.

Theorem 3.10. *If μ is an $(\in, \in \vee q_k)$-fuzzy filter satisfying $\mu(1) < \frac{1-k}{2}$, then μ is a fuzzy filter.*

Proof. Let μ be an $(\in, \in \vee q_k)$-fuzzy filter of X such that $\mu(1) < \frac{1-k}{2}$, using (c_3) we have $min\{\mu(x), \frac{1-k}{2}\} \leq \mu(1) < \frac{1-k}{2}$ and so $\mu(x) \leq \frac{1-k}{2}$ for all $x \in X$. It follows from (c_3) and (c_4) that $\mu(1) \geq \mu(x)$ and $\mu(y) \geq min\{\mu(x), \mu(y * x)\}$ for all $x, y \in X$. Hence μ is a fuzzy filter of X. □

Corollary 3.11. *If μ is an $(\in, \in \vee q)$-fuzzy filter satisfying $\mu(1) < 0.5$, then μ is a fuzzy filter.*

Proposition 3.12. *For any $k_1, k_2 \in (0, 1]$ with $k_1 < k_2$, every $(\in, \in \vee q_{k_1})$-fuzzy filter is an $(\in, \in \vee q_{k_2})$-fuzzy filter.*

Example 3.13. Let $X = \{1, a, b, c\}$ be a set with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	1	a	b	1

Then $(X, *, 0)$ is a CI-algebra. Define a fuzzy set $\mu : X \to [0, 1]$ on X, by $\mu(1) = \mu(a) = \mu(b) = 0.6$ and $\mu(c) = 0.5$ and $k_1 = 0.1$. Then μ is a $(\in, \in \vee q_{k_1})$-fuzzy filter of X. Also by $\mu(1) = \mu(a) = \mu(b) = 0.6$ and $\mu(c) = 0.4$ and $k_2 = 0.4$, then μ is a $(\in, \in \vee q_{k_2})$-fuzzy filter of X, but $(\in, \in \vee q_{k_2})$-fuzzy filter is not an $(\in, \in \vee q_{k_1})$-fuzzy filter.

For any fuzzy set μ in X and any $t \in (0, 1]$, we consider for subsets:

$$Q(\mu; t) := \{x \in X \mid (x, t)q\mu\}, [\mu]_t := \{x \in X | (x, t) \in \vee q\mu\},$$

$$Q^k(\mu; t) := \{x \in X | (x, t)q_k\mu\}, [\mu]_t^k := \{x \in X | (x, t) \in \vee q_k\mu\}$$

It is clear that $[\mu]_t = U(\mu; t) \bigcup Q(\mu; t)$ and $[\mu]_t^k = U(\mu; t) \bigcup Q^k(\mu; t)$.

Theorem 3.14. *If μ is an $(\in, \in \vee q_k)$-fuzzy filter of X, then $Q^k(\mu; t)$ is a filter of X whenever $Q^k(\mu; t) \neq \varnothing$, for all $t \in (\frac{1-k}{2}, 1]$.*

Proof. Assume that μ is an $(\in, \in \vee q_k)$-fuzzy filter of X and let $t \in (\frac{1-k}{2}, 1]$ be such that $Q^k(\mu; t) \neq \varnothing$.Then there exists $x \in Q^k(\mu; t)$, and so $\mu(x) + t + k > 1$. It follows (c_3) that $\mu(1) \geq min\{\mu(x), \frac{1-k}{2}\} \geq min\{1 - t - k, \frac{1-k}{2}\} = 1 - t - k$

so that $1 \in Q^k(\mu; t)$. Let $x, y \in X$ be such that $x \in Q^k(\mu; t)$ and $x * y \in Q^k(\mu; t)$. Then $(x, t)q_k\mu$ and $(x + y, t)q_k\mu$,i.e, $\mu(x) + t + k > 1$ and $\mu(x * y) + t + k > 1$. Using (c_4), we have $\mu(y) \geq min\{\mu(x), \mu(x * y), \frac{1-k}{2}\} \geq min\{1 - t - k, \frac{1-k}{2}\} = 1 - t - k$ and so $(y, t)q_k\mu$, that is , $y \in Q^k(\mu; t)$. Therefore $Q^k(\mu; t)$ is a filter of X. □

Corollary 3.15. *If μ is an $(\in, \in \vee q)$ -fuzzy filter of X, then $\forall t \in (0.5, 1], Q(\mu; t) \neq \varnothing \Rightarrow Q(\mu; t)$ is a filter of X.*

Proof. It is obvious by taking $k = 0$ in Theorem 3.14. □

Corollary 3.16. *Let $k, r \in (0, 1]$ with $k < r$. If μ is an $(\in, \in \vee q_k)$-fuzzy filter of X, then $Q^r(\mu; t)$ is an filter of X whenever $Q^r(\mu; t) \neq \varnothing$ for all $t \in (\frac{1-r}{1}, 1]$.*

Proof. It is clear by Proposition 3.12 and Theorem 3.14. □

Theorem 3.17. *For any fuzzy μ in X , the following are equivalent:*
(1) *μ is an $(\in, \in \vee q_k)$-fuzzy filter of X.*
(2) *$\forall t \in (0, 1]; [\mu]^k_t \neq \varnothing \Rightarrow [\mu]^k_t$ is a filter of X.*

Proof. Assume that μ is an $(\in, \in \vee q_k)$-fuzzy filter of X and let $t \in (0, 1]$ be such that $[\mu]^k_t \neq \varnothing$. Then there exists $x \in [\mu]^k_t = U(\mu; t) \bigcup Q^k(\mu; t)$, and so $x \in U(\mu; t)$ or $x \in Q^k(\mu; t)$, If $x \in U(\mu; t)$. then (c_3) implies that

$$\mu(1) \geq min\{\mu(x), \tfrac{1-k}{2}\} \geq min\{t, \tfrac{1-k}{2}\}$$
$$= \begin{cases} t & if \quad t \leq \frac{1-k}{2}, \\ \frac{1-k}{2} > 1 - t - k & if \quad t > \frac{1-k}{2} \end{cases}$$

Thus $1 \in U(\mu; t) \bigcup Q^k(\mu; t) = [\mu]^k_t$.

Assume that $x \in Q^k(\mu; t)$. Then $(x, t)q_k\mu$, i.e, $\mu(x) + t + k > 1$. Thus if $t > \frac{1-k}{2}$, then

$$\mu(1) \geq min\{\mu(x), \tfrac{1-k}{2}\} := \begin{cases} \mu(x) > 1 - t - k & if \quad \mu(x) < \frac{1-k}{2}, \\ \frac{1-k}{2} > 1 - k - t & if \quad \mu(x) \geq \frac{1-k}{2} \end{cases}$$

and so $1 \in Q^k(\mu; t) \subseteq [\mu]^k_t$. If $t \leq \frac{1-k}{2}$, then

$$\mu(1) \geq min\{\mu(x), \tfrac{1-k}{2}\} := \begin{cases} \mu(x) > 1 - t - k & if \quad \mu(x) < \frac{1-k}{2}, \\ \frac{1-k}{2} \geq t & if \quad \mu(x) \geq \frac{1-k}{2} \end{cases}$$

which implies that $1 \in U(\mu; t) \bigcup Q^k(\mu; t) = [\mu]^k_t$. Let $x, y \in X$ be such $x \in [\mu]^k_t$ and $x + y \in [\mu]^k_t$. Then
$\mu(x) \geq t$ or $\mu(x) + t + k > 1$.
and
$\mu(x + y) \geq t$ or $\mu(x + y) + t + k > 1$
we can consider four cases:
(1) $\mu(x) \geq t$ and $\mu(x * y) \geq t$
(2) $\mu(x) \geq t$ and $\mu(x * y) + t + k > 1$,
(3) $\mu(x) + t + k > 1$ and $\mu(x * y) \geq t$,
(4) $\mu(x) + t + k > 1$ and $\mu(x * y) + t + k > 1$.
for the first case, (c_4) implies that

$$\mu(y) \geq min\{\mu(x), \mu(x * y), \tfrac{1-k}{2}\}$$
$$\geq min\{t, \tfrac{1-k}{2}\} := \begin{cases} \frac{1-k}{2} & if \quad t > \frac{1-k}{2}, \\ t & if \quad t \leq \frac{1-k}{2} \end{cases}$$

and so $\mu(y)+t+k > \frac{1-k}{2} + \frac{1-k}{2} + k = 1$, i.e, $(y,t)q_k\mu$, or $y \in U(\mu;t)$. Therefore $y \in U(\mu;t) \bigcup Q^k(\mu;t) = [\mu]_t^k$. For the case (3), $\mu(y) \geq min\{\mu(x), \mu(x*y), \frac{1-k}{2}\} = min\{\mu(x*y), \frac{1-k}{2}\} > 1-t-k$

we have $min\{\mu(x*y), \frac{1-k}{2}\} \leq \mu(x)$; and $\mu(y) \geq min\{\mu(x), \mu(x*y), \frac{1-k}{2}\} = \mu(x) \geq t$

whenever $min\{\mu(x*y), \frac{1-k}{2}\} \leq \mu(x)$; and $\mu(y) \geq min\{\mu(x), \mu(x*y), \frac{1-k}{2}\} = \mu(x) \geq t$

whenever $min\{\mu(x*y), \frac{1-k}{2}\} > \mu(x)$. Thus $y \in U(\mu;t) \bigcup Q^k(\mu;t) = [\mu]_t^k$. suppose that $t \leq \frac{1-k}{2}$. Then $1-t \geq \frac{1-k}{2}$, which implies that $\mu(y) \geq min\{\mu(x), \mu(x*y), \frac{1-k}{2}\} = min\{\mu(x), \frac{1-k}{2}\} \geq t$ whenever $min\{\mu(x), \frac{1-k}{2}\} > \mu(x*y)$ and thus $y \in U(\mu;t) \bigcup Q^k(\mu;t) = [\mu]_t^k$.

We have similar result of the case (4), for the final case , if

$t > \frac{1-k}{2}$, then $1-t-k \leq 1-t < \frac{1-k}{2}$. Hence

$\mu(y) \geq min\{\mu(x), \mu(x*y), \frac{1-k}{2}\} = min\{\mu(x), \mu(x*y)\} > 1-t-k$

whenever $min\{\mu(x), \mu(x*y)\} < \frac{1-k}{2}$. Hence $y \in Q^k(\mu;t) \subseteq [\mu]_t^k$. If $t \leq \frac{1-k}{2}$. then

$\mu(y) \geq min\{\mu(x), \mu(x*y), \frac{1-k}{2}\} = \frac{1-k}{2} \geq t$

whenever $min\{\mu(x), \mu(x*y)\} \geq \frac{1-k}{2}$; and

$\mu(y) \geq min\{\mu(x), \mu(x*y), \frac{1-k}{2}\} = min\{\mu(x), \mu(x*y)\} > 1-t-k$

whenever $min\{\mu(x), \mu(x*y)\} < \frac{1-k}{2}$. Thus $y \in U(\mu;t) \bigcup Q^k(\mu;t) = [\mu]_t^k$, therefore $[\mu]_t^k$ is a filter of X.

Conversely, let μ be a fuzzy set in X such that $[\mu]_t^k$ is a filter of X for all $t \in (0,1]$. If there exists $a \in X$ such that $\mu(1) < min\{\mu(a), \frac{1-k}{2}\}$, then $\mu(1) < t_a \leq min\{\mu(a), \frac{1-k}{2}\}$ for some $t_a \in (0, \frac{1-k}{2}]$. It follows that $a \in U(\mu;t_a) \subseteq [\mu]_{t_a}^k$ but $1 \notin U(\mu;t_a)$. Also, $\mu(1)+t_a < 2t_a \leq 1-k$, and so $(1,t_a)\overline{q_k}\mu$, i.e, $1 \notin Q^k(\mu;t_a)$. Therefore $1 \notin [\mu]_{t_a}^k$, which is a contradiction.

Hence $\mu(1) \geq min\{\mu(x), \frac{1-k}{2}\}$ for all $x \in X$. Suppose that there exist $a, b \in X$ such that $\mu(b) < min(\mu(a), \mu(a*b), \frac{1-k}{2}\}$. Then

(5) $\mu(b) < t_b \leq min\{\mu(a), \mu(a*b), \frac{1-k}{2}\}$

for some $t_b \in (0, \frac{1-k}{2}]$, which implies $a, a*b \in U(\mu;t_b) \subseteq [\mu]_{t_b}^k$ so from (b_2) that $b \in [\mu]_{t_b}^k = U(\mu;t_b) \bigcup Q^k(\mu;t_b)$ since $[\mu]_{t_b}^k$ is a filter of X. But , (5) implies that $b \notin U(\mu;t_b)$ and $\mu(b)+t_b < 2t_b \leq 1-k$ i.e, $b \notin Q^k(\mu;t_b)$. Which is a contradiction. Therefore $\mu(y) \geq min\{\mu(x), \mu(x*y), \frac{1-k}{2}\}$ for all $x, y \in X$. By Theorem 3.3, we conclude that μ is an $(\in, \in \vee q_k)$-fuzzy filter of X. □

Corollary 3.18. *For any fuzzy set μ in X, the following conditions are equivalent*

(1) *μ is an $(\in, \in \vee q)$-fuzzy filter of X,*

(2) *$\forall \in [0,1]; [\mu]_t \neq \varnothing \Rightarrow [\mu]_t$ is a filter of X.*

4. Conclusions

In this paper, we defined some types of $(\in, \in \vee q_k)$-fuzzy filter of CI-algebra and investigated the relationship between filters and $(\in, \in \vee q_k)$-fuzzy filter which seems to be a good support for future study. The other direction of future study is an investigation of $(\in, \in \vee q)$-fuzzy filter on interval valued.

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