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On soft fuzzy* topological groups

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Abstract. In this paper, the concepts of soft fuzzy* topological spaces and soft fuzzy* groups are introduced. In this connection, the concept of soft fuzzy* topological group is introduced. The concepts of Homomorphic images and inverse images of soft fuzzy* topological groups are studied.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh [7]. Fuzzy sets have applications in many fields such as information [4] and control [5]. The theory of fuzzy topological spaces was introduced and developed by Chang [1] and since then various notions in classical topology has been extended to fuzzy topological spaces. A. Rosenfeld [3] formulated the elements of a theory of fuzzy groups. David Foster [2] introduced the concept of fuzzy topological groups. The concept of soft fuzzy topological space was introduced by Ismail U. Triyaki [6]. In this paper, the concepts of soft fuzzy* topological groups are introduced and some basic properties are studied.

2. Preliminaries

Definition 2.1 ([2])**.** Let X be a group and G be a fuzzy set in X with membership function μ_G. Then G is a ***fuzzy group*** in X iff the following conditions are satisfied:
(i) $\mu_G(xy) \geq min\{\mu_G(x), \mu_G(y)\}$, for all $x, y \in X$;
(ii) $\mu_G(x^{-1}) \geq \mu_G(x)$, for all $x \in X$.

Definition 2.2 ([6])**.** Let X be a non-empty set. A ***soft fuzzy set*** (in short, SFS) A have the form $A = (\lambda, M)$ where the function $\lambda : X \to I$ denotes the degree of membership and M is the subset of X. The set of all soft fuzzy set will be denoted by ***SF(X)***.

Definition 2.3 ([6]). The relation $\sqsubseteq$ on $SF(X)$ is given by $(\mu, N) \sqsubseteq (\lambda, M) \Leftrightarrow \mu(x) \leq \lambda(x), \forall x \in X$ and $M \subseteq N$.

Proposition 2.4 ([6]). *If $(\mu_j, N_j) \in SF(X), j \in J$, then the family $\{(\mu_j, N_j)|j \in J\}$ has a meet, ie., g.l.b., in $(SF(X), \sqsubseteq)$ denoted by $\sqcap_{j\in J}(\mu_j, N_j)$ and given by $\sqcap_{j\in J}(\mu_j, N_j) = (\mu, N)$ where*

$$\mu(x) = \wedge_{j\in J}\mu_j(x)\forall x \in X$$

and

$$M = \cap M_j \text{ for } j \in J.$$

Proposition 2.5 ([6]). *If $(\mu_j, N_j) \in SF(X)$, $j \in J$, then the family $\{(\mu_j, N_j)|j \in J\}$ has a join, ie., l.u.b., in $(SF(X), \sqsubseteq)$ denoted by $\sqcup_{j\in J}(\mu_j, N_j)$ and given by $\sqcup_{j\in J}(\mu_j, N_j) = (\mu, N)$ where*

$$\mu(x) = \vee_{j\in J}\mu_j(x)\forall x \in X$$

and

$$M = \cup M_j \text{ for } j \in J.$$

Definition 2.6 ([6]). Let X be a set. Let T be family of soft fuzzy subsets of X. Then T is called a soft fuzzy topology on X if T satisfies the following conditions:
(i) $(0, \varnothing)$ and $(1, X) \in T$.
(ii) If $(\mu_j, N_j) \in T, j = 1, 2......n$ then $\sqcap_{j=1}^{n}(\mu_j, N_j) \in T$.
(iii) If $(\mu_j, N_j) \in T, j \in J$ then $\sqcup_{j\in J}(\mu_j, N_j) \in T$.
The pair (X, T) is called a ***soft fuzzy topological space*** (in short, *SFTS*). The members of T are ***soft fuzzy open sets*** and its complement are ***soft fuzzy closed sets***.

Proposition 2.7 ([4]). *Let $\varphi : X \to Y$ be a point function.*
(i) *The mapping $\varphi^{\rightharpoonup}$ from $SF(X)$ to $SF(Y)$ corresponding to the* **image operator** *of the difunction (f, F) is given by*

$$\varphi^{\rightharpoonup}(\mu, N) = (\nu, L) \text{ where } \nu(y) = sup\{\mu(x)|y = \varphi(x)\}, \text{ and}$$

$$L = \{\varphi(x)|x \in N \text{ and } \nu(\varphi(x)) = \mu(x)\}.$$

(ii) *The mapping $\varphi^{\leftharpoonup}$ from $SF(X)$ to $SF(Y)$ corresponding to the* **inverse image** *of the difunction (f, F) is given by*

$$\varphi^{\leftharpoonup}(\nu, L) = (\nu \circ \varphi, \varphi^{-1}[L]).$$

3. Soft fuzzy* set

Definition 3.1. Let X be a non empty set and $M \subseteq X$. Then the pair (λ, M) is said to be ***soft fuzzy* set*** if $\lambda : M \to I = [0, 1]$. The collection of all soft fuzzy* sets is denoted by ***SF*(X)***.

Definition 3.2. Let $(\lambda, M), (\mu, N) \in SF^*(X)$. Then (λ, M) is called a ***soft fuzzy* subset*** of (μ, N) if $M \subseteq N$ and $\lambda(e) \leq \mu(e)$ for each $e \in M$.

Definition 3.3. The ***union*** of two soft fuzzy* sets (λ, M) and (μ, N) over X is the soft fuzzy* set $(\gamma, L) = (\lambda, M) \sqcup (\mu, N)$ where $L = M \cup N$ and $\gamma(e)=$
$$\begin{cases} \lambda(e) & \text{if} \quad e \in M \backslash N \\ \mu(e) & \text{if} \quad e \in N \backslash M \\ \lambda(e) \vee \mu(e) & \text{if} \quad e \in M \cap N \end{cases}$$

Definition 3.4. If (λ, M) and (μ, N) be two soft fuzzy* sets then the ***intersection*** of (λ, N) and (μ, N) is a soft fuzzy* set $(\gamma, L) = (\lambda, M) \sqcap (\mu, N)$ where $L = M \cap N$and $\gamma(e) = \lambda(e) \wedge \mu(e)$ for all $e \in L$.

Definition 3.5. Let $(\lambda, M) \in SF^*(X)$. Then the ***complement*** of (λ, M) is denoted by $(\lambda, M)'$ is the soft fuzzy* set defined by $(\lambda, M)' = (1, X) - (\lambda, M)$ where $\lambda'(e) = 1 - \lambda(e)$ for all $e \in M$ and $M' = X \backslash M$.

Definition 3.6. Let $(\lambda, M) \in SF^*(X)$. Then the soft fuzzy* set (λ, M) is the ***soft fuzzy* null set*** denoted by $(0, \varnothing)$ if $\lambda(e) = 0$ for every $e \in M$.

Definition 3.7. Let $(\lambda, M) \in SF^*(X)$. Then the soft fuzzy* set (λ, M) is the ***soft fuzzy* universal set*** denoted by $(1, X)$ if $\lambda(e) = 1$ for every $e \in M$.

Definition 3.8. Let $f : X \to Y$ be a function.

(i) the mapping f from $SF^*(X) \to SF^*(Y)$ corresponding to the ***image*** operator of the difunction (ϕ, ψ) is given by $f(\mu, N) = (\gamma, L)$ where $\gamma(y) = sup\{\mu(x) | y = f(x), x \in N\}, y \in L$.

(ii) the mapping f^{-1} from $SF^*(Y) \to SF^*(X)$ corresponding to the ***inverse image*** of the difunction (ϕ, ψ) is given by $f^{-1}(\mu, N) = \mu \circ f$.

Property 3.9. *Let f be a mapping from a set X to a set Y. Let $\{(\lambda_j, M_j)\}_{j \in J}$ be a family of soft fuzzy* sets in X and $\{(\mu_j, N_j)\}_{j \in J}$ a family of soft fuzzy* sets in Y. Then*

(i) $f^{-1}(\sqcup_{j \in J}(\mu_j, N_j)) = \sqcup_{j \in J} f^{-1}(\mu_j, N_j)$
(ii) $f^{-1}(\sqcap_{j \in J}(\mu_j, N_j)) = \sqcap_{j \in J} f^{-1}(\mu_j, N_j)$
(iii) $f(\sqcup_{j \in J}(\lambda_j, M_j)) = \sqcup_{j \in J} f(\lambda_j, M_j)$
(iv) $f(\sqcap_{j \in J}(\lambda_j, M_j)) \sqsubseteq \sqcap_{j \in J} f(\lambda_j, M_j)$

Proof. (i) For all $e \in \cup_{j \in J} N_j$ then
$$\begin{aligned} f^{-1}(\sqcup_{j \in J}(\mu_j, N_j)) &= \vee_{j \in J} \mu_j \circ f(e) \\ &= \vee_{j \in J} (\mu_j \circ f)(e) \\ &= \vee_{j \in J} \nu_j \circ f \\ &= \sqcup_{j \in J} f^{-1}(\mu_j, N_j) \end{aligned}$$

(ii) The Proof is similar for (i).
(iii) The proof is immediately from the Definition 3.8.
(iv) Let $f(\sqcap_{j \in J}(\lambda_j, M_j)) = (\gamma, L)$. The membership function of (γ, L) is given by
$$\begin{aligned} \gamma(y) &= sup\{\wedge_{j \in J} \lambda_j(x) | y = f(x), x \in \wedge_{j \in J} f(M_j)\} \\ &\sqsubseteq \wedge_{j \in J} sup\{\lambda_j(x) | y = f(x), x \in \wedge_{j \in J} f(M_j)\} \\ &= \sqcap_{j \in J} f(\lambda_j, M_j) \end{aligned}$$
for all $y \in L$. □

4. Soft fuzzy* topological spaces and subspaces

Definition 4.1. A subset $T \subseteq SF^*(X)$ is called soft fuzzy* topology on X if

(i) For all $c \in I$ and $H \subseteq X$, $(k_c, H) \in T$,

(ii) $(\mu_j, N_j) \in T, j = 1, ..., n \Rightarrow \sqcap_{j=1}^{n} (\mu_j, N_j) \in T$,

(iii) $(\mu_j, N_j) \in T, j \in J \Rightarrow \sqcup_{j \in J} (\mu_j, N_j) \in T$

As usual the elements of T are soft fuzzy* open and the complement T' of T is called soft fuzzy* closed.

If T is a soft fuzzy* topology on X we call the pair (X, T) an ***soft fuzzy* topological space***.

Note 4.2. We denote by (k_c, H) the soft fuzzy* set in X with membership function $k_c(x) = c$ for all $x \in H$. The soft fuzzy* set (k_1, X) corresponds to the set $(1, X)$ and the soft fuzzy* set $(k_0, \varnothing)$ to the empty set $(0, \varnothing)$.

Definition 4.3. Let (X, T) be a soft fuzzy* topological space. Let (λ, M) be a soft fuzzy* set. Then

$$T_{(\lambda, M)} = \{(\lambda, M) \sqcap (\delta, P); (\delta, P) \in T\}$$

is called an ***induced soft fuzzy* topology*** on (λ, M) and $((\lambda, M), T_{(\lambda, M)})$ is called a ***soft fuzzy* subspace topology*** on (X, T).

Definition 4.4. Let (λ, M) and (μ, N) be any soft fuzzy* set in X. Then the ***product*** of (λ, M) and (μ, N) is defined by

$$(\lambda \times \mu)_{M \times N}(x) = sup_{x=(x_1, x_2)} min\{\lambda(x_1), \mu(x_2)\}$$

for all $x_1 \in M, x_2 \in N, x \in M \times N$.

Note 4.5. The induced soft fuzzy* topology does not in general satisfy condition (i) of Definition 4.3. Condition (ii), however is satisfied and so is condition (iii). Thus if $(\delta'_j, P'_j) \in T_{(\lambda, M)}$ for all $j \in J$ then there exists $(\delta_j, P_j) \in T, j \in J$ such that $(\delta'_j, P'_j) = (\delta_j, P_j) \sqcap (\lambda, M)$ for each $j \in J$. The union $(\delta', P') = \sqcup_{j \in J} (\delta'_j, P'_j) = \sqcup_{j \in J} ((\delta_j, P_j) \sqcap (\lambda, M))$ has the soft fuzzy* membership function is given by

$$\begin{aligned} \delta'(x) &= sup_{j \in J} \delta'_j(x) \\ &= sup_{j \in J} min\{\delta_j(x), \lambda(x)\} \\ &= min\{sup_{j \in J} \delta_j(x), \lambda(x)\} \\ &= (\sqcup_{j \in J} \delta_j \sqcap \lambda)(x) \end{aligned}$$

for all $x \in P'$. Hence $(\delta', P') = (\sqcup_{j \in J} (\delta_j, P_j)) \sqcap (\lambda, M))$.

Definition 4.6. If $((\lambda, M), T_{(\lambda, M)})$ and $((\mu, N), S_{(\mu, N)})$ are soft fuzzy* subspaces of soft fuzzy topological spaces (X, T) and (Y, S) respectively. If f is a mapping of (X, T) into (Y, S) then f is a mapping of $((\lambda, M), T_{(\lambda, M)})$ into $((\mu, N), S_{(\mu, N)})$ if $f(\lambda, M) \sqsubseteq (\mu, N)$.

Definition 4.7. Let $((\lambda, M), T_{(\lambda, M)})$ and $((\mu, N), S_{(\mu, N)})$ be any two soft fuzzy* subspaces of soft fuzzy* topological spaces (X, T) and (Y, S) respectively. Then a mapping f of $((\lambda, M), T_{(\lambda, M)})$ into $((\mu, N), S_{(\mu, N)})$ is said to be ***soft fuzzy* relatively continuous*** iff for each soft fuzzy* open set (γ', L') in $S_{(\mu, N)}$, $f^{-1}(\gamma', L') \sqcap (\lambda, M)$ is soft fuzzy* open in $T_{(\lambda, M)}$.

Definition 4.8. Let $((\lambda, M), T_{(\lambda,M)})$ and $((\mu, N), S_{(\mu,N)})$ be any two soft fuzzy* subspaces of soft fuzzy* topological spaces (X, T) and (Y, S) respectively. Then a mapping f of $((\lambda, M), T_{(\lambda,M)})$ into $((\mu, N), S_{(\mu,N)})$ is said to be ***soft fuzzy* relatively open*** iff for each soft fuzzy* open set (γ', L') in $T_{(\lambda,M)}$, $f(\gamma', L')'$ is soft fuzzy* open in $S_{(\mu,N)}$.

Property 4.9. *Let $((\lambda, M), T_{(\lambda,M)})$ and $((\mu, N), S_{(\mu,N)})$ be any two soft fuzzy* subspaces of soft fuzzy* topological spaces (X, T) and (Y, S) respectively. Let f be soft fuzzy* continuous mapping of (X, T) into (Y, S) such that $f(\lambda, M) \sqsubseteq (\mu, N)$. Then f is soft fuzzy* relatively continuous mapping of $((\lambda, M), T_{(\lambda,M)})$ into $((\mu, N), S_{(\mu,N)})$.*

Proof. Let (γ', L') be soft fuzzy* open in $S_{(\mu,N)}$. Then there exists soft fuzzy* open (γ, L) in S such that $(\gamma', L') = (\gamma, L) \sqcap (\mu, N)$. The inverse image $f^{-1}(\gamma, L)$ is soft fuzzy* open in T. Hence

$$\begin{aligned} f^{-1}(\gamma', L') \sqcap (\lambda, M) &= f^{-1}(\gamma, L) \sqcap f^{-1}(\mu, N) \sqcap (\lambda, M) \\ &= f^{-1}(\gamma, L) \sqcap (\lambda, M) \end{aligned}$$

is soft fuzzy* open in $T_{(\lambda,M)}$. Therefore f is soft fuzzy* relatively continuous. □

Definition 4.10. A bijective mapping f of a soft fuzzy* topological space (X, T) into (Y, S) is said to be ***soft fuzzy* homeomorphism*** iff it is soft fuzzy* continuous and soft fuzzy* open.

Definition 4.11. A bijective mapping f of a soft fuzzy* subspaces of $((\lambda, M), T_{(\lambda,M)})$ of (X, T) into $((\mu, N), S_{(\mu,N)})$ of (Y, S) is said to be ***soft fuzzy* relatively homeomorphism*** iff $f(\lambda, M) = (\mu, N)$ and f is soft fuzzy* relatively continuous and soft fuzzy* relatively open.

Property 4.12. *Let f be a soft fuzzy* continuous(resp. soft fuzzy* open) mapping of a soft fuzzy* topological space (X, T) into a soft fuzzy* topological space (Y, S) and g a soft fuzzy* continuous(resp. soft fuzzy* open) mapping of (Y, S) into a soft fuzzy* topological space (Z, R). Then the composition $g \circ f$ is a soft fuzzy* continuous(resp. soft fuzzy* open) mapping of (X, T) into (Z, R).*

Proof. It is obvious. □

Property 4.13. *Let $((\lambda, M), T_{(\lambda,M)})$, $((\mu, N), S_{(\mu,N)})$, $((\gamma, L), S_{(\gamma,L)})$ be any three soft fuzzy* subspaces of soft fuzzy* topological spaces (X, T), (Y, S), (Z, R) respectively. Let f be a soft fuzzy* relatively continuous(resp. soft fuzzy* open) mapping of $((\lambda, M), T_{(\lambda,M)})$ into $((\mu, N), S_{(\mu,N)})$ and g be a soft fuzzy* relatively continuous(resp. soft fuzzy* open) mapping of $((\mu, N), S_{(\mu,N)})$ into $((\gamma, L), S_{(\gamma,L)})$. Then the composition $g \circ f$ is soft fuzzy* relatively continuous(resp. soft fuzzy* relatively open) mapping of $((\lambda, M), T_{(\lambda,M)})$ into $((\gamma, L), S_{(\gamma,L)})$.*

Proof. Let (γ', L') be soft fuzzy* open in $Z_{(\gamma,L)}$. Then $g^{-1}((\gamma', L') \sqcap (\mu, N))$ is soft fuzzy* open in $S_{(\mu,N)}$ and $(f^{-1}(g^{-1}(\gamma', L')) \sqcap (\mu, N)) \sqcap (\lambda, M)$. But $(g \circ f)^{-1}(\gamma', L') \sqcap (\lambda, M) = f^{-1}(g^{-1}(\gamma', L') \sqcap (\mu, N)) \sqcap (\lambda, M)$. Since $f(\lambda, M) \sqsubseteq (\mu, N)$ and so $g \circ f$ is soft fuzzy* relatively continuous. The proof is trivial for soft fuzzy* relatively open mappings. □

Definition 4.14. Let (X, T) be a soft fuzzy* topological space. A subfamily $\mathfrak{B}$ of T is a soft fuzzy* base for T iff each member of T can be expressed as the union of members of $\mathfrak{B}$.

Definition 4.15. Let (X, T) be a soft fuzzy* topological space. Let $T_{(\lambda,M)}$ the induced soft fuzzy* topology on a soft fuzzy* open set (λ, M) of (X, T). A subfamily $\mathfrak{B}'$ of $T_{(\lambda,M)}$ is soft fuzzy* base for $T_{(\lambda,M)}$ iff each member of $T_{(\lambda,M)}$ can be expressed as the union of members of $\mathfrak{B}'$.

Note 4.16. If $\mathfrak{B}$ is a soft fuzzy base for a soft fuzzy* topology T on a set X, then

$$\mathfrak{B}_{(\lambda,M)} = \{(\delta, P) \sqcap (\lambda, M) : (\delta, P) \in \mathfrak{B}\}$$

is a soft fuzzy* base for the induced soft fuzzy* topology $T_{(\lambda,M)}$ on the soft fuzzy* open set (λ, M).

Property 4.17. *Let f be a mapping from soft fuzzy* topological space (X, T) to a soft fuzzy* topological space (Y, S). Let $\mathfrak{B}$ be a soft fuzzy* base for S. Then f is soft fuzzy* continuous iff for each soft fuzzy* open (λ, M) in $\mathfrak{B}$ the inverse image $f^{-1}(\lambda, M)$ is soft fuzzy* open is in T.*

Proof. Proof is obvious. □

Property 4.18. *Let $((\lambda, M), T_{(\lambda,M)})$, $((\mu, N), S_{(\mu,N)})$ be soft fuzzy* subspaces of soft fuzzy* topological spaces (X, T), (Y, S) respectively. Let $\mathfrak{B}'$ be a soft fuzzy* base for $S_{(\mu,N)}$. Then a mapping f of $((\lambda, M), T_{(\lambda,M)})$ into $((\mu, N), S_{(\mu,N)})$ is soft fuzzy* relatively continuous iff for each (μ', N') in $\mathfrak{B}'$ the intersection $f^{-1}((\mu', N') \sqcap (\lambda, M))$ is in $T_{(\lambda,M)}$.*

Proof. Proof is obvious. □

Definition 4.19. Let T_1 and T_2 be two soft fuzzy* topologies on the same set X. Then we say that T_1 is ***finer*** that T_2 (and that T_2 is ***coarser*** that T_1) if the identity mapping of (X, T_1) into (X, T_2) is soft fuzzy* continuous.

Definition 4.20. Let $f : X \to Y$. Let T be a soft fuzzy* topology on X. The finest soft fuzzy* topology S on Y for which f is soft fuzzy* continuous is called the image under f of T. A soft fuzzy* set (μ, N) in Y is soft fuzzy* open in S iff $f^{-1}(\mu, N)$ is a soft fuzzy* open set in X.

Definition 4.21. Let $f : X \to Y$ be a mapping. Let S be a soft fuzzy* topology on Y. The coarsest soft fuzzy* topology T on X for which f is soft fuzzy* continuous is called the inverse image under f of S. The soft fuzzy* open sets in X are the inverse images of soft fuzzy* open sets in Y.

Definition 4.22. Given a family $\{(X_j, T_j)\}_{j\in J}$ of a soft fuzzy* topological spaces. Define their product $\Pi_{j\in J}(X_j, T_j)$ to be the soft fuzzy* topological space (X, T) where $X = \Pi_{j\in J} X_j$ is the usual set product and T is the coarsest soft fuzzy* topology on X for which the projection p_j of X onto X_j are soft fuzzy* continuous for each $j \in J$. The soft fuzzy* topology T is called ***product soft fuzzy* topology*** on X and (X, T) a ***product soft fuzzy* topological space***.

Property 4.23. *Let $\{(X_j, T_j)\}_{j\in J}$ be a family of soft fuzzy* topological spaces and (X, T) the product soft fuzzy* topological space. The product soft fuzzy* topology T on X has a soft fuzzy* base the set of finite intersections of soft fuzzy* sets of the form $p_j^{-1}(\lambda_j, M_j)$ where $(\lambda_j, M_j) \in T_j$, $j \in J$.*

Proof. Let $\{X_j\}$, $j = 1, 2, ..., n$ be a finite family of soft fuzzy* sets and for each $j = 1, 2, .., n$, let (λ_j, M_j) be a soft fuzzy* sets in X_j. Define the product $(\lambda, M) = \Pi_{j=1}^n(\lambda_j, M_j)$ of the family $\{(\lambda_j, M_j)\}_{j=1,2,...,n}$ as the soft fuzzy* set in $X = \Pi_{j=1}^n X_j$ that has the membership function given by

$$\lambda(x_1, x_2, ..., x_n) = min\{\lambda_1(x_1), ..., \lambda_n(x_n)\}$$

for all $(x_1, ..., x_n) \in M$.

For each $j = 1, 2, ..., n$, $p_j(\lambda, M) \sqsubseteq (\lambda_j, M_j)$, since the membership function of $p_j(\lambda, M) = (\gamma, L)$ is given by

$$\begin{aligned}\gamma(x_j) &= sup_{(x_1,...,x_n)\in p_j^{-1}(x_j)}\lambda(x_1, ..., x_n)\\ &= sup_{(x_1,...,x_n)\in p_j^{-1}(x_j)}min\{\lambda_1(x_1), ..., \lambda_n(x_n)\}\\ &= min\{\sup_{x_1\in M_1}\lambda_1(x_1), ..\lambda_j(x_j), ..., \sup_{x_n\in M_n}\lambda_n(x_n)\}\\ &\leq \lambda_j(x_j) \text{ for all } x_j \in L\end{aligned}$$

□

Remark 4.24. By Property 4.23, if X_j has soft fuzzy* topology T_j, $j = 1, 2, .., n$ the product soft fuzzy* topology on X has a soft fuzzy* base the set of product soft fuzzy* sets of the form $\Pi_{j=1}^n(\lambda_j, M_j)$ where $(\lambda_j, M_j) \in T_j$, $j = 1, 2, .., n$.

Property 4.25. *Let $\{(X_j, T_j)\}$, $j = 1, 2, .., n$ be a finite family of soft fuzzy* topological spaces and (X, T) the product soft fuzzy* topological space. For each $j = 1, 2, .., n$ let (λ_j, M_j) be a soft fuzzy* set in X_j and (λ, M) be the product soft fuzzy* set in X. Then the induced soft fuzzy* topology $T_{(\lambda,M)}$ has a soft fuzzy* base the set of product soft fuzzy* sets of the form $\Pi_{j=1}^n(\alpha'_j, A'_j)$ where $(\alpha'_j, A'_j) \in (T_j)_{(\lambda_j,M_j)}$, $j = 1, 2, .., n$.*

Proof. By Remark 4. 24, T has a soft fuzzy* base

$$\mathfrak{B} = \{\Pi_{j=1}^n(\alpha_j, A_j) : (\alpha_j, A_j) \in T_j, j = 1, 2, .., n\}$$

A soft fuzzy* base for $T_{(\lambda,M)}$ is therefore given by

$$\mathfrak{B}_{(\lambda,M)} = \{(\Pi_{j=1}^n(\alpha_j, A_j)) \sqcap (\lambda, M) : (\lambda_j, M_j) \in T_j, j = 1, 2, .., n\}.$$

But $(\Pi_{j=1}^n(\alpha_j, A_j)) \sqcap (\lambda, M) = \Pi_{j=1}^n((\alpha_j, A_j) \sqcap (\lambda, M))$. Hence the property follows with $(\alpha'_j, A'_j) = (\alpha_j, A_j) \sqcap (\lambda, M)$. □

Property 4.26. *Let $\{(X_j, T_j)\}_{j\in J}$ be a family of soft fuzzy* topological spaces (X, T) the product soft fuzzy* topological space. Let f be a mapping of a soft fuzzy* topological space (Y, S) into (X, T). Then f is soft fuzzy* continuous iff $p_j \circ f$ is soft fuzzy* continuous for each $j \in J$.*

Proof. Proof is obvious. □

Corollary 4.27. *Let $\{(X_j, T_j)\}$, $\{(Y_j, S_j)\}$, $j \in J$ be two families of soft fuzzy* topological spaces and (X, T) (Y, S) the respective product soft fuzzy* topological spaces. For each $j \in J$, let f_j be a mapping of (X_j, T_j) into (Y_j, S_j). Then the product mapping $f : \Pi_{j\in J} f_j : (x_j) \to (f_j(x_j))$ of (X, T) into (Y, S) is soft fuzzy* continuous if f_j is soft fuzzy* continuous for each $j \in J$.*

Proof. The mapping f can be written as $x \to (f_j(P_j(x)))$ where $x = (x_j)$ and is therefore soft fuzzy* continuous by Property 4.26. □

Property 4.28. *Let $\{(X_j, T_j)\}$, $j = 1, 2, .., n$ be a finite family of soft fuzzy* topological spaces and (X, T) the product soft fuzzy* topological spaces. For each $j = 1, 2, .., n$, let (λ_j, M_j) be a soft fuzzy* set in X_j and (λ, M) the product soft fuzzy* set in X. Let (Y, S) be a soft fuzzy* topological space, (μ, N) be a soft fuzzy* set in (Y, S) and f a mapping of the soft fuzzy* subspace $((\mu, N), S_{(\mu,N)})$ into the soft fuzzy* subspace $((\lambda, M), T_{(\lambda,M)})$. Then f is soft fuzzy* relatively continuous iff $p_i \circ f$ is soft fuzzy* relatively continuous for each $j = 1, 2, .., n$.*

Proof. By Property 4.13, the soft fuzzy* continuity of p_j implies the soft fuzzy* relatively continuity of p_j for each $j = 1, 2, .., n$. The composition $p_j \circ f$ is therefore soft fuzzy* relatively continuous for each $j = 1, 2, .., n$.

Conversely, let $(\lambda', M') = (\lambda'_1, M'_1) \times ... \times (\lambda'_n, M'_n)$ where $(\lambda'_j, M'_j) \in (T_j)_{(\lambda_j,M_j)}$, $j = 1, 2, ..., n$. By Property 4. 25, the set of such (λ', M') form a soft fuzzy base of $T_{(\lambda,M)}$. Since

$$f^{-1}(\lambda', M') \sqcap (\mu, N) = f^{-1}(p_1^{-1}(\lambda'_1, M'_1) \sqcap ... \sqcap p_n^{-1}(\lambda'_1, M'_1) \sqcap (\mu, N)$$
$$= \sqcap_{j=1}^{n}((p_j \circ f)^{-1}(\lambda'_j, M'_j) \sqcap (\mu, N))$$

is soft fuzzy* open in $S_{(\mu,N)}$, as $p_j \circ f$ is soft fuzzy* relatively continuous for each $j = 1, 2, .., n$ it follows that from Property 4.17, that f is soft fuzzy* relatively continuous. □

Corollary 4.29. *Let $\{(X_j, T_j)\}$, $\{(Y_j, S_j)\}$ $j = 1, 2, .., n$ be two finite families of soft fuzzy* topological spaces and (X, T), (Y, S) the respective product soft fuzzy* topological spaces. For each $j = 1, 2, .., n$, let (λ_j, M_j) be a soft fuzzy* set in X_j, (μ_j, N_j) be a soft fuzzy* set in Y_j and f_j a mapping of the soft fuzzy* subspaces $((\lambda_j, M_j), T_{(\lambda_j,M_j)})$ into the soft fuzzy* subspace $((\mu_j, N_j), S_{(\mu_j,N_j)})$. Let $(\lambda, M) = \Pi_{j=1}^{n}(\lambda_j, M_j)$ and $(\mu, N) = \Pi_{j=1}^{n}(\mu_j, N_j)$ be the product mapping $f = \Pi_{j=1}^{n} f_j$: $(x_1, .., x_n) \to (f_1(x_1), ...f_n(x_n))$ of the soft fuzzy* subspace $((\lambda, M), T_{(\lambda,M)})$ into the soft fuzzy* subspace $((\mu, N), S_{(\mu,N)})$ is soft fuzzy* relatively continuous if f_j is soft fuzzy* relatively continuous for each $j = 1, 2, .., n$.*

Proof. By Corollary 4.27, the proof is obvious. □

Property 4.30. *Let $\{(X_j, T_j)\}$, $\{(Y_j, S_j)\}$ $j = 1, 2, .., n$ be two finite families of soft fuzzy* topological spaces and (X, T), (Y, S) the respective product soft fuzzy* topological spaces. For each $j = 1, 2, .., n$, let f_j be a mapping of (X_j, T_j) into (Y_j, S_j). Then the product mapping $f : \Pi_{j=1}^{n} f_j : (x_1, ..., x_n) \to (f_1(x_1), .., f_n(x_n))$ of (X, T) into (Y, S) is soft fuzzy* open if f_j is soft fuzzy* open for each $j = 1, .., n$.*

Proof. Let (λ, M) be soft fuzzy* open in (X, T). Then there exists soft fuzzy* open set (λ_{ja}, M_{ja}) $a \in A$, $j = 1, ..., n$ such that $(\lambda, M) = \sqcup_{a\in A}\Pi_{j=1}^{n}(\lambda_{ja}, M_{ja})$.

The image $f(\lambda, M)$ of (λ, M) has the membership function $f(\lambda, M) = (\gamma, L)$ where for all $y \in L \subseteq S$.

$$\begin{aligned}\gamma(y) &= \sqcup_{a\in A} sup_{z\in f^{-1}(y)}\Pi_{j=1}^{n}\lambda_{ja}(z)\\ &= sup_{a\in A} sup_{z_1\in f_1^{-1}(y_1)}...sup_{z_n\in f_n^{-1}(y_n)} min\{\lambda_{1a}(z_1)...\lambda_{na}(z_n)\}\\ &= sup_{a\in A}(min\{sup_{z_1\in f_1^{-1}(y_1)}\lambda_{1a}(z_1)...sup_{z_n\in f_n^{-1}(y_n)}\lambda_{na}(z_n)...\}\\ &= \sqcup_{a\in A}\Pi_{j=1}^{n}(f_j(\lambda_{ja}, M_{ja}))\end{aligned}$$

Thus $f(\lambda, M) = \sqcup_{a\in A}\Pi_{j=1}^{n}(f_j(\lambda_{ja}, M_{ja})$. Since f_j is soft fuzzy* open for each $j = 1, .., n$, $f(\lambda, M)$ is soft fuzzy* open in (Y, S). □

Property 4.31. *Let $\{(X_j, T_j)\}$, $\{(Y_j, S_j)\}$ $j = 1, 2, .., n$ be two finite families of soft fuzzy* topological spaces and (X, T), (Y, S) the respective product soft fuzzy* topological spaces. For each $j = 1, 2, .., n$, let (λ_j, M_j) be a soft fuzzy* set in X_j, (μ_j, N_j) be a soft fuzzy* set in Y_j and f_j a mapping of the soft fuzzy* subspace $((\lambda_j, M_j), (T_j)_{(\lambda_j,M_j)})$ into the soft fuzzy* subspace $((\mu_j, N_j), (S_j)_{(\mu_j,N_j)})$. Let $(\lambda, M) = \Pi_{j=1}^{n}(\lambda_j, M_j)$, $(\mu, N) = \Pi_{j=1}^{n}(\mu_j, N_j)$ be the product soft fuzzy* sets in X, Y respectively. Then the product soft fuzzy* mapping $f = \Pi_{j=1}^{n}f_j : (x_1, .., x_n) \to (f_1(x_1), .., f_n(x_n))$ of the soft fuzzy* subspace $((\lambda, M), T_{(\lambda,M)})$ into the soft fuzzy* subspace $((\mu, N), S_{(\mu,N)})$ is soft fuzzy* relatively open if f_j is soft fuzzy* relatively open for each $j = 1, 2, .., n$.*

Proof. Let (λ', M') be soft fuzzy* open in $T_{(\lambda,M)}$. By Property 4.25, there exists soft fuzzy* open sets $(\lambda'_{ja}, M'_{ja}) \in (T_j)_{(\lambda_j,M_j)}$, $a \in A$, $j = 1, .., n$ such that $(\lambda', M') = \sqcup_{a\in A}\Pi_{j=1}^{n}(\lambda'_{ja}, M'_{ja})$. By Property 4.30, $f(\lambda', M') = \sqcup_{a\in A}\Pi_{j=1}^{n}(f_j(\lambda'_{ja}, M'_{ja}))$. Since f_j is soft fuzzy* relatively open for each $j = 1, .., n$, $f(\lambda', M')$ is soft fuzzy* open in $S_{(\mu,N)}$. □

Property 4.32. *Let (X_1, T_1) and (X_2, T_2) be soft fuzzy* topological spaces and (X, T) the product soft fuzzy* topological space. Then for each $a_1 \in X_1$, the mapping $i : x_2 \to (a_1, x_2)$ of (X_2, T_2) into (X, T) is soft fuzzy* continuous.*

Proof. The constant mapping $i_1 : x_2 \to a_1$ from (X_2, T_2) into (X_1, T_1) is soft fuzzy* continuous. For if (λ_1, M_1) is soft fuzzy* open in T_1, the inverse image $f^{-1}(\lambda_1, M_1)$ has the membership function is given by

$$\begin{aligned}i_1^{-1}(\lambda_1)(x_2) &= \lambda_1 \circ i(x_2)\\ &= \lambda_1(a_1)\\ &= k_c(x_2) \text{ for all } x_2 \in M_2\end{aligned}$$

where (k_c, H) is the soft fuzzy* open set in X_2 which has the constant membership function with value $c = \lambda_1(a_1)$. since the identity mapping $i_2 : x_2 \to x_2$ of (X_2, T_2) into itself is soft fuzzy continuous, the mapping i is soft fuzzy continuous by Property 4.26. □

Property 4.33. *Let (X_1, T_1) and (X_2, T_2) be soft fuzzy* topological spaces and (X, T) the product soft fuzzy* topological space. Let (λ_1, M_1), (λ_2, M_2) be a soft* *fuzzy* open set in X_1, X_2 respectively. Let (λ, M) be the product soft fuzzy* set in X. Then for each $a_1 \in M_1$ such that $\lambda_1(a_1) \geq \lambda_2(x_2)$ for all $x_2 \in M_2$, the mapping $i : x_2 \to (a_1, x_2)$ of the soft fuzzy* subspace $((\lambda_2, M_2), (T_2)_{(\lambda_2, M_2)})$ into the soft fuzzy* subspace $((\lambda, M), T_{(\lambda, M)})$ is soft fuzzy* relatively continuous.*

Proof. Since $i(\lambda_2, M_2) \sqsubseteq (\lambda, M)$, since the membership function of $i(\lambda_2, M_2) = (\gamma, L)$ is given by

$$\gamma(x_1, x_2) = sup_{x_2 \in f^{-1}(x_1, x_2)} \lambda_2(x_2)$$

$$= \begin{cases} \lambda_2(x_2) & \text{if} \quad x_1 = a_1 \\ 0 & otherwise \end{cases}$$

and that of (λ, M) by

$$\begin{aligned} \lambda(x_1, x_2) &= (min\{\lambda_1(x_1), \lambda_2(x_2)\} \\ &\geq \lambda(x_2) \text{ for all } (x_1, x_2) \in M, x_1 \in M_1. \end{aligned}$$

The proof of the soft fuzzy* relative continuity of i is analogous to the proof of the soft fuzzy* continuity of i in Property 4.32. □

5. Soft fuzzy* group

Definition 5.1. Let X be a group and M be a subgroup of X. Let (λ, M) be any soft fuzzy* set in X. Then (λ, M) is said to be soft fuzzy* group in X satisfies the follwoing conditions

(i) $\lambda(xy) \geq min\{\lambda(x), \lambda(y)\}$ for every $x, y \in M$;
(ii) $\lambda(x^{-1}) = \lambda(x)$ for every $x \in M$.

Property 5.2. *If (λ, M) is a soft fuzzy* group then*
(i) $\lambda(x) \leq \lambda(e)$ for $x, e \in M$.
(ii) $\lambda(xy^{-1}) \geq min\{\lambda(x), \lambda(y)\}$ for every $x, y \in M$.

Proof. (i) Let $x, e \in M$. Now

$$\begin{aligned} \lambda(e) &= \lambda(xx^{-1}) \\ &\geq min\{\lambda(x), \lambda(x^{-1})\} \\ &= \lambda(x)[\text{ By Definition 5.1(ii) }] \end{aligned}$$

Therefore, $\lambda(x) \leq \lambda(e)$ for $x, e \in M$.
(ii) Let $x, y \in M$. Now

$$\begin{aligned} \lambda(xy^{-1}) &\geq min\{\lambda(x), \lambda(y^{-1})\} \\ &= min\{\lambda(x), \lambda(y)\} \end{aligned}$$

Therefore, $\lambda(xy^{-1}) \geq min\{\lambda(x), \lambda(y)\}$ for every $x, y \in M$. □

Property 5.3. *Let X, Y be groups and f a homomorphism of X into Y. Let (λ, M) be a soft fuzzy* group in Y. Then the inverse image $f^{-1}(\lambda, M)$ of (λ, M) is a soft fuzzy* group in X.*

Proof. For all $x, y \in f^{-1}(M)$,

$$\begin{aligned} f^{-1}(\lambda)(xy^{-1}) &= \lambda(f(xy^{-1})) \\ &= \lambda(f(x)f(y^{-1})) \\ &= \lambda(f(x)(f(y))^{-1}) \\ &\geq min\{\lambda(f(x)), \lambda(f(y))\} \\ &= (min\{f^{-1}(\lambda(x)), f^{-1}(\lambda(y))\}. \end{aligned}$$

Therefore, $f^{-1}(\lambda, M)$ of (λ, M) is a soft fuzzy* group in X. □

Definition 5.4. A soft fuzzy* set (λ, M) of X is said to have ***soft fuzzy* sup property*** if for any subset $S \subseteq X$ there exists $t_0 \in S$ such that $\lambda(t_0) = sup_{t \in S}\lambda(t)$.

Property 5.5. *Let X, Y be groups and f a homomorphism of X into Y. Let (λ, M) be a soft fuzzy* group in X that has soft fuzzy* sup property. Then the image $f(\lambda, M)$ of (λ, M) is a soft fuzzy* group in Y.*

Proof. Let $f(\lambda, M) = (\gamma, L)$. Let $u, v \in L$. If either $f^{-1}(u), f^{-1}(v)$ is empty then the inequality in Property 4.1(ii) is trivially satisfied. Suppose neither $f^{-1}(u)$ nor $f^{-1}(v)$ is empty.

Let $r_0 \in f^{-1}(u)$, $s_0 \in f^{-1}(v)$ such that

$$\lambda(r_0) = sup_{t \in f^{-1}(u)}\lambda(t) \text{ and } \lambda(s_0) = sup_{t \in f^{-1}(v)}\lambda(t).$$

Then

$$\begin{aligned} \gamma(uv^{-1}) &= sup_{w \in f^{-1}(uv^{-1})}\lambda(w) \\ &\geq min\{\lambda(r_0), \lambda(s_0)\} \\ &= min\{sup_{t \in f^{-1}(u)}\lambda(t), sup_{t \in f^{-1}(v)}\lambda(t)\} \\ &= min\{\gamma(u), \gamma(v)\}. \end{aligned}$$

Therefore, the image $f(\lambda, M)$ of (λ, M) is a soft fuzzy* group in Y. □

Note 5.6. The membership function λ of a soft fuzzy* group (λ, M) in a group X is ***soft fuzzy* invariant*** if for all $x_1, x_2 \in M$, $f(x_1) = f(x_2)$ implies $\lambda(x_1) = \lambda(x_2)$. Clearly a homomorphic image $f(\lambda, M)$ of (λ, M) is then a soft fuzzy* group.

Remark 5.7. Given a soft fuzzy* group (λ, M) in a group X where M denote the set $\{x|\lambda(x) = \lambda(e)\}$ is a subgroup of X. For $a \in X$, let $\rho_a : x \to xa$ and $\sigma_a : x \to ax$ denote respectively, the right and left translation of X into itself.

Property 5.8. *Let (λ, M) be a soft fuzzy* group in a group X then for all $a \in M$, $\rho_a(\lambda, M) = \sigma_a(\lambda, M) = (\lambda, M)$.*

Proof. Let $a \in M$. Then the membership function of $\rho_a(\lambda, M) = (R_a, R)$ is given by

$$\begin{aligned} R_a(x) &= sup_{t \in f^{-1}(x)}\lambda(t) \text{ for all } t \in M \\ &= \lambda(xa^{-1}) \text{ [by soft fuzzy* sup property, there exists } xa^{-1} \in M] \\ &\geq (min\{\lambda(x), \lambda(e)\} \text{ [by Propert 5.2(ii)]} \\ &= \lambda(x) \end{aligned}$$

Conversely,

$$\begin{aligned} \lambda(x) &= \lambda(xa^{-1}a) \\ &\geq min\{\lambda(xa^{-1}), \lambda(a)\} \\ &= min\{\lambda(xa^{-1}), \lambda(e)\} \text{ [since } a \in M] \\ &= \lambda(xa^{-1}) \\ &= sup_{t\in M}\lambda(t) \\ &= R_a(x) \text{ for all } x \in L \subseteq M \end{aligned}$$

Therefore, $\rho_a(\lambda, M) = (\lambda, M)$. Similarly, $\sigma_a(\lambda, M) = (\lambda, M)$. □

6. Soft fuzzy* topological groups

Definition 6.1. Let X be a group and T a soft fuzzy* topology on X. Let (λ, M) be a soft fuzzy* group in X and let (λ, M) be endowed with induced soft fuzzy* topology $T_{(\lambda,M)}$. Then (λ, M) is a soft fuzzy* topological group in X iff it satisfies the following two conditions

(i) the mapping $\alpha : (x, y) \to xy$ of $((\lambda, M), T_{(\lambda,M)}) \times ((\lambda, M), T_{(\lambda,M)})$ into $((\lambda, M), T_{(\lambda,M)})$ defined by $\alpha(x, y) = xy$ is soft fuzzy* relatively continuous.

(ii) the mapping $\beta : x \to x^{-1}$ of $((\lambda, M), T_{(\lambda,M)})$ into $((\lambda, M), T_{(\lambda,M)})$ is defined by $\beta(x) = x^{-1}$ is soft fuzzy* relatively continuous.

Note 6.2. A soft fuzzy* group structure and an induced soft fuzzy* topology are said to be ***compatible*** if they satisfy (i) and (ii).

Property 6.3. *Let X be a group having soft fuzzy* topology T. A soft fuzzy* group (λ, M) in X is a soft fuzzy* topological group iff the mapping $f : (x, y) \to xy^{-1}$ of $((\lambda, M), T_{(\lambda,M)} \times ((\lambda, M), T_{(\lambda,M)}))$ into itself is soft fuzzy* relatively continuous.*

Proof. The mapping $(x, y) \to (x, y^{-1})$ of $((\lambda, M), T_{(\lambda,M)} \times ((\lambda, M), T_{(\lambda,M)}))$ into itself is soft fuzzy* continuous. By the Corollary 4.33. Hence the composition $(x, y) \to (x, y^{-1}) \to xy^{-1}$ is soft fuzzy* relatively continuous.

Conversely, by Property 5.2(i), $\lambda(e) \geq \lambda(x)$ for all $x \in M$ and therefore by Property 4.33, the canonical injection $i : y \to (e, y) \to ey^{-1}$ of $((\lambda, M), T_{(\lambda,M)})$ into $((\lambda, M), T_{(\lambda,M)} \times ((\lambda, M), T_{(\lambda,M)}))$ is soft fuzzy* relatively continuous. Hence the composition $\beta : y \to (e, y) \to ey^{-1}$ is soft fuzzy* continuous. The mapping $\alpha : (x, y) \to xy$ of $((\lambda, M), T_{(\lambda,M)}) \times ((\lambda, M), T_{(\lambda,M)})$ into $((\lambda, M), T_{(\lambda,M)})$ is soft fuzzy* relatively continuous since it is the composition $(x, y) \to (x, y^{-1}) \to x(y^{-1})^{-1}$ of soft fuzzy* relatively continuous mappings. □

Remark 6.4. If (λ, M) is a soft fuzzy* group in a group X carrying soft fuzzy* topology T. Then in general, the translations ρ_a, σ_a, $a \in X$ are not soft fuzzy* relatively continuous mappings of $((\lambda, M), T_{(\lambda,M)})$ into itself. However the follwoing special case $M = \{x|\lambda(e) = \lambda(x)\}$.

Property 6.5. *Let X be a group having having soft fuzzy* topology T and let (λ, M) be a soft fuzzy* topological group in X. For each $a \in M$ the translations ρ_a, σ_a are soft fuzzy* relative homeomorphism of $((\lambda, M), T_{(\lambda,M)})$ into itself.*

Proof. By Property 5.8, $\rho_a(\lambda, M) = (\lambda, M)$ and $\sigma_a(\lambda, M) = (\lambda, M)$ for all $a \in M$. The mapping σ_a is the composition of the injection $i : y \to (a, y)$ and the mapping $\alpha : (x, y) \to xy$. Since $\lambda(a) \geq \lambda(y)$ for every $y \in M \subseteq Y$.It follows from Property 4.33, i is a soft fuzzy* relative continuous mapping of $((\lambda, M), T_{(\lambda,M)})$ into $((\lambda, M), T_{(\lambda,M)}) \times ((\lambda, M), T_{(\lambda,M)})$. The mapping α is soft fuzzy* relative continuous by hypothesis. Hence σ_a is soft fuzzy* relatively open. Then σ_a is soft fuzzy* relative homeomorphism. Similarly we proved ρ_a and ρ_a^{-1} is soft fuzzy* relative homeomorphism. □

Suppose that X and Y are groups and that f is a homoemorphism of X into Y. Let Y have soft fuzzy* topology Y and let (λ, M) be a soft fuzzy* topological group in Y. The mapping f gives rise to a soft fuzzy* topology T on X, the inverse image under f of S, and by Property 5.3, it also gives rise to a soft fuzzy* group in X, the inverse image $f^{-1}(\lambda, M)$ of (λ, M). The following property shows that the induced soft fuzzy* topology on $f^{-1}(\lambda, M)$ and the soft fuzzy* group structure are compatible.

Property 6.6. *Given groups X, Y a homomorphism f of X into Y and a soft fuzzy* topology S on Y, let X have soft fuzzy* topology T, where T is the inverse image under f of S and let (λ, M) be a soft fuzzy* topological in Y. Then the inverse image $f^{-1}(\lambda, M)$ of (λ, M) is a soft fuzzy* topological group in X.*

Proof. To show that the mapping $\gamma_X : (x_1, x_2) \to (x_1 x_2^{-1})$ of $(f^{-1}(\lambda, M), T_{(f^{-1}(\lambda,M))})$ $\times (f^{-1}(\lambda, M), T_{(f^{-1}(\lambda,M))})$ into $(f^{-1}(\lambda, M), T_{(f^{-1}(\lambda,M))})$ is soft fuzzy* relatively continuous. Let (α', A') be an soft fuzzy* open in the induced soft fuzzy* topology $T_{f^{-1}(\lambda,M)}$ on $f^{-1}(\lambda, M)$. Since f is a soft fuzzy* continuous mapping of (X, T) into (Y, S) it is , by Property 4.9, a soft fuzzy* relatively continuous mapping of $(f^{-1}(\lambda, M), T_{(f^{-1}(\lambda,M))})$ into $((\lambda, M), T_{(\lambda,M)})$. Also that there exists an soft fuzzy* open set (δ', P') in $S_{(\lambda,M)}$ such that $f^{-1}(\delta', P') = (\alpha', A')$. The membership function of

$$\begin{aligned}
\gamma_X^{-1}(\alpha')(x_1, x_2) &= \alpha' \circ \gamma_X(x_1, x_2) \\
&= \alpha'(x_1 x_2^{-1}) \\
&= f^{-1}(\delta')(x_1 x_2^{-1}) \\
&= \delta'(f(x_1 x_2^{-1})) \\
&= \delta'(f(x_1)(f(x_2))^{-1})
\end{aligned}$$

for all $(x_1, x_2) \in \gamma_X^{-1}(A') \times \gamma_X^{-1}(A')$ where $A' = f^{-1}(P')$.

By hypothesis, the mapping $\gamma_Y : (y_1, y_2) \to y_1 y_2^{-1}$ of $((\lambda, M), S_{(\lambda,M)}) \times ((\lambda, M), S_{(\lambda,M)})$ into $((\lambda, M), S_{(\lambda,M)})$ is soft fuzzy* relatively continuous and by Corollary 4.29, so is the product mapping $f \times f$ of $(f^{-1}(\lambda, M), T_{(f^{-1}(\lambda,M))}) \times (f^{-1}(\lambda, M), T_{(f^{-1}(\lambda,M))})$ into $((\lambda, M), S_{(\lambda,M)})$. But

$$\begin{aligned}
\delta'(f(x_1)(f(x_2))^{-1}) &= \gamma_Y^{-1}(\delta')(f(x_1), f(x_2)) \text{ where } y_1 = f(x_1) \text{ and } y_2 = (f(x_2))^{-1} \\
&= (f \times f)^{-1}(\gamma_Y^{-1}(\delta'))(x_1, x_2)
\end{aligned}$$

for every $(x_1, x_2) \in \gamma_X^{-1}(A') \times \gamma_X^{-1}(A')$. Hence $\gamma_X^{-1}(\alpha', A') \sqcap (f^{-1}(\lambda, M) \times f^{-1}(\lambda, M)) = (f \times f)^{-1}(\gamma_Y^{-1}(\delta', P')) \sqcap (f^{-1}(\lambda, M) \times f^{-1}(\lambda, M))$ is open in the induced soft fuzzy* topology on $(f^{-1}(\lambda, M) \times f^{-1}(\lambda, M)) \times (f^{-1}(\lambda, M) \times f^{-1}(\lambda, M))$. □

Property 6.7. *Given groups X, Y a homomorphism f of X into Y and a soft fuzzy* topology T on X, let Y have soft fuzzy* topology S, where S is the image under f of T and let (λ, M) be a soft fuzzy* topological group in X. If the membership function λ of (λ, M) is soft fuzzy* f invariant then the image $f(\lambda, M)$ of (λ, M) is a soft fuzzy* topological group in Y.*

Proof. To show that the mapping $\gamma_Y : (y_1, y_2) \to (y_1 y_2^{-1})$ of $(f(\lambda, M), S_{f(\lambda,M)}) \times (f(\lambda, M), S_{f(\lambda,M)})$ into $(f(\lambda, M), S_{f(\lambda,M)})$ is soft fuzzy* relatively continuous. Note that f is soft fuzzy* open, for if $(\gamma, L) \in T$, then $(f(\gamma, L)) \in S$. Since the inverse image $f^{-1}(f(\gamma, L))$ is the union of soft fuzzy* open sets and thus soft fuzzy* open in T. It follows that f is soft fuzzy* relatively open. Since if $(\delta', P') \in T_{(\lambda,M)}$ there exists (γ, L) in T such that $(\delta', P') = (\gamma, L) \sqcap (\lambda, M)$ and by the soft fuzzy* f invariance of λ,$f(\delta', P') = f(\gamma, L) \sqcap f(\lambda, M) \in S_{(\lambda,M)}$. By Property 4.25, the product mapping $f \times f$ is soft fuzzy* relatively open mapping of $((\lambda, M), T_{(\lambda,M)}) \times ((\lambda, M), T_{(\lambda,M)})$ into $(f(\lambda, M), S_{(f(\lambda,M))}) \times (f(\lambda, M), S_{(f(\lambda,M))})$.

Let (α', A') be a soft fuzzy* open set in $S_{f(\lambda,M)}$. The membership function of $(f \times f)^{-1}(\gamma_Y^{-1}(\alpha', A'))$ is given by

$$\begin{aligned}(f \times f)^{-1}(\gamma_Y^{-1}(\alpha')) &= (\gamma_Y^{-1}(\alpha')) \circ (f \times f)(x_1, x_2) \\ &= (\gamma_Y^{-1}(\alpha'))(f(x_1)(f(x_2))^{-1}) \\ &= (\gamma_X^{-1} \circ f^{-1})(\delta^{-1})(x_1, x_2)\end{aligned}$$

for all $(x_1, x_2) \in (f \times f)^{-1}(\gamma_Y^{-1}(A'))$ where $\gamma_X : (x_1, x_2) \to x_1 x_2^{-1}$. But, by hypothesis, γ_X is a soft fuzzy* relatively continuous mapping of $((\lambda, M), T_{(\lambda,M)}) \times ((\lambda, M), T_{(\lambda,M)})$ into $((\lambda, M), T_{(\lambda,M)})$ and f is a soft fuzzy* relatively continuous mapping of $((\lambda, M), T_{(\lambda,M)})$ into $(f(\lambda, M), S_{(f(\lambda,M))})$. Hence, by the soft fuzzy* f invariance of λ,

$$(f \times f)^{-1}(\gamma_Y^{-1}(\alpha', A')) \sqcap (f(\lambda, M) \times f(\lambda, M)) = (f \times f)^{-1}(\gamma_Y^{-1}(\delta', P')) \sqcap (\lambda, M) \times (\lambda, M)$$

is open in the induced soft fuzzy* topology on $(\lambda, M) \times (\lambda, M)$. As $f \times f$ is soft fuzzy* relatively open,

$$(f \times f)(f \times f)^{-1}((\gamma_Y^{-1}(\delta', P') \sqcap f(\lambda, M) \times f(\lambda, M)) = \gamma_Y^{-1}(\delta', P') \sqcap (f(\lambda, M) \times (f(\lambda, M)))$$

is open in the induced soft fuzzy* topology on $(f(\lambda, M)) \times (f(\lambda, M))$. □

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References

[1] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182–190.
[2] D. Foster, Fuzzy topological groups, J. Math. Anal. Appl. 67 (1979) 549–564.
[3] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512–517.
[4] P. Smets, The degree of belief in a fuzzy event, Inform. Sci. 25 (1981) 1–19.

[5] M. Sugeno, An introductory survey of fuzzy control, Inform. Sci. 36 (1985) 59–83.
[6] I. U. Triyaki, Fuzzy sets over the poset $\mathbb{I}$, Hacet. J. Math. Stat. 37(2) (2008) 143–166.
[7] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

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