On soft fuzzy* topological groups

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Abstract. In this paper, the concepts of soft fuzzy* topological spaces and soft fuzzy* groups are introduced. In this connection, the concept of soft fuzzy* topological group is introduced. The concepts of Homomorphic images and inverse images of soft fuzzy* topological groups are studied.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh [7]. Fuzzy sets have applications in many fields such as information [4] and control [5]. The theory of fuzzy topological spaces was introduced and developed by Chang [1] and since then various notions in classical topology have been extended to fuzzy topological spaces. A. Rosenfeld [3] formulated the elements of a theory of fuzzy groups. David Foster [2] introduced the concept of fuzzy topological groups. The concept of soft fuzzy topological space was introduced by Ismail U. Triyaki [6]. In this paper, the concepts of soft fuzzy* topological groups are introduced and some basic properties are studied.

2. Preliminaries

Definition 2.1 ([2]). Let $X$ be a group and $G$ be a fuzzy set in $X$ with membership function $\mu_G$. Then $G$ is a fuzzy group in $X$ iff the following conditions are satisfied:
(i) $\mu_G(xy) \geq \min\{\mu_G(x), \mu_G(y)\}$, for all $x, y \in X$;
(ii) $\mu_G(x^{-1}) \geq \mu_G(x)$, for all $x \in X$.

Definition 2.2 ([6]). Let $X$ be a non-empty set. A soft fuzzy set (in short, SFS) $A$ have the form $A = (\lambda, M)$ where the function $\lambda : X \to I$ denotes the degree of membership and $M$ is the subset of $X$. The set of all soft fuzzy set will be denoted by $SF(X)$. 
Definition 2.3 ([5]). The relation \( \sqsubseteq \) on \( SF(X) \) is given by \( (\mu, N) \sqsubseteq (\lambda, M) \) if \( \mu(x) \leq \lambda(x), \forall x \in X \) and \( M \subseteq N \).

Proposition 2.4 ([5]). If \( (\mu_j, N_j) \in SF(X), j \in J \), then the family \( \{(\mu_j, N_j)\mid j \in J\} \) has a meet, ie., g.l.b., in \( SF(X) \) denoted by \( \bigcap_{j \in J}(\mu_j, N_j) \) and given by
\[
\mu(x) = \bigwedge_{j \in J} \mu_j(x) \quad \forall x \in X
\]
and
\[
M = \bigcap M_j \text{ for } j \in J.
\]

Proposition 2.5 ([5]). If \( (\mu_j, N_j) \in SF(X), j \in J \), then the family \( \{(\mu_j, N_j)\mid j \in J\} \) has a join, ie., l.u.b., in \( SF(X) \) denoted by \( \bigvee_{j \in J}(\mu_j, N_j) \) and given by
\[
\mu(x) = \bigvee_{j \in J} \mu_j(x) \quad \forall x \in X
\]
and
\[
M = \bigvee M_j \text{ for } j \in J.
\]

Definition 2.6 ([5]). Let \( X \) be a set. Let \( T \) be family of soft fuzzy subsets of \( X \). Then \( T \) is called a soft fuzzy topology on \( X \) if \( T \) satisfies the following conditions:
(i) \( (0, \emptyset) \) and \( (1, X) \in T \).
(ii) If \( (\mu_j, N_j) \in T, j = 1, 2, ..., n \) then \( \bigcap_{j=1}^n(\mu_j, N_j) \in T \).
(iii) If \( (\mu_j, N_j) \in T, j \in J \) then \( \sqcup_{j \in J}(\mu_j, N_j) \in T \).

The pair \((X, T)\) is called a soft fuzzy topological space (in short, SFTS). The members of \( T \) are soft fuzzy open sets and its complement are soft fuzzy closed sets.

Proposition 2.7 ([5]). Let \( \varphi : X \to Y \) be a point function.
(i) The mapping \( \varphi^{-} \) from \( SF(X) \) to \( SF(Y) \) corresponding to the image operator of the difunction \((f, F)\) is given by
\[
\varphi^{-}(\mu, N) = (\nu, L) \text{ where } \nu(y) = \sup\{\mu(x)\mid y = \varphi(x)\}, \text{ and } \]
\[
L = \{\varphi(x)\mid x \in N \text{ and } \nu(\varphi(x)) = \mu(x)\}.
\]
(ii) The mapping \( \varphi^{\ast} \) from \( SF(X) \) to \( SF(Y) \) corresponding to the inverse image of the difunction \((f, F)\) is given by
\[
\varphi^{\ast}(\nu, L) = (\nu \circ \varphi, \varphi^{-1}[L]).
\]

3. Soft fuzzy* set

Definition 3.1. Let \( X \) be a non empty set and \( M \subseteq X \). Then the pair \((\lambda, M)\) is said to be soft fuzzy* set if \( \lambda : M \to I = [0, 1] \). The collection of all soft fuzzy* sets is denoted by \( SF^{\ast}(X) \).

Definition 3.2. Let \((\lambda, M), (\mu, N) \in SF^{\ast}(X) \). Then \((\lambda, M)\) is called a soft fuzzy* subset of \((\mu, N)\) if \( M \subseteq N \) and \( \lambda(e) \leq \mu(e) \) for each \( e \in M \).
Definition 3.3. The union of two soft fuzzy* sets \((\lambda, M)\) and \((\mu, N)\) over \(X\) is the soft fuzzy* set \((\gamma, L) = (\lambda, M) \cup (\mu, N)\) where \(L = M \cup N\) and \(\gamma(e) = \begin{cases} \lambda(e) & \text{if } e \in M \setminus N \\ \mu(e) & \text{if } e \in N \setminus M \\ \lambda(e) \vee \mu(e) & \text{if } e \in M \cap N \end{cases}\)

Definition 3.4. If \((\lambda, M)\) and \((\mu, N)\) be two soft fuzzy* sets then the intersection of \((\lambda, N)\) and \((\mu, N)\) is a soft fuzzy* set \((\gamma, L) = (\lambda, M) \cap (\mu, N)\) where \(L = M \cap N\) and \(\gamma(e) = \lambda(e) \wedge \mu(e)\) for all \(e \in L\).

Definition 3.5. Let \((\lambda, M) \in SF^*(X).\) Then the complement of \((\lambda, M)\) is denoted by \((\lambda, M)'\) is the soft fuzzy* set defined by \((\lambda, M)' = (1, X) - (\lambda, M)\) where \(\lambda'(e) = 1 - \lambda(e)\) for all \(e \in M\) and \(M' = X \setminus M\).

Definition 3.6. Let \((\lambda, M) \in SF^*(X).\) Then the soft fuzzy* set \((\lambda, M)\) is the soft fuzzy* null set denoted by \((0, \emptyset)\) if \(\lambda(e) = 0\) for every \(e \in M\).

Definition 3.7. Let \((\lambda, M) \in SF^*(X).\) Then the soft fuzzy* set \((\lambda, M)\) is the soft fuzzy* universal set denoted by \((1, X)\) if \(\lambda(e) = 1\) for every \(e \in M\).

Definition 3.8. Let \(f : X \to Y\) be a function.

(i) the mapping \(f\) from \(SF^*(X) \to SF^*(Y)\) corresponding to the image operator of the difunction \((\phi, \psi)\) is given by \(f(\mu, N) = (\gamma, L)\) where \(\gamma(y) = \sup\{\mu(x)|y = f(x), x \in N\}, y \in L\).

(ii) the mapping \(f^{-1}\) from \(SF^*(Y) \to SF^*(X)\) corresponding to the inverse image of the difunction \((\phi, \psi)\) is given by \(f^{-1}(\mu, N) = \mu \circ f\).

Property 3.9. Let \(f\) be a mapping from a set \(X\) to a set \(Y.\) Let \(\{(\lambda_j, M_j)\}_{j \in J}\) be a family of soft fuzzy* sets in \(X\) and \(\{\mu_j, N_j\}\) \(j \in J\) a family of soft fuzzy* sets in \(Y.\) Then

\begin{align*}
\text{(i)} & \quad f^{-1}(\bigcup_{j \in J}(\mu_j, N_j)) = \bigcup_{j \in J}f^{-1}(\mu_j, N_j) \\
\text{(ii)} & \quad f^{-1}(\bigcap_{j \in J}(\mu_j, N_j)) = \bigcap_{j \in J}f^{-1}(\mu_j, N_j) \\
\text{(iii)} & \quad f(\bigcup_{j \in J}(\lambda_j, M_j)) = \bigcup_{j \in J}f(\lambda_j, M_j) \\
\text{(iv)} & \quad f(\bigcap_{j \in J}(\lambda_j, M_j)) \subseteq \bigcap_{j \in J}f(\lambda_j, M_j)
\end{align*}

Proof. (i) For all \(e \in \bigcup_{j \in J}N_j\) then

\[
f^{-1}(\bigcup_{j \in J}(\mu_j, N_j)) = \bigcup_{j \in J}\mu_j \circ f(e)
\]

\[
= \bigvee_{j \in J}(\mu_j \circ f)(e)
\]

\[
= \bigvee_{j \in J}\mu_j \circ f
\]

\[
= \bigcup_{j \in J}f^{-1}(\mu_j, N_j)
\]

(ii) The proof is similar for (i).

(iii) The proof is immediately from the Definition 3.8.

(iv) Let \(f(\bigcap_{j \in J}(\lambda_j, M_j)) = (\gamma, L).\) The membership function of \((\gamma, L)\) is given by

\[
\gamma(y) = \sup\{\Lambda_{j \in J}\lambda_j(x)|y = f(x), x \in \Lambda_{j \in J}f(M_j)\}
\]

\[
\subseteq \Lambda_{j \in J}\sup\{\lambda_j(x)|y = f(x), x \in \Lambda_{j \in J}f(M_j)\}
\]

\[
= \Lambda_{j \in J}f(\lambda_j, M_j)
\]

for all \(y \in L.\)
4. Soft fuzzy* topological spaces and subspaces

Definition 4.1. A subset $T \subseteq SF^*(X)$ is called soft fuzzy* topology on $X$ if

(i) For all $c \in I$ and $H \subseteq X$, $(k_c,H) \in T$,
(ii) $(\mu_j, N_j) \in T, j = 1, \ldots, n \Rightarrow \cap_{j=1}^n (\mu_j, N_j) \in T$,
(iii) $(\mu_j, N_j) \in T, j \in J \Rightarrow \cup_{j \in J} (\mu_j, N_j) \in T$

As usual the elements of $T$ are soft fuzzy* open and the complement $T'$ of $T$ is called soft fuzzy* closed.

If $T$ is a soft fuzzy* topology on $X$ we call the pair $(X,T)$ an **soft fuzzy* topological space**.

Note 4.2. We denote by $(k_c, H)$ the soft fuzzy* set in $X$ with membership function $k_c(x) = c$ for all $x \in H$. The soft fuzzy* set $(k_1, X)$ corresponds to the set $(1, X)$ and the soft fuzzy* set $(k_0, \emptyset)$ to the empty set $(0, \emptyset)$.

Definition 4.3. Let $(X, T)$ be a soft fuzzy* topological space. Let $(\lambda, M)$ be a soft fuzzy* set. Then

$$T_{(\lambda, M)} = \{ (\lambda, M) \cap (\delta, P); (\delta, P) \in T \}$$

is called an **induced soft fuzzy* topology** on $(\lambda, M)$ and $((\lambda, M), T_{(\lambda, M)})$ is called a soft fuzzy* subspace topology on $(X, T)$.

Definition 4.4. Let $(\lambda, M)$ and $(\mu, N)$ be any soft fuzzy* set in $X$. Then the **product** of $(\lambda, M)$ and $(\mu, N)$ is defined by

$$((\lambda \times \mu)_{M \times N}(x) = \sup_{x=(x_1, x_2)} \min\{\lambda(x_1), \mu(x_2)\}$$

for all $x_1 \in M, x_2 \in N$.

Note 4.5. The induced soft fuzzy* topology does not in general satisfy condition (i) of Definition 4.3. Condition (ii), however is satisfied and so is condition (iii).

Thus if $(\delta_j', P_j') \in T_{(\lambda, M)}$ for all $j \in J$ then there exists $(\delta_j, P_j) \in T, j \in J$ such that $(\delta_j', P_j') = (\delta_j, P_j) \cap (\lambda, M)$ for each $j \in J$. The union $(\delta', P') = \cup_{j \in J} (\delta_j', P_j') = \cup_{j \in J} ((\delta_j, P_j) \cap (\lambda, M))$ has the soft fuzzy* membership function is given by

$$\delta'(x) = \sup_{j \in J} \delta_j'(x) = \sup_{j \in J} \min\{\delta_j(x), \lambda(x)\} = \min\{\sup_{j \in J} \delta_j(x), \lambda(x)\} = \min\{\cup_{j \in J} \delta_j \cap \lambda\}(x)$$

for all $x \in P'$. Hence $(\delta', P') = (\cup_{j \in J} (\delta_j, P_j)) \cap (\lambda, M))$.

Definition 4.6. If $((\lambda, M), T_{(\lambda, M)})$ and $((\mu, N), S_{(\mu, N)})$ are soft fuzzy* subspaces of soft fuzzy topological spaces $(X, T)$ and $(Y, S)$ respectively. If $f$ is a mapping of $(X, T)$ into $(Y, S)$ then $f$ is a mapping of $((\lambda, M), T_{(\lambda, M)})$ into $((\mu, N), S_{(\mu, N)})$ if $f(\lambda, M) \subseteq (\mu, N)$.

Definition 4.7. Let $((\lambda, M), T_{(\lambda, M)})$ and $((\mu, N), S_{(\mu, N)})$ be any two soft fuzzy* subspaces of soft fuzzy* topological spaces $(X, T)$ and $(Y, S)$ respectively. Then a mapping $f$ of $((\lambda, M), T_{(\lambda, M)})$ into $((\mu, N), S_{(\mu, N)})$ is said to be soft fuzzy* relatively continuous iff for each soft fuzzy* open set $(\gamma', L')$ in $S_{(\mu, N)}$, $f^{-1}(\gamma', L') \cap (\lambda, M)$ is soft fuzzy* open in $T_{(\lambda, M)}$. 794
Definition 4.8. Let \((\lambda, M), T_{(\lambda,M)}\) and \(((\mu, N), S_{(\mu,N)})\) be any two soft fuzzy* subspaces of soft fuzzy* topological spaces \((X, T)\) and \((Y, S)\) respectively. Then a mapping \(f\) of \(((\lambda, M), T_{(\lambda,M)})\) into \(((\mu, N), S_{(\mu,N)})\) is said to be soft fuzzy* relatively open iff for each soft fuzzy* open set \((\gamma', L')\) in \(T_{(\lambda,M)}\), \(f(\gamma', L')'\) is soft fuzzy* open in \(S_{(\mu,N)}\).

Property 4.9. Let \(((\lambda, M), T_{(\lambda,M)})\) and \(((\mu, N), S_{(\mu,N)})\) be any two soft fuzzy* subspaces of soft fuzzy* topological spaces \((X, T)\) and \((Y, S)\) respectively. Let \(f\) be soft fuzzy* continuous mapping of \((X, T)\) into \((Y, S)\) such that \(f(\lambda, M) \subseteq (\mu, N)\). Then \(f\) is soft fuzzy* relatively continuous mapping of \(((\lambda, M), T_{(\lambda,M)})\) into \(((\mu, N), S_{(\mu,N)})\).

Proof. Let \((\gamma', L')\) be soft fuzzy* open in \(S_{(\mu,N)}\). Then there exists soft fuzzy* open \((\gamma, L)\) in \(S\) such that \((\gamma', L') = (\gamma, L) \cap (\mu, N)\). The inverse image \(f^{-1}(\gamma, L)\) is soft fuzzy* open in \(T_{(\lambda,M)}\). Hence \(f^{-1}(\gamma', L') \cap (\lambda, M) = f^{-1}(\gamma, L) \cap f^{-1}(\mu, N) \cap (\lambda, M) = f^{-1}(\gamma, L) \cap (\lambda, M)\) is soft fuzzy* open in \(T_{(\lambda,M)}\). Therefore \(f\) is soft fuzzy* relatively continuous.

Definition 4.10. A bijective mapping \(f\) of a soft fuzzy* topological space \((X, T)\) into \((Y, S)\) is said to be soft fuzzy* homeomorphism iff it is soft fuzzy* continuous and soft fuzzy* open.

Definition 4.11. A bijective mapping \(f\) of a soft fuzzy* subspace \(((\lambda, M), T_{(\lambda,M)})\) of \((X, T)\) into \(((\mu, N), S_{(\mu,N)})\) of \((Y, S)\) is said to be soft fuzzy* relatively homeomorphism iff \(f(\lambda, M) = (\mu, N)\) and \(f\) is soft fuzzy* relatively continuous and soft fuzzy* relatively open.

Property 4.12. Let \(f\) be soft fuzzy* continuous (resp. soft fuzzy* open) mapping of a soft fuzzy* topological space \((X, T)\) into a soft fuzzy* topological space \((Y, S)\) and \(g\) a soft fuzzy* continuous (resp. soft fuzzy* open) mapping of \((Y, S)\) into a soft fuzzy* topological space \((Z, R)\). Then the composition \(g \circ f\) is soft fuzzy* continuous (resp. soft fuzzy* open) mapping of \((X, T)\) into \((Z, R)\).

Proof. It is obvious.

Property 4.13. Let \(((\lambda, M), T_{(\lambda,M)}), ((\mu, N), S_{(\mu,N)}), ((\gamma, L), S_{(\gamma,L)}))\) be any three soft fuzzy* subspaces of soft fuzzy* topological spaces \((X, T), (Y, S), (Z, R)\) respectively. Let \(f\) be soft fuzzy* relatively continuous (resp. soft fuzzy* open) mapping of \(((\lambda, M), T_{(\lambda,M)})\) into \(((\mu, N), S_{(\mu,N)})\) and \(g\) be soft fuzzy* relatively continuous (resp. soft fuzzy* open) mapping of \(((\mu, N), S_{(\mu,N)})\) into \(((\gamma, L), S_{(\gamma,L)})\). Then the composition \(g \circ f\) is soft fuzzy* relatively continuous (resp. soft fuzzy* relatively open) mapping of \(((\lambda, M), T_{(\lambda,M)})\) into \(((\gamma, L), S_{(\gamma,L)})\).

Proof. Let \((\gamma', L')\) be soft fuzzy* open in \(Z_{(\gamma,L)}\). Then \(g^{-1}(\gamma', L') \cap (\mu, N)\) is soft fuzzy* open in \(S_{(\mu,N)}\) and \((f^{-1}(g^{-1}(\gamma', L'))) \cap (\mu, N)\) \(\cap (\lambda, M)\). But \(g \circ f\)^{-1}(\gamma', L') \(\cap (\lambda, M) = f^{-1}(g^{-1}(\gamma', L') \cap (\mu, N)) \cap (\lambda, M)\). Since \(f(\lambda, M) \subseteq (\mu, N)\) and so \(g \circ f\) is soft fuzzy* relatively continuous. The proof is trivial for soft fuzzy* relatively open mappings.
Definition 4.14. Let \((X, T)\) be a soft fuzzy* topological space. A subfamily \(\mathfrak{B}\) of \(T\) is a soft fuzzy* base for \(T\) iff each member of \(T\) can be expressed as the union of members of \(\mathfrak{B}\).

Definition 4.15. Let \((X, T)\) be a soft fuzzy* topological space. Let \(T_{(\lambda, M)}\) be the induced soft fuzzy* topology on \((\lambda, M)\) of \((X, T)\). A subfamily \(\mathfrak{B}'\) of \(T_{(\lambda, M)}\) is soft fuzzy* base for \(T_{(\lambda, M)}\) iff each member of \(T_{(\lambda, M)}\) can be expressed as the union of members of \(\mathfrak{B}'\).

Note 4.16. If \(\mathfrak{B}\) is a soft fuzzy base for a soft fuzzy* topology \(T\) on a set \(X\), then
\[\mathfrak{B}_{(\lambda, M)} = \{(\delta, P) \cap (\lambda, M) : (\delta, P) \in \mathfrak{B}\}\]
is a soft fuzzy* base for the induced soft fuzzy* topology \(T_{(\lambda, M)}\) on the soft fuzzy* open set \((\lambda, M)\).

Property 4.17. Let \(f\) be a mapping from soft fuzzy* topological space \((X, T)\) to a soft fuzzy* topological space \((Y, S)\). Let \(\mathfrak{B}\) be a soft fuzzy* base for \(S\). Then \(f\) is soft fuzzy* continuous iff for each soft fuzzy* open set \((\lambda, M)\) in \(\mathfrak{B}\) the inverse image \(f^{-1}(\lambda, M)\) is soft fuzzy* open is in \(T\).

Proof. Proof is obvious.

Property 4.18. Let \(((\lambda, M), T_{(\lambda, M)}), ((\mu, N), S_{(\mu, N)})\) be soft fuzzy* subspaces of soft fuzzy* topological spaces \((X, T), (Y, S)\) respectively. Let \(\mathfrak{B}'\) be a soft fuzzy* base for \(S_{(\mu, N)}\). Then a mapping \(f\) of \(((\lambda, M), T_{(\lambda, M)})\) into \(((\mu, N), S_{(\mu, N)})\) is soft fuzzy* relatively continuous iff for each \((\mu', N')\) in \(\mathfrak{B}'\) the intersection \(f^{-1}((\mu', N') \cap (\lambda, M))\) is in \(T_{(\lambda, M)}\).

Proof. Proof is obvious.

Definition 4.19. Let \(T_1\) and \(T_2\) be two soft fuzzy* topologies on the same set \(X\). Then we say that \(T_1\) is finer that \(T_2\) (and that \(T_2\) is coarser than \(T_1\)) if the identity mapping of \((X, T_1)\) into \((X, T_2)\) is soft fuzzy* continuous.

Definition 4.20. Let \(f : X \to Y\). Let \(T\) be a soft fuzzy* topology on \(X\). The finest soft fuzzy* topology \(S\) on \(Y\) for which \(f\) is soft fuzzy* continuous is called the image under \(f\) of \(T\). A soft fuzzy* set \((\mu, N)\) in \(Y\) is soft fuzzy* open in \(S\) iff \(f^{-1}(\mu, N)\) is a soft fuzzy* open set in \(X\).

Definition 4.21. Let \(f : X \to Y\) be a mapping. Let \(S\) be a soft fuzzy* topology on \(Y\). The coarsest soft fuzzy* topology \(T\) on \(X\) for which \(f\) is soft fuzzy* continuous is called the inverse image under \(f\) of \(S\). The soft fuzzy* open sets in \(X\) are the inverse images of soft fuzzy* open sets in \(Y\).

Definition 4.22. Given a family \(\{(X_j, T_j)\}_{j \in J}\) of a soft fuzzy* topological spaces. Define their product \(\Pi_{j \in J}(X_j, T_j)\) to be the soft fuzzy* topological space \((X, T)\) where \(X = \Pi_{j \in J}X_j\) is the usual set product and \(T\) is the coarsest soft fuzzy* topology on \(X\) for which the projection \(p_j\) of \(X\) onto \(X_j\) are soft fuzzy* continuous for each \(j \in J\). The soft fuzzy* topology \(T\) is called product soft fuzzy* topology on \(X\) and \((X, T)\) a product soft fuzzy* topological space.
Property 4.23. Let \( \{ (X_j, T_j) \} \in J \) be a family of soft fuzzy* topological spaces and \((X, T)\) the product soft fuzzy* topological space. The product soft fuzzy* topology \( T \) on \( X \) has a soft fuzzy* base the set of finite intersections of soft fuzzy* sets of the form \( p_j^{-1}(\lambda_j, M_j) \) where \( (\lambda_j, M_j) \in T_j, j \in J \).

Proof. Let \( \{ X_j \}, j = 1, 2, ..., n \) be a finite family of soft fuzzy* sets and for each \( j = 1, 2, ..., n \), let \( (\lambda_j, M_j) \) be a soft fuzzy* set in \( X_j \). Define the product \( (\lambda, M) = \prod_{j=1}^{n}(\lambda_j, M_j) \) of the family \( \{ (\lambda_j, M_j) \}_j=1,2,...,n \) as the soft fuzzy* set in \( X = \prod_{j=1}^{n}X_j \) that has the membership function given by

\[
\lambda(x_1, x_2, ..., x_n) = \min\{\lambda_1(x_1), ..., \lambda_n(x_n)\}
\]

for all \( (x_1, ..., x_n) \in M \).

For each \( j = 1, 2, ..., n \), \( p_j(\lambda, M) \subseteq (\lambda_j, M_j) \), since the membership function of \( p_j(\lambda, M) = (\gamma, L) \) is given by

\[
\gamma(x_j) = \sup\{x_1, ..., x_n)\in p_j^{-1}(x_j)\lambda(x_1, ..., x_n)
\]

\[
= \sup\{x_1, ..., x_n)\in p_j^{-1}(x_j)\min\{\lambda_1(x_1), ..., \lambda_n(x_n)\}
\]

\[
= \min\{\sup x_1, ..., \lambda_j(x_j), ..., \sup x_n)\in M_n\}
\]

\[
\leq \lambda_j(x_j) \text{ for all } x_j \in L
\]

□

Remark 4.24. By Property 4.23, if \( X_j \) has soft fuzzy* topology \( T_j, j = 1, 2, ..., n \) the product soft fuzzy* topology on \( X \) has a soft fuzzy* base the set of product soft fuzzy* sets of the form \( \Pi_{j=1}^{n}(\lambda_j, M_j) \) where \( (\lambda_j, M_j) \in T_j, j = 1, 2, ..., n \).

Property 4.25. Let \( \{ (X_j, T_j) \}, j = 1, 2, ..., n \) be a finite family of soft fuzzy* topological spaces and \((X, T)\) the product soft fuzzy* topological space. For each \( j = 1, 2, ..., n \) let \( (\lambda_j, M_j) \) be a soft fuzzy* set in \( X_j \) and \( (\lambda, M) \) be the product soft fuzzy* set in \( X \). Then the induced soft fuzzy* topology \( T_{(\lambda, M)}(\lambda, M) \) has a soft fuzzy* base the set of product soft fuzzy* sets of the form \( \Pi_{j=1}^{n}(\alpha_j', A_j') \) where \( (\alpha_j', A_j') \in (T_j)_{(\lambda_j, M_j)}, j = 1, 2, ..., n \).

Proof. By Remark 4.24, \( T \) has a soft fuzzy* base

\[
\mathcal{B} = \{ \Pi_{j=1}^{n}(\alpha_j, A_j) : (\alpha_j, A_j) \in T_j, j = 1, 2, ..., n \}
\]

A soft fuzzy* base for \( T_{(\lambda, M)}(\lambda, M) \) is therefore given by

\[
\mathcal{B}(\lambda, M) = \{ (\Pi_{j=1}^{n}(\alpha_j, A_j)) \cap (\lambda, M) : (\lambda_j, M_j) \in T_j, j = 1, 2, ..., n \}.
\]

But \( (\Pi_{j=1}^{n}(\alpha_j, A_j)) \cap (\lambda, M) = \Pi_{j=1}^{n}(\alpha_j, A_j) \cap (\lambda, M) \). Hence the property follows with \( (\alpha_j', A_j') = (\alpha_j, A_j) \cap (\lambda, M) \). □

Property 4.26. Let \( \{ (X_j, T_j) \} \in J \) be a family of soft fuzzy* topological spaces \((X, T)\) the product soft fuzzy* topological space. Let \( f \) be a mapping of a soft fuzzy* topological space \((Y, S)\) into \((X, T)\). Then \( f \) is soft fuzzy* continuous iff \( p_j \circ f \) is soft fuzzy* continuous for each \( j \in J \).

Proof. Proof is obvious. □
Corollary 4.27. Let \( \{(X_j, T_j)\}, \{(Y_j, S_j)\}, j \in J \) be two families of soft fuzzy* topological spaces and \( (X, T) \) \((Y, S)\) the respective product soft fuzzy* topological spaces. For each \( j \in J \), let \( f_j \) be a mapping of \((X_j, T_j)\) into \((Y_j, S_j)\). Then the product mapping \( f : \Pi_j \in J f_j : (x_j) \mapsto (f_j(x_j)) \) of \((X, T)\) into \((Y, S)\) is soft fuzzy* continuous if \( f_j \) is soft fuzzy* continuous for each \( j \in J \).

Proof. The mapping \( f \) can be written as \( x \mapsto (f_j(P_j(x))) \) where \( x = (x_j) \) and is therefore soft fuzzy* continuous by Property 4.26. \( \square \)

Property 4.28. Let \( \{(X_j, T_j)\}, j = 1, 2, \ldots, n \) be a finite family of soft fuzzy* topological spaces and \((X, T)\) the product soft fuzzy* topological spaces. For each \( j = 1, 2, \ldots, n \), let \((\lambda_j, M_j)\) be a soft fuzzy* set in \( X_j \) and \((\lambda, M)\) the product soft fuzzy* set in \( X \). Let \((Y, S)\) be a soft fuzzy* topological space, \((\mu, N)\) be a soft fuzzy* set in \((Y, S)\) and \( f \) a mapping of the soft fuzzy* subspace \((\mu, N), S_{(\mu,N)}\) into the soft fuzzy* subspace \((\lambda, M), T_{(\lambda,M)}\). Then \( f \) is soft fuzzy* relatively continuous iff \( p_j \circ f \) is soft fuzzy* relatively continuous for each \( j = 1, 2, \ldots, n \).

Proof. By Property 4.13, the soft fuzzy* continuity of \( p_j \) implies the soft fuzzy* relatively continuity of \( p_j \) for each \( j = 1, 2, \ldots, n \). The composition \( p_j \circ f \) is therefore soft fuzzy* relatively continuous for each \( j = 1, 2, \ldots, n \).

Conversely, let \((\lambda', M') = (\lambda'_1, M'_1) \times \ldots \times (\lambda'_n, M'_n)\) where \((\lambda'_j, M'_j)\in(T_j)_{(\lambda_j,M_j)}, j = 1, 2, \ldots, n\). By Property 4.25, the set of such \((\lambda', M')\) form a soft fuzzy base of \( T_{(\lambda,M)} \). Since

\[
f^{-1}(\lambda', M') \cap (\mu, N) = f^{-1}(p_1^{-1}(\lambda'_1, M'_1) \cap \ldots \cap p_n^{-1}(\lambda'_n, M'_n) \cap (\mu, N)) = \cap_{j=1}^n ((p_j \circ f)^{-1}(\lambda'_j, M'_j) \cap (\mu, N)) \]

is soft fuzzy* open in \( S_{(\mu,N)} \), as \( p_j \circ f \) is soft fuzzy* relatively continuous for each \( j = 1, 2, \ldots, n \) it follows that from Property 4.17, that \( f \) is soft fuzzy* relatively continuous. \( \square \)

Corollary 4.29. Let \( \{(X_j, T_j)\}, \{(Y_j, S_j)\}, j = 1, 2, \ldots, n \) be two finite families of soft fuzzy* topological spaces and \((X, T), (Y, S)\) the respective product soft fuzzy* topological spaces. For each \( j = 1, 2, \ldots, n \), let \((\lambda_j, M_j)\) be a soft fuzzy* set in \( X_j \), \((\mu_j, N_j)\) be a soft fuzzy* set in \( Y_j \) and \( f_j \) a mapping of the soft fuzzy* subspaces \(((\lambda_j, M_j), T_{(\lambda_,M_j)})\) into the soft fuzzy* subspace \(((\mu_j, N_j), S_{(\mu_,N_j)})\). Let \((\lambda, M) = \Pi_{j=1}^n(\lambda_j, M_j)\) and \((\mu, N) = \Pi_{j=1}^n(\mu_j, N_j)\) be the product mapping \( f = \Pi_{j=1}^n f_j : (x_1, \ldots, x_n) \mapsto (f_1(x_1), \ldots, f_n(x_n)) \) of the soft fuzzy* subspace \(((\lambda,M), T_{(\lambda_,M)})\) into the soft fuzzy* subspace \(((\mu,N), S_{(\mu,N)})\) is soft fuzzy* relatively continuous if \( f_j \) is soft fuzzy* relatively continuous for each \( j = 1, 2, \ldots, n \).

Proof. By Corollary 4.27, the proof is obvious. \( \square \)

Property 4.30. Let \( \{(X_j, T_j)\}, \{(Y_j, S_j)\}, j = 1, 2, \ldots, n \) be two finite families of soft fuzzy* topological spaces and \((X, T), (Y, S)\) the respective product soft fuzzy* topological spaces. For each \( j = 1, 2, \ldots, n \), let \( f_j \) be a mapping of \((X_j, T_j)\) into \((Y_j, S_j)\). Then the product mapping \( f : \Pi_{j=1}^n f_j : (x_1, \ldots, x_n) \mapsto (f_1(x_1), \ldots, f_n(x_n)) \) of \((X, T)\) into \((Y, S)\) is soft fuzzy* open if \( f_j \) is soft fuzzy* open for each \( j = 1, \ldots, n \).
Proof. Let \((\lambda, M)\) be soft fuzzy* open in \((X, T)\). Then there exists soft fuzzy* open set \((\lambda_{ja}, M_{ja})\) \(a \in A, j = 1, \ldots, n\) such that \((\lambda, M) = \sqcup_{a \in A} \prod_{j=1}^{n} (\lambda_{ja}, M_{ja})\).

The image \(f(\lambda, M)\) of \((\lambda, M)\) has the membership function \(f(\lambda, M) = (\gamma, L)\) for all \(y \in L \subseteq S\).

\[
\gamma(y) = \sqcup_{a \in A} \sup_{z \in f^{-1}(y)} \prod_{j=1}^{n} \lambda_{ja}(z)
\]

Thus \(f(\lambda, M) = \sqcup_{a \in A} \prod_{j=1}^{n} (f_{j}(\lambda_{ja}, M_{ja}))\). Since \(f_{j}\) is soft fuzzy* open for each \(j = 1, \ldots, n\), \(f(\lambda, M)\) is soft fuzzy* open in \((Y, S)\).

Property 4.31. Let \(\{(X_j, T_j)\}, \{(Y_j, S_j)\}\) \(j = 1, 2, \ldots, n\) be two finite families of soft fuzzy* topological spaces and \((X, T), (Y, S)\) the respective product soft fuzzy* topological spaces. For each \(j = 1, 2, \ldots, n\), let \((\lambda_j, M_j)\) be a soft fuzzy* set in \(X_j\), \((\mu_j, N_j)\) be a soft fuzzy* set in \(Y_j\) and \(f_j\) a mapping of the soft fuzzy* subspace \((\lambda_j, M_j) \times (\mu_j, N_j)\) into the soft fuzzy* subspace \((\lambda, M) \times (\mu, N)\) of \(X\) \times \(Y\) respectively. Then the product soft fuzzy* mapping \(f = \prod_{j=1}^{n} f_j : (x_1, \ldots, x_n) \to (f_1(x_1), \ldots, f_n(x_n))\) of the soft fuzzy* subspace \((\lambda, M), T_{(\lambda, M)}\) into the soft fuzzy* subspace \((\mu, N), S_{(\mu, N)}\) is soft fuzzy* relatively open if \(f_j\) is soft fuzzy* relatively open for each \(j = 1, 2, \ldots, n\).

Proof. Let \((X', M')\) be soft fuzzy* open in \(T_{(\lambda, M)}\). By Property 4.25, there exists soft fuzzy* open sets \((\lambda_{ja}, M_{ja}) \in (T_j)_{(\lambda_j, M_j)}, a \in A, j = 1, \ldots, n\) such that \((X', M') = \sqcup_{a \in A} \prod_{j=1}^{n} (\lambda_{ja}, M_{ja})\). By Property 4.30, \(f(X', M') = \sqcup_{a \in A} \prod_{j=1}^{n} (f_j(\lambda_{ja}, M_{ja}))\). Since \(f_j\) is soft fuzzy* relatively open for each \(j = 1, \ldots, n\), \(f(X', M')\) is soft fuzzy* open in \(S_{(\mu, N)}\).

Property 4.32. Let \((X_1, T_1)\) and \((X_2, T_2)\) be soft fuzzy* topological spaces and \((X, T)\) the product soft fuzzy* topological space. Then for each \(a_1 \in X_1\), the mapping \(i : x_2 \to (a_1, x_2)\) of \((X_2, T_2)\) into \((X, T)\) is soft fuzzy* continuous.

Proof. The constant mapping \(i_1 : x_2 \to a_1\) from \((X_2, T_2)\) into \((X_1, T_1)\) is soft fuzzy* continuous. For if \((\lambda_1, M_1)\) is soft fuzzy* open in \(T_1\), the inverse image \(f^{-1}(\lambda_1, M_1)\) has the membership function is given by

\[
i_1^{-1}(\lambda_1)(x_2) = \lambda_1 \circ i(x_2)
\]

where \((k_c, H)\) is the soft fuzzy* open set in \(X_2\) which has the constant membership function with value \(c = \lambda_1(a_1)\), since the identity mapping \(i_2 : x_2 \to x_2\) of \((X_2, T_2)\) into itself is soft fuzzy continuous, the mapping \(i\) is soft fuzzy continuous by Property 4.26.

Property 4.33. Let \((X_1, T_1)\) and \((X_2, T_2)\) be soft fuzzy* topological spaces and \((X, T)\) the product soft fuzzy* topological space. Let \((\lambda_1, M_1)\), \((\lambda_2, M_2)\) be a soft
fuzzy* open set in $X_1, X_2$ respectively. Let $(\lambda, M)$ be the product soft fuzzy* set in $X$. Then for each $a_1 \in M_1$ such that $\lambda_1(a_1) \geq \lambda_2(x_2)$ for all $x_2 \in M_2$, the mapping $i : x_2 \rightarrow (a_1, x_2)$ of the soft fuzzy* subspace $((\lambda_2, M_2), (T_2)(\lambda_2, M_2))$ into the soft fuzzy* subspace $((\lambda, M), T(\lambda, M))$ is soft fuzzy* relatively continuous.

Proof. Since $i(\lambda_2, M_2) \subseteq (\lambda, M)$, since the membership function of $i(\lambda_2, M_2) = (\gamma, L)$ is given by

$$\gamma(x_1, x_2) = \sup_{x_2 \in f^{-1}(x_1, x_2)} \lambda_2(x_2)$$

and that of $(\lambda, M)$ by

$$\lambda(x_1, x_2) = (\min\{\lambda_1(x_1), \lambda_2(x_2)\})$$

$$\geq \lambda(x_2)$$

for all $(x_1, x_2) \in M, x_1 \in M_1$.

The proof of the soft fuzzy* relative continuity of $i$ is analogous to the proof of the soft fuzzy* continuity of $i$ in Property 4.32. □

5. Soft fuzzy* group

Definition 5.1. Let $X$ be a group and $M$ be a subgroup of $X$. Let $(\lambda, M)$ be any soft fuzzy* set in $X$. Then $(\lambda, M)$ is said to be soft fuzzy* group in $X$ satisfies the following conditions

(i) $\lambda(xy) \geq \min\{\lambda(x), \lambda(y)\}$ for every $x, y \in M$;
(ii) $\lambda(x^{-1}) = \lambda(x)$ for every $x \in M$.

Property 5.2. If $(\lambda, M)$ is a soft fuzzy* group then

(i) $\lambda(x) \leq \lambda(e)$ for $x, e \in M$.
(ii) $\lambda(xy^{-1}) \geq \min\{\lambda(x), \lambda(y)\}$ for every $x, y \in M$.

Proof. (i) Let $x, e \in M$. Now

$$\lambda(e) = \lambda(xx^{-1})$$

$$\geq \min\{\lambda(x), \lambda(x^{-1})\}$$

$$= \lambda(x)$$

By Definition 5.1(ii) [

Therefore, $\lambda(x) \leq \lambda(e)$ for $x, e \in M$.

(ii) Let $x, y \in M$. Now

$$\lambda(xy^{-1}) \geq \min\{\lambda(x), \lambda(y^{-1})\}$$

$$= \min\{\lambda(x), \lambda(y)\}$$

Therefore, $\lambda(xy^{-1}) \geq \min\{\lambda(x), \lambda(y)\}$ for every $x, y \in M$. □

Property 5.3. Let $X, Y$ be groups and $f$ a homomorphism of $X$ into $Y$. Let $(\lambda, M)$ be a soft fuzzy* group in $X$. Then the inverse image $f^{-1}(\lambda, M)$ of $(\lambda, M)$ is a soft fuzzy* group in $X$. 800
Proof. For all \(x, y \in f^{-1}(M)\),
\[
f^{-1}(\lambda)(xy^{-1}) = \lambda(f(xy^{-1}))
\]
\[
= \lambda(f(x)f(y^{-1}))
\]
\[
= \lambda(f(x)(f(y))^{-1})
\]
\[
\geq \min\{\lambda(f(x)), \lambda(f(y))\}
\]
\[
= \{\min\{f^{-1}(\lambda(x)), f^{-1}(\lambda(y))\}\}.
\]
Therefore, \(f^{-1}(\lambda, M)\) of \((\lambda, M)\) is a soft fuzzy* group in \(X\). \(\square\)

**Definition 5.4.** A soft fuzzy* set \((\lambda, M)\) of \(X\) is said to have **soft fuzzy* sup property** if for any subset \(S \subseteq X\) there exists \(t_0 \in S\) such that \(\lambda(t_0) = \sup_{t \in S} \lambda(t)\).

**Property 5.5.** Let \(X, Y\) be groups and \(f\) a homomorphism of \(X\) into \(Y\). Let \((\lambda, M)\) be a soft fuzzy* group in \(X\) that has soft fuzzy* sup property. Then the image \(f(\lambda, M)\) of \((\lambda, M)\) is a soft fuzzy* group in \(Y\).

**Proof.** Let \(f(\lambda, M) = (\gamma, L)\). Let \(u, v \in L\). If either \(f^{-1}(u), f^{-1}(v)\) is empty then the inequality in Property 4.1(ii) is trivially satisfied. Suppose neither \(f^{-1}(u)\) nor \(f^{-1}(v)\) is empty.

Let \(r_0 \in f^{-1}(u)\), \(s_0 \in f^{-1}(v)\) such that 
\[
\lambda(r_0) = \sup_{t \in f^{-1}(u)} \lambda(t)\) and \(\lambda(s_0) = \sup_{t \in f^{-1}(u)} \lambda(t)\).
Then
\[
\gamma(uv^{-1}) = \sup_{w \in f^{-1}(uv^{-1})} \lambda(w)
\]
\[
\geq \min\{\lambda(r_0), \lambda(s_0)\}
\]
\[
= \min\{\sup_{t \in f^{-1}(u)} \lambda(t), \sup_{t \in f^{-1}(v)} \lambda(t)\}
\]
\[
= \min\{\gamma(u), \gamma(v)\}.
\]
Therefore, the image \(f(\lambda, M)\) of \((\lambda, M)\) is a soft fuzzy* group in \(Y\). \(\square\)

**Note 5.6.** The membership function \(\lambda\) of a soft fuzzy* group \((\lambda, M)\) in a group \(X\) is **soft fuzzy* invariant** if for all \(x_1, x_2 \in M\), \(f(x_1) = f(x_2)\) implies \(\lambda(x_1) = \lambda(x_2)\).

Clearly a homomorphic image \(f(\lambda, M)\) of \((\lambda, M)\) is then a soft fuzzy* group.

**Remark 5.7.** Given a soft fuzzy* group \((\lambda, M)\) in a group \(X\) where \(M\) denote the set \(\{x | \lambda(x) = \lambda(e)\}\) is a subgroup of \(X\). For \(a \in X\), let \(\rho_a : x \rightarrow ax\) and \(\sigma_a : x \rightarrow ax\) denote respectively, the right and left translation of \(X\) into itself.

**Property 5.8.** Let \((\lambda, M)\) be a soft fuzzy* group in a group \(X\) then for all \(a \in M\), 
\[
\rho_a(\lambda, M) = \sigma_a(\lambda, M) = (\lambda, M).
\]

**Proof.** Let \(a \in M\). Then the membership function of \(\rho_a(\lambda, M) = (R_a, R)\) is given by 
\[
R_a(x) = \sup_{t \in f^{-1}(x)} \lambda(t)\) for all \(t \in M
\]
\[
= \lambda(xa^{-1})\) [by soft fuzzy* sup property, there exists \(xa^{-1} \in M]\)
\[
\geq \{\min\{\lambda(x), \lambda(e)\}\) [by Propert 5.2(ii)]]
\[
= \lambda(x)
\]
Conversely,

\[ \lambda(x) = \lambda(xa^{-1}) \]

\[ \geq \min\{\lambda(xa^{-1}), \lambda(a)\} \]

\[ = \min\{\lambda(xa^{-1}), \lambda(e)\} \] [since \( a \in M \)]

\[ = \lambda(xa^{-1}) \]

\[ = \sup_{t \in M} \lambda(t) \]

\[ = R_a(x) \] for all \( x \in L \subseteq M \)

Therefore, \( \rho_a(\lambda, M) = (\lambda, M) \). Similarly, \( \sigma_a(\lambda, M) = (\lambda, M) \).

\( \square \)

6. Soft fuzzy* topological groups

**Definition 6.1.** Let \( X \) be a group and \( T \) a soft fuzzy* topology on \( X \). Let \( (\lambda, M) \) be a soft fuzzy* group in \( X \) and let \( (\lambda, M) \) be endowed with induced soft fuzzy* topology \( T_{(\lambda,M)} \). Then \( (\lambda, M) \) is a soft fuzzy* topological group in \( X \) iff it satisfies the following two conditions

1. the mapping \( \alpha : (x, y) \rightarrow xy \) of \( (\lambda, M), T_{(\lambda,M)} \times (\lambda, M), T_{(\lambda,M)} \) into \( (\lambda, M), T_{(\lambda,M)} \) defined by \( \alpha(x, y) = xy \) is soft fuzzy* relatively continuous.

2. the mapping \( \beta : x \rightarrow x^{-1} \) of \( (\lambda, M), T_{(\lambda,M)} \) into \( (\lambda, M), T_{(\lambda,M)} \) is defined by \( \beta(x) = x^{-1} \) is soft fuzzy* relatively continuous.

**Note 6.2.** A soft fuzzy* group structure and an induced soft fuzzy* topology are said to be compatible if they satisfy (i) and (ii).

**Property 6.3.** Let \( X \) be a group having soft fuzzy* topology \( T \). A soft fuzzy* group \( (\lambda, M) \) in \( X \) is a soft fuzzy* topological group iff the mapping \( f : (x, y) \rightarrow xy^{-1} \) of \( ((\lambda, M), T_{(\lambda,M)} \times (\lambda, M), T_{(\lambda,M)} ) \) into itself is soft fuzzy* relatively continuous.

**Proof.** The mapping \( (x, y) \rightarrow (x, y^{-1}) \) of \( ((\lambda, M), T_{(\lambda,M)} \times (\lambda, M), T_{(\lambda,M)} ) \) into itself is soft fuzzy* continuous. By the Corollary 4.33. Hence the composition \( (x, y) \rightarrow (x, y^{-1}) \rightarrow xy^{-1} \) is soft fuzzy* relatively continuous.

Conversely, by Property 5.2(i), \( \lambda(e) \geq \lambda(x) \) for all \( x \in M \) and therefore by Property 4.33, the canonical injection \( i : y \rightarrow (e, y) \rightarrow ey^{-1} \) of \( ((\lambda, M), T_{(\lambda,M)} ) \) into \( ((\lambda, M), T_{(\lambda,M)} \times (\lambda, M), T_{(\lambda,M)} ) \) is soft fuzzy* relatively continuous. Hence the composition \( \beta : y \rightarrow (e, y) \rightarrow ey^{-1} \) is soft fuzzy* continuous. The mapping \( \alpha : (x, y) \rightarrow xy \) of \( (\lambda, M), T_{(\lambda,M)} \times (\lambda, M), T_{(\lambda,M)} ) \) into \( ((\lambda, M), T_{(\lambda,M)} ) \) is soft fuzzy* relatively continuous since it is the composition \( (x, y) \rightarrow (x, y^{-1}) \rightarrow x(y^{-1})^{-1} \) of soft fuzzy* relatively continuous mappings.

**Remark 6.4.** If \( (\lambda, M) \) is a soft fuzzy* group in a group \( X \) carrying soft fuzzy* topology \( T \). Then in general, the translations \( \rho_a, \sigma = x \in X \) are not soft fuzzy* relatively continuous mappings of \( ((\lambda, M), T_{(\lambda,M)} ) \) into itself. However the following special case \( M = \{ x | \lambda(e) = \lambda(x) \} \).

**Property 6.5.** Let \( X \) be a group having soft fuzzy* topology \( T \) and let \( (\lambda, M) \) be a soft fuzzy* topological group in \( X \). For each \( a \in M \) the translations \( \rho_a, \sigma_a \) are soft fuzzy* relative homeomorphism of \( ((\lambda, M), T_{(\lambda,M)} ) \) into itself.
Proof. By Property 5.8, \( \rho_a(\lambda, M) = (\lambda, M) \) and \( \sigma_a(\lambda, M) = (\lambda, M) \) for all \( a \in M \). The mapping \( \sigma_a \) is the composition of the injection \( i : y \rightarrow (a, y) \) and the mapping \( \alpha : (x, y) \rightarrow xy \). Since \( \lambda(a) \geq \lambda(y) \) for every \( y \in M \subseteq Y \). It follows from Property 4.33, \( i \) is a soft fuzzy* relative continuous mapping of \((\lambda, M), T_{(\lambda, M)}(i)\) into \((\lambda, M), T_{(\lambda, M)}(i)\). The mapping \( \alpha \) is soft fuzzy* relative continuous by hypothesis. Hence \( \sigma_a \) is soft fuzzy* relatively open. Then \( \sigma_a \) is soft fuzzy* relative homeomorphism. Similarly we proved \( \rho_a \) and \( \rho_a^{-1} \) is soft fuzzy* relative homeomorphism.

Suppose that \( X \) and \( Y \) are groups and that \( f \) is a homomorphism of \( X \) into \( Y \). Let \( Y \) have soft fuzzy* topology \( Y \) and let \( (\lambda, M) \) be a soft fuzzy* topological group in \( Y \). The mapping \( f \) gives rise to a soft fuzzy* topology \( T \) on \( X \), the inverse image under \( f \) of \( S \), and by Property 5.3, it also gives rise to a soft fuzzy* group in \( X \), the inverse image \( f^{-1}(\lambda, M) \) of \((\lambda, M)\). The following property shows that the induced soft fuzzy* topology on \( f^{-1}(\lambda, M) \) and the soft fuzzy* group structure are compatible.

**Property 6.6.** Let \( X, Y \) be a homomorphism \( f \) of \( X \) into \( Y \) and a soft fuzzy* topology \( S \) on \( Y \), let \( X \) have soft fuzzy* topology \( T \), where \( T \) is the inverse image under \( f \) of \( S \) and \( (\lambda, M) \) be a soft fuzzy* topological in \( Y \). Then the inverse image \( f^{-1}(\lambda, M) \) of \((\lambda, M)\) is a soft fuzzy* topological group in \( X \).

**Proof.** To show that the mapping \( \gamma_X : (x_1, x_2) \rightarrow (x_1x_2^{-1}) \) of \((f^{-1}(\lambda, M), T_{(f^{-1}(\lambda, M))}) \times (f^{-1}(\lambda, M), T_{(f^{-1}(\lambda, M))}) \) into \((f^{-1}(\lambda, M), T_{(f^{-1}(\lambda, M))}) \) is soft fuzzy* relatively continuous. Let \((\alpha', A')\) be an soft fuzzy* open in the induced soft fuzzy* topology \( T_{f^{-1}(\lambda, M)} \) on \( f^{-1}(\lambda, M) \). Since \( f \) is a soft fuzzy* continuous mapping of \((X, T) \) into \((Y, S)\) it is, by Property 4.9, a soft fuzzy* relatively continuous mapping of \((f^{-1}(\lambda, M), T_{(f^{-1}(\lambda, M))}) \) into \((\lambda, M), T_{(\lambda, M)}\). Also that there exists an soft fuzzy* open set \((\delta', P')\) in \( S_{(\lambda, M)} \) such that \( f^{-1}(\delta', P') = (\alpha', A') \). The membership function of

\[
\gamma_X^{-1}(\alpha')(x_1, x_2) = \alpha' \circ \gamma_X(x_1, x_2) = \alpha'(x_1x_2^{-1}) = f^{-1}(\delta')(x_1x_2^{-1}) = \delta'(f(x_1x_2^{-1})) = \delta'(f(x_1)(f(x_2))^{-1})
\]

for all \((x_1, x_2) \in \gamma_X^{-1}(A') \times \gamma_X^{-1}(A') \) where \( A' = f^{-1}(P') \).

By hypothesis, the mapping \( \gamma_Y : (y_1, y_2) \rightarrow y_1y_2^{-1} \) of \((\lambda, M), S_{(\lambda, M)} \times (\lambda, M), S_{(\lambda, M)} \) into \((\lambda, M), S_{(\lambda, M)} \) is soft fuzzy* relatively continuous and by Corollary 4.29, so is the product mapping \( f \times f \) of \((f^{-1}(\lambda, M), T_{(f^{-1}(\lambda, M))}) \times (f^{-1}(\lambda, M), T_{(f^{-1}(\lambda, M))}) \) into \((\lambda, M), S_{(\lambda, M)} \). But

\[
\delta'(f(x_1)(f(x_2))^{-1}) = \gamma_Y^{-1}(\delta')(f(x_1), f(x_2)) \text{ where } y_1 = f(x_1) \text{ and } y_2 = (f(x_2))^{-1} = (f \times f)^{-1}(\gamma_Y^{-1}(\delta'))(x_1, x_2)
\]

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for every \((x_1, x_2) \in \gamma_X^{-1}(A')\times \gamma_X^{-1}(A')\). Hence \(\gamma_X^{-1}(\alpha', A') \cap (f^{-1}(\lambda, M) \times f^{-1}(\lambda, M)) = (f \times f)^{-1}(\gamma_Y^{-1}(\delta', P')) \cap (f^{-1}(\lambda, M) \times f^{-1}(\lambda, M))\) is open in the induced soft fuzzy* topology on \((f^{-1}(\lambda, M) \times f^{-1}(\lambda, M)) \times (f^{-1}(\lambda, M) \times f^{-1}(\lambda, M))\). \(\Box\)

**Property 6.7.** Given groups \(X, Y\) a homomorphism \(f\) of \(X\) into \(Y\) and a soft fuzzy* topology \(T\) on \(X\), let \(Y\) have soft fuzzy* topology \(S\), where \(S\) is the image under \(f\) of \(T\) and let \((\lambda, M)\) be a soft fuzzy* topological group in \(X\). If the membership function \(\lambda\) of \((\lambda, M)\) is soft fuzzy* \(f\) invariant then the image \(f(\lambda, M)\) of \((\lambda, M)\) is a soft fuzzy* topological group in \(Y\).

**Proof.** To show that the mapping \(\gamma_Y : (y_1, y_2) \rightarrow (y_1y_2^{-1})\) of \((f(\lambda, M), S_{f(\lambda, M)})\times (f(\lambda, M), S_{f(\lambda, M)})\) into \((f(\lambda, M), S_{f(\lambda, M)})\) is soft fuzzy* relatively continuous. Note that \(f\) is soft fuzzy* open, for if \((\gamma, L) \in T\), then \((f(\gamma, L)) \in S\). Since the inverse image \(f^{-1}(f(\gamma, L))\) is the union of soft fuzzy* open sets and thus soft fuzzy* open in \(T\). It follows that \(f\) is soft fuzzy* relatively open. Since if \((\delta', P') \in T_{f(\lambda, M)}\), there exists \((\gamma, L) \in T\) such that \((\delta', P') = (\gamma, L) \cap (\lambda, M)\) and by the soft fuzzy* \(f\) invariance of \(\lambda, f(\delta', P') = f(\gamma, L) \cap f(\lambda, M) \in S_{f(\lambda, M)}\). By Property 4.25, the product mapping \(f \times f\) is soft fuzzy* relatively open mapping of \(((\lambda, M), T_{f(\lambda, M)})\times ((\lambda, M), T_{f(\lambda, M)})\) into \((f(\lambda, M), S_{f(\lambda, M)})\times (f(\lambda, M), S_{f(\lambda, M)})\).

Let \((\alpha', A')\) be a soft fuzzy* open set in \(S_{f(\lambda, M)}\). The membership function of \((f \times f)^{-1}(\gamma_Y^{-1}(\alpha', A'))\) is given by

\[
(f \times f)^{-1}(\gamma_Y^{-1}(\alpha')) = (\gamma_X^{-1}(\alpha'))(f(x_1)(f(x_2)^{-1})
\]

for all \((x_1, x_2) \in (f \times f)^{-1}(\gamma_Y^{-1}(A'))\) where \(\gamma_X : (x_1, x_2) \rightarrow x_1x_2^{-1}\). But, by hypothesis, \(\gamma_X\) is a soft fuzzy* relatively continuous mapping of \(((\lambda, M), T_{f(\lambda, M)})\times ((\lambda, M), T_{f(\lambda, M)})\) into \(((\lambda, M), T_{f(\lambda, M)})\) and \(f\) is a soft fuzzy* relatively continuous mapping of \(((\lambda, M), T_{f(\lambda, M)})\) into \((f(\lambda, M), S_{f(\lambda, M)})\). Hence, by the soft fuzzy* \(f\) invariance of \(\lambda\),

\[
(f \times f)^{-1}(\gamma_Y^{-1}(\alpha', A')) \cap (f(\lambda, M) \times f(\lambda, M)) = (f \times f)^{-1}(\gamma_Y^{-1}(\delta', P')) \cap (f(\lambda, M) \times f(\lambda, M))
\]

is open in the induced soft fuzzy* topology on \((\lambda, M) \times (\lambda, M)\). As \(f \times f\) is soft fuzzy* relatively open,

\[
(f \times f)(f \times f)^{-1}(\gamma_Y^{-1}(\delta', P')) \cap (f(\lambda, M) \times f(\lambda, M)) = (f \times f)^{-1}(\delta', P') \cap (f(\lambda, M) \times f(\lambda, M))
\]

is open in the induced soft fuzzy* topology on \((f(\lambda, M) \times f(\lambda, M))\). \(\Box\)

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