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Intuitionistic fuzzy soft matrix theory and its application in medical diagnosis

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ABSTRACT. Today, is a world of uncertainty with its associated problems, which can well be handled by Soft set theory, which is a new emerging mathematical tool. In this paper, we focus on developing an algorithm which is a new problem solving strategy in the medical diagnosis domain by implementing a novel term " intuitionistic fuzzy soft matrices". In order to attain this, we define a Value matrix and Score matrix and the output is obtained, based on the maximum score in the Score matrix.

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Keywords: Soft set, Fuzzy soft set, Intuitionistic fuzzy soft set, Intuitionistic fuzzy soft matrix, Value matrix, Score matrix.

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1. INTRODUCTION

In real world, we face so many uncertainties in all walks of life fields. However most of the existing mathematical tools for formal modeling, reasoning and computing are crisp deterministic and precise in character. There are theories viz, theory of probability, evidence, fuzzy set, intuitionistic fuzzy set, vague set, interval mathematics, rough set for dealing with uncertainties. These theories have their own difficulties as pointed out by Molodtsov([6]). In 1999, Molodtsov([6]) initiated a novel concept of soft set theory, which is completely new approach for modeling vagueness and uncertainties. Soft set theory has a rich potential for application in solving practical problems in economics, social science, medical science etc. Later on Maji et al. ([3]) have proposed the theory of fuzzy soft set.Majumdar et al([5]) have further generalised the concept of fuzzy soft sets.Maji et al. [4] extended soft sets to intuitionistic fuzzy soft sets.

Matrices play an important role in the broad area of science and engineering. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties. Fuzzy matrices play a vital role in Scientific Development. In ([9]), Yong et al. initiated a matrix representation of a fuzzy soft set and applied it in certain decision making problems. In([1]) Borah et al. and in ([7]) Neog et al. extended fuzzy soft matrix theory and its application. In ([2]), Chetia et al. proposed intuitionistic fuzzy soft matrix theory. Accordingly,([8]), Rajarajeswari et al proposed new definitions for intuitionistic fuzzy soft matrices and its types. Also extended and applied some operations on it.

In this paper, a new approach for medical diagnosis is proposed by employing intuitionistic fuzzy soft matrices. In order to attain this, Value matrix and Score matrix are employed. The Solution is obtained based on the maximum score in the score matrix.

2. Preliminaries

In this section, we recall some basic notion of fuzzy soft set theory and fuzzy soft matrices.

Definition 2.1 ([6]). Suppose that U is an initial Universe set and E is a set of parameters, let P(U) denotes the power set of U.A pair(F, E) is called a soft set over U where F is a mapping given by $F : E \to P(U)$.Clearly, a soft set is a mapping from parameters to P(U), and it is not a set, but a parameterized family of subsets of the Universe.

Example 2.2. Suppose that $U = \{s_1, s_2, s_3, s_4\}$ is a set of students and $E = \{e_1, e_2, e_3\}$ is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set E to the set of all subsets of power set U.Then soft set (F, E) describes the character of the students with respect to the given parameters, for finding the best student of an academic year.

 $(F, E) = \{ \{ \text{result} = s_1, s_3, s_4 \}, \{ \text{conduct} = s_1, s_2 \},$ $\{ \text{sports performances} = s_2, s_3, s_4 \} \}$

We can represent a soft set in the form of table given below

U	$\operatorname{Result}(e_1)$	$\operatorname{Conduct}(e_2)$	$Sports(e_3)$
s_1	1	1	0
s_2	0	1	1
s_3	1	0	1
s_4	1	0	1

Definition 2.3 ([3]). Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called fuzzy soft set over U where F is a mapping given by $F: A \to I^U$, where I^U denotes the collection of all fuzzy subsets of U.

Example 2.4. Consider the example [2.2], in soft set (F, E), if s_1 is medium in studies, we cannot expressed with only the two numbers 0 and 1, we can characterize it by a membership function instead of the crisp number 0 and 1, which associates with each element a real number in the interval [0, 1]. Then fuzzy soft set can describe as

$$(F, A) = \{F(e_1) = \{(s_1, 0.9), (s_2, 0.3), (s_3, 0.8), (s_4, 0.9)\}$$

$$F(e_2) = \{(s_1, 0.8), (s_2, 0.9), (s_3, 0.4), (s_4, 0.3)\}\}$$

$$766$$

where $A = \{e_1, e_2\}.$

We can represent a fuzzy soft set in the form of table given below

U	$\operatorname{Result}(e_1)$	$\operatorname{Conduct}(e_2)$
s_1	0.9	0.8
s_2	0.3	0.9
s_3	0.8	0.4
s_4	0.9	0.3

Definition 2.5 ([6]). Let $U = \{c_1, c_2, c_3, \ldots, c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \ldots, e_n\}$.Let $A \subseteq E$ and (F, A) be a fuzzy soft set in the fuzzy soft class (U, E).Then fuzzy soft set (F, A) in a matrix form as $A_{m \times n} = [a_{ij}]_{m \times n}$ or $A = [a_{ij}]$ $i = 1, 2, \ldots, m, j = 1, 2, 3, \ldots, n$

where $a_{ij} = \begin{cases} \mu_j(c_i) & \text{if } e_j \in A \\ 0 & \text{if } e_j \notin A \end{cases} \mu_j(c_i)$ represents the membership of c_i in the fuzzy

set $F(e_j)$.

Example 2.6. Consider the example [2.4], the matrix representation is

/ 0.9 0.8	0	
0.3 0.9	0	
0.8 0.4	0	
0.9 0.3	0	Ϊ

Definition 2.7 ([4]). Let U be the Universal set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called intuitionistic fuzzy soft set over U where F is a mapping given by $F : A \to I^U$, where I^U denotes the collection of all intuitionistic fuzzy subsets of U.

Example 2.8. Suppose that $U = \{s_1, s_2, s_3, s_4\}$ is a set of students and $E = \{e_1, e_2, e_3\}$ is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set $A \subseteq E$ to the set of all intuitionistic fuzzy subsets of power set U. Then soft set (F, A) describes the character of the students with respect to the given parameters, for finding the best student of an academic year. Consider, $A = \{e_1, e_2\}$ then intuitionistic fuzzy soft set is

$$\begin{array}{lll} (F,A) = \{F(e_1) &=& \{(s_1,0.8,0.1),(s_2,0.3,0.6),(s_3,0.8,0.2),(s_4,0.9,0.0)\},\\ F(e_2) &=& \{(s_1,0.8,0.1),(s_2,0.9,0.1),(s_3,0.4,0.5),(s_4,0.3,0.6)\}\}. \end{array}$$

Definition 2.9 ([9]). Let $U = \{c_1, c_2, c_3, \ldots, c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \ldots, e_n\}$.Let $A \subseteq E$ and (F, A) be a intuitionistic fuzzy soft set in the fuzzy soft class (U, E).Then intuitionistic fuzzy soft set (F, A) in a matrix form as $A_{m \times n} = [a_{ij}]_{m \times n}$ or $A = [a_{ij}] \ i = 1, 2, \ldots m$,

$$j = 1, 2, 3, \dots, n \text{ where } a_{ij} = \begin{cases} (\mu_j)(e_i), \nu_j(e_i) & \text{if } e_j \notin A \\ (0, 1) & \text{if } e_j \notin A \end{cases}$$

 $\mu_j(c_i)$ represents the membership of c_i in the intuitionistic fuzzy set $F(e_j)$. $\nu_j(c_i)$ represents the non-membership of c_i in the intuitionistic fuzzy set $F(e_j)$. **Example 2.10.** Suppose that $U = \{s_1, s_2, s_3, s_4\}$ is a set of students and $E = \{e_1, e_2, e_3\}$ is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set $A \subseteq E$ to the set of all intuitionistic fuzzy subsets of power set U. Then soft set (F, A) describes the character of the students with respect to the given parameters, for finding the best student of an academic year. Consider, $A = \{e_1, e_2\}$ then intuitionistic fuzzy soft set is

$$(F,A) = \{F(e_1) = \{(s_1, 0.8, 0.1), (s_2, 0.3, 0.6), (s_3, 0.8, 0.2), (s_4, 0.9, 0.0)\}, \\ F(e_2) = \{(s_1, 0.8, 0.1), (s_2, 0.9, 0.1), (s_3, 0.4, 0.5), (s_4, 0.3, 0.6)\}\}.$$

We would represent in matrix form as

$$\left(\begin{array}{ccccc} (0.8,0.1) & (0.8,0.1) & (0.0,1.0) \\ (0.3,0.6) & (0.9,0.1) & (0.0,1.0) \\ (0.8,0.2) & (0.4,0.5) & (0.0,1.0) \\ (0.9,0.0) & (0.3,0.6) & (0.0,1.0) \end{array}\right)$$

Definition 2.11 ([9]). If $A = [a_{ij}] \in \mathbf{IFSM}_{m \times n}, B = [b_{jk}] \in \mathbf{IFSM}_{n \times p}$, then we define A * B, product of A and B as

$$A * B = [c_{ik}]_{m \times p}$$

= (max min(μ_{Aj}, μ_{Bj}), min max(ν_{Aj}, ν_{Bj})) $\forall i, j$

Example 2.12. Let $A = \begin{pmatrix} (0.8, 0.1) & (0.4, 0.5) \\ (0.7, 0.3) & (0.4, 0.6) \end{pmatrix}_{2 \times 2}$ and $B = \begin{pmatrix} (0.6, 0.3) & (0.8, 0.2) \\ (0.7, 0.3) & (0.5, 0.5) \end{pmatrix}_{2 \times 2}$ are two intuitionistic fuzzy soft matrices,

then the product of these two matrices is

$$A * B = \begin{pmatrix} (0.6, 0.3) & (0.8, 0.2) \\ (0.6, 0.3) & (0.7, 0.3) \end{pmatrix}_{2 \times 2}$$
$$B * A = \begin{pmatrix} (0.7, 0.3) & (0.4, 0.5) \\ (0.7, 0.3) & (0.4, 0.5) \end{pmatrix}_{2 \times 2}$$

$$(0.7, 0.3)$$
 (0.4,

Remark: $A * B \neq B * A$.

Definition 2.13 ([9]). If $A = [a_{ij}] \in \mathbf{IFSM}_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. Then A^c is called a Intuitionistic Fuzzy Soft Complement Matrix if $A^c = [b_{ij}]_{mxn}$

$$b_{ij} = (
u_j(c_i), \mu_j(c_i)) \, orall i, j$$

Example 2.14. Let

$$A = \left(\begin{array}{cc} (0.8, 0.1) & (0.4, 0.5) \\ (0.7, 0.3) & (0.4, 0.6) \end{array}\right)_{2 \times 2}$$

be intuitionistic fuzzy soft matrix, then the complement of this matrix is

$$\mathbf{4}^{c} = \left(\begin{array}{cc} (0.1, 0.8) & (0.5, 0.4) \\ (0.3, 0.7) & (0.6, 0.4) \end{array}\right)_{2 \times 2}$$

3. Major section

Definition 3.1. If $A = [a_{ij}] \in \mathbf{IFSM}_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$, then we define the Value matrix of Intuitionistic Fuzzy Soft Matrix A is $V(A) = [a_{ij}] = [\mu_j(c_i) - \nu_j(c_i)] i = 1, 2, ..., n, j = 1, 2, ... n.$

Definition 3.2. If $A = [a_{ij}] \in \mathbf{IFSM}_{m \times n}, B = [b_{ij}] \in \mathbf{IFSM}_{m \times n}$, then we define Score matrix of A and B as $S(A, B) = [d_{ij}]_{m \times n}$ where $[d_{ij}] = V(A) - V(B) \forall i, j$.

Methodology. Let us assume that there is a set of m patients $P = \{p_1, p_2, p_3, \ldots, p_n\}$ p_m with a set of n symptoms $S = \{s_1, s_2, s_3, s_4, \dots, s_n\}$ related to a set of k diseases $D = \{d_1, d_2, d_3, \dots, d_k\}$. We apply intuitionistic fuzzy soft set technology to diagnose which patient is suffering from what disease. we construct a intuitionistic fuzzy soft set (F, P) over S where F is a mapping $F : P \to IF^S, IF^S$ is the collection of all intuitionistic fuzzy subsets of S. This intuitionistic fuzzy soft set gives a relation matrix A called patient symptom matrix, then construct another intuitionistic fuzzy soft set (G,S) over D where G is a mapping $G: S \to IF^D$, IF^D is the collection of all intuitionistic fuzzy subsets of D. This intuitionistic fuzzy soft set gives a relation matrix B called symptom-disease matrix, where each element denote the weight of the symptoms for a certain disease. We compute the complements $(F, P)^c$ and $(G, S)^c$ and their matrices A^c and B^c . Compute A * B which is the maximum membership of occurrence of Symptoms of the diseases. Compute $A^c * B^c$ which is the maximum membership of non occurrence of Symptoms of the diseases. Using def (3.1) and (3.2), Compute V(A * B), $V(A^c * B^c)$ and the Score matrix. Finally find max (S_i) , then conclude that the patient p_i is suffering from disease d_i . Incase $max(S_i)$ occurs for more than one value, then reassess the symptoms to break the tie.

Algorithm.

Step1: Input the intuitionistic fuzzy soft set (F, E), (G, E) and obtain the intuitionistic fuzzy soft matrices A, B corresponding to (F, E) and (G, E) respectively.

Step2: Write the intuitionistic fuzzy soft complement set $(F, E)^c$, $(G, E)^c$ and obtain the intuitionistic fuzzy soft matrices A^c, B^c corresponding to $(F, E)^c$ and $(G, E)^c$ respectively.

Step3: Compute $(A * B), (A^c * B^c), V(A * B), V(A^c * B^c).$

Step4: Compute the score matrix.

Step5: Find p for which $max(S_i)$.

Then we conclude that the patient p_i is suffering from disease d_j . Incase $\max(S_i)$ occurs for more than one value, then reassess the symptoms to break the tie.

Case Study. Suppose there are four patients Ramya,Mani,Sumi,Nekha in a hospital with symptoms temperature,headache,cough, stomach problem and body pain. Let the possible diseases related to the above symptoms be viral fever, typhoid and malaria. Now take $P = \{p_1, p_2, p_3, p_4\}$ as the universal set where p_1, p_2, p_3 and p_4 represents patients Ramya,Mani,Sumi, Nekha respectively.Let $S = \{s_1, s_2, s_3, s_4, s_5\}$ as the set of symptoms where s_1, s_2, s_3, s_4, s_5 represents symptoms temperature, headache,cough,stomach problem and body pain respectively. Suppose that LESE(C, S) gump P_2 where C is a manning $C \neq S$ with P_2 gives a self-action of an

IFSS(G,S) over P, where G is a mapping $G:S \to I^P,$ gives a collection of an 769

approximate description of patient symptoms in the hospital.

This intuitionistic fuzzy soft set is represented by the following intuitionistic fuzzy soft matrix

$$A = \begin{array}{ccccccccc} & s_1 & s_2 & s_3 & s_4 & s_5 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{pmatrix} (0.8, 0.1) & (0.6, 0.1) & (0.2, 0.8) & (0.6, 0.1) & (0.1, 0.6) \\ (0.0, 0.8) & (0.4, 0.4) & (0.6, 0.1) & (0.1, 0.7) & (0.1, 0.8) \\ (0.8, 0.1) & (0.8, 0.1) & (0.0, 0.6) & (0.2, 0.7) & (0.0, 0.5) \\ (0.6, 0.1) & (0.5, 0.4) & (0.3, 0.4) & (0.7, 0.2) & (0.3, 0.4) \\ \end{pmatrix}$$

Next consider the set $S = \{s_1, s_2, s_3, s_4, s_5\}$ as universal set where s_1, s_2, s_3, s_4, s_5 represents symptoms temperature, headache, cough, stomach problem and body pain respectively and the set $D = \{d_1, d_2, d_3\}$ where d_1, d_2 and d_3 represent the diseases viral fever, typhoid and malaria respectively. Suppose that IFSS(F, D) over S, where F is a mapping $F : D \to I^S$, gives an approximate description of intuitionistic fuzzy soft medical knowledge of the three diseases and their symptoms. Let

$$\begin{aligned} &(F,D) = \\ &\{F(d_1) = \{(s_1,0.6,0.2)), (s_2,0.3,0.5), (s_3,0.1,0.8), (s_4,0.4,0.5), (s_5,0.1,0.7)\} \\ &F(d_2) = \{(s_1,0.6,0.2)), (s_2,0.2,0.6), (s_3,0.2,0.7), (s_4,0.7,0.2), (s_5,0.1,0.8)\} \\ &F(d_3) = \{(s_1,0.3,0.4)), (s_2,0.7,0.2), (s_3,0.7,0.2), (s_4,0.3,0.4), (s_5,0.2,0.7)\}\} \end{aligned}$$

This intuitionistic fuzzy soft set is represented by the following intuitionistic fuzzy soft matrix

$$B = \begin{cases} d_1 & d_2 & d_3 \\ s_1 & (0.6, 0.2) & (0.6, 0.2) & (0.3, 0.4) \\ (0.3, 0.5) & (0.2, 0.6) & (0.7, 0.2) \\ (0.1, 0.8) & (0.2, 0.7) & (0.7, 0.2) \\ (0.4, 0.5) & (0.7, 0.2) & (0.3, 0.4) \\ s_5 & (0.1, 0.7) & (0.1, 0.8) & (0.2, 0.7) \end{cases}$$

Then the intuitionistic fuzzy soft complement matrices are

$$A^{c} = \begin{pmatrix} s_{1} & s_{2} & s_{3} & s_{4} & s_{5} \\ p_{2} \\ p_{3} \\ p_{4} \end{pmatrix} \begin{pmatrix} (0.1, 0.8) & (0.1, 0.6) & (0.8, 0.2) & (0.1, 0.6) & (0.6, 0.1) \\ (0.8, 0.0) & (0.4, 0.4) & (0.1, 0.6) & (0.7, 0.1) & (0.8, 0.1) \\ (0.1, 0.8) & (0.1, 0, 8) & (0.6, 0.0) & (0.7, 0.2) & (0.5, 0.0) \\ (0.1, 0.6) & (0.4, 0.5) & (0.4, 0.3) & (0.2, 0.7) & (0.4, 0.3) \end{pmatrix}$$

$$770$$

$$B^{c} = \begin{cases} d_{1} & d_{2} & d_{3} \\ s_{2} \\ s_{3} \\ s_{4} \\ s_{5} \end{cases} \begin{pmatrix} (0.2, 0.6) & (0.2, 0.6) & (0.4, 0.3) \\ (0.5, 0.3) & (0.6, 0.2) & (0.2, 0.7) \\ (0.8, 0.1) & (0.7, 0.2) & (0.2, 0.7) \\ (0.5, 0.4) & (0.2, 0.7) & (0.4, 0.3) \\ (0.7, 0.1) & (0.8, 0.1) & (0.7, 0.2) \end{pmatrix}$$

Then the prodcut matrices are

$$A * B = \begin{cases} p_1 \\ p_2 \\ p_3 \\ p_4 \end{cases} \begin{pmatrix} (0.6, 0.2) & (0.6, 0.2) & (0.6, 0.2) \\ (0.3, 0.5) & (0.2, 0.6) & (0.6, 0.2) \\ (0.6, 0.2) & (0.6, 0.2) & (0.7, 0.2) \\ (0.6, 0.2) & (0.7, 0.2) & (0.5, 0.4) \end{pmatrix} \\ A^c * B^c = \begin{cases} p_1 \\ p_2 \\ p_3 \\ p_4 \end{cases} \begin{pmatrix} (0.8, 0.1) & (0.7, 0.1) & (0.6, 0.2) \\ (0.7, 0.1) & (0.8, 0.1) & (0.7, 0.2) \\ (0.6, 0.1) & (0.6, 0.1) & (0.5, 0.2) \\ (0.4, 0.3) & (0.4, 0.3) & (0.4, 0.3) \end{pmatrix}$$

$$V(A * B) = \begin{pmatrix} d_1 & d_2 & d_3 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} \begin{pmatrix} 0.4 & 0.4 & 0.4 \\ -0.2 & -0.4 & 0.4 \\ 0.4 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.1 \end{pmatrix}$$

$$V(A^c * B^c) = \begin{pmatrix} d_1 & d_2 & d_3 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} \begin{pmatrix} 0.7 & 0.6 & 0.4 \\ 0.6 & 0.7 & 0.5 \\ 0.5 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.1 \end{pmatrix}$$

Calculate the score matrix

$$S = \begin{array}{ccc} & d_1 & d_2 & d_3 \\ p_1 & -0.3 & -0.2 & 0.0 \\ p_2 & -0.8 & -1.1 & -0.1 \\ p_3 & -0.1 & -0.1 & 0.2 \\ 0.3 & 0.4 & 0.0 \end{array}$$

It is clear from the above matrix that patients Ramya, Mani, Sumi (p_1, p_2, p_3) is suffering from malaria (d_3) and p_4 is suffering from typhoid (d_2) .

4. Conclusions

In this paper, we have developed an algorithm which is a new approach in medical diagnosis , by implementing intuitionistic fuzzy soft matrices. This algorithm is more flexible and adjustable. Solution is obtained by looking for the maximum score in

the score matrix. As far as, future directions are concerned, there would be required to study whether the notion put forward in this paper yield a fruitful result.

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