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Intuitionistic fuzzy rough centred texture wallman type compactification

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Abstract. In this paper, the concepts of intuitionistic fuzzy rough centred texture di - structure space and intuitionistic fuzzy rough centred texture di - function are introduced. Some interesting properties are discussed. In this connection, the concept of intuitionistic fuzzy rough centred texture di - structure compactification is established.

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh [14]. Fuzzy sets have applications in many fields such as information [11] and control [12]. Pawlak [9] introduced the concept of rough sets. Nanda and Manjumdar [8] introduced and studied fuzzy rough sets. Atanassov [2] introduced and studied intuitionistic fuzzy sets. T.K.Mandal and S.K.Samanta [10] introduced the concept of intuitionistic fuzzy rough sets and studied some related concepts. The theory of fuzzy topological spaces was introduced and developed by Chang [5]. H.Hazra, S.K.Samanta and K.C.Chattopadhyay [6] introduced the topological space in an intuitionistic fuzzy rough sets. The method of centered system in the theory of topology was introduced by S.Iliadis and S.Fomin in [10]. In 2007, the above concept was extended to fuzzy topological spaces by M.K.Uma, E.Roja and G.Balasubramanian [13]. L.M.Brown [3] is introduced the concept of texture space. Later M.Diker [4] discussed the concept of compact space in Ditopological texture space. Also the concept of a wallman-type compactification of texture spaces was introducted by Aysegul Altay and M. Diker [1]. In this paper, the concepts of intuitionistic fuzzy rough centred texture di - structure space and intuitionistic fuzzy rough centred texture di - function are introduced. Some interesting properties are discussed. In this connection, the concept of intuitionistic fuzzy rough centred texture di - structure compactification is established.

2. Preliminaries

Let $(V, \mathcal{B})$ be an rough universe V is an non empty set and $\mathcal{B}$ is a Boolean sub algebra of an Boolean algebra of all subsets of V. Let a rough set $X = (X_L, X_U) \in \mathcal{B}^2$ with $X_L \subseteq X_U$.

Definition 2.1 ([8]). A fuzzy rough set (briefly FRS) in X is an object of the form $A = (A_L, A_U)$ where A_L and A_U are characterized by a pair of maps $A_L : X_L \to \mathscr{L}$ and $A_L : X_L \to \mathscr{L}$ with $A_L(x) \leq A_U(x)$, $\forall x \in X_L$ where $(\mathscr{L}, \leq)$ is a fuzzy lattice(i.e completed and completely distributive lattice whose least and greatest elements are denoted by 0 and 1 respectively with an involutive order reversing operation$\prime : \mathscr{L} \to \mathscr{L}$.

Definition 2.2 ([8]). For any two fuzzy rough sets $A = (A_L, A_U)$ and $B = (B_L, B_U)$ in X,
(i) $A \subset B$ iff $A_L(x) \leq B_L(x)$, $\forall x \in X_L$ and $A_U(x) \leq B_U(x)$ $\forall x \in X_L$
(ii) $A = B$ iff $A \subset B$ and $B \subset A$.
If $\{A_i : i \in J\}$ be any family of fuzzy rough sets in X, where $A_i = (A_{iL}, A_{iU})$ then
(i) $E = \bigcup_i A_i$ where $E_L(x) = \vee A_{iL}(x)$, $\forall x \in X_L$ and $E_U(x) = \vee A_{iU}(x)$, $\forall x \in X_U$
(ii) $E = \bigcap_i A_i$ where $F_L(x) = \wedge A_{iL}(x)$, $\forall x \in X_L$ and $F_U(x) = \wedge A_{iU}(x)$, $\forall x \in X_U$

Definition 2.3 ([10]). If A and B are fuzzy sets in X_L and X_U respectively where $X_L \subset X_U$. Then the restriction of B on X_L and the extension of A on X_U (denoted by $B_{>L}$ and $A_{<U}$ respectively) are defined by complement of an FRS $A = (A_L, A_U)$ in X are denoted by $\overline{A} = ((\overline{A})_L, (\overline{A})_U)$ and is defined by $(\overline{A})_L(x) = (A_{U>L})'(x)$, $\forall x \in X_L$ and $(\overline{A})_U(x) = (A_{L<U})'(x)$, $\forall x \in X_U$. For simplicity we write $(\overline{A}_L, \overline{A}_U)$ instead of $((\overline{A})_L, (\overline{A})_U)$.

Definition 2.4 ([10]). If A and B are two FRSs in X with $B \subset \overline{A}$ and $A \subset \overline{B}$, then the ordered pair (A, B) is called an intuitionistic fuzzy rough set (briefly $IFRS$) in X. The condition $A \subset \overline{B}$ and $B \subset \overline{A}$ are called intuitionistic condition (briefly IC)

Definition 2.5 ([10]). Let $P = (A, B)$ and $Q = (C, D)$ be two $IFRS$s in X. Then
(i) $P \subset Q$ iff $A \subset C$ and $B \supset D$.
(ii) $P = Q$ iff $P \subset Q$ and $Q \subset P$.
(iii) The complement of $P = (A, B)$ is denoted by P' , is defined by $P' = (B, A)$.
(iv) For $IFRS$s $P_i = (A_i, B_i)$ in X, $i \in J$, define $\bigcup_{i \in J} P_i = (\bigcup_{i \in J} A_i, \bigcap_{i \in J} B_i)$ and $\bigcap_{i \in J} P_i = (\bigcap_{i \in J} A_i, \bigcup_{i \in J} B_i)$.

Theorem 2.6 ([10]). *Let $P = (A, B)$, $Q = (C, D)$, $R = (E, F)$ and $P_i = (A_i, B_i)$, $i \in J$ be $IFRS$s in X. Then*
(i) $P \cap P = P = P \cup P$

(ii) $P \cap Q = Q \cap P$, $P \cup Q = Q \cup P$,
(iii) $(P \cap Q) \cap R = P \cap (Q \cap R)$, $(P \cup Q) \cup R = P \cup (Q \cup R)$,
(iv) $P \cap Q \subset P$, $Q \subset P \cup Q$
(v) $P \subset Q$ *and* $Q \subset R \Rightarrow P \subset R$
(vi) $P_i \subset Q, \forall i \in J \Rightarrow \bigcup_{i \in J} P_i \subset Q$
(vii) $Q \subset P_i \forall i \in J \Rightarrow Q \subset \bigcap_{i \in J} P_i$
(viii) $Q \cup (\bigcap_{i \in J} P_i) = \bigcap_{i \in J}(Q \cup P_i)$
(ix) $Q \cap (\bigcup_{i \in J} P_i) = \bigcup_{i \in J}(Q \cap P_i)$
(x) $(P')' = P$
(xi) $P \subset Q \Leftrightarrow P' \subset Q'$
(xii) $\bigcup_{i \in J}(P_i)' = \bigcap_{i \in J} P'_i$ *and* $\bigcap_{i \in J}(P_i)' = \bigcup_{i \in J} P'_i$

Theorem 2.7 ([10]). *If A be any FRS in X, $\tilde{0} = (\tilde{0}_L, \tilde{0}_U)$ be the null FRS and $\tilde{1} = (\tilde{1}_L, \tilde{1}_U)$ be the whole FRS in X, then (i) $\tilde{0} \subset A \subset \tilde{1}$ and (ii) $\overline{\tilde{0}} = \tilde{1}, \overline{\tilde{1}} = \tilde{0}$.*

Definition 2.8 ([10]). $0^* = (\tilde{0}, \tilde{0})$ and $1^* = (\tilde{1}, \tilde{1})$ are respectively called null $IFRS$ and whole $IFRS$ in X. Clearly $(0^*)' = 1^*$ and $(1^*)' = 0^*$.

Theorem 2.9 ([10]). *If P be any $IFRS$ in X, then $0^* \subset P \subset 1^*$*

Definition 2.10 ([6]). Let $X = (X_L, X_U)$ be a rough set and τ be a family of $IFRS$s in X such that
(i) $0^*, 1^* \in \tau$;
(ii) $P \cap Q \in \tau$ for any $P, Q \in \tau$;
(iii) $P_i \in \tau, i \in \Delta \Rightarrow \bigcup_{i \in \Delta} P_i \in \tau$

Then τ is called a topology of $IFRS$s in X and the pair (X, τ) topological space of $IFRS$s in X. Every member of τ is called open $IFRS$. An $IFRS$ C is called closed $IFRS$ if $C' \in \tau$

Definition 2.11 ([7]). Let R be a fuzzy Hausdorff space.A system $p = \{\lambda_\alpha\}$ of fuzzy open sets of R is called fuzzy centred if any finite number of fuzzy sets of the system has a non zero intersection.

Definition 2.12 ([3]). Let S be a set.Then $\mathscr{L} \subseteq \wp(S)$ is called a texturing of S and S is said to be textured by $\mathscr{L}$ if

(1) $\mathscr{L} \subseteq$ is a complete lattice containing S and ϕ and for any index set I and $A_i \in \mathscr{L}$, the meet $\bigwedge_{i \in I} A_i$ and the join $\bigvee_{i \in I} A_i$ in $\mathscr{L}$ are related with the intersection and union in $\wp(S)$ by the equalities $\bigwedge_{i \in I} A_i = \bigcap_{i \in I} A_i$, for all I, while $\bigvee_{i \in I} A_i = \bigcap_{i \in I} A_i$ for all finite I.

(2) $\mathscr{L}$ is completely distributive.

(3) $\mathscr{L}$ separates the points of S.That is, given $s_1 \neq s_2$ in S we have $L \in \mathscr{L}$ with $s_1 \in \mathscr{L}$, $s_2 \notin \mathscr{L}$, or $L \in \mathscr{L}$ with $s_2 \in \mathscr{L}$ $s_1 \notin \mathscr{L}$

If S is textured by $\mathscr{L}$ then $(S, \mathscr{L})$ is called a texture space or simply a texture. Hence a texturing $\mathscr{L}$ on S is a set of ordinary crisp subsets of S satisfying the above properties.

Definition 2.13 ([3]). $(\mathscr{L}, \tau, \mathcal{K})$ is called a ditopological texture space on S if $\tau \subseteq \mathscr{L}$ satisfies the following conditions
(i) $S, \phi \in \tau$
(ii) If $G_1, G_2 \in \tau$ then $G_1 \cap G_2 \in \tau$

(iii) If $G_i \in \tau$ for $i \in J$ then $\bigvee_{i\in J} G_i \in \tau$
and $\mathcal{K} \subseteq \mathscr{L}$ satisfies the following conditions
(i) $S, \phi \in \mathcal{K}$
(ii) If $F_1, F_2 \in \mathcal{K}$ then $F_1 \cup F_2 \in \mathcal{K}$
(iii) If $F_i \in \mathcal{K}$ for $i \in J$ then $\bigwedge_{i\in J} F_i \in \mathcal{K}$

3. Intuitionistic fuzzy rough centred texture di - structure compactification

Definition 3.1. An intuitionistic fuzzy rough topological space (X, T) is called an intuitionistic fuzzy rough Hausdorff space (or) intuitionistic fuzzy rough T_2 space if for each pair of non zero intuitionistic fuzzy rough sets $A = (P, Q)$ and $B = (L, M)$ such that $A \neq B$, where $P = (P_L, P_U)$, $Q = (Q_L, Q_U)$, $L = (L_L, L_U)$ and $M = (M_L, M_U)$ are fuzzy rough sets of X , then there exist open intuitionistic fuzzy rough sets $C = (U, V)$ and $D = (G, H)$ where $U = (U_L, U_U)$, $V = (V_L, V_U)$ $G = (G_L, G_U)$ and $H = (H_L, H_U)$ are fuzzy rough sets of X such that $A \subseteq C$, $B \subseteq D$ and $C \cap D = 0^*$.

Definition 3.2. Let (X, T) be an intuitionistic fuzzy rough Hausdorff space and a system $p = \{A_i\}$ where each $A_i = (P_i, Q_i)$ is an open intuitionistic fuzzy rough set and $P_i = (P_{i_L}, P_{i_U})$ and $Q_i = (Q_{i_L}, Q_{i_U})$. Then p is said to be an intuitionistic fuzzy rough centred system if any finite collection of A_i such that $A_i \cap A_j \neq 0^*$, for $i \neq j$. The system p is said to be an intuitionistic fuzzy maximal rough centred system (or) intuitionistic fuzzy rough end if it cannot be included in any larger intutionistic fuzzy rough centered system.

Notation 3.1. Let $\mathfrak{D}_X = \{p_i / i \in J\}$ be a non empty set where each p_i is an intuitionistic fuzzy rough centred system in an intuitionistic fuzzy rough Hausdorff space (X, T) and J is an index set. Now, $\wp(\mathfrak{D}_X$ denotes the power set of $\mathfrak{D}_X$.

Definition 3.3. Let $\mathfrak{D}_X = \{p_i / i \in J\}$ be a non empty set where each p_i is an intuitionistic fuzzy rough centred system in an intuitionistic fuzzy rough Hausdorff space (X, T) and J is an index set. Then $\mathscr{L} \subseteq \wp(\mathfrak{D}_X)$ is said to be an intuitionistic fuzzy rough centred texturing of $\mathfrak{D}_X$ and $\mathfrak{D}_X$ is said to be an intuitionistic fuzzy rough centred textured by $\mathscr{L}$ if

(i) $(\mathscr{L}, \subseteq)$ is a complete lattice containing $\mathfrak{D}_X$ and ϕ, for any index set J and $A_i \in \mathscr{L}$, $i \in J$ the meet $\bigwedge_{i\in J} A_i$ and the join $\bigvee_{i\in J} A_i$ in $\mathscr{L}$ are related with the intersection and union in $\wp(\mathfrak{D}_X)$ by the equalities $\bigwedge_{i\in J} A_i = \bigcap_{i\in J} A_i$ for all J, while $\bigvee_{i\in J} A_i = \bigcup_{i\in J} A_i$ for all finite J.

(ii) $\mathscr{L}$ is completely distributive.

(iii) $\mathscr{L}$ separates the points of $\mathfrak{D}_X$. That is, if $p_1 \neq p_2$ in $\mathfrak{D}_X$, then $L \in \mathscr{L}$ with $p_1 \in L$, $p_2 \notin L$ or $L \in \mathscr{L}$ with $p_2 \in L$, $p_1 \notin L$.

If $\mathfrak{D}_X$ is intuitionistic fuzzy rough centred textured by $\mathscr{L}$ then $(\mathfrak{D}_X, \mathscr{L})$ is said to be an intuitionistic fuzzy rough centred texture space. Every member of $(\mathfrak{D}_X, \mathscr{L})$ is said to be an intuitionistic fuzzy rough centred texture open set and the complement of intuitionistic fuzzy rough centred texture open set is said to be an intuitionistic fuzzy rough centred texture closed set.

Definition 3.4. Let $(\mathfrak{D}_X, \mathscr{L})$ be an intuitionistic fuzzy rough centred texture space. then $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ is said to be an intuitionistic fuzzy rough centred texture di - structure space on $\mathfrak{D}_X$ if the collection of intuitionistic fuzzy rough centred texture open sets $\mathfrak{T}$ satisfies the following conditions.

(i) $\mathfrak{D}_X, \phi \in \mathfrak{T}$.
(ii) If $G_1, G_2 \in \mathfrak{T}$ then $G_1 \cap G_2 \in \mathfrak{T}$.
(iii) If $G_i \in \mathfrak{T}$ for $i \in J$ then $\bigvee_{i \in J} G_i \in \mathfrak{T}$.

and if the collection of intuitionistic fuzzy rough centred texture closed sets $\mathfrak{K}$ satisfies the following conditions.

(i) $\mathfrak{D}_X, \phi \in \mathfrak{K}$.
(ii) If $F_1, F_2 \in \mathfrak{K}$ then $F_1 \cup F_2 \in \mathfrak{K}$.
(iii) If $F_i \in \mathfrak{K}$ for $i \in J$ then $\bigwedge_{i \in J} F_i \in \mathfrak{K}$.

Definition 3.5. Let $(\mathfrak{D}_X, \mathscr{L}_1)$ and $(\mathfrak{D}_Y, \mathscr{L}_2)$ be intuitionistic fuzzy rough centred texture spaces and let $v : \mathfrak{D}_X \to \mathfrak{D}_Y$ be a function. Then v is said to be an intuitionistic fuzzy rough centred texture

(i) one to one function if each pair of distinct points of $\mathfrak{D}_X$, their images under v are distinct.

(ii) onto function if $p_1 \in \mathfrak{D}_Y$, then $p_2 = v(p_1)$ for at least one $p_2 \in \mathfrak{D}_X$.

(iii) bijective function, if v is both intuitionistic fuzzy rough centred texture one to one function and intuitionistic fuzzy rough centred texture onto function.

Definition 3.6. Let $(\mathfrak{D}_X, \mathscr{L})$ be an intuitionistic fuzzy rough centred texture space and $\gamma : \mathscr{L} \to \mathscr{L}$ be an intuitionistic fuzzy rough centred texture bijective function. Then γ is said to be an intuitionistic fuzzy rough centred texture complementation if

(i) $\gamma(\gamma(P)) = \gamma^2(P) = P$ for all $P \in \mathscr{L}$.
(ii) If $P \subseteq Q$ then $\gamma(Q) \subseteq \gamma(P)$, for $P, Q \in \mathscr{L}$.

Note 3.1. (i) An intuitionistic fuzzy rough centred texture space $(\mathfrak{D}_X, \mathscr{L})$ with an intuitionistic fuzzy rough centred texture complementation is said to be an intuitionistic fuzzy rough centred complemented texture space and is denoted by $(\mathfrak{D}_X, \mathscr{L}, \gamma)$.

(ii) In general there is no relation between intuitionistic fuzzy rough centred texture open set and intuitionistic fuzzy rough centred texture closed set in an intuitionistic fuzzy rough centred texture di - structure space $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$. If γ is an intuitionistic fuzzy rough centred texture complementation, then $\mathfrak{K} = \{\gamma(G)/G \in \mathfrak{T}\}$.

Definition 3.7. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space. Then

(i) $\sigma \subseteq \mathfrak{T}$ is said to be an intuitionistic fuzzy rough centred texture di - structure subbase of $\mathfrak{T}$ if every element of $\mathfrak{T}$ is a supremum of finite intersection of sets of σ.

(ii) $\delta \subseteq \mathfrak{K}$ is said to be an intuitionistic fuzzy rough centred texture di - structure subbase of $\mathfrak{K}$ if every element of $\mathfrak{K}$ is an intersection of finite unions of sets of δ.

Definition 3.8. Let $(\mathfrak{D}_X, \mathscr{L})$ be an intuitionistic fuzzy rough centred texture space. Then for each $p_1, p_2 \in \mathfrak{D}_X$ the P - set and Q - set are defined and denoted by $P_{p_1} = \cap\{L \in \mathscr{L}/p_1 \in L\}$ and $Q_{p_2} = \cup\{P_{p_1}/p_2 \notin P_{p_1}\}$ respectively.

Note 3.2. The set P_{p_1} is the smallest element of $\mathscr{L}$ containing p_1.

Definition 3.9. Let $(\mathfrak{D}_{X_i}, \mathscr{L}_i)$ be an intuitionistic fuzzy rough centred texture spaces with $\mathfrak{D}_{X_i} \cap \mathfrak{D}_{X_j} = \phi$ for $i \neq j$. Let $\mathfrak{D}_X = \bigcup_{i\in J} \mathfrak{D}_{X_i}$ and $\mathscr{L} = \{A/A \subseteq \mathfrak{D}_X, A \cap \mathfrak{D}_{X_i} \in \mathscr{L}_i, i \in J\}$. Then the intuitionistic fuzzy rough centred texture space $(\mathfrak{D}_X, \mathscr{L})$ is said to be the sum of the disjoint intuitionistic fuzzy rough centred texture spaces $(\mathfrak{D}_{X_i}, \mathscr{L}_i)$ and it is denoted by $\oplus_{i\in J}\mathfrak{D}_{X_i}$.

Note 3.3. Let $(\mathfrak{D}_X, \wp(\mathfrak{D}_X))$, $(\mathfrak{D}_Y, \mathcal{Q})$ be an intuitionistic fuzzy rough centred texture spaces and $\wp(\mathfrak{D}_X) \otimes \mathcal{Q} = \{A_i \times B_i/i \in J, A_i \in \wp(\mathfrak{D}_X)\text{and}B_i \in \mathcal{Q}\}$. Now, $(\mathfrak{D}_X \times \mathfrak{D}_Y, \wp(\mathfrak{D}_X) \otimes \mathcal{Q})$ denotes the product intuitionistic fuzzy rough centred texture space.

Definition 3.10. Let $(\mathfrak{D}_X \times \mathfrak{D}_Y, \wp(\mathfrak{D}_X) \otimes \mathscr{L})$ be the product intuitionistic fuzzy rough centred texture space. Then

(i) P-set is denoted and defined by $\overline{P}_{(p_1,q_1)} = \{p_1\} \times P_{q_1}$.

(ii) Q-set is denoted and defined by $\overline{Q}_{(p_1,q_1)} = (\mathfrak{D}_X \setminus \{p_1\} \times \mathfrak{D}_Y) \cup (\mathfrak{D}_X \times Q_{q_1})$ where $p_1 \in \mathfrak{D}_X$ and $q_1 \in \mathfrak{D}_Y$.

Definition 3.11. Let $(\mathfrak{D}_X \times \mathfrak{D}_Y, \wp(\mathfrak{D}_X) \otimes \mathcal{Q}\mathcal{L})$ be the product intuitionistic fuzzy rough centred texture space. Then r is said to be an intuitionistic fuzzy rough centred texture relation from $(\mathfrak{D}_X, \mathscr{L})$ to $(\mathfrak{D}_Y, \mathcal{Q})$ if

(i) $r \not\subseteq \overline{Q}_{(p_1,q_1)}$ and $P_{p_2} \not\subseteq Q_{p_1}$, then $r \not\subseteq \overline{Q}_{(p_2,q_1)}$,

(ii) $r \not\subseteq \overline{Q}_{(p_1,q_1)}$, then there exists $p_2 \in \mathfrak{D}_X$ such that $P_{p_1} \not\subseteq Q_{p_2}$ and $r \not\subseteq \overline{Q}_{(p_2,q_1)}$.

Definition 3.12. Let $(\mathfrak{D}_X \times \mathfrak{D}_Y, \wp(\mathfrak{D}_X) \otimes \mathcal{Q}\mathcal{L})$ be the product intuitionistic fuzzy rough centred texture space. Then R is said to be an intuitionistic fuzzy rough centred texture co - relation from $(\mathfrak{D}_X, \mathscr{L})$ to $(\mathfrak{D}_Y, \mathcal{Q})$ if

(i) $\overline{P}_{(p_1,q_1)} \not\subseteq R$ and $P_{p_1} \not\subseteq Q_{p_2}$, then $\overline{P}_{(p_2,q_1)} \not\subseteq R$,

(ii) $\overline{P}_{(p_1,q_1)} \not\subseteq R$, then there exists $p_2 \in \mathfrak{D}_X$ such that $P_{p_2} \not\subseteq Q_{p_1}$ and $\overline{P}_{(p_2,q_1)} \not\subseteq R$.

Definition 3.13. Let $(\mathfrak{D}_X, \wp(\mathfrak{D}_X))$ and $(\mathfrak{D}_Y, \mathcal{Q})$ be any two intuitionistic fuzzy rough centred texture spaces. If r is an intuitionistic fuzzy rough centred texture relation and R is an intuitionistic fuzzy rough centred texture co-relation, then the pair (r, R) is an intuitionistic fuzzy rough centred texture di - relation from $(\mathfrak{D}_X, \wp(\mathfrak{D}_X))$ to $(\mathfrak{D}_Y, \mathcal{Q})$.

Note 3.4. Let $(\mathfrak{D}_X, \mathscr{L})$ and $(\mathfrak{D}_Y, \mathcal{Q})$ be any two intuitionistic fuzzy rough centred texture spaces and the pair (r, R) is an intuitionistic fuzzy rough centred texture di - relation from $(\mathfrak{D}_X, \wp(\mathfrak{D}_X))$ to $(\mathfrak{D}_Y, \mathcal{Q})$. Then an

(i) intuitionistic fuzzy rough centred texture inverse di - relation of an intuitionistic fuzzy rough centred texture di - relation (r, R) is also an intuitionistic fuzzy rough centred texture di - relation from $(\mathfrak{D}_Y, \mathcal{Q})$ to $(\mathfrak{D}_X, \mathscr{L})$ which is denoted as $(R^{\leftarrow}, r^{\leftarrow})$ where $r^{\leftarrow} = \bigcap\{\overline{Q}_{(q_1,p_1)}/r \not\subseteq \overline{Q}_{(p_1,q_1)}\}$ and $R^{\leftarrow} = \bigvee\{\overline{P}_{(q_1,p_1)}/\overline{P}_{(p_1,q_1)} \not\subseteq R\}$.

(ii) intuitionistic fuzzy rough centred texture image and an intuitionistic fuzzy rough centred texture co-image are denoted and defined as $r^{\rightarrow}A = \bigcap\{Q_{q_1}/$for every p_1, if $r \not\subseteq \overline{Q}_{(p_1,q_1)}$, then $A \subseteq Q_{p_1}\}$ and $R^{\rightarrow}A = \bigvee\{P_{q_1}/$ for every p_1, if $\overline{P}_{(p_1,q_1)} \not\subseteq R$, then $P_{p_1} \subseteq A\}$ for all $A \in \mathscr{L}$ respectively.

Definition 3.14. Let $(\mathfrak{D}_X, \mathscr{L})$ and $(\mathfrak{D}_Y, \mathcal{Q})$ be any two intuitionistic fuzzy rough centred texture spaces and the pair (f, F) be an intuitionistic fuzzy rough centred texture di - relation from $(\mathfrak{D}_X, \mathscr{L})$ to $(\mathfrak{D}_Y, \mathcal{Q})$. Then (f, F) is said to be an intuitionistic fuzzy rough centred texture di - function from $(\mathfrak{D}_X, \mathscr{L})$ to $(\mathfrak{D}_Y, \mathcal{Q})$ if

(i) for $p_1, p_2 \in \mathfrak{D}_X$ and $P_{p_1} \nsubseteq Q_{p_2}$, then there exists $q_1 \in \mathfrak{D}_Y$ with $f \nsubseteq Q_{(p_1,q_1)}$ and $P_{(p_2,q_1)} \nsubseteq F$.

(ii) for $q_1, q_2 \in \mathfrak{D}_Y$ and $p_1 \in \mathfrak{D}_X$, $f \nsubseteq \overline{Q}_{(p_1,q_1)}$ and $\overline{P}_{(p_1,q_2)} \nsubseteq F$, then $P_{q_2} \nsubseteq Q_{p_1}$.

Definition 3.15. Let $(\mathfrak{D}_X, \mathscr{L})$ be an intuitionistic fuzzy rough centred texture space and the pair $(i, \mathcal{I})$ be an intuitionistic fuzzy rough centred texture di - relation from $(\mathfrak{D}_X, \mathscr{L})$ to $(\mathfrak{D}_Y, \mathcal{Q})$. Then $(i, \mathcal{I})$ is said to be an intuitionistic fuzzy rough centred texture identity di - function from $(\mathfrak{D}_X, \mathscr{L})$ to $(\mathfrak{D}_Y, \mathcal{Q})$ if $i = \bigvee\{P_{(p_1,p_1)}/p_1 \in \mathfrak{D}_X\}$ and $\mathcal{I} = \bigcap\{Q_{(p_1,p_1)}/p_1 \in \mathfrak{D}_X\}$.

Definition 3.16. Let $(\mathfrak{D}_X, \mathscr{L}_1)$, $(\mathfrak{D}_Y, \mathscr{L}_2)$ and $(\mathfrak{D}_Z, \mathscr{L}_3)$ be any three intuitionistic fuzzy rough centred texture spaces and for any intuitionistic fuzzy rough centred texture relation f and intuitionistic fuzzy rough centred texture co-relation F from $(\mathfrak{D}_X, \mathscr{L}_1)$ to $(\mathfrak{D}_Y, \mathscr{L}_2)$ and for any intuitionistic fuzzy rough centred texture relation g and intuitionistic fuzzy rough centred texture co-relation G from $(\mathfrak{D}_Y, \mathscr{L}_2)$ to $(\mathfrak{D}_Z, \mathscr{L}_3)$, an

(i) intuitionistic fuzzy rough centred texture composition $g \circ f$ from $(\mathfrak{D}_X, \mathscr{L}_1)$ to $(\mathfrak{D}_Z, \mathscr{L}_3)$ is denoted and defined as $g \circ f = \bigvee\{P_{(p_1,u_1)}/\text{there exists } q_1 \in \mathfrak{D}_Y \text{ with } f \nsubseteq Q_{(p_1,q_1)} \text{ and } g \nsubseteq Q_{(t,u)}\}$

(ii) intuitionistic fuzzy rough centred texture composition $G \circ F$ from $(\mathfrak{D}_X, \mathscr{L}_1)$ to $(\mathfrak{D}_Z, \mathscr{L}_3)$ is denoted and defined as $G \circ F = \bigcap\{Q_{(p_1,u_1)}/ \text{ there exists } q_1 \in \mathfrak{D}_Y \text{ with } P_{(p_1,q_1)} \nsubseteq F \text{ and } P_{(t,u)} \nsubseteq G\}$.

Note 3.5. Let (f, F) and (g, G) be any two intuitionistic fuzzy rough centred texture di - relations from $(\mathfrak{D}_X, \mathscr{L}_1)$ to $(\mathfrak{D}_Y, \mathscr{L}_2)$ and from $(\mathfrak{D}_Y, \mathscr{L}_2)$ to $(\mathfrak{D}_Z, \mathscr{L}_3)$ respectively. Then the composition of intuitionistic fuzzy rough centred texture di - relations (f, F) and (g, G) is also an intuitionistic fuzzy rough centred texture di - relation $(g \circ f, G \circ F)$ from $(\mathfrak{D}_X, \mathscr{L}_1)$ to $(\mathfrak{D}_Z, \mathscr{L}_3)$.

Definition 3.17. Let $(\mathfrak{D}_X, \mathscr{L})$ be an intuitionistic fuzzy rough centred texture space and a family $\mathscr{H} \subseteq \mathscr{L}$ is said to be an intuitionistic fuzzy rough centred texture filter if

(i) $\phi \notin \mathscr{H}$,
(ii) for all $A, B \in \mathscr{H}$, then $A \cap B \in \mathscr{H}$,
(iii) $A \in \mathscr{H}$, $B \in \mathscr{L}$ and $A \subseteq B$ then $B \in \mathscr{H}$.

Definition 3.18. Let $(\mathfrak{D}_X, \mathscr{L})$ be an intuitionistic fuzzy rough centred texture space and a family $\mathscr{G} \subseteq \mathscr{L}$ is said to be an intuitionistic fuzzy rough centred texture co - filter if

(i) $\mathfrak{D}_X \notin \mathscr{G}$,
(ii) If $A, B \in \mathscr{G}$, then $A \cup B \in \mathscr{G}$,
(iii) If $A \in \mathscr{G}$, $B \in \mathscr{L}$ and $B \subseteq A$ then $B \in \mathscr{G}$.

Definition 3.19. Let $(\mathfrak{D}_X, \mathscr{L}, \gamma, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred complemented di - structure texture space. Then a set $\mathfrak{A} \subseteq \mathscr{L} \times \mathscr{L}$ is said to be an intuitionistic fuzzy rough centred texture di - family on $\mathfrak{D}_X$.

Note 3.6. An intuitionistic fuzzy rough centred texture di - family $\mathfrak{A}$ satisfying $\mathfrak{A} \subseteq \mathfrak{T} \times \mathfrak{K}$ is an intuitionistic fuzzy rough centred texture open and co-closed di - family, and $\mathfrak{A} \subseteq \mathfrak{K} \times \mathfrak{T}$ is an intuitionistic fuzzy rough centred texture closed and co-open di - family.

Definition 3.20. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space. Then an intuitionistic fuzzy rough centred texture di - family $\mathscr{F}$ is an intuitionistic fuzzy rough centred texture di - filter on $\mathfrak{D}_X$ if

(i) $\mathscr{F} = \mathscr{H} \times \mathscr{G}$ where $\mathscr{H}$ is an intuitionistic fuzzy rough centred texture filter and $\mathscr{G}$ is an intuitionistic fuzzy rough centred texture co-filter on $\mathfrak{D}_X$.

(ii) for every $(H, G) \in \mathscr{F}$, $H \not\subseteq G$.

Definition 3.21. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space. Then an intuitionistic fuzzy rough centred texture di - filter $\mathscr{F} = \mathscr{H} \times \mathscr{G}$ (resp.,$\mathscr{F} = \mathscr{G} \times \mathscr{H}$) is said to be an intuitionistic fuzzy rough centred texture closed co-open (resp., open co - closed) di - filter on $\mathfrak{D}_X$ if $\mathscr{H} \subseteq \mathfrak{K}$ and $\mathscr{G} \subseteq \mathfrak{T}$ (resp.$\mathscr{G} \subseteq \mathfrak{T}$ and $\mathscr{H} \subseteq \mathfrak{K}$.

Definition 3.22. Let $(\mathfrak{D}_X, \mathscr{L}, \gamma, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred complemented di - structure texture space. Then an intuitionistic fuzzy rough centred texture di - family $\mathfrak{A}$ has the finite exclusion property (in short.,fep) if $\bigcap_{i=1}^{n} F_i \not\subseteq \bigcup_{i=1}^{n} G_i$, for every$(F_i, G_i) \in \mathfrak{A}$, $i = 1, 2, ..., n$.

Definition 3.23. Let $(\mathfrak{D}_X, \mathscr{L}, \gamma, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred complemented texture di - structure space and $\mathscr{F}$ be an intuitionistic fuzzy rough centred texture difilter.Then $\mathscr{F}$ is said to be an intuitionistic fuzzy rough centred texture complemented difilter if the following condition holds .That is, $(H, G) \in \mathscr{F}$ if $(\gamma(G), \gamma(H)) \in \mathscr{F}$.

Definition 3.24. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred di - structure texture space $\mathscr{F}$ be an intuitionistic fuzzy rough centred texture maximal (closed co-open) complemented di - filter on $\mathfrak{D}_X$. If $(H, G) \notin \mathscr{F}$, then there exists $(E, F) \in \mathscr{F}$ such that $H \cap E \subseteq G \cup F$.

Proposition 3.25. *Let $(\mathfrak{D}_X, \mathscr{L}, \gamma, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred di - structure texture space and $\mathscr{F}$ be an intuitionistic fuzzy rough centred texture maximal (closed co-open) complemented di - filter on $\mathfrak{D}_X$. Then*

(i) *If $(H, \phi) \notin \mathscr{F}$ and $(E, \phi) \notin \mathscr{F}$, then $(H \cup E, \phi) \notin \mathscr{F}$.*

(ii) *If $(H, \phi) \notin \mathscr{F}$ and $(\mathfrak{D}_X, G) \notin \mathscr{F}$, then there exists $(E, F) \in \mathscr{F}$ such that $H \cap E \subseteq F$ and $E \subseteq G \cup F$.*

Proof. (i) Assume that for an intuitionistic fuzzy rough centred texture maximal (closed co-open) complemented di - filter $\mathscr{F}$ on $\mathfrak{D}_X$, $(H, \phi) \notin \mathscr{F}$ and $(E, \phi) \notin \mathscr{F}$ where $H, E \in \mathfrak{K}$ and $\phi \in \mathfrak{T}$. By Definition 3.24., $H \cap K \subseteq L \cup \phi$ and $E \cap M \subseteq N \cup \phi$ for some (K, L) and $(M, N) \in \mathscr{F}$ where $K, M \in \mathfrak{K}$ and $L, N \in \mathfrak{T}$. Clearly, $(H \cap K) \cup (E \cap M) \subseteq L \cup N$ and $(K \cap M, L \cup N) \in \mathscr{F}$. If $(H \cup E, \phi) \in \mathscr{F}$, then $(((K \cap M) \cap (H \cup E)) and (L \cup N)) \in \mathscr{F}$. Hence, $((K \cap M) \cap (H \cup E)) \not\subseteq (L \cup N)$. But, $((K \cap M) \cap (H \cup E)) \subseteq ((H \cap K) \cup (E \cap M)) \subseteq (L \cup N)$ is a contradiction. Hence, $(H \cup E, \phi) \notin \mathscr{F}$.

(ii) By Definition 3.24., $H \cap K \subseteq L$ and $M \subseteq G \cup N$ for some (K, L) and $(M, N) \in \mathscr{F}$ $K, M \in \mathfrak{K}$ and $L, N \in \mathfrak{T}$. Clearly, $K \cap M \subseteq L \cup N \cup G$. Let $E = K \cap M$ and $F = L \cup N$. Then $E \subseteq G \cup F$. Since $H \cap K \subseteq L$, $H \cap K \cap M \subseteq L \cup N$. Therefore, $H \cap E \subseteq F$. □

Definition 3.26. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space and $\mathscr{F}$ be an intuitionistic fuzzy rough centred texture di - filter on $\mathfrak{D}_X$. Then $\mathscr{F}$ is said to be an intuitionistic fuzzy rough centred texture maximal di - filter on $\mathfrak{D}_X$ if there exists no intuitionistic fuzzy rough centred texture di - filter on $\mathfrak{D}_X$ which is finer than $\mathscr{F}$.

Definition 3.27. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space and $\mathscr{F}$ be an intuitionistic fuzzy rough centred texture maximal di - filter on $\mathfrak{D}_X$. Then $\mathscr{F}$ is said to be an intuitionistic fuzzy rough centred texture free maximal di - filter on $\mathfrak{D}_X$ if $\mathscr{F} \neq \mathscr{F}_{p_1}$ for every $p_1 \in \mathfrak{D}_X$ where $\mathscr{F}_{p_1} = \{(K, L)/P_{p_1} \subseteq K$ and $L \subseteq \bigcup_{p_1 \not\in P_{p_2}} P_{p_2}$ where $K \in \mathfrak{K}$ and $L \in \mathfrak{T}\}$.

Remark 3.28. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space and C be the family of all intuitionistic fuzzy rough centred texture free maximal closed co-open di - filters on $\mathfrak{D}_X$ and $T = \{(K, L)/P_{p_1} \subseteq K$ and $L \subseteq \bigcup_{p_1 \not\in P_{p_2}} P_{p_2}, K \in \mathfrak{K}, L \subseteq \mathfrak{T}\}$ be the family of all maximal closed co-open di - filters on $\mathfrak{D}_X$. Consider the intuitionistic fuzzy rough centred texture space $(C, \mathscr{C})$ where $\wp(C) = \mathscr{C}$ is intuitionistic fuzzy rough centred texture, $\mathcal{T} = \{\psi(B)/B \in \mathscr{L}\}$, $\psi(B) = \{\mathscr{F}_{p_1}/p_1 \in B \in \mathscr{L}\}$ and $\psi : \mathscr{L} \to \mathscr{L}$ be an intuitionistic fuzzy rough centred textural isomorphism. Now , $(\xi\mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$ be an intuitionistic fuzzy rough centred texture di - structure space where $\xi\mathfrak{D}_X = T \cup C$, $\mathcal{T} \oplus \mathscr{C} = \mathcal{T} \cup \mathscr{C}$. Now, for every $K \in \mathfrak{K}$ and for every $G \in \mathfrak{T}$ define the sets $K^*_{\mathfrak{K}} = \{\mathscr{F}/(K, \phi) \in \mathscr{F} \in \xi\mathfrak{D}_X\}$ and $G^*_{\mathfrak{T}} = \{\mathscr{F}/(\mathfrak{D}_X, G) \notin \mathscr{F} \in \xi\mathfrak{D}_X\}$. Every member of $\mathfrak{T}_*$ can be written as the union of elements of $G^*_{\mathfrak{K}}$ and every member of $\mathfrak{K}_*$ can be written as the intersection of elements of $K^*_{\mathfrak{K}}$.

Definition 3.29. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred di - structure texture space and $V \in \mathscr{L}$ and let $\mathscr{L}^V = \{A \cap V/A \in \mathscr{L}\}$. Then $\mathscr{V} = \mathscr{L}^V$ is called an intuitionistic fuzzy rough centred texture induced structure on V and $(V, \mathscr{V})$ is said to be an intuitionistic fuzzy rough centred principal subtexture of $(\mathfrak{D}_X, \mathscr{L})$. If $\mathfrak{T}^V = \{G \cap V/G \in \mathfrak{T}\}$ and $\mathfrak{K}^V = \{K \cap V/K \in \mathfrak{K}\}$ then $(\mathfrak{T}^V, \mathfrak{K}^V)$ is said to be an intuitionistic fuzzy rough centred texture induced di - structure on V and $(V, \mathscr{L}, \mathfrak{T}^V, \mathfrak{K}^V)$ is said to be an intuitionistic fuzzy rough centred principal subtexture di - structure space of $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$.

Definition 3.30. Let (f, F) be an intuitionistic fuzzy rough centred texture di - function from $(\mathfrak{D}_X, \mathscr{L}_1, \mathfrak{T}_1, \mathfrak{K}_1)$ to $(\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$ and $(V, \mathscr{L}, \mathfrak{T}^V, \mathfrak{K}^V)$ be an intuitionistic fuzzy rough centred principal subtexture di - structure space of $(\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$. Then the intuitionistic fuzzy rough centred texture di - function $(f_{\mathfrak{D}_X \times V}, F_{\mathfrak{D}_Y \times V}) : (\mathfrak{D}_X, \mathscr{L}_1, \mathfrak{T}_1, \mathfrak{K}_1) \to (V, \mathscr{L}, \mathfrak{T}^V, \mathfrak{K}^V)$ is said to be an intuitionistic fuzzy rough centred texture restriction di - function of (f, F) if $f_{\mathfrak{D}_X \times V} = \bigvee\{\overline{P}^{\mathfrak{D}_X \times V}_{(p_1, q_1)}/$ there exists $p_2 \in \mathfrak{D}_X$, $P_{p_1} \not\subseteq Q_{p_2}$ and $f \not\subseteq \overline{Q}_{(p_2, q_1)}$ $\}$ and $F_{\mathfrak{D}_X \times V} = \bigcap\{\overline{Q}^{\mathfrak{D}_X \times V}_{(p_1, q_1)}/$ there exists $p_2 \in A$, $P_{p_2} \not\subseteq Q_{p_1}$ and $\overline{P}_{(p_2, q_1)} \not\subseteq F$ $\}$.

Note 3.7. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space. Let $(i_A, \mathcal{I}_A)$ be an intuitionistic fuzzy rough centred texture identity di - function of the intuitionistic fuzzy rough centred principle subtexture $(A, \mathscr{L}^A)$. Then $(i_{\mathfrak{D}_X \times A}, \mathcal{I}_{\mathfrak{D}_X \times A})$ is the intuitionistic fuzzy rough centred texture restriction of $(i_{\mathfrak{D}_X}, \mathcal{I}_{\mathfrak{D}_X})$ to the intuitionistic fuzzy rough centred principal subtexture $(A, \mathscr{L}^A)$

Definition 3.31. Let $(\mathfrak{D}_X, \mathscr{L}_1)$, $(\mathfrak{D}_Y, \mathscr{L}_2)$ and $(\mathfrak{D}_Z, \mathscr{L}_3)$ be any three intuitionistic fuzzy rough centred texture spaces. Let (f, F) be an intuitionstic fuzzy centred texture di - function from $(\mathfrak{D}_X, \mathscr{L}_1)$ to $(\mathfrak{D}_Y, \mathscr{L}_2)$, (g, G) be an intuitionstic fuzzy centred texture di - function from $(\mathfrak{D}_Y, \mathscr{L}_2)$ to $(\mathfrak{D}_Z, \mathscr{L}_3)$, $A \in \mathscr{L}_1$ and $B \in \mathscr{L}_3$. Then

(i) $g_{\mathfrak{D}_Y \times B} \circ f_{A \times \mathfrak{D}_Y} = (g \circ f)_{A \times B}$ and $G_{\mathfrak{D}_Y \times B} \circ F_{A \times \mathfrak{D}_Y} = (G \circ F)_{A \times B}$.
(ii) $(f^{\leftarrow})_{C \times A} = (f_{A \times C})^{\leftarrow}$ and $(F^{\leftarrow})_{C \times A} = (F_{A \times C})^{\leftarrow}$ where $A \in \mathscr{L}_1$ and $C \in \mathscr{L}_2$.
(iii) $f^{\leftarrow}_{B \times A}(M) = f^{\leftarrow}(M \cap A)$ for every $M, A \in \mathscr{L}_2$ and $B \in \mathscr{L}_2$.
(iv) $(f, F) = (f_{\mathfrak{D}_X, \times \mathfrak{D}_Y}, F_{\mathfrak{D}_X, \times \mathfrak{D}_Y})$.

Definition 3.32. Let $(\mathfrak{D}_X, \mathscr{L}_1)$ and $(\mathfrak{D}_Y, \mathscr{L}_2)$ be any two intuitionistic fuzzy rough centred texture spaces. Let (f, F) be an intuitionstic fuzzy centred texture di - function from $(\mathfrak{D}_X, \mathscr{L}_1)$ to $(\mathfrak{D}_Y, \mathscr{L}_2)$ and $(i, \mathcal{I})$ be an intuitionistic fuzzy rough centred texture identity di - function from $(\mathfrak{D}_X, \mathscr{L}_1)$ to $(\mathfrak{D}_X, \mathscr{L}_1)$.Then

(i) $(f, F) \circ (i, \mathcal{I}) = (f \circ i, F \circ \mathcal{I}) = (f, F)$.
(ii) $i^{\leftarrow} = I$ and $I^{\leftarrow} = i$.
(iii) $(f, F)^{\leftarrow} = (F^{\leftarrow}, f^{\leftarrow})$.
(iv) $f \circ F^{\leftarrow} \supseteq i_{\mathfrak{D}_Y}$ and $f^{\leftarrow} \circ F \supseteq \mathcal{I}_{\mathfrak{D}_Y}$.

Definition 3.33. Let $(\mathfrak{D}_X, \mathscr{L}_1, \mathfrak{T}_1, \mathfrak{K}_1)$ and $(\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$ be any two intuitionistic fuzzy rough centred texture di - structure spaces and $(f, F) : (\mathfrak{D}_X, \mathscr{L}_1, \mathfrak{T}_1, \mathfrak{K}_1) \to (\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$ be an intuitionistic fuzzy centred texture di - function. Then it is said to be an intuitionistic fuzzy centred texture

(i) surjective di - function if $F \circ f^{\leftarrow} \subseteq \mathcal{I}_{\mathscr{L}_2}$ and $i_{\mathscr{L}_2} \subseteq f \circ F^{\leftarrow}$.
(ii) injective di - function if $F^{\leftarrow} \circ f \subseteq i_{\mathscr{L}_2}$ and $i_{\mathscr{L}_2} \subseteq f^{\leftarrow} \circ F$.

Definition 3.34. Let $(\mathfrak{D}_X, \mathscr{L}_1, \mathfrak{T}_1, \mathfrak{K}_1)$ and $(\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$ be any two intuitionistic fuzzy centred di - structure texture spaces and (f, F) be an intuitionistic fuzzy rough centred texture di - function from $\mathscr{L}_1$ to $\mathscr{L}_2$. Then (f, F) is said to be an intuitionistic fuzzy rough centred texture

(i) continuous di - function if $G \in \mathfrak{T}_2$, then $F^{\leftarrow}(G) \in \mathfrak{T}_1$.
(ii) co-continiuous difunction if $K \in \mathfrak{K}_2$, then $f^{\leftarrow}(K) \in \mathfrak{K}_1$.
(iii) bicontinuous difunction if it is both intuitionistic fuzzy rough centred texture continuous difunction and intuitionistic fuzzy rough centred texture co continuous difunction.
(iv) di - homoemorphism if it is intuitionistic fuzzy rough centred texture bijective di - function, intuitionistic fuzzy rough centred texture bi - continuous di - function, and its inverse is an intuitionistic fuzzy rough centred texture bicontinuous di - function.
(v) open di - function if $G \in \mathfrak{T}_2$, then $f^{\rightarrow}(G) \in \mathfrak{T}_1$.
(vi) closed di - function if $K \in \mathfrak{K}_2$, then $f^{\rightarrow}(K) \in \mathfrak{K}_1$.

Definition 3.35. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred di - structure texture space. Let $G \in \mathfrak{T} \setminus \{\phi\}$ and $K \in \mathfrak{K} \setminus \{\mathfrak{D}_X\}$.

(i) If $G \cap A \neq \phi$ for every $G \in \mathfrak{T} \setminus \{\phi\}$, then A is said to be an intuitionistic fuzzy rough centred texture dense in $\mathfrak{D}_X$.

(ii) If $A \nsubseteq K$ for every $K \in \mathfrak{K} \setminus \{\mathfrak{D}_X\}$,then A is said to be an intuitionistic fuzzy rough centred texture co - dense in $\mathfrak{D}_X$.

(iii) If $G \cap A \nsubseteq K$ for every $G \in \mathfrak{T}$ and $K \in \mathfrak{K} \setminus \{\phi\}$ where $G \nsubseteq K$, then A is said to be an intuitionistic fuzzy rough centred texture bi - dense in $\mathfrak{D}_X$.

Definition 3.36. Let $(\mathfrak{D}_X, \mathscr{L}_1, \mathfrak{T}_1, \mathfrak{K}_1)$ and $(\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$ be an intuitionistic fuzzy rough centred texture di - structure spaces and (f, F) be an intuitionistic fuzzy rough centred texture di - function from $\mathscr{L}_1$ to $\mathscr{L}_2$. Then (f, F) is said to be an intuitionistic fuzzy rough centred texture di - embedding from $\mathscr{L}_1$ to $\mathscr{L}_2$ if there exists an intuitionistic fuzzy rough centred texture bi-dense di - structure pricipal subtexture $(N, \mathscr{L}_2^N, \mathfrak{T}_2^N, \mathfrak{V}_2^N)$ of $(\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$ such that $(f_{\mathfrak{D}_X \times N}, F_{\mathfrak{D}_X \times N}) : (\mathfrak{D}_X, \mathscr{L}_1) \to (N, \mathscr{L}_2^N)$ is an intuitionistic fuzzy rough centred texture di - homeomorphism. If $(\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$ is an intuitionistic fuzzy centred texture di - compact space, then $((f, F), (\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2))$ is said to be an intuitionistic fuzzy rough centred texture di - compactification of $(\mathfrak{D}_X, \mathscr{L}_1, \mathfrak{T}_1, \mathfrak{K}_1)$.

Definition 3.37. Let $(\mathfrak{D}_X, \mathscr{L}_1, \mathfrak{T}_1, \mathfrak{K}_1)$, $(\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$ and $(\mathfrak{D}_Z, \mathscr{L}_3, \mathfrak{T}_3, \mathfrak{K}_3)$ be an intuitionistic fuzzy rough centred texture di - structure spaces. Let (f, F) is an intuitionistic fuzzy rough centred texture di-function from $(\mathfrak{D}_X, \mathscr{L}_1, \mathfrak{T}_1, \mathfrak{K}_1)$ to $(\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$ and (g, G) be an intuitionistic fuzzy rough centred texture di-function from $(\mathfrak{D}_Y, \mathscr{L}_2, \mathfrak{T}_2, \mathfrak{K}_2)$ to $(\mathfrak{D}_Z, \mathscr{L}_3, \mathfrak{T}_3, \mathfrak{K}_3)$. If there exists an intuitionistic fuzzy rough centred texture restriction di-function $(h, H) : (\mathfrak{D}_X, \mathscr{L}_1, \mathfrak{T}_1, \mathfrak{K}_1) \leftarrow (\mathfrak{D}_Y, \mathscr{L}_3, \mathfrak{T}_3, \mathfrak{K}_3)$ of $(g \circ f, G \circ F)$, then

(i) $(g_{\mathfrak{D}_Y \times \mathfrak{D}_Z} \circ f_{\mathfrak{D}_X \times \mathfrak{D}_Y})_{\mathfrak{D}_X \times \mathfrak{D}_Y} = (g_{\mathfrak{D}_Y \times \mathfrak{D}_Y}, f_{\mathfrak{D}_X \times \mathfrak{D}_Y}) = (g_{\mathfrak{D}_Y}, f_{\mathfrak{D}_X \times \mathfrak{D}_Y})$

(ii) $(G_{\mathfrak{D}_Y \times \mathfrak{D}_Z} \circ F_{\mathfrak{D}_X \times \mathfrak{D}_Y})_{\mathfrak{D}_X \times \mathfrak{D}_Y} = (G_{\mathfrak{D}_Y \times \mathfrak{D}_Y}, F_{\mathfrak{D}_X \times \mathfrak{D}_Y}) = (G_{\mathfrak{D}_Y}, F_{\mathfrak{D}_X \times \mathfrak{D}_Y})$.

(iii) $(g_{\mathfrak{D}_Y \times \mathfrak{D}_Z} \circ f_{\mathfrak{D}_X \times \mathfrak{D}_Y})^{\leftarrow}_{\mathfrak{D}_X \times \mathfrak{D}_Y} = (f^{\leftarrow}_{\mathfrak{D}_Y \times \mathfrak{D}_X}, g^{\leftarrow}_{\mathfrak{D}_Z \times \mathfrak{D}_Y})_{\mathfrak{D}_Y \times \mathfrak{D}_X}$
$= (f^{\leftarrow}_{\mathfrak{D}_Y \times \mathfrak{D}_X}, g^{\leftarrow}_{\mathfrak{D}_Y \times \mathfrak{D}_Y})$.

(iv) $(G_{\mathfrak{D}_Y \times \mathfrak{D}_Z} \circ F_{\mathfrak{D}_X \times \mathfrak{D}_Y})^{\leftarrow}_{\mathfrak{D}_X \times \mathfrak{D}_Y} = (F^{\leftarrow}_{\mathfrak{D}_Y \times \mathfrak{D}_X}, G^{\leftarrow}_{\mathfrak{D}_Z \times \mathfrak{D}_Y})_{\mathfrak{D}_Y \times \mathfrak{D}_X}$
$= (F^{\leftarrow}_{\mathfrak{D}_Y \times \mathfrak{D}_X}, G^{\leftarrow}_{\mathfrak{D}_Y \times \mathfrak{D}_Y})$.

Proposition 3.38. *$(i_{T \times \xi \mathfrak{D}_X} \circ f_\psi, \mathcal{I}_{T \times \xi \mathfrak{D}_X} \circ F_\psi)$ is an intuitionistic fuzzy rough centred texture di - embedding from $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ to $(\xi \mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$*

Proof. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space and $(\xi \mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$ be an intuitionistic fuzzy rough centred texture di - structure space as in Remark 3.28., Assume an intuitionistic fuzzy rough centred principal subtexture di - structure space $(T, (\mathcal{T} \oplus \mathscr{C})^T, \mathfrak{T}_*^T, \mathfrak{K}_*^T)$ of $(\xi \mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$. Suppose that (f_ψ, F_ψ) is an intuitionistic fuzzy rough centred texture di - function from $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ to $(T, (\mathcal{T} \oplus \mathscr{C})^T, \mathfrak{T}_*^T, \mathfrak{K}_*^T)$ and $(i_{T \times \xi \mathfrak{D}_X}, \mathcal{I}_{T \times \xi \mathfrak{D}_X}) : (T, (\mathcal{T} \oplus \mathscr{C})^T, \mathfrak{T}_*^T, \mathfrak{K}_*^T) \to (\xi \mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$ are any two intuitionistic fuzzy centred texture di - functions. Clearly, $(i_{T \times \xi \mathfrak{D}_X} \circ f_\psi, \mathcal{I}_{T \times \xi \mathfrak{D}_X} \circ F_\psi) : (\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K}) \to (\xi \mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$ is an intuitionistic fuzzy rough centred texture di - function. Assume an intuitionistic fuzzy rough centred texture restriction di - function $((i_{T \times \xi \mathfrak{D}_X} \circ f_\psi)_{\mathfrak{D}_X \times T}, (\mathcal{I}_{T \times \xi \mathfrak{D}_X} \circ F_\psi)_{\mathfrak{D}_X \times T}) : (\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K}) \to (T, (\mathcal{T} \oplus \mathscr{C})^T, \mathfrak{T}_*^T, \mathfrak{K}_*^T)$.

by Definitions 3.3., 3.4., 3.9., $(i_{T\times\xi\mathfrak{D}_X}\circ f_\psi)_{\mathfrak{D}_X\times T}\circ((\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)_{\mathfrak{D}_X\times T})^{\leftarrow}$

$$\begin{aligned}
&= (i_{T\times\xi\mathfrak{D}_X}\circ f_\psi)_{\mathfrak{D}_X\times T}\circ(\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)^{\leftarrow}_{T\times\mathfrak{D}_X}\\
&= (i_{T\times\xi\mathfrak{D}_X}\circ f_\psi)_{\mathfrak{D}_X\times T}\circ(F_\psi^{\leftarrow}\circ(\mathcal{I}_{T\times\xi\mathfrak{D}_X})^{\leftarrow})_{T\times\mathfrak{D}_X}\\
&= (i_{T\times\xi\mathfrak{D}_X}\circ f_\psi)_{\mathfrak{D}_X\times T}\circ(F_\psi^{\leftarrow}\circ(\mathcal{I}^{\leftarrow}_{\xi\mathfrak{D}_X\times T}))_{T\times\mathfrak{D}_X}\\
&= (i_T\circ f_\psi)\circ(F_\psi^{\leftarrow}\circ\mathcal{I}_T^{\leftarrow})\\
&= (i_T\circ f_\psi)\circ(F_\psi^{\leftarrow}\circ i_T)\\
&= f_\psi\circ F_\psi^{\leftarrow}\supseteq i_T.
\end{aligned}$$

hence, by Definition 3.31., $(i_{T\times\xi\mathfrak{D}_X}\circ f_\psi,\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)$ is an intuitionistic fuzzy rough centred texture surjective di - function.
similarly, $((i_{T\times\xi\mathfrak{D}_X}\circ f_\psi)_{\mathfrak{D}_X\times T})^{\leftarrow}\circ(\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)_{\mathfrak{D}_X\times T}$

$$\begin{aligned}
&= (i_{T\times\xi\mathfrak{D}_X}\circ f_\psi)^{\leftarrow}_{T\times\mathfrak{D}_X}\circ(\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)_{\mathfrak{D}_X\times T}\\
&= (f_\psi^{\leftarrow}\circ(i_{T\times\xi\mathfrak{D}_X})^{\leftarrow}_{\mathfrak{D}_X\times T}\circ(\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)_{T\times\mathfrak{D}_X}\\
&= (f_\psi^{\leftarrow}\circ(i^{\leftarrow}_{\xi\mathfrak{D}_X\times T}))_{\mathfrak{D}_X\times T}\circ(\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)_{T\times\mathfrak{D}_X}\\
&= (f_\psi^{\leftarrow}\circ i_T^{\leftarrow})\circ(\mathcal{I}_T\circ F_\psi)\\
&= (f_\psi^{\leftarrow}\circ\mathcal{I}_T)\circ(\mathcal{I}_T\circ F_\psi)\\
&= f_\psi^{\leftarrow}\circ F_\psi\supseteq\mathcal{I}_{\mathfrak{D}_X}
\end{aligned}$$

Therefore, $(i_{T\times\xi\mathfrak{D}_X}\circ f_\psi,\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)$ is an intuitionistic fuzzy rough centred texture injective di - function. Now let $H\cap T\in T^*_{\mathfrak{T}_*}$. Therefore $H=\bigvee_{j\in J}(H_j)^*_{\mathfrak{T}}$ where $H_j\in\mathfrak{T}$ for every $j\in J$.
Then

$$\begin{aligned}
((\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)_{\mathfrak{D}_X\times T})^{\leftarrow}(H\cap T) &= ((\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)^{\leftarrow}(H\cap T))\\
&= ((F_\psi^{\leftarrow}\circ(\mathcal{I}_{T\times\xi\mathfrak{D}_X})^{\leftarrow})(H\cap T))\\
&= F_\psi^{\leftarrow}((\mathcal{I}_{\xi\mathfrak{D}_X\times T})^{\leftarrow})\bigvee_{j\in J}((H_j)^*_{\mathfrak{T}}\cap T)\\
&= F_\psi^{\leftarrow}(\bigvee_{j\in J}((\mathcal{I}_{\xi\mathfrak{D}_X\times T})^{\leftarrow})(H_j)^*_{\mathfrak{T}}\cap T)\\
&= F_\psi^{\leftarrow}(\bigvee_{j\in J}(H_j)^*_{\mathfrak{T}}\cap T)\\
&= F_\psi^{\leftarrow}(\bigvee_{j\in J}F_\psi(H_j))\\
&= \bigvee_{j\in J}F_\psi^{\leftarrow}(F_\psi(H_j))\\
&= \bigvee_{j\in J}H_j\in\mathfrak{T}.
\end{aligned}$$

Hence, $(i_{T\times\xi\mathfrak{D}_X}\circ f_\psi,\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)_{\mathfrak{D}_X\times T}$ is an intuitionistic fuzzy rough centred texture continuous function. Similarly, it is also an intuitionistic fuzzy rough centred texture co-continuous function. Therefore, $(i_{T\times\xi\mathfrak{D}_X}\circ f_\psi,\mathcal{I}_{T\times\xi\mathfrak{D}_X}\circ F_\psi)_{\mathfrak{D}_X\times T}$ is an intuitionistic fuzzy rough centred texture di-continuous function.

Take $H \in \mathfrak{T}$. Then

$$\begin{aligned}(i_{T\times\xi\mathfrak{D}_X} \circ f_\psi)_{\mathfrak{D}_X\times T}(H) &= (i_{T\times\xi\mathfrak{D}_X} \circ f_\psi)(H) = i_{T\times\xi\mathfrak{D}_X}(f_\psi(H)) \\ &= i_{\xi\mathfrak{D}_X}(f_\psi(H)) = f_\psi(H) = H^*_{\mathfrak{T}} \cap T.\end{aligned}$$

Hence, $(i_{T\times\xi\mathfrak{D}_X} \circ f_\psi, \mathcal{I}_{T\times\xi\mathfrak{D}_X} \circ F_\psi)$ is an intuitionistic fuzzy rough centred texture open di - function. Similarly, it is also an intuitionistic fuzzy rough centred texture closed di - function from $\mathfrak{D}_X$ to T.

Let $G = \bigvee_{i\in J_1} G^*_{i_{\mathfrak{T}}}$ and $K = \bigcap_{j\in J_2} K^*_{j_{\mathfrak{K}}}$ where $G^*_{j_{\mathfrak{T}}} \in \mathfrak{T}^*$,$K^*_{j_{\mathfrak{K}}} \in \mathfrak{K}^*, G \in \mathfrak{T}^T_*$ and $K \in \mathfrak{K}^T_*$ such that $G \not\subseteq K$. Then for some $i \in J_1$, $G^*_{i_{\mathfrak{T}}} \not\subseteq \bigcap_{j\in J_2} K^*_{j_{\mathfrak{K}}}$ and hence , for some $i \in J_1$, $G^*_{i_{\mathfrak{T}}} \not\subseteq K^*_{j_{\mathfrak{K}}}$. Then $G^*_{i_{\mathfrak{T}}} \not\subseteq Q_{\mathscr{F}}$ and $P_{\mathscr{F}} \not\subseteq K^*_{j_{\mathfrak{K}}}$ and this gives $(\mathfrak{D}_X, G_i) \notin \mathscr{F}$ and $(K_j, \phi) \notin \mathscr{F}$. By Proposition 3.1 (ii), there exists $(K, H) \in \mathscr{F}$ such that $K \subseteq G_i \cup H$ and $K_j \cap K \subseteq H$ where $\mathscr{F}$ is an intuitionistic fuzzy rough centred texture maximal closed co - open di - filters on $\mathfrak{D}_X$ where $K \in \mathfrak{K}$ and $H \in \mathfrak{T}$. Since $K \not\subseteq H$, choose some $p_1 \in \mathfrak{D}_X$ where $K \not\subseteq Q_{p_1}$ and $P_{p_1} \not\subseteq H$. Then $P_{p_1} \subseteq G_i$ and hence, $G_j \not\subseteq \bigcup_{p_1 \notin P_{p_2}} P_{p_2}$. That is, $(\mathfrak{D}_X, G_i) \notin \mathscr{F}_{p_1}$ and so $P_{\mathscr{F}_{p_1}} \in G^*_{i_{\mathfrak{T}}} \cap T$. Further, since $P_{p_1} \not\subseteq H$ and $P_{p_1} \not\subseteq K_j \cap K$, $P_{p_1} \not\subseteq K_j$ implies that $(K_j, \phi) \notin \mathscr{F}_{p_1}$ where $\mathscr{F}_{p_1} = \{(K, L)/P_{p_1} \subseteq K$ and $L \subseteq \bigcup_{p_1 \notin P_{p_2}} P_{p_2}$ where $K \in \mathfrak{K}$ and $L \in \mathfrak{T}\}$. That is , $P_{\mathscr{F}_{p_1}} \notin K^*_{j_{\mathfrak{K}}}$. Hence, $P_{\mathscr{F}_p} \notin \bigcap_{j\in J} K^*_{j_{\mathfrak{K}}}$. Therefore , $G^*_{j_{\mathfrak{T}}} \cap T \not\subseteq \bigcap_{j\in J} K^*_{j_{\mathfrak{K}}}$ and so $\bigvee_{i\in J_1} G^*_{i_{\mathfrak{T}}} \cap T \not\subseteq \bigcap_{j\in J_2} K^*_{j_{\mathfrak{K}}}$. As a result, by complete distributivity of the direct sum $\mathcal{T} \oplus \mathscr{C}$, $(\bigvee_{i\in J} G^*_{i_{\mathfrak{T}}}) \cap T \not\subseteq \bigcap_{j\in J} K^*_{j_{\mathfrak{K}}}$. That is, $G \cap T \not\subseteq K$. Hence, $(T, (\mathcal{T} \oplus \mathscr{C})^T, \mathfrak{T}^T_*, \mathfrak{K}^T_*)$ is an intuitionistic fuzzy rough centred texture di - dense space. Hence, $(i_{T\times\xi\mathfrak{D}_X} \circ f_\psi, \mathcal{I}_{T\times\xi\mathfrak{D}_X} \circ F_\psi)$ is an intuitionistic fuzzy rough centred texture di - embedding from $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ to $(\xi\mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$. □

Definition 3.39. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space. Then $(\mathfrak{D}_X, \mathscr{L}, \gamma, \mathfrak{T}, \mathfrak{K})$ is said to be an intuitionistic fuzzy rough centred texture semi-weakly T_0 space if $P_{p_1} \neq P_{p_2}$ implies that there exists $B \in \mathfrak{T} \cup \mathfrak{K}$ with $P_{p_1} \subseteq B \subseteq Q_{p_2}$ or $P_{p_2} \subseteq B \subseteq Q_{p_1}$.

Definition 3.40. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space and $\{(F_i, G_i)/i \in J\} = \mathcal{D} \subseteq \mathscr{L} \times \mathscr{L}$ be an intuitionistic fuzzy rough centred texture di - family on $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$. Then $\mathcal{D}$ is said to be an intuitionistic fuzzy rough centred texture di - cover of $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ if for all partitions J_1 and J_2 of J, then $\bigcap_{i\in J_1} F_i \subseteq \bigvee_{i\in J_2} G_i$. If $F_i \in \mathfrak{T}$ (resp., $F_i \in \mathfrak{T}$) and $G_i \in \mathfrak{K}$ (resp., $G_i \in \mathfrak{T}$) then $\mathcal{D}$ is said to be an intuitionistic fuzzy rough centred texture open co - closed (resp., closed co - open) di - cover.

Definition 3.41. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space and $\{(F_i, G_i)/i \in J\} = \mathcal{D} \subseteq \mathscr{L} \times \mathscr{L}$ be an intuitionistic fuzzy rough centred texture di - family on $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$. If $\mathcal{D}$ is an intuitionistic fuzzy rough centred texture di - cover of $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$, then for every finite sets $\{i_1, i_2, ..., i_n\}$ and $\{j_1, j_2, ..., j_m\}$, $\bigcap_{k=1}^{n} K_{i_k} \subseteq \bigcup_{j=1}^{m} H_{i_j}$.

Definition 3.42. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space and $\mathcal{D} = \{(F_i, G_i)/i \in J\}$ be an intuitionistic fuzzy rough centred texture di - family on $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$. Then

(i) domain of $\mathcal{D}$ is denoted and defined as $\{F_i/i \in J\}$ and range of $\mathcal{D}$ is denoted and defined as $\{G_i/i \in J\}$. (ii) $\mathcal{D}$ is said to be an intuitionistic fuzzy rough centred texture finite co - finite difamily if $dom\mathcal{D}$ and $ran\mathcal{D}$ are finite.

Note 3.8. Let $(\mathfrak{D}_X, \mathscr{L})$ be an intuitionistic fuzzy rough centred texture space. Then

(i) For $A, B \in \mathscr{L}$ if $A \not\subseteq B$, then there exists $p_1 \in \mathfrak{D}_X$ with $A \not\subseteq Q_{p_1}$ and $P_{p_1} \not\subseteq B$.

(ii) $A = \bigcap\{Q_{p_1}/P_{p_1} \not\subseteq A\}$ for all $A \in \mathscr{L}$.

(iii) $A = \bigvee\{P_{p_1}/A \not\subseteq P_{p_1}\}$ for all $A \in \mathscr{L}$.

Definition 3.43. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space. Then $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ is said to be an

(i) intuitionistic fuzzy rough centred texture compact space if whenever $\mathfrak{D}_X = \bigvee_{i\in J} G_i$, $G_i \in \mathfrak{T}$, $i \in J$, there is a finite subset J_o of J with $\mathfrak{D}_X = \bigcup_{j\in J_o} G_j$.

(ii) intuitionistic fuzzy rough centred texture co - compact space if whenever $\bigcap_{i\in J} F_i = \phi$, $F_i \in \mathfrak{K}$, $i \in J$, there is a finite subset J_o of J with $\bigcap_{j\in J_o} F_j = \phi$.

(iii) intuitionistic fuzzy rough centred texture di - compact space if it is both an intuitionistic fuzzy rough centred texture compact spac and an intuitionistic fuzzy rough centred texture co-compact space.

Definition 3.44. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space and $\mathscr{F}$ be an intuitionistic fuzzy rough centred texture di - filter in $\mathfrak{D}_X$. If $\mathscr{F}$ has the finite exclusion property, then there exists an intuitionistic fuzzy rough centred texture maximal di - filter $\mathscr{G}$ in $\mathfrak{D}_X$ such that $\mathscr{F} \subseteq \mathscr{G}$.

Proposition 3.45. *Let $(\xi\mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$ be an intuitionistic fuzzy centred di - structure texture space. Then $(\xi\mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$ is an intuitionistic fuzzy rough centred texture semi-weakly T_0 space and intuitionistic fuzzy rough centred texture dicompact space.*

Proof. Let $(\mathfrak{D}_X, \mathscr{L}, \mathfrak{T}, \mathfrak{K})$ be an intuitionistic fuzzy rough centred texture di - structure space , $\mathscr{F}$ and $\mathscr{G}$ are intuitionistic fuzzy rough centred texture maximal closed co - open filters in $(\xi\mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$ and $P_{\mathscr{F}} \neq P_{\mathscr{G}}$. Then $\mathscr{F} \neq \mathscr{G}$. Since $\mathscr{F}$ and $\mathscr{G}$ are intuitionistic fuzzy rough centred texture maximal filters, $\mathscr{F} \not\subseteq \mathscr{G}$ and $\mathscr{G} \not\subseteq \mathscr{F}$. Hence, for some $(K, G) \in \mathscr{F}$, $(K, G) \notin \mathscr{G}$ where $K \in \mathfrak{K}$ and $G \in \mathfrak{T}$. Now, $(\mathfrak{D}_X, G) \notin \mathscr{G}$ or $(K, \phi) \notin \mathscr{G}$. If $(K, \phi), (\mathfrak{D}_X, G) \in \mathscr{F}$, then $P_{\mathscr{G}} \subseteq G^*_{\mathfrak{T}} \subseteq Q_{\mathscr{F}}$ or $P_{\mathscr{F}} \subseteq K^*_{\mathfrak{K}} \subseteq Q_{\mathscr{G}}$ for some $K \in \mathfrak{K}$ and $G \in \mathfrak{T}$. By the similar way, $\mathscr{G} \not\subseteq \mathscr{F}$ implies that $P_{\mathscr{F}} \subseteq G'^*_{\mathfrak{T}} \subseteq Q_{\mathscr{G}}$ or $P_{\mathscr{G}} \subseteq K'^*_{\mathfrak{K}} \subseteq Q_{\mathscr{F}}$, for some $G' \in \mathfrak{T}$ and $K' \in \mathfrak{K}$. Therefore, $(\xi\mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$ is an intuitionistic fuzzy rough centred texture semi-weakly T_0 space.

Let $\mathcal{B}$ be an intuitionistic fuzzy rough centred texture closed co - open di - filter in $(\xi\mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$. Consider the intuitionistic fuzzy rough centred texture closed co - open di - family $\mathscr{F} = \{(K, G) \mid K \in \mathfrak{K}, G \in \mathfrak{T}$ and $(K^*_{\mathfrak{K}}, G^*_{\mathfrak{T}}) \in \mathcal{B}\}$. Since $\mathcal{B}$ is an intuitionistic fuzzy rough centred texture closed co-open di-filter , $(K^*_{\mathfrak{K}}, G^*_{\mathfrak{T}}) \in \mathcal{B}$. Then, $K^*_{\mathfrak{K}} \not\subseteq G^*_{\mathfrak{T}}$. Therefore, $\bigcap K^*_{\mathfrak{K}} \not\subseteq G^*_{\mathfrak{T}} \subseteq \bigcup G^*_{\mathfrak{T}}$. Therefore, $\bigcap K^*_{\mathfrak{K}} \not\subseteq \bigcup G^*_{\mathfrak{T}}$. for every $K \in \mathfrak{K}, G \in \mathfrak{T}$. Clearly, $\mathscr{F}$ has finite-exclusion property. Hence, there exists an intuitionistic fuzzy rough centred texture maximal closed co-open di - filter $\mathscr{F}_1 \in \xi\mathfrak{D}_X$ such that $\mathscr{F} \subseteq \mathscr{F}_1$. Let $(K, H) \in \mathcal{B}$ where $K \in \mathfrak{K}, G \in \mathfrak{T}$. Suppose that, $P_{\mathscr{F}_1} \not\subseteq K$ or $P_{\mathscr{F}_1} \subseteq H$. If $P_{\mathscr{F}_1} \not\subseteq K$,then there exists $F \in \mathfrak{K}$ such that $K \subseteq F^*_{\mathfrak{K}}$ and

$P_{\mathscr{F}_1} \nsubseteq F^*_{\mathfrak{K}}$. Clearly, $(F^*_{\mathfrak{K}}, \phi^*{}_{\mathfrak{T}}) \in \mathcal{B}$ and therefore, $(F, \phi) \in \mathscr{F} \subseteq \mathscr{F}_1$ and so $P_{\mathscr{F}} \subseteq F^*_{\mathfrak{K}}$ is a contradiction. Now, let $\mathscr{F}_1 \in H$. For some $G \in \mathfrak{T}$, $P_{\mathscr{F}_1} \subseteq G^*_{\mathfrak{T}} \subseteq H$ and hence, $(\mathfrak{D}_X{}^*_{\mathfrak{K}}, G^*_{\mathfrak{T}}) \in \mathcal{B}$. Then $(\mathfrak{D}_X, G) \in \mathscr{F} \subseteq \mathscr{F}_1$ gives the contradiction $P_{\mathscr{F}_1} \nsubseteq G^*_{\mathfrak{T}}$.

Let $\mathscr{M} = \{(H_i, K_i) \mid i \in J\}$ be an intuitionistic fuzzy rough centred texture open co-closed dicover where $H_i \in \mathfrak{T}$ and $K_i \in \mathfrak{K}$ and assume that $\xi\mathfrak{D}_X \subseteq \bigvee_{i\in J}(H_i, K_i)$. Suppose that $\mathscr{M}$ has no intuitionistic fuzzy rough centred texture finite co-finite subdicover such that $\xi\mathfrak{D}_X \nsubseteq \bigvee_{i\in J_0}(H_i, K_i)$. Since $\mathscr{M}$ is an intuitionistic fuzzy rough centred texture dicover, then for all partitions $\{J_1, J_2\}$ of J, $\bigcap_{i\in J_1} K_i \subseteq \bigvee_{i\in J_2} H_i$. By our assumption, for every finite sets $\{i_1, i_2, ..., i_n\}$ and $\{j_1, j_2, ..., j_m\}$, $\bigcap_{k=1}^n K_{i_k} \nsubseteq \bigcup_{j=1}^m H_{i_j}$. Hence, the intuitionistic fuzzy rough centred texture di - family $\mathcal{A} = \{(K_i, H_i) \mid i \in J\}$ has finite exclusion property. Then, by Definition 3.44., there exists an intuitionistic fuzzy rough centred texture closed co-open maximal di - filter $\mathcal{B}$ containing $\mathcal{A}$. But there exists an intuitionistic fuzzy rough centred texture closed co-open maximal di - filter $\mathscr{F} \in \xi\mathfrak{D}_X$ such that for all $i \in J$, $P_{\mathscr{F}} \subseteq K_i$ and $P_{\mathscr{F}} \nsubseteq H_i$. If $\{J_1, J_2\}$ is the partition where $J_1 = J \setminus \{j\}$ and $J_2 = \{j\}$, then $\bigcap_{i\in J_1} K_i \nsubseteq H_j$, which is a contradiction. Hence, $\mathscr{M}$ has no intuitionistic fuzzy rough centred texture finite co-finite sub-dicover such that $\xi\mathfrak{D}_X \subseteq \bigvee_{i\in J_0}(H_i, K_i)$. Therefore, $(\xi\mathfrak{D}_X, \mathcal{T} \oplus \mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*)$ is an intuitionistic fuzzy rough centred texture dicompact space. □

Remark 3.46. From Propositions 3.25 and 3.28, it is clear that $((i_{T\times\xi\mathfrak{D}_X} \circ f_\psi, \mathcal{I}_{T\times\xi\mathfrak{D}_X} \circ F_\psi), (\xi\mathfrak{D}_X, \mathcal{T}\oplus\mathscr{C}, \mathfrak{T}_*, \mathfrak{K}_*))$ is an intuitionistic fuzzy rough centred texture di - structure compactification.

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