Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 7, No. 4, (April 2014), pp. 669–686
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

New similarity measures of fuzzy soft sets based on distance measures

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Received 14 July 2013; Revised 14 August 2013; Accepted 5 September 2013

Abstract. Similarity measure is a very important problem in fuzzy soft set theory. In this paper, seven similarity measures of fuzzy soft sets are introduced, which are based on the normalized Hamming distance, the normalized Euclidean distance, the generalized normalized distance, the Type-2 generalized normalized distance, the Type-2 normalized Euclidean distance, the Hausdorff distance and the Chebyshev distance. Secondly, some properties of these similarity measures are analyzed. Thirdly, comparison analysis on these similarity measures is provided. Moreover, an example is given to illustrate the application of these different similarity measures of fuzzy soft sets in decision making.

2010 AMS Classification: 03E72, 08A72

Keywords: Soft set, Fuzzy soft set, Similarity measures, Distance measures, Decision making.

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1. Introduction

There are many uncertain problems in our real life. In order to deal with these uncertainties, rough sets, fuzzy sets, D-S evidence theory, vague sets, intuitionistic fuzzy sets, interval mathematics were introduced. But all these theories have their difficulties as pointed out in [15]. Molodtsov initiated soft set theory [15] as a new mathematical tool for dealing with uncertainties.

Fuzzy soft set, which is an extension of classical soft set, was introduced by Maji [13]. A lot of works about fuzzy soft set theory have been studied in operators and decision making. Ahmad et al. [1] defined arbitrary fuzzy soft union and fuzzy soft intersection, and proved Demorgan laws in fuzzy soft set theory. Rehamn et al. [18] studied some operations of fuzzy soft sets and gave fundamental properties of fuzzy soft sets. Roy and Maji [19] proposed a novel method of object recognition from an imprecise multi-observer data and a decision making application of fuzzy soft set. Feng et al. [8] presented an adjustable approach to fuzzy soft set based decision making and gave some illustrations. Neog et al. ([16], [3]) studied an application of fuzzy soft sets in decision making problems using fuzzy soft matrices. Kong [11] researched application of fuzzy soft set in decision making based on grey theory. N. Cagman et al. [4] studied fuzzy parameterized soft set theory, their related properties and its application in decision making. Yao [24] introduced the granularity of fuzzy soft set based on α-dominance class and α-covering approximation space. Guan et al. [9] introduced a new order relation on fuzzy soft sets, which is called soft information order. Cagman [5] redefined the fuzzy soft set and their operations, and gave an example which showed that the fuzzy soft aggregation operator was successfully applied to many problems containing uncertainties. Zhang [25] studied the parameter reduction of fuzzy soft sets based on fuzzy rough sets. Basu et al. [2] used a balanced solution of a fuzzy soft set based decision making problem in medical science. Deng et al. [7] introduced an object-parameter approach to predict unknown data in incomplete fuzzy soft sets. Celik et al. [6] researched applications of fuzzy soft sets in ring theory.

As we all known, similarity measure is a very important issue in fuzzy soft sets. In recent years, similarity measures between two fuzzy soft sets have been studied from different aspects and applied in various fields, such as decision making, pattern recognition, region extraction, coding theory, image processing and so on. For example, similarity measures [14] have been researched in fuzzy soft sets, which were based on distance, set theoretic approach and matching function. D.K.Sut [21] and Dr. P. Rajarajeswari [17] used the notion of similarity measure in [14] to make decision. Several similarity measures [12] based on four types quasi-metrics were introduced in fuzzy soft sets. Nor Hashimah Sulaiman [20] researched a set theoretic similarity measure for fuzzy soft sets and applied it in group decision making. However, the literature of [14] and [12] didn't systematically study the similarity measures of fuzzy soft sets from the point of distance, and the computational costs of similarity measures in [14] and [12] are higher than that in this paper.

We all know that there are various distance measures in mathematics. So in this paper, we will study some new similarity measures of fuzzy soft sets based on different distance measures systematically, and analyze some properties of them and compare these similarity measures in different contexts. Moreover, an example is given to illustrate the application of these similarity measures in decision making.

The rest of this paper is organized as follows. In section 2, we will review some notions of fuzzy soft set. In section 3, firstly, some new similarity measures of fuzzy soft sets are introduced based on the normalized Hamming distance, the normalized Euclidean distance, the generalized normalized distance, the Type-2 generalized normalized distance, the Type-2 normalized Euclidean distance, the Hausdorff distance and the Chebyshev distance. Secondly, some properties of these similarity measures are analyzed. Thirdly, applicable scope of these similarity measures is studied. In section 4, an example is given to illustrate the application of these different similarity measures of fuzzy soft sets in decision making. Finally, conclusions are stated in section 5.

2. Preliminaries

In this section, we will review some related definitions of fuzzy soft set. Given an initial universe $U = \{x_1, x_2, \cdots, x_n\}$ and a parameter set $E = \{e_1, e_2, \cdots, e_m\}$.

Definition 2.1 ([15]). A pair (F, E) is called a soft set over U, if F is a mapping of E into the set of all subsets of U.

In other words, a soft set is a parameterized family of subsets of U. Every set $F(e)$ $(e \in E)$ from this family may be considered as the set of e-elements of the soft set, or as the set of e-approximate elements of the soft set.

Maji firstly study on hybrid structures involving both fuzzy set and soft set. Fuzzy soft set which was regarded as a generalized model of soft set was introduced.

Definition 2.2 ([13]). Let U be the universe and A be the parameter set. $P(U)$ denotes the set of all fuzzy subsets of U, a pair (F, A) is called a fuzzy soft set over U, where $F : A \to P(U)$ is a mapping from A into $P(U)$.

In order to explain the concept of fuzzy soft set clearly, let us see the following example.

Example 2.3 ([14]). Suppose a fuzzy soft set (F, E) describes attractiveness of the shirts which the consumers are going to wear. $U=\{x_1, x_2, x_3, x_4, x_5\}$, where $\{x_i, i = 1, 2, 3, 4, 5\}$ represents the set of all shirts under consideration. Let $P(U)$ be the set of all fuzzy subsets of U. Also let $E = \{e_1, e_2, e_3, e_4\}$, where e_1 denotes colorful, e_2 denotes bright, e_3 denotes cheap, e_4 denotes warm. Let

$F(e_1) = \left\{\frac{x_1}{0.5}, \frac{x_2}{0.9}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0}\right\}$. $F(e_2) = \left\{\frac{x_1}{1.0}, \frac{x_2}{0.8}, \frac{x_3}{0.7}, \frac{x_4}{0}, \frac{x_5}{0}\right\}$.

$F(e_3) = \left\{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}, \frac{x_4}{0.6}, \frac{x_5}{0}\right\}$. $F(e_4) = \left\{\frac{x_1}{0}, \frac{x_2}{1.0}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0.3}\right\}$.

So, fuzzy soft set (F, E) is a family $\{F(e_i), i = 1, 2, 3, 4\}$.

According to the characters of the fuzzy soft set, we know that every fuzzy soft set can be represented in the form of a tabular. So we can represent the fuzzy soft set in example 2.3 by tabular as follows.

Table 1. fuzzy soft set (F, E)

(F, E)	e_1	e_2	e_3	e_4
x_1	0.5	1.0	0	0
x_2	0.9	0.8	0	1.0
x_3	0	0.7	0	0
x_4	0	0	0.6	0
x_5	0	0	0	0.3

Definition 2.4 ([1]). Let U be a universe and E be a set of parameters. Then the pair (U, E) denotes the set of all fuzzy soft sets on U with parameters from E and is called a fuzzy soft space.

Definition 2.5 ([13]). A soft set (F, A) over U is said to be null fuzzy soft set denoted by Φ, if $\forall e \in A$, $F(e) = \varnothing$.

Definition 2.6 ([13]). A soft set (F, A) over U is said to be absolute fuzzy soft set denoted by $\widetilde{A}$, if $\forall e \in A$, $F(e) = U$.

Definition 2.7 ([1]). For two fuzzy soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a fuzzy soft subset of (G, B), if (i)$A \subseteq B$, (ii)$\forall e \in A$, $F(e) \subseteq G(e)$, and is written as $(F, A)\widetilde{\subseteq}(G, B)$.

Definition 2.8 ([1]). Let (F, A) and (G, B) be two fuzzy soft sets, (F, A) and (G, B) are said to be fuzzy soft equal if and only if (F, A) is a fuzzy soft subset of (G, B) and (G, B) is a fuzzy soft subset of (F, A), it can be represented as $(F, A) = (G, B)$.

Definition 2.9 ([1]). Union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is a fuzzy soft set (H, C), where $C = A \cup B$ and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & if \quad e \in A - B; \\ G(e), & if \quad e \in B - A; \\ F(e) \cup G(e), & if \quad e \in A \cap B. \end{cases}$$

and it is written as $(F, A)\widetilde{\bigcup}(G, B) = (H, C)$.

Definition 2.10 ([1]). Let (F, A) and (G, B) be two fuzzy soft sets over a common universe U with $A \cap B \neq \varnothing$. Restricted intersection of two fuzzy soft sets (F, A) and (G, B) is a fuzzy soft set (H, C), where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. We write $(F, A)\widetilde{\bigcap}(G, B) = (H, C)$.

Definition 2.11 ([13]). The complement of a fuzzy soft set (F, A) is denoted by $(F, A)^C$ and is defined by $(F, A)^C = \left(F^C, A\right)$ where $F^C : A \to P(U)$ is a mapping given by $F^C(\alpha) = [F(\alpha)]^C$, $\forall \alpha \in A$.

Definition 2.12 ([23]). A and B be two fuzzy sets on $X = \{x_1, x_2, \cdots, x_n\}$, $A = \{\mu_A(x_i), i = 1, 2, \cdots n.\}$, $B = \{\mu_B(x_i), i = 1, 2, \cdots n.\}$, where $\mu_A(x_i)$ represents the membership degree of x_i in A, $\mu_B(x_i)$ represents the membership degree of x_i in B.

The Hamming distance is defined as
$d_h(A, B) = \sum\limits_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|$.

The normalized Hamming distance is defined as
$d_{nh}(A, B) = \frac{1}{n} \sum\limits_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|$.

The Euclidean distance is defined as
$d_e(A, B) = (\sum\limits_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|^2)^{\frac{1}{2}}$.

The normalized Euclidean distance is defined as
$d_{ne}(A, B) = \frac{1}{n}(\sum\limits_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|^2)^{\frac{1}{2}}$.

The Hausdorff distance is defined as
$d_h(A, B) = \max |\mu_A(x_i) - \mu_B(x_i)|$.

Definition 2.13 ([12]). (F, A) and (G, B) be two fuzzy soft sets, the Chebyshev quasi-distance of them is defined as

$$d_c((F, A), (G, B)) = \frac{\|A \Delta B\|}{2\,\|A \cup B\|} + \frac{1}{2} \min_{e_i \in A \cap B} \{\max_{x_j \in U} |F(e_i)(x_j) - G(e_i)(x_j)|\} \tag{2.1}$$

where $A\Delta B$ represents the symmetric difference between two sets A and B, $A\Delta B = (A \cup B)\backslash(A \cap B)$. $\|\bullet\|$ denotes the cardinality of a set.

3. New similarity measures based on distance measures

Distance and similarity measures have attracted a lot of attentions in the last few decades due to the fact that they can be applied to many areas such as pattern recognition, cluster analysis, approximate reasoning, image processing, medical diagnosis and decision making. A lot of distance and similarity measures have been developed for fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets and so on, but there is little research on fuzzy soft sets. Consequently, it is very necessary to develop some distance and similarity measures under fuzzy soft sets. We address this issue based on the axiomatic definitions of the distance and similarity measures.

In this section, we firstly give the axiomatic definitions of the distance and similarity measures as follows.

Definition 3.1. Let (F, A) and (G, B) be two fuzzy soft sets over (U, E). Then distance measure between (F, A) and (G, B) is defined as $d\left((F, A), (G, B)\right)$, which satisfies the following properties.

(D1) $0 \leq d\left((F, A), (G, B)\right) \leq 1$;

(D2) $d\left((F, A), (G, B)\right) = 0$, if $(F, A) = (G, B)$;

(D3) $d\left((F, A), (G, B)\right) = d\left((G, B), (F, A)\right)$;

(D4) Let (H, C) be a fuzzy soft set, if $(F, A) \tilde{\subseteq} (G, B) \tilde{\subseteq} (H, C)$.
then $d((F, A), (G, B)) \leq d((F, A), (H, C))$ and $d((G, B), (H, C)) \leq d((F, A), (H, C))$.

Definition 3.2 ([12]). Let (F, A) and (G, B) be two fuzzy soft sets over (U, E). Then similarity measure between (F, A) and (G, B) is defined as $s\left((F, A), (G, B)\right)$, which satisfies the following properties.

(S1) $0 \leq s\left((F, A), (G, B)\right) \leq 1$;

(S2) $s\left((F, A), (G, B)\right) = 1$, if $(F, A) = (G, B)$;

(S3) $s\left((F, A), (G, B)\right) = s\left((G, B), (F, A)\right)$;

(S4) Let(H, C) be a fuzzy soft set, if $(F, A) \tilde{\subseteq} (G, B) \tilde{\subseteq} (H, C)$,
then $s((F, A), (H, C)) \leq s((F, A), (G, B))$ and $s((F, A), (H, C)) \leq s((G, B), (H, C))$.

3.1 Distance and new similarity measures of fuzzy soft sets

In this paper, we assume that the fuzzy soft sets (F, A) and (G, B) have the same parameter set, namely, $A = B$.

Drawing on the well-known Hamming distance and Euclidean distance, we define a normalized Hamming distance and a normalized Euclidean distance in fuzzy soft sets as follows.

The normalized Hamming distance in fuzzy soft sets is defined as

$$d_1\left((F, A), (G, B)\right) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right) - G\left(e_i\right)\left(x_j\right)\right| \tag{3.1}$$

The normalized Euclidean distance in fuzzy soft sets is defined as

$$d_2\left((F, A), (G, B)\right) = \frac{1}{mn}\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right) - G\left(e_i\right)\left(x_j\right)\right|^2\right)^{\frac{1}{2}} \tag{3.2}$$

Example 3.3. Let $U = \{x_1, x_2, x_3\}$ be a set of three houses under consideration of a decision maker to purchase. $E = \{e_1, e_2, e_3\}$ be a parameter set, where e_1, e_2 and e_3 represent cheap, big and beautiful, respectively. The fuzzy soft set (F, E) describes attractiveness of the houses to the first decision maker, and the fuzzy soft set (G, E) describes attractiveness of the houses to the second decision maker. Let (F, E) and (G, E) are represented by two tables as follows.

TABLE 2. fuzzy soft set (F, E)

(F, E)	e_1	e_2	e_3
x_1	0.6	0.9	0.9
x_2	0.5	0.8	0.7
x_3	0.8	0.7	0.6

TABLE 3. fuzzy soft set (G, E)

(G, E)	e_1	e_2	e_3
x_1	0.4	0.7	1.0
x_2	0.6	0.7	0.5
x_3	0.5	0.7	0.8

We use Eq.(3.1) and Eq.(3.2) to calculate the distance between (F, E) and (G, E).

$$d_1((F, E), (G, E))$$
$$= \tfrac{1}{3\times 3} \sum_{i=1}^{3} \sum_{j=1}^{3} (0.2 + 0.1 + 0.3 + 0.2 + 0.1 + 0 + 0.1 + 0.2 + 0.2)$$
$$\approx 0.156$$
$$d_2((F, E), (G, E))$$
$$= \tfrac{1}{3\times 3} (\sum_{i=1}^{3} \sum_{j=1}^{3} (0.2^2 + 0.1^2 + 0.3^2 + 0.2^2 + 0.1^2 + 0^2 + 0.1^2 + 0.2^2 + 0.2^2))^{\frac{1}{2}}$$
$$\approx 0.069$$

We will further extend Eq.(3.1) and Eq.(3.2) into a generalized normalized distance in fuzzy soft sets as follows

$$d_3\left((F, A), (G, B)\right) = \frac{1}{mn} (\sum_{i=1}^{m} \sum_{j=1}^{n} \left|F\left(e_i\right)\left(x_j\right) - G\left(e_i\right)\left(x_j\right)\right|^p)^{\frac{1}{p}}, (p \in N_+) \tag{3.3}$$

Clearly, if $p = 1$, then Eq.(3.3) is reduced to Eq.(3.1). If $p = 2$, Eq.(3.3) is reduced to Eq.(3.2).

From Eq.(3.1), we know that

$$d' = \frac{1}{n} \sum_{j=1}^{n} \left|F\left(e_i\right)\left(x_j\right) - G\left(e_i\right)\left(x_j\right)\right|$$

indicates the distance between the $i - th$ parameter of (F, A) and (G, B), and $d_1\left((F, A), (G, B)\right)$ indicates the distance among all parameters of (F, A) and (G, B).

From this perspective, we define another generalized normalized distance of (F, A) and (G, B) as follows

$$(3.4) \qquad d_4\left(\left(F,A\right),\left(G,B\right)\right)=\frac{1}{m}\sum_{i=1}^{m}[\frac{1}{n}(\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right)-G\left(e_i\right)\left(x_j\right)\right|^p)^{\frac{1}{p}}], p\in N_+$$

Eq.(3.4) is called a Type-2 generalized normalized distance in fuzzy soft sets.

It is clear that Eq.(3.4) is different from Eq.(3.3). But if $p = 1$, Eq.(3.4) is also reduced to Eq.(3.1). If $p = 2$, Eq.(3.4) becomes Type-2 normalized Euclidean distance Eq.(3.5).

$$(3.5) \qquad d_5\left(\left(F,A\right),\left(G,B\right)\right)=\frac{1}{m}\sum_{i=1}^{m}[\frac{1}{n}(\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right)-G\left(e_i\right)\left(x_j\right)\right|^2)^{\frac{1}{2}}]$$

Example 3.4. Consider the fuzzy soft sets given in example 3.3, we use Eq.(3.5) to calculate the distance between the fuzzy soft sets (F, E) and (G, E).

$$\begin{aligned} & d_5((F,E),(G,E)) \\ &= \tfrac{1}{3}\sum_{i=1}^{3}[\tfrac{1}{3}(\sum_{j=1}^{3}|F(e_i)(x_j)-G(e_i)(x_j)|^2)] \\ &= \tfrac{1}{9}(\sqrt{0.02+.001+0.09}+\sqrt{0.04+0.01+0}+\sqrt{0.01+0.04+0.04}) \\ &\approx 0.097 \end{aligned}$$

Theorem 3.5. *$d_4\left(\left(F,A\right),\left(G,B\right)\right)$ is a normalized distance measure between fuzzy soft sets (F, A) and (G, B).*

Proof. It is easy to see that $d_4\left(\left(F,A\right),\left(G,B\right)\right)$ satisfies the properties (D1)-(D3). Therefore we only prove it satisfies the property (D4).

Let$(F,A)\,\tilde{\subseteq}\,(G,B)\,\tilde{\subseteq}\,(H,C)$, then for $\forall e_i \in E$, $\forall x_j \in U$,

$$0\le F\left(e_i\right)\left(x_j\right)\le G\left(e_i\right)\left(x_j\right)\le H\left(e_i\right)\left(x_j\right).$$

It follows that

$$\left|F\left(e_i\right)\left(x_j\right)-G\left(e_i\right)\left(x_j\right)\right|\le\left|F\left(e_i\right)\left(x_j\right)-H\left(e_i\right)\left(x_j\right)\right|,$$

$$\left|G\left(e_i\right)\left(x_j\right)-H\left(e_i\right)\left(x_j\right)\right|\le\left|F\left(e_i\right)\left(x_j\right)-H\left(e_i\right)\left(x_j\right)\right|,$$

thus

$$\frac{1}{n}(\sum_{j=1}^{n}|F(e_i)(x_j)-G(e_i)(xj)|^p)^{\frac{1}{p}}\le\frac{1}{n}(\sum_{j=1}^{n}|F(e_i)(x_j)-H(e_i)(xj)|^p)^{\frac{1}{p}}, p\in N_+.$$

$$\frac{1}{n}(\sum_{j=1}^{n}|G(e_i)(x_j)-H(e_i)(xj)|^p)^{\frac{1}{p}}\le\frac{1}{n}(\sum_{j=1}^{n}|F(e_i)(x_j)-H(e_i)(xj)|^p)^{\frac{1}{p}}, p\in N_+$$

So we have

$$d_4\left(\left(F,A\right),\left(G,B\right)\right) \le d_4\left(\left(F,A\right),\left(H,C\right)\right),$$
$$d_4\left(\left(G,B\right),\left(H,C\right)\right) \le d_4\left(\left(F,A\right),\left(H,C\right)\right).$$

Thus the property (D4) is obtained. □

Except the normalized Hamming distance, the normalized Euclidean distance, the generalized normalized distance, the Type-2 generalized normalized distance and the Type-2 normalized Euclidean distance, we will next take the Hausdorff distance and Chebyshev distance into consideration.

If we apply the Hausdorff metric to the distance measure, then a generalized normalized Hausdorff distance of fuzzy soft sets (F, A) and (G, B) is given as

$$d_6 = \frac{1}{m}(\sum_{i=1}^{m} \max_{e_i \in A\cap B, x\in U}\{|F(e_i)(x) - G(e_i)(x)|^p\})^{\frac{1}{p}}, p \in N_+ \tag{3.6}$$

Now we will discuss a special case of the generalized normalized Hausdorff distance of fuzzy soft sets.

If $p = 1$, then Eq.(3.6) becomes a normalized Hamming-Hausdorff distance as

$$d_7((F,A),(G,B)) = \frac{1}{m}\sum_{i=1}^{m} \max_{e_i \in A\cap B, x\in U}\{|F(e_i)(x) - G(e_i)(x)|\}, m = \|A\cap B\| \tag{3.7}$$

Theorem 3.6. *$d_7\left(\left(F,A\right),\left(G,B\right)\right)$ is a normalized distance measure between fuzzy soft sets (F, A) and (G, B).*

Proof. It is easy to see that $d_7\left(\left(F,A\right),\left(G,B\right)\right)$ satisfies the properties (D1)-(D3). Therefore we only prove it satisfies the property (D4).

Since $(F, A) \,\tilde{\subseteq}\, (G, B) \,\tilde{\subseteq}\, (H, C)$, then for $\forall e_i \in E$, $\forall x \in U$,

$$0 \le F\left(e_i\right)\left(x\right) \le G\left(e_i\right)\left(x\right) \le H\left(e_i\right)\left(x\right).$$

It follows that

$$\left|F\left(e_i\right)\left(x\right) - G\left(e_i\right)\left(x\right)\right| \le \left|F\left(e_i\right)\left(x\right) - H\left(e_i\right)\left(x\right)\right|,$$

$$\left|G\left(e_i\right)\left(x\right) - H\left(e_i\right)\left(x\right)\right| \le \left|F\left(e_i\right)\left(x\right) - H\left(e_i\right)\left(x\right)\right|,$$

thus

$$\max_{e_i \in A\cap B, x\in U}\{|F(e_i)(x) - G(e_i)(x)|\} \le \max_{e_i \in A\cap B, x\in U}\{|F(e_i)(x) - H(e_i)(x)|\},$$

$$\max_{e_i \in A\cap B, x\in U}\{|G(e_i)(x) - H(e_i)(x)|\} \le \max_{e_i \in A\cap B, x\in U}\{|F(e_i)(x) - H(e_i)(x)|\}.$$

So we have

$$d_7\left((F,A),(G,B)\right) \le d_7\left((F,A),(H,C)\right),$$

$$d_7\left((G,B),(H,C)\right) \le d_7\left((F,A),(H,C)\right).$$

According to the definition 3.1, (D4) for Eq.(3.7) is obtained. □

Example 3.7. Consider the fuzzy soft sets given in example 3.3, we use Eq.(3.7) to calculate the distance between the fuzzy soft sets (F,E) and (G,E).

$$\begin{aligned} & d_7((F,E),(G,E)) \\ &= \frac{1}{3}\sum_{i=1}^{3} \max_{e_i\in E, x\in U}\{|F(e_i)(x) - G(e_i)(x)|\} \\ &= \frac{1}{3}(\max\{0.2,0.1,0.3\} + \max\{0.2,0.1,0\} + \max\{0.1,0.2,0.2\}) \\ &\approx 0.233 \end{aligned}$$

Next, we will take the distance of fuzzy soft sets (F,A) and (G,B) based on Chebyshev distance into consideration.

$$d_8((F,A),(G,B)) = \min_{e_i\in A\cap B}\{\max_{x\in U}|F(e_i)(x) - G(e_i)(x)| \tag{3.8}$$

Theorem 3.8. *$d_8\left((F,A),(G,B)\right)$ is a normalized distance measure between fuzzy soft sets (F,A) and (G,B).*

Proof. The proof is similar to that of Theorem 3.6. □

Example 3.9. Consider the fuzzy soft sets given in example 3.3, we use Eq.(3.8) to calculate the distance between the fuzzy soft sets (F,E) and (G,E).

$$\begin{aligned} & d_8((F,E),(G,E)) \\ &= \min_{e_i\in E}\{\max_{x\in U}|F(e_i)(x) - G(e_i)(x)|\} \\ &= \min_{e_i\in E}(\max\{0.2,0.1,0.3\}, \max\{0.2,0.1,0\}, \max\{0.1,0.2,0.2\}) \\ &= 0.2 \end{aligned}$$

It is well known that the similarity measure and distance measure are dual concept. The larger the distance is, the smaller the similarity measure is. Hence we may use distance measures to define similarity measures.

According to [26], let f be a monotone decreasing function and $d_{\max}$ be the maximal distance. Because

$$0 \le d\left((F,A),(G,B)\right) \le d_{\max},$$

$$f\left(d_{\max}\right) \le f(d\left((F,A),(G,B)\right)) \le f(0),$$

this implies

$$0 \le \frac{f\left(d\left((F,A),(G,B)\right)\right) - f\left(d_{\max}\right)}{f\left(0\right) - f\left(d_{\max}\right)} \le 1,$$

therefore we will define the similarity measure between fuzzy soft sets (F,A) and (G,B) as follows

$$s\left((F,A),(G,B)\right) = \frac{f\left(d\left((F,A),(G,B)\right)\right) - f\left(d_{\max}\right)}{f\left(0\right) - f\left(d_{\max}\right)} \tag{3.9}$$

If we choose a monotone decreasing function $f(x) = 1-x$, then the corresponding similarity measures between (F, A) and (G, B) will be obtained.

The similarity measures based on the normalized Hamming distance and the normalized Euclidean distance of soft sets were proposed in [10]. In this paper, these similarity measures of soft sets were extended into fuzzy soft sets.

Definition 3.10. Suppose (F, A) and (G, B) be two fuzzy soft sets over U, the similarity measure based on the normalized Hamming distance between them is defined as

$$s_1\left((F, A), (G, B)\right) = 1 - \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right) - G\left(e_i\right)\left(x_j\right)\right| \tag{3.10}$$

Definition 3.11. Suppose (F, A) and (G, B) be two fuzzy soft sets over U, the similarity measure based on the normalized Euclidean distance between them is defined as

$$s_2\left((F, A), (G, B)\right) = 1 - \frac{1}{mn}(\sum_{i=1}^{m}\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right) - G\left(e_i\right)\left(x_j\right)\right|^2)^{\frac{1}{2}} \tag{3.11}$$

According to the generalized normalized distance, the Type-2 generalized normalized distance, the Type-2 normalized Euclidean distance, the Hamming-Hausdorff distance and the Chebyshev distance are proposed. Based on these distance measures, we will define five similarity measures of fuzzy soft sets as follows.

Definition 3.12. Suppose (F, A) and (G, B) be two fuzzy soft sets over U, the similarity measure based on the generalized normalized distance between them is defined as

$$s_3\left((F, A), (G, B)\right) = 1 - \frac{1}{mn}(\sum_{i=1}^{m}\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right) - G\left(e_i\right)\left(x_j\right)\right|^p)^{\frac{1}{p}}, p \in N_+ \tag{3.12}$$

Definition 3.13. Suppose (F, A) and (G, B) be two fuzzy soft sets over U, the similarity measure based on the Type-2 generalized normalized distance between them is defined as

$$s_4\left((F, A), (G, B)\right) = 1 - \frac{1}{m}\sum_{i=1}^{m}[\frac{1}{n}(\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right) - G\left(e_i\right)\left(x_j\right)\right|^p)^{\frac{1}{p}}], p \in N_+ \tag{3.13}$$

Definition 3.14. Suppose (F, A) and (G, B) be two fuzzy soft sets over U, the similarity measure based on the Type-2 normalized Euclidean distance between them is defined as

$$s_5\left((F, A), (G, B)\right) = 1 - \frac{1}{m}\sum_{i=1}^{m}[\frac{1}{n}(\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right) - G\left(e_i\right)\left(x_j\right)\right|^2)^{\frac{1}{2}}] \tag{3.14}$$

Definition 3.15. Suppose (F, A) and (G, B) be two fuzzy soft sets over U, the similarity measure based on the Hamming-Hausdorff distance between them is defined as

$$s_6((F, A), (G, B)) = 1 - \frac{1}{m}\sum_{i=1}^{m}\max_{e_i \in A\cap B, x \in U}\{|F(e_i)(x) - G(e_i)(x)|\} \tag{3.15}$$

Definition 3.16. Suppose (F,A) and (G,B) be two fuzzy soft sets over U, the similarity measure based on the Chebyshev distance between them is defined as

$$s_7((F,A),(G,B)) = 1 - \min_{e_i \in A\cap B}\{\max_{x\in U}|F(e_i)(x) - G(e_i)(x)|\} \tag{3.16}$$

3.2 Properties of distance and new similarity measures of fuzzy soft sets

We have proposed several distance and similarity measures of fuzzy soft sets, in this subsection, we will research their properties.

Proposition 3.17. *If (F,E) is a soft set, then $d_i\left((F,E),\left(F^C,E\right)\right) = 1,(i = 1,7,8)$.*

Proposition 3.18. *$d_i\left((F,A),\left(F^C,A\right)\right) = 0,(i = 1,2,3,4,5,6,7,8)$, if (F,A) is a 0.5 fuzzy soft set.*

Remark 3.19. Let (F,A) is a fuzzy soft set. If $\forall e_i \in A$, $\forall x_j \in U$, $F(e_i)(x_j) = 0.5$, (F,A) is called a 0.5 fuzzy soft set.

Proposition 3.20. *(F,A) and (G,B) are two fuzzy soft sets, according to Eq.(3.4) and Eq.(3.5), the larger value of p is, the smaller distance of (F,A) and (G,B) is.*

Proposition 3.21. *If (F,E) is a soft set, then $s_i\left((F,E),\left(F^C,E\right)\right) = 0,(i = 1,6,7)$.*

Proposition 3.22. *(F,A) and (G,B) are two fuzzy soft sets, according to Eq.(3.12) and Eq.(3.13), the larger value of p is, the larger similarity of (F,A) and (G,B) is.*

Proposition 3.23. *$s_i\left((F,A),\left(F^C,A\right)\right) = 1,(i = 1,2,3,4,5,6,7)$, if (F,A) is a 0.5 fuzzy soft set.*

Theorem 3.24. *$s_2 \geq s_5 \geq s_1 \geq s_6$.*

Proof. We only need prove $s_1 \geq s_6$, $s_5 \geq s_1$ and $s_2 \geq s_5$.

Firstly, we prove $s_1 \geq s_6$.

$$\frac{1}{n}\sum_{j=1}^{n}|F(e_i)(x_j) - G(e_i)(x_j)| \leq \max_{e_i\in A\cap B, x\in U}\{|F(e_i)(x) - G(e_i)(x)|\}$$

if and only if

$$\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}|F(e_i)(x_j) - G(e_i)(x_j)| \leq \frac{1}{m}\sum_{i=1}^{m}\max_{e_i\in A\cap B, x\in U}\{|F(e_i)(x) - G(e_i)(x)|\}$$

thus

$$1-\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}|F(e_i)(x_j) - G(e_i)(x_j)| \geq 1-\frac{1}{m}\sum_{i=1}^{m}\max_{e_i\in A\cap B, x\in U}\{|F(e_i)(x) - G(e_i)(x)|\}$$

So we have $s_1 \geq s_6$.

$s_5 \geq s_1$ and $s_2 \geq s_5$ can be proved similarly. □

Remark 3.25. We don't consider the order of s_3, s_4 and s_7. Because according to Eq.(3.12) and Eq.(3.13), the values of s_3 and s_4 depend on the value of p, according to Eq.(3.16), the order of s_7 depends on the value of every object about every parameter in fuzzy soft sets (i.e. because the value of every object about every parameter is different, the order of s_7 is variable).

3.3 Comparison of these new similarity measures

In this paper, seven similarity measures of fuzzy soft sets based on seven distance measures are proposed, but these similarity measures of fuzzy soft sets are not always fit to every example. Then what are these similarity measures of fuzzy soft sets suitable for? We will next analyze these similarity measures of fuzzy soft sets through some examples.

It is hard to say which similarity measure is the best, but we obtain the following results.

(1) s_1, s_2, s_5, s_6 are suitable for general fuzzy soft sets that they have the same parameter set. According to theorem 3.24, the value of the similarity measure s_2 is the largest and the value of the similarity measure s_6 is the smallest for the same example.

(2) s_2 and s_5 are not suitable for computing the similarity measure between classical soft set and its complement. Because the similarity measure between classical soft set and its complement is 0, which represents that they are completely dissimilar. If we use s_2 or s_5 to calculate the similarity measure between classical soft set and its complement,the value of it is not 0. But if we use s_1, s_6 or s_7 to calculate the similarity measure between classical soft set and its complement, the value of it is 0.

(3) s_6 is not suitable for computing the similarity measure of fuzzy soft sets (F, A) and (G, B), if (F, A) and (G, B) satisfy the condition as follows, for $\forall e \in A \bigcap B$, $\max\limits_{e \in A \cap B, x \in U} \{|F(e_i)(x) - G(e_i)(x)|\} = 1$.

Because it is unreasonable that if the maximal difference value between two fuzzy soft sets about every parameter is 1, then two fuzzy soft sets are completely dissimilar.

Next, we use an example to illustrate this result.

Example 3.26. (F, E) and (G, E) are two fuzzy soft sets, we will calculate similarity measure between (F, E) and (G, E) by s_6.

Table 4. fuzzy soft set (F, E)

(F, E)	e_1	e_2	e_3
h_1	0.5	0	1
h_2	0.5	0	1
h_3	0	1	1

Table 5. fuzzy soft set (G, E)

(G,E)	e_1	e_2	e_3
h_1	0.5	0	1
h_2	0.5	0	1
h_3	1	0	0

$$\begin{aligned} s_6 &= 1 - \tfrac{1}{m}\sum_{i=1}^{m} \max_{e_i \in E, x \in U}\{|F(e_i)(x) - G(e_i)(x)|\} \\ &= 1 - \tfrac{1}{3}(\max\{0,0,1\} + \max\{0,0,1\} + \max\{0,0,1\}) \\ &= 1 - \tfrac{1}{3}(1+1+1) \\ &= 0 \end{aligned}$$

We find that the values of h_1 and h_2 in (F, E) are the same as the values of h_1 and h_2 in (G, E), but the similarity measure of them is 0, this is not reasonable.

(4) s_7 is not suitable for computing the similarity measure of fuzzy soft sets (F, A) and (G, B), if (F, A) and (G, B) satisfy the condition as follows, for $\exists e \in A \bigcap B$, $\forall x \in U$, such that $F(e)(x) = G(e)(x)$.

Because it is unreasonable that if the values of two fuzzy soft sets about a certain parameter are equal, then two fuzzy soft sets are completely similar.

Next, we give an example to illustrate this result.

Example 3.27 ([22]). Suppose that one who suffers from certain pneumonia has seven symptoms as follows: fever, cough, laryngological, body pain, headache, chill and fatigue. Let $U = \{yes, no\}$ be the Universe. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ be the parameter set of symptoms, where e_1 = fever, e_2 = cough, e_3 = laryngological, e_4 = body pain, e_5 = headache, e_6 = chill, e_7 = fatigue. The fuzzy soft set for ill person suffering from the pneumonia is given in table 6. The fuzzy soft set for another diagnosed person is given in table 7.

Table 6. Model fuzzy soft set for ill person

(F,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7
yes	1	1	1	1	1	1	1
no	0	0	0	0	0	0	0

Table 7. fuzzy soft set for another person

(G,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7
yes	1	0	0	0	0	0	0
no	0	1	1	1	1	1	1

We use s_7 to calculate the similarity measure of two patients.

$$\begin{aligned} s_7 &= 1 - \min_{e_i \in E}\{\max_{x \in U} |F(e_i)(x) - G(e_i)(x)|\} \\ &= 1 - \min\{0, 1, 1, 1, 1, 1, 1\} \\ &= 1 - 0 \\ &= 1 \end{aligned}$$

Similarity of (F, E) and (G, E) is 1, it means that the second patient also suffered from pneumonia, we know that one who suffers from certain pneumonia has seven symptoms: fever, cough, laryngological, body pain, headache, chill and fatigue, but the second patient has only one symptom, so he can't suffer from pneumonia. Consequently, using s_7 to calculate the similarity measure of two patients is unreasonable.

(5) If $A \neq B$, s_1, s_2, s_3, s_4, s_5 is not suitable for computing the similarity measure of (F, A) and (G, B). Because $A \neq B$, we don't determine the number of parameter set so that we can't use s_1, s_2, s_3, s_4, s_5 to calculate the similarity measure of fuzzy soft sets (F, A) and (G, B). But s_6 or s_7 can be used to compute the similarity measure of (F, A) and (G, B).

(6) Suppose (F, A) and (G, B) be two fuzzy soft sets, if $A \subseteq B$, to compute the similarity measure of them, we can extend A until both of them have the same parameter set. The values of the added parameters are all zero in A.

For example, $A = \{e_1, e_2, e_3\}$, $B = \{e_1, e_2, e_3, e_4, e_5\}$, if we want to calculate the similarity measure of (F, A) and (G, B), we can add e_4, e_5 into A such that they have the same parameter set, and for fuzzy soft set (F, A), $\forall x \in U$, $F(e_4)(x) = 0$, $F(e_5)(x) = 0$. But the approaches in [14] can only compute similarity measures of fuzzy soft sets with the same parameter set.

(7) The similarity measure based on Hamming quasi-metric between two fuzzy soft sets was studied in [12]. Next we will compare it with the similarity measure in this paper.

The similarity measure based on Hamming quasi-metric [12] is defined as follows.

(3.17)
$$S_H((F, A), (G, B)) = 1 - \frac{\|A \Delta B\|}{2\,\|A \cup B\|} - \frac{1}{2\,\|U\|} \min_{e \in A \cap B}\{\sum_{u \in U} |F(e)(u) - G(e)(u)|\}$$

where $A \Delta B$ represents the symmetric difference between two sets A and B, $A \Delta B = (A \cup B) \backslash (A \cap B)$. $\|\bullet\|$ denotes the cardinality of a set.

In order to illustrate the limitation of similarity measure in [12], an example is given as follows.

Example 3.28. Suppose (F, A) and (G, B) be two fuzzy soft sets, we use Eq.(3.17) and Eq.(3.10) to calculate similarity measure between two fuzzy soft sets (F, A) and (G, B).

$$\begin{aligned} & S_H((F, A), (G, B)) \\ &= 1 - \tfrac{\|A \Delta B\|}{2\|A \cup B\|} - \tfrac{1}{2\|U\|} \min_{e \in A \cap B}\{\sum_{u \in U} |F(e)(u) - G(e)(u)|\} \\ &= 1 - \tfrac{0}{2 \times 3} - \tfrac{1}{2 \times 3} \min\{0, 1, 1.4\} \\ &= 1 \end{aligned}$$

The result shows that (F, A) and (G, B) are complete similar (the similarity measure is 1 between two objects). However, (F, A) and (G, B) aren't complete similar. The reason is that $(F, A) \neq (G, B)$.

TABLE 8. fuzzy soft set (F, A)

(F, E)	e_1	e_2	e_3
x_1	0.7	0.8	1.0
x_2	0.9	0.3	0.7
x_3	0.5	0.6	0.2

TABLE 9. fuzzy soft set (G, B)

(G, E)	e_1	e_2	e_3
x_1	0.7	0.2	0.3
x_2	0.9	0.5	0.4
x_3	0.5	0.4	0.6

We use the similarity measure Eq.(3.10) to calculate similarity measure between two fuzzy soft sets (F, A) and (G, B) as follows.

$$s_1\left(\left(F, A\right), \left(G, B\right)\right)$$
$$= 1 - \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right) - G\left(e_i\right)\left(x_j\right)\right|$$
$$= 1 - \frac{1}{3\times3}(0 + 0 + 0 + 0.6 + 0.2 + 0.2 + 0.7 + 0.3 + 0.4)$$
$$\approx 0.733$$

The similarity measure is 0.733 between two fuzzy soft sets (F, A) and (G, B). Obviously, (F, A) and (G, B) aren't complete similar. It shows that the similarity measure based on Hamming distance in this paper is more reasonable than that in [12].

4. Application of similarity measures based on distances of fuzzy soft sets in decision making

We know that similarity measures are applied to many areas such as pattern recognition, cluster analysis, approximate reasoning, image processing, medical diagnosis and decision making. In this section, a numerical example [21] is given to illustrate the application of these proposed different similarity measures of fuzzy soft sets to make decision.

Suppose the authority of an institution wants to give award to the performing students in an academic year. We assume that after some screening rounds, three students are available for the award. Let our universe set contain only two elements "yes(y) and no(n)", i.e. $U = \{y, n\}$. Here the set of parameters E is the set of certain approximations determined by the authority. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$, where e_1 = sincerity, e_2 = extracurricular activity, e_3 = pleasing personality, e_4 = good moral character, e_5 = sports activity, e_6 = communication skills, e_7 = examination performance. Our model fuzzy soft set for the performing students is given in Table 10. Similarly, we construct the fuzzy soft sets for the three students under consideration as given in Table 11, 12 and 13.

TABLE 10. Model fuzzy soft set for performing student

(F,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7
y	1	1	0	1	0	0	1
n	0	0	1	0	1	1	0

TABLE 11. Fuzzy soft set for the first student under consideration

(F_1,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7
y	0.3	0	0.7	0.2	0.9	0.1	0
n	0.6	0.8	0.2	0.3	0.5	0.2	0.1

TABLE 12. Fuzzy soft set for the second student under consideration

(F_2,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7
y	0.1	0.4	0.5	0.2	0.3	0.1	0.7
n	0.3	0.4	0.2	0.1	0.9	0.7	0.1

TABLE 13. Fuzzy soft set for the third student under consideration

(F_3,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7
y	0.3	0	0.2	0.5	0.6	0.1	0
n	0.5	0.7	0.1	0.2	0.5	0.4	0.7

We will use similarity measures s_1, s_2, s_5, s_6, s_7 to evaluate these students.

$$s_1\left(\left(F,A\right),\left(G,B\right)\right)=1-\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right)-G\left(e_i\right)\left(x_j\right)\right| \tag{4.1}$$

$$s_2\left(\left(F,A\right),\left(G,B\right)\right)=1-\frac{1}{mn}(\sum_{i=1}^{m}\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right)-G\left(e_i\right)\left(x_j\right)\right|^2)^{\frac{1}{2}} \tag{4.2}$$

$$s_5\left(\left(F,A\right),\left(G,B\right)\right)=1-\frac{1}{m}\sum_{i=1}^{m}[\frac{1}{n}(\sum_{j=1}^{n}\left|F\left(e_i\right)\left(x_j\right)-G\left(e_i\right)\left(x_j\right)\right|^2)^{\frac{1}{2}}] \tag{4.3}$$

$$s_6((F,A),(G,B))=1-\frac{1}{m}\sum_{i=1}^{m}\max_{e_i\in A\cap B,x\in U}\{|F(e_i)(x)-G(e_i)(x)|\} \tag{4.4}$$

$$s_7((F,A),(G,B))=1-\min_{e_i\in A\cap B}\{\max_{x\in U}|F(e_i)(x)-G(e_i)(x)|\} \tag{4.5}$$

The evaluate results are showed by table 14.

Similarity measures that we introduced in this paper were used to give the order of these students. The result is invariable (the second student > the third student

Table 14. Evaluate Results

Similarity Eq.	first student	second student	third student
s_1	0.350	0.586	0.414
s_2	0.810	0.871	0.827
s_5	0.486	0.687	0.561
s_6	0.143	0.429	0.243
s_7	0.300	0.700	0.500

$>$ the first student) by using different similarity measures to calculate the similarity between the model student and the candidate student. So the authority of an institution should give award to the second student.

From this example, we find that all similarity measures proposed in this paper are reasonable. For the same example, the results of decision making which are based on different distance measures in this paper and based on set-theoretic in reference [21] are consistent, but the costs of computation of similarity measures in this paper is lower than that in [21].

5. Conclusions

In this paper, firstly, some new similarity measures which are based on different distance measures are introduced in fuzzy soft sets. Secondly, some properties of these similarity measures are analyzed. Thirdly, applicable scope of these similarity measures is studied. Moreover, a numerical example is given to illustrate the application of these different similarity measures of fuzzy soft sets in decision making.

In the future, we will use the similarity measures which are proposed in this paper in group decision making. And we will study the distance measures and similarity measures of the generalized models of fuzzy soft sets, such as vague soft sets, intuitionistic fuzzy soft sets, probability fuzzy soft sets and so on.

Acknowledgements. This work was partially supported by the Natural Science Foundation of Shanxi Province (No.2012011011-1) and the Natural Science Foundation of Shanxi Normal University (No.ZR1109).

References

[1] B. Ahmad and A. Kharal, On fuzzy soft sets, Adv. Fuzzy Syst. 2009, Art. ID 586507, 6 pp.

[2] T. M. Basu, N. K. Mahapatra and S. K. Mondal, A balanced solution of a fuzzy soft set based decision making problem in medical science, Appl. Soft Comput. 12(10) (2012) 3260–3275.

[3] M. J. Borah, T. J. Neog and D. K. Sut, Fuzzy soft matrix theory and its decision making, Int. J. Modern Engineering Research 2(2) (2012) 121–127.

[4] N. Cagman, F. Citak and S. Enginoglu, FP-soft set theory and its applications, Ann. Fuzzy Math. Inform. 2(2) (2011) 219–226.

[5] N. Cagman, S. Enginoglu and F. Citak, Fuzzy soft set theory and its application, Iran. J. Fuzzy Syst. 8(3) (2011) 137–147.

[6] Y. Celik, C. Ekiz and S. Yamak, Applications of fuzzy soft sets in ring theory, Ann. Fuzzy Math. Inform. 5(3) (2013) 451–462.

[7] T. Deng and X. Wang, An object-parameter approach to predicting unknown data in incomplete fuzzy soft sets, Appl. Math. Model. 37(6) (2013) 4139–4146.

[8] F. Feng, Y. B. Jun, X. Liu and L. Li, An adjustable approach to fuzzy soft sets based decision making, J. Comput. Appl. Math. 234(1) (2010) 10–20.
[9] X. Guan, Y. Li and F. Feng, A new order relation on fuzzy soft sets and its application, Soft Comput. 17(1) (2013) 63–70.
[10] A. Kharal, Distance and similarity measures for soft sets, New Math. Nat. Comput. 6(3) (2010) 321–334.
[11] Z. Kong, L. Wang and Z. Wu, Application of fuzzy soft set in decision making problems based on grey theory, J. Comput. Appl. Math. 236(6) (2011) 1521–1530.
[12] H. Li and Y. Shen, Similarity measures of fuzzy soft sets based on different distance, In: Proceedings of the Fifth International Symposium on Computational Intelligence and Design (2012) 527–529.
[13] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9(3) (2001) 589–602.
[14] P. Majumdar and S. K. Samanta, On similarity measures of fuzzy soft sets, Int. J. Advance Soft Comput. Appl. 3(2) (2011) 1–8.
[15] D. Molodtsov, Soft set theory-first result, Comput. Math. Appl. 37 (1999) 19–31.
[16] T. J. Neog and D. K. Sut, An application of fuzzy soft sets in decision making problems using fuzzy soft matrices, Int. J. Math. Arch. 2(11) (2011) 2258–2263.
[17] Dr. P. Rajarajeswari and P. Dhanalakshmi, An application of similarity measure of fuzzy soft set based on distance, IOSR Journal of Mathematics 4 (2012) 27–30.
[18] A. Rehman, S. Abdullah, M. Aslam and M. S. Kamran, A study on fuzzy soft set and its operations, Ann. Fuzzy Math. Inform. 6(2) (2013) 339–362.
[19] A. R. Roy and P. K. Maji, A fuzzy soft set theoretic approach to decision making problems, J. Comput. Appl. Math. 203(2) (2007) 412–418.
[20] N. H. Sulaiman and D. Mohamad, A set theoretic similarity measure for fuzzy soft sets and its application in group decision making, AIP Conf. Proc. 1522 (2013) 237–244.
[21] D. K. Sut, An application of similarity of fuzzy soft sets in decision making, Int. J. Computer Tech. Appl. 3(2) (2012) 742–745.
[22] Z. Xiao, X. Yang, Q. Niu, Y. Dong, K. Gong, S. Xia and Y. Pang, A new evaluation method based on D-S generalized fuzzy soft sets and its application in medical diagnosis problem, Appl. Math. Model. 36(10) (2012) 4592–4604.
[23] Z. Xu and M. Xia, Distance and similarity measures for hesitant fuzzy sets, Inform. Sci. 181(11) (2011) 2128–2138.
[24] D. Yao, C. Wang, J. Mao and Y. Zhang, Granularity analysis of fuzzy soft set, LNCS 7414 (2012) 409–415.
[25] Z. Zhang, The parameter reduction of fuzzy soft sets based on soft fuzzy rough sets, Adv. Fuzzy Syst. 2013, Art. ID 197435, 12 pp.
[26] X. Zhou and Q. Li, Some new similarity measures for hesitant fuzzy sets and their applications in multiple attribute decision making, (2012), http://arXiv.org/pdf/1211.4125.pdf.

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