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Intuitionistic fuzzy prime ideals of BCK-algebras

Saleem Abdullah

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Abstract. We consider the intuitionistic fuzzification of the concept of prime ideals in commutative BCK-algebras, and investigate some of their properties. We show that if P is a prime ideal of commutative BCK-algebra X iff $\widetilde{P} = \langle X_P, \bar{X}_P \rangle$ is an intuitionistic fuzzy prime ideal of X. We also prove that An IFS $A = \langle \mu_A, \lambda_A \rangle$ of commutative BCK-algebra X is an intuitionistic fuzzy prime ideal of X if and only if for all $s, t \in [0, 1]$, the set's $U(\mu_A, t)$ and $L(\lambda_A, s)$ are prime ideals of X.

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Corresponding Author: Saleem Abdullah (saleemabdullah81@yahoo.com)

1. Introduction

The concept of fuzzy set was defined by Zadeh [25]. Since then these ideas have been applied to other algebraic structures such as semigroup, group, ring, etc. The idea of "intuitionistic fuzzy set" was introduced by Atanassov [5, 6], as generalization of the notion of fuzzy set. In 1966, Imai and Iseki introduced two classes of abstract algebras, BCK-algebras and BCI-algebras [12, 13]. BCI-algebras are generalizations of BCK-algebras which were studied by many researchers [3, 10, 11, 14, 15, 20, 21, 2, 4, 1]. In 1995, Jun [17] applied the concept of fuzzy set to BCK-algebras. He got some interesting results. Yaqoob, Mostafa and Ansari [24] applied the theory of cubic sets to KU-ideals of KU-algebras and obtained some results on cubic KU-ideals. Jun et al. [18] studied further properties of fuzzy ideals and fuzzy sub-algebras of BCK-algebras. In 1990, Biswas introduced the concept of anti fuzzy subgroup of group [8], also see [9, 22, 23]. Recently, Hong and Jun, modifying Biswas idea, applied the concept to BCK-algebras. So, they defined the notion of anti fuzzy ideal of BCK algebras and obtain some useful results on it. In 1999, Jeong applied the Biswas concept to prime ideal in BCK-algebras. So, he defined the notion of anti fuzzy prime ideal of BCK-algebras and obtained some useful results [19]. In 2000, Jun and Kim, using the Atanassov's idea to BCK-algebras [17]. So, they established the intuitionistic fuzzification of the concept of subalgebras and ideals in BCK-algebras, and investigated some of their properties.

In this paper, we introduce the notion of intuitionistic fuzzy prime ideals of commutative BCK-algebras. We establish the intuitionistic fuzzification of the concept of prime ideals in BCK-algebras, and investigate some of their properties. We show that every intuitionistic fuzzy prime ideal of commutative BCK-algebra X is an intuitionistic fuzzy ideal of X. We also show that if P is a prime ideal of commutative BCK-algebra X iff $\widetilde{P} = \langle X_P, \bar{X}_P \rangle$ is an intuitionistic fuzzy prime ideal of X. We also prove that an IFS $A = \langle \mu_A, \lambda_A \rangle$ of commutative BCK-algebra X is an intuitionistic fuzzy prime ideal of X if and only if for all $s, t \in [0,1]$, the set's $U(\mu_A, t)$ and $L(\lambda_A, s)$ are prime ideals of X.

2. Preliminaries

Definition 2.1 ([13, 14]). Algebra $(X, *, 0)$ of type $(2, 0)$ is called BCK-algebra, if for all $x, y \in X$, the following axioms hold:

(1) $(x * y) * (x * z) \leq (z * y)$
(2) $(x * (x * y) \leq y$
(3) $x \leq x$
(4) $x \leq y, y \leq x \Longrightarrow x = y$
(5) $0 \leq x$
Where $x \leq y$ is defined by $x * y = 0$.

Definition 2.2 ([14]). A subset I of BCK-algebra $(X, *, 0)$ is called an ideal of X, if for any $x, y \in X$

(i) $0 \in I$
(ii) $x * y$ and $y \in I \Longrightarrow x \in I$

Definition 2.3 ([14]). An ideal I of BCK-algebra $(X, *, 0)$ is called closed ideal, if $0 * x \in I$, for all $x \in I$.

Definition 2.4 ([7]). An ideal of commutative BCK-algebras X is said to be prime if $x \wedge y \in I$ implies $x \in I$ or $y \in I$.

Definition 2.5. Let X be non empty set. A fuzzy subset μ of the set X is a mapping $\mu : X \longrightarrow [0,1]$. The complement of fuzzy set μ of a set X is denoted by $\mu(x) = 1 - \mu(x)$, for all $x \in X$.

Definition 2.6. A fuzzy ideal μ of commutative algebra X is called anti fuzzy prime ideal of X if

$$\mu(x \wedge y) \geq \min\{\mu(x), \mu(y)\}$$

for all $x, y \in X$.

Definition 2.7 ([5, 6]). An intuitionistic fuzzy set (in short, IFS) in a non-empty set X is an object having the form $A = \{\langle x, \mu_A(x), \lambda_A(x) \rangle / x \in X\}$, where the function $\mu_A : X \longrightarrow [0,1]$ and $\lambda_A : X \longrightarrow [0,1]$ denoted the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. For the simplicity, we use the symbol form $A = \langle \mu_A, \lambda_A \rangle$.

Definition 2.8 ([16]). Let X be a BCK-algebra. An intuitionistic fuzzy set $A = \langle \mu_A, \lambda_A \rangle$ of BCK-algebra X is called an intuitionistic fuzzy subalgebra of X if.

(i) $\mu_A(x * y) \geq \min\{ \mu_A(x), \mu_A(y)\}$
(ii) $\lambda_A(x * y) \leq \max\{ \lambda_A(x), \lambda_A(y)\}$
for all $x, y \in X$.

Definition 2.9 ([16]). An intuitionistic fuzzy set $A = \langle \mu_A, \lambda_A \rangle$ in X is called an intuitionistic fuzzy ideal of X, if it satisfies the following axioms:

(IF1) $\mu_A(0) \geq \mu_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$
(IF2) $\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\}$
(IF3) $\lambda_A(x) \leq \min\{\lambda_A(x * y), \lambda_A(y)\}$,
for all $x, y \in X$.

Definition 2.10 ([16]). An intuitionistic fuzzy ideal $A = \langle \mu_A, \lambda_A \rangle$ of a BCK-algebra X is called an intuitionistic fuzzy closed-ideal of X, if the following axiom satisfies

(IF4) $\mu_A(0 * x) \geq \mu_A(x)$ and $\lambda_A(0 * x) \leq \lambda_A(x)$
for all $x \in X$.

Theorem 2.11 ([16]). *Every intuitionistic fuzzy ideal of BCK-algebra X is an intuitionistic fuzzy subalgebra of BCK-algebra X.*

Theorem 2.12. *A non empty subset I of BCK-algebra X is an ideal of BCK-algebra X if and only if $\widetilde{P} = \langle X_P, \bar{X}_P \rangle$ is an intuitionistic fuzzy ideal.*

Proposition 2.13. *Every Prime ideal of commutative BCK-algebra X is an ideal of X.*

3. Major Section

Definition 3.1. An intuitionistic fuzzy ideal $A = \langle \mu_A, \lambda_A \rangle$ of a BCK-algebra X is called an intuitionistic fuzzy prime ideal of X if

(IFP1) $\mu_A(x \wedge y) \leq \max\{ \mu_A(x), \mu_A(y)\}$
(IFP2) $\lambda_A(x \wedge y) \geq \min\{ \lambda_A(x), \lambda_A(y)\}$
for all $x, y \in X$.

Example 3.2. Let $X = \{0, x, y, z\}$ with Cayley table as follows:

$*$	0	x	y	z
0	0	0	0	0
x	x	0	0	0
y	y	x	0	0
z	z	y	x	0

It is easy to verify that $(X, *, 0)$ is commutative BCK-algebra. Define an IFS $A = \langle \mu_A, \lambda_A \rangle$ as: $\mu_A(0) = 1, \mu_A(x) = 0.9, \mu_A(y) = 0.5, \mu_A(z) = 0$ and $\lambda_A(0) = 0, \lambda_A(x) = 0.1, \lambda_A(y) = 0.5, \lambda_A(z) = 1$. By routine calculations $A = \langle \mu_A, \lambda_A \rangle$ is an intuitionistic fuzzy prime ideal of X.

Theorem 3.3. *If $A = \langle \mu_A, \lambda_A \rangle$ is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X, then the sets $J = \{x \in X/\mu_A(x) = \mu_A(0)\}$ and $K = \{x \in X/\lambda_A(x) = \lambda_A(0)\}$ are prime ideals of X.*

Proof. Straightforward. □

Corollary 3.4. *If $A = \langle \mu_A, \lambda_A \rangle$ is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X, then the sets $P_1 = \{x \in X/\mu_A(x) = 0\}$ and $P_2 = \{x \in X/\lambda_A(x) = 0\}$ are either empty or prime ideals of X.*

Proposition 3.5. *If $A = \langle \mu_A, \lambda_A \rangle$ is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X, then the sets $I = \{x \in X/\mu_A(x) = 1\}$ and $I = \{x \in X/\lambda_A(x) = 1\}$ are either empty or prime ideals of X.*

Proposition 3.6. *Every intuitionistic fuzzy prime ideal of a commutative BCK-algebra X is an intuitionistic fuzzy ideal of commutative BCK-algebra X.*

Proof. Straightforward. □

Remark 3.7. An intuitionistic fuzzy ideal of a commutative BCK-algebra X need not be an intuitionistic fuzzy prime ideal of X. Shown in the following example.

Example 3.8. Let $X = \{0, x, y, z\}$ be a BCK-algebra with Cayley table as follows:

$*$	0	x	y	z
0	0	0	0	0
x	x	0	0	x
y	y	x	0	y
z	z	z	z	0

Then the BCK-algebra X is commutative. Define an IFS $A = \langle \mu_A, \lambda_A \rangle$, $\mu_A : X \longrightarrow [0, 1]$ by $\mu_A(0) = 1, \mu_A(x) = \mu_A(y) = 0.5, \mu_A(z) = 0$ and $\lambda_A(0) = 0, \lambda_A(x) = \lambda_A(y) = 0.5, \lambda_A(z) = 1$. Routine calculations give that $A = \langle \mu_A, \lambda_A \rangle$ is an intuitionistic fuzzy ideal of X but not intuitionistic fuzz prime ideal of X.

Proposition 3.9. *Let P be an ideal of a commutative BCK-algebra X. Then, P is a prime ideal of X iff $\widetilde{P} = \langle X_P, \bar{X}_P \rangle$ is an intuitionistic fuzzy prime ideal of X.*

Proof. Suppose that P is a prime ideal of X. Then, by Proposition 2.13, P is an ideal of X. Since by Theorem 2.12, $\widetilde{P} = \langle X_P, \bar{X}_P \rangle$ is an intuitionistic fuzzy ideal of X. Let for any $x, y \in X$. Then, we have two case's (i) $x \wedge y \in P$ and (ii) $x \wedge y \notin P$.

Case (i) if $x \wedge y \in P$, then $X_P(x \wedge y) = 1$, since P is a prime ideal of X, so either $x \in P$ or $y \in P$ this implies $X_P(x) = 1$ or $X_P(y) = 1$. Thus, $X_P(x \wedge y) = 1 = \max\{X_P(x), X_P(y)\}$ and $\bar{X}_P(x \wedge y) = 1 - X_P(x \wedge y) = 0$, and $\bar{X}_P(x) = 1 - X_P(x) = 0$ or $\bar{X}_P(y) = 1 - X_P(y) = 0$ this implies $\bar{X}_P(x \wedge y) = 0 = \min\{\bar{X}_P(x), \bar{X}_P(y)\}$.

Case (ii) if $x \wedge y \notin P$, then $X_P(x \wedge y) = 0$ and $X_P(x) \geq 0$ and $X_P(y) \geq 0$ this imply $X_P(x \wedge y) = 0 \leq \max\{X_P(x), X_P(y)\}$, and $\bar{X}_P(x \wedge y) = 1$ and $X_P(x) \geq 0$, $X_P(y) \geq 0$ this imply $\bar{X}_P(x) \leq 1$ and $\bar{X}_P(y) \leq 1$ this imply $\bar{X}_P(x \wedge y) = 1 \geq$

$\min\{\bar{X}_P(x), \bar{X}_P(y)\}$. Hence $\widetilde{P} = \langle X_P, \bar{X}_P\rangle$ be an intuitionistic fuzzy prime ideal of X.

Conversely, suppose that $x \wedge y \in P$ and $x \notin P$, for any $x, y \in X$. Then,

$$\begin{aligned} 0 &= \bar{X}_P(x \wedge y) = 1 - X_P(x \wedge y) \geq 1 - \max\{X_P(x), X_P(y)\} \\ &= \min\{1 - X_P(x), 1 - X_P(y)\} = \min\{\bar{X}_P(x), \bar{X}_P(y)\} = \bar{X}_P(y) \end{aligned}$$

this implies $0 \geq \bar{X}_P(y)$ this implies $\bar{X}_P(y) = 0$, so that $X_P(y) = 1$. Thus, $y \in P$. Similarly, if $x \wedge y \in P$ and $y \notin P$, for any $x, y \in X$. Hence, P is a prime ideal of X. This completes the proof. □

Proposition 3.10. *Let $A = \langle\mu_A, \lambda_A\rangle$ be an intuitionistic fuzzy set of a commutative BCK-algebra X. Then, $A = \langle\mu_A, \lambda_A\rangle$ is an intuitionistic fuzzy prime ideal of X iff μ_A and $\overset{c}{\lambda}_A$ are fuzzy prime ideals of X, where $\overset{c}{\lambda}_A = 1 - \lambda_A$.*

Proof. Let $A = \langle\mu_A, \lambda_A\rangle$ be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X. Since by Proposition 3.6, $A = \langle\mu_A, \lambda_A\rangle$ is an intuitionistic fuzzy ideal, so by [16, Page 843, Lemma 3.11], μ_A and $\overset{c}{\lambda}_A$ are fuzzy ideals of X. Let for any $x, y \in X$. Then,

$$\begin{aligned} \mu_A(x \wedge y) &\leq \max\{\mu_A(x), \mu_A(y)\} \\ \lambda_A(x \wedge y) &\geq \min\{\lambda_A(x), \lambda_A(y)\} \end{aligned}$$

Now

$$\begin{aligned} 1 - \lambda_A(x \wedge y) &\leq 1 - \min\{\lambda_A(x), \lambda_A(y)\} \\ \overset{c}{\lambda}_A(x \wedge y) &\leq \max\{1 - \lambda_A(x), 1 - \lambda_A(y)\} \\ \overset{c}{\lambda}_A(x \wedge y) &\leq \max\{\overset{c}{\lambda}_A(x), \overset{c}{\lambda}_A(y)\} \end{aligned}$$

Hence μ_A and $\overset{c}{\lambda}_A$ are fuzzy prime ideal of X.

The converse part is easy, we omit the proof. □

Proposition 3.11. *Let $A = \langle\mu_A, \lambda_A\rangle$ be an intuitionistic fuzzy ideal of a commutative BCK-algebra X. Then $A = \langle\mu_A, \lambda_A\rangle$ is an intuitionistic fuzzy prime ideal of X iff $\overset{c}{\mu}_A$ and λ_A are anti fuzzy prime ideals of X, where $\overset{c}{\mu}_A = 1 - \mu_A$.*

Proof. Proof is same as above Proposition. □

Proposition 3.12. *Let $A = \langle\mu_A, \lambda_A\rangle$ be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X. Then, $\Box A = \langle\mu_A, \overset{c}{\mu}_A\rangle$ is an intuitionistic fuzzy prime ideal of X, where $\overset{c}{\mu}_A = 1 - \mu_A$.*

Proof. Let $A = \langle\mu_A, \lambda_A\rangle$ be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X. Then since by Proposition 3.6, $A = \langle\mu_A, \lambda_A\rangle$ is an intuitionistic fuzzy ideal of X, so by [16, Page 843, Theorem 3.12], $\Box A = \langle\mu_A, \overset{c}{\mu}_A\rangle$ is an intuitionistic fuzzy ideal of X. For any $x, y \in X$, then

$$\mu_A(x \wedge y) \leq \max\{\mu_A(x), \mu_A(y)\}$$

Now

$$\begin{aligned} 1-\mu_A(x\wedge y) &\geq 1-\max\{\mu_A(x),\mu_A(y)\} \\ \overset{c}{\mu}_A(x\wedge y) &\geq \min\{1-\mu_A(x),1-\mu_A(y)\} \\ \overset{c}{\mu}_A(x\wedge y) &\geq \min\{\overset{c}{\mu}_A(x),\overset{c}{\mu}_A(y)\} \end{aligned}$$

Hence $\Box A=\langle\mu_A,\overset{c}{\mu}_A\rangle$ is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X. □

Proposition 3.13. *Let $A=\langle\mu_A,\lambda_A\rangle$ be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X. Then $\Diamond A=\langle\overset{c}{\lambda}_A,\lambda_A\rangle$ is an intuitionistic fuzzy prime ideal of X, where $\overset{c}{\lambda}_A=1-\lambda_A$.*

Proof. Let $A=\langle\mu_A,\lambda_A\rangle$ be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X. Then since by Proposition 3.6 $A=\langle\mu_A,\lambda_A\rangle$ is an intuitionistic fuzzy ideal of X, so by [16, Page 843, Theorem 3.12] $\Diamond A=\langle\overset{c}{\lambda}_A,\lambda_A\rangle$ is an intuitionistic fuzzy ideal of X. Let for any $x,y\in X$. Then,

$$\begin{aligned} \mu_A(x\wedge y) &\leq \max\{\mu_A(x),\mu_A(y)\} \\ \lambda_A(x\wedge y) &\geq \min\{\lambda_A(x),\lambda_A(y)\} \end{aligned}$$

Now

$$\begin{aligned} 1-\lambda_A(x\wedge y) &\leq 1-\min\{\lambda_A(x),\lambda_A(y)\} \\ \overset{c}{\lambda}_A(x\wedge y) &\leq \max\{1-\lambda_A(x),1-\lambda_A(y)\} \\ \overset{c}{\lambda}_A(x\wedge y) &\leq \max\{\overset{c}{\lambda}_A(x),\overset{c}{\lambda}_A(y)\} \end{aligned}$$

Hence, $\Diamond A=\langle\overset{c}{\lambda}_A,\lambda_A\rangle$ is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X. □

Theorem 3.14. *Let $A=\langle\mu_A,\lambda_A\rangle$ be an IFS in a commutative BCK-algebra X. Then, $A=\langle\mu_A,\lambda_A\rangle$ is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X if and only if $\Box A=\langle\mu_A,\overset{c}{\mu}_A\rangle$ and $\Diamond A=\langle\overset{c}{\lambda}_A,\lambda_A\rangle$ are an intuitionistic fuzzy prime ideals of X.*

Definition 3.15. Let $A=\langle\mu_A,\lambda_A\rangle$ be an intuitionistic fuzzy set of a BCK-algebra X. Then for $s,t\in[0,1]$, the set $U(\mu_A,t)=\{x\in X/\mu_A(x)\geq t\}$ is called upper t-level cut of μ_A and the set $L(\lambda_A,s)=\{x\in X/\lambda_A(x)\leq s\}$ is called lower s-level cut of λ_A.

Theorem 3.16. *An IFS $A=\langle\mu_A,\lambda_A\rangle$ is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X if and only if for all $s,t\in[0,1]$, the set's $U(\mu_A,t)$ and $L(\lambda_A,s)$ are prime ideals of X.*

Proof. Suppose $A=\langle\mu_A,\lambda_A\rangle$ is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X. Then since by Proposition 3.6 $A=\langle\mu_A,\lambda_A\rangle$ is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X. So by [16, Page 843, Theorem 3.13] $U(\mu_A,t)$ and $L(\lambda_A,t)$ are ideal of X. Let $x\wedge y\in U(\mu_A,t)$ this implies $\mu_A(x\wedge y)\geq t$ and $\mu_A(x\wedge y)\leq\max\{\mu_A(x),\mu_A(y)\}$ this implies $\max\{\mu_A(x),\mu_A(y)\}\geq\mu_A(x\wedge$

$y) \geq t$ implies that $\max\{\mu_A(x), \mu_A(y)\} \geq t$ this implies $\mu_A(x) \geq t$ or $\mu_A(y) \geq t$. so that $x \in U(\mu_A, t)$ and $y \in U(\mu_A, t)$. Hence $U(\mu_A, t)$ is a prime ideal of X. Similarly $L(\lambda_A, s)$ is a prime of X.

Conversely, let $U(\mu_A, t)$ and $L(\lambda_A, s)$ be prime ideals of X. Then by [7] $U(\mu_A, t)$ and $L(\lambda_A, s)$ are ideals of X. Since by [16, Page 843, Theorem 3.13] $A = \langle \mu_A, \lambda_A \rangle$ is an intuitionistic fuzzy ideal of X. On contrary $A = \langle \mu_A, \lambda_A \rangle$ is not an intuitionistic fuzzy prime ideal of X. Then there exist $x, y \in X$ such that

$$\begin{aligned} \mu_A(x \wedge y) &> \max\{\mu_A(x), \mu_A(y)\} \\ \text{Let } t &= \frac{1}{2}\{\mu_A(x \wedge y) + \max\{\mu_A(x), \mu_A(y)\}\} \\ \text{this implies } \mu_A(x \wedge y) &> t > \max\{\mu_A(x), \mu_A(y)\} \\ \text{this implies } x \wedge y &\in U(\mu_A, t) \text{ but } x \notin U(\mu_A, t) \text{ and } y \notin U(\mu_A, t) \end{aligned}$$

which is a contradiction. Hence, $A = \langle \mu_A, \lambda_A \rangle$ is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra X. □

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Saleem Abdullah (saleemabdullah81@yahoo.com)
Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan