Intuitionistic fuzzy prime ideals of BCK-algebras

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ABSTRACT. We consider the intuitionistic fuzzification of the concept of prime ideals in commutative BCK-algebras, and investigate some of their properties. We show that if \( P \) is a prime ideal of commutative BCK-algebra \( X \) iff \( \sim P = \langle X_P, X_P \rangle \) is an intuitionistic fuzzy prime ideal of \( X \). We also prove that An IFS \( A = \langle \mu_A, \lambda_A \rangle \) of commutative BCK-algebra \( X \) is an intuitionistic fuzzy prime ideal of \( X \) if and only if for all \( s, t \in [0, 1] \), the set’s \( U(\mu_A, t) \) and \( L(\lambda_A, s) \) are prime ideals of \( X \).

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1. Introduction

The concept of fuzzy set was defined by Zadeh [25]. Since then these ideas have been applied to other algebraic structures such as semigroup, group, ring, etc. The idea of “intuitionistic fuzzy set” was introduced by Atanassov [5, 6], as generalization of the notion of fuzzy set. In 1966, Imai and Iseki introduced two classes of abstract algebras, BCK-algebras and BCI-algebras [12, 13]. BCI-algebras are generalizations of BCK-algebras which were studied by many researchers [3, 10, 11, 14, 15, 20, 21, 22, 23]. In 1995, Jun [17] applied the concept of fuzzy set to BCK-algebras. He got some interesting results. Yaqoob, Mostafa and Ansari [24] applied the theory of cubic sets to KU-ideals of KU-algebras and obtained some results on cubic KU-ideals. Jun et al. [18] studied further properties of fuzzy ideals and fuzzy sub-algebras of BCK-algebras. In 1990, Biswas introduced the concept of anti fuzzy subgroup of group [8], also see [9, 22, 23]. Recently, Hong and Jun, modifying Biswas idea, applied the concept to BCK-algebras. So, they defined the notion of anti fuzzy ideal of BCK algebras and obtain some useful results on it. In 1999, Jeong applied the Biswas concept to prime ideal in BCK-algebras. So, he defined the notion of anti fuzzy
prime ideal of BCK-algebras and obtained some useful results [19]. In 2000, Jun
and Kim, using the Atanassov’s idea to BCK-algebras [17]. So, they established the
intuitionistic fuzzification of the concept of subalgebras and ideals in BCK-algebras,
and investigated some of their properties.

In this paper, we introduce the notion of intuitionistic fuzzy prime ideals of com-
mutable BCK-algebras. We establish the intuitionistic fuzzification of the concept
of prime ideals in BCK-algebras, and investigate some of their properties. We show
that every intuitionistic fuzzy prime ideal of commutative BCK-algebra \( X \) is an
intuitionistic fuzzy ideal of \( X \). We also show that if \( P \) is a prime ideal of commutative
BCK-algebra \( X \) iff \( \sim P = \langle X_P, X_P \rangle \) is an intuitionistic fuzzy prime ideal of \( X \).

We also prove that an IFS \( A = \langle \mu_A, \lambda_A \rangle \) of commutative BCK-algebra
\( X \) is an intuitionistic fuzzy prime ideal of \( X \) if and only if for all \( s, t \in [0, 1] \), the set’s \( U(\mu_A, t) \) and \( L(\lambda_A, s) \) are prime ideals of \( X \).

2. Preliminaries

Definition 2.1 ([13, 14]). Algebra \((X, *, 0)\) of type \((2, 0)\) is called BCK-algebra, if
for all \( x, y \in X \), the following axioms hold:

1. \((x * y) * (x * z) \leq (z * y)\)
2. \((x * (x * y)) \leq y\)
3. \(x \leq x\)
4. \(x \leq y, y \leq x \implies x = y\)
5. \(0 \leq x\)

Where \(x \leq y\) is defined by \(x * y = 0\).

Definition 2.2 ([14]). A subset \( I \) of BCK-algebra \((X, *, 0)\) is called an ideal of \( X \),
if for any \( x, y \in X \)

(i) \(0 \in I\)
(ii) \(x * y \) and \( y \in I \implies x \in I\)

Definition 2.3 ([14]). An ideal \( I \) of BCK-algebra \((X, *, 0)\) is called closed ideal, if
\(0 * x \in I\), for all \( x \in I\).

Definition 2.4 ([7]). An ideal of commutative BCK-algebras \( X \) is said to be prime
if \(x \land y \in I\) implies \(x \in I\) or \(y \in I\).

Definition 2.5. Let \( X \) be non empty set. A fuzzy subset \( \mu \) of the set \( X \) is a
mapping \( \mu : X \rightarrow [0, 1] \). The complement of fuzzy set \( \mu \) of a set \( X \) is denoted by
\(\mu(x) = 1 - \mu(x)\), for all \( x \in X \).

Definition 2.6. A fuzzy ideal \( \mu \) of commutative algebra \( X \) is called anti fuzzy prime
ideal of \( X \) if

\(\mu(x \land y) \geq \min\{\mu(x), \mu(y)\}\)

for all \( x, y \in X \).

Definition 2.7 ([5, 6]). An intuitionistic fuzzy set (in short, IFS) in a non-empty set
\( X \) is an object having the form \( A = \{\langle x, \mu_A(x), \lambda_A(x) \rangle / x \in X \}\), where the function
\(\mu_A : X \rightarrow [0, 1]\) and \(\lambda_A : X \rightarrow [0, 1]\) denoted the degree of membership (namely
\(\mu_A(x)\)) and the degree of nonmembership (namely \(\lambda_A(x)\)) of each element \( x \in X \) to

the set $A$ respectively and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. For the simplicity, we use the symbol form $A = \langle \mu_A, \lambda_A \rangle$.

**Definition 2.8 (16).** Let $X$ be a BCK-algebra. An intuitionistic fuzzy set $A = \langle \mu_A, \lambda_A \rangle$ of BCK-algebra $X$ is called an intuitionistic fuzzy subalgebra of $X$ if:

(i) $\mu_A(x \ast y) \geq \min \{ \mu_A(x), \mu_A(y) \}$
(ii) $\lambda_A(x \ast y) \leq \max \{ \lambda_A(x), \lambda_A(y) \}$

for all $x, y \in X$.

**Definition 2.9 (16).** An intuitionistic fuzzy set $A = \langle \mu_A, \lambda_A \rangle$ in $X$ is called an intuitionistic fuzzy prime ideal of $X$, if it satisfies the following axioms:

(IF1) $\mu_A(0) \geq \mu_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$

(IF2) $\mu_A(x) \geq \min \{ \mu_A(x \ast y), \mu_A(y) \}$

(IF3) $\lambda_A(x) \leq \min \{ \lambda_A(x \ast y), \lambda_A(y) \}$

for all $x, y \in X$.

**Definition 2.10 (16).** An intuitionistic fuzzy prime ideal $A = \langle \mu_A, \lambda_A \rangle$ of a BCK-algebra $X$ is called an intuitionistic fuzzy closed-ideal of $X$, if it follows axiom satisfies

(IF4) $\mu_A(0 \ast x) \geq \mu_A(x)$ and $\lambda_A(0 \ast x) \leq \lambda_A(x)$

for all $x \in X$.

**Theorem 2.11 (16).** Every intuitionistic fuzzy ideal of BCK-algebra $X$ is an intuitionistic fuzzy subalgebra of BCK-algebra $X$.

**Theorem 2.12.** A non empty subset $I$ of BCK-algebra $X$ is an ideal of BCK-algebra $X$ if and only if $\tilde{I} = \langle X_I, \overline{\ast} \rangle$ is an intuitionistic fuzzy ideal.

**Proposition 2.13.** Every Prime ideal of commutative BCK-algebra $X$ is an ideal of $X$.

3. MAJOR SECTION

**Definition 3.1.** An intuitionistic fuzzy ideal $A = \langle \mu_A, \lambda_A \rangle$ of a BCK-algebra $X$ is called an intuitionistic fuzzy prime ideal of $X$ if

(IFP1) $\mu_A(x \ast y) \leq \max \{ \mu_A(x), \mu_A(y) \}$

(IFP2) $\lambda_A(x \ast y) \geq \min \{ \lambda_A(x), \lambda_A(y) \}$

for all $x, y \in X$.

**Example 3.2.** Let $X = \{0, x, y, z\}$ with Cayley table as follows:

<table>
<thead>
<tr>
<th>*</th>
<th>0</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>x</td>
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<td>0</td>
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<tr>
<td>y</td>
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<td>0</td>
</tr>
<tr>
<td>z</td>
<td>x</td>
<td>0</td>
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</tbody>
</table>

It is easy to verify that $(X, \ast, 0)$ is commutative BCK-algebra. Define an IFS $A = \langle \mu_A, \lambda_A \rangle$ as: $\mu_A(0) = 1, \mu_A(x) = 0.9, \mu_A(y) = 0.5, \mu_A(z) = 0$ and $\lambda_A(0) = 0, \lambda_A(x) = 0.1, \lambda_A(y) = 0.5, \lambda_A(z) = 1$. By routine calculations $A = \langle \mu_A, \lambda_A \rangle$ is an intuitionistic fuzzy prime ideal of $X$. 663
Theorem 3.3. If \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \), then the sets \( J = \{ x \in X/\mu_A(x) = \mu_A(0) \} \) and \( K = \{ x \in X/\lambda_A(x) = \lambda_A(0) \} \) are prime ideals of \( X \).

Proof. Straightforward. \( \square \)

Corollary 3.4. If \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \), then the sets \( P_1 = \{ x \in X/\mu_A(x) = 0 \} \) and \( P_2 = \{ x \in X/\lambda_A(x) = 0 \} \) are either empty or prime ideals of \( X \).

Proposition 3.5. If \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \), then the sets \( I = \{ x \in X/\mu_A(x) = 1 \} \) and \( I = \{ x \in X/\lambda_A(x) = 1 \} \) are either empty or prime ideals of \( X \).

Proposition 3.6. Every intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \) is an intuitionistic fuzzy ideal of commutative BCK-algebra \( X \).

Proof. Straightforward. \( \square \)

Remark 3.7. An intuitionistic fuzzy ideal of a commutative BCK-algebra \( X \) need not be an intuitionistic fuzzy prime ideal of \( X \). Shown in the following example.

Example 3.8. Let \( X = \{ 0, x, y, z \} \) be a BCK-algebra with Cayley table as follows:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>x</th>
<th>y</th>
<th>z</th>
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</tr>
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</table>

Then the BCK-algebra \( X \) is commutative. Define an IFS \( A = \langle \mu_A, \lambda_A \rangle \), \( \mu_A : X \rightarrow [0, 1] \) by \( \mu_A(0) = 1, \mu_A(x) = \mu_A(y) = 0.5, \mu_A(z) = 0 \) and \( \lambda_A(0) = 0, \lambda_A(x) = \lambda_A(y) = 0.5, \lambda_A(z) = 1 \). Routine calculations give that \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy ideal of \( X \) but not intuitionistic fuzzy prime ideal of \( X \).

Proposition 3.9. Let \( P \) be an ideal of a commutative BCK-algebra \( X \). Then, \( P \) is a prime ideal of \( X \) iff \( \widetilde{P} = (X_P, \overline{X}_P) \) is an intuitionistic fuzzy prime ideal of \( X \).

Proof. Suppose that \( P \) is a prime ideal of \( X \). Then, by Proposition 2.13 \( P \) is an ideal of \( X \). Since by Theorem 2.12 \( \widetilde{P} = (X_P, \overline{X}_P) \) is an intuitionistic fuzzy ideal of \( X \). Let for any \( x, y \in X \). Then, we have two case’s (i) \( x \wedge y \in P \) and (ii) \( x \wedge y \notin P \).

Case (i) if \( x \wedge y \in P \), then \( X_P(x \wedge y) = 1 \), since \( P \) is a prime ideal of \( X \), so either \( x \in P \) or \( y \in P \) this implies \( X_P(x) = 1 \) or \( X_P(y) = 1 \). Thus, \( X_P(x \wedge y) = 1 = \max\{X_P(x), X_P(y)\} \) and \( \overline{X}_P(x \wedge y) = 1 - X_P(x \wedge y) = 0 \), and \( \overline{X}_P(x) = 1 - X_P(x) = 0 \) or \( X_P(y) = 1 - X_P(y) = 0 \) this implies \( \overline{X}_P(x \wedge y) = 0 = \min\{X_P(x), X_P(y)\} \).

Case (ii) if \( x \wedge y \notin P \), then \( X_P(x \wedge y) = 0 \) and \( X_P(x) \geq 0 \) and \( X_P(y) \geq 0 \) this imply \( X_P(x \wedge y) = 0 \leq \max\{X_P(x), X_P(y)\} \), and \( \overline{X}_P(x \wedge y) = 1 \) and \( X_P(x) \geq 0 \), \( X_P(y) \geq 0 \) this imply \( X_P(x) \leq 1 \) and \( X_P(y) \leq 1 \) this imply \( \overline{X}_P(x \wedge y) = 1 \geq \max\{X_P(x), X_P(y)\} \).
min\{\bar{X}_P(x), \bar{X}_P(y)\}. Hence \(\bar{P} = \langle X_P, \bar{X}_P \rangle\) be an intuitionistic fuzzy prime ideal of \(X\).

Conversely, suppose that \(x \land y \in P\) and \(x \not\in P\), for any \(x, y \in X\). Then,
\[
0 = \bar{X}_P(x \land y) = 1 - X_P(x \land y) \geq 1 - \max\{X_P(x), X_P(y)\} = \min\{1 - X_P(x), 1 - X_P(y)\} = \min\{\bar{X}_P(x), \bar{X}_P(y)\} = \bar{X}_P(y)
\]
this implies \(0 \geq \bar{X}_P(y)\) this implies \(\bar{X}_P(y) = 0\), so that \(X_P(y) = 1\). Thus, \(y \in P\). Similarly, if \(x \land y \in P\) and \(y \not\in P\), for any \(x, y \in X\). Hence, \(P\) is a prime ideal of \(X\). This completes the proof. \(\square\)

**Proposition 3.10.** Let \(A = \langle \mu_A, \lambda_A \rangle\) be an intuitionistic fuzzy set of a commutative BCK-algebra \(X\). Then, \(A = \langle \mu_A, \lambda_A \rangle\) is an intuitionistic fuzzy prime ideal of \(X\) iff \(\mu_A\) and \(\lambda_A\) are fuzzy prime ideals of \(X\), where \(\lambda_A = 1 - \lambda_A\).

**Proof.** Let \(A = \langle \mu_A, \lambda_A \rangle\) be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \(X\). Since by Proposition 3.6, \(A = \langle \mu_A, \lambda_A \rangle\) is an intuitionistic fuzzy ideal, so by [16, Page 843, Lemma 3.11], \(\mu_A\) and \(\lambda_A\) are fuzzy ideals of \(X\). Let for any \(x, y \in X\). Then,
\[
\begin{align*}
\mu_A(x \land y) & \leq \max\{\mu_A(x), \mu_A(y)\} \\
\lambda_A(x \land y) & \geq \min\{\lambda_A(x), \lambda_A(y)\}
\end{align*}
\]
Now
\[
1 - \lambda_A(x \land y) \leq 1 - \min\{\lambda_A(x), \lambda_A(y)\}
\]
\[
\begin{align*}
\tilde{\lambda}_A(x \land y) & \leq \max\{1 - \lambda_A(x), 1 - \lambda_A(y)\} \\
\tilde{\lambda}_A(x \land y) & \leq \max\{\tilde{\lambda}_A(x), \tilde{\lambda}_A(y)\}
\end{align*}
\]
Hence \(\mu_A\) and \(\tilde{\lambda}_A\) are fuzzy prime ideals of \(X\).

The converse part is easy, we omit the proof. \(\square\)

**Proposition 3.11.** Let \(A = \langle \mu_A, \lambda_A \rangle\) be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \(X\). Then \(A = \langle \mu_A, \lambda_A \rangle\) is an intuitionistic fuzzy prime ideal of \(X\) iff \(\mu_A\) and \(\lambda_A\) are anti fuzzy prime ideals of \(X\), where \(\mu_A = 1 - \mu_A\).

**Proof.** Proof is same as above Proposition. \(\square\)

**Proposition 3.12.** Let \(A = \langle \mu_A, \lambda_A \rangle\) be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \(X\). Then, \(\square A = \langle \mu_A, \tilde{\mu}_A \rangle\) is an intuitionistic fuzzy prime ideal of \(X\), where \(\tilde{\mu}_A = 1 - \mu_A\).

**Proof.** Let \(A = \langle \mu_A, \lambda_A \rangle\) be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \(X\). Then since by Proposition 3.6, \(A = \langle \mu_A, \lambda_A \rangle\) is an intuitionistic fuzzy ideal of \(X\), so by [16, Page 843, Theorem 3.12], \(\square A = \langle \mu_A, \tilde{\mu}_A \rangle\) is an intuitionistic fuzzy ideal of \(X\). For any \(x, y \in X\), then
\[
\mu_A(x \land y) \leq \max\{\mu_A(x), \mu_A(y)\}
\]
Now
\[ 1 - \mu_A(x \land y) \geq 1 - \max\{\mu_A(x), \mu_A(y)\} \]
\[ \hat{\mu}_A(x \land y) \geq \min\{1 - \mu_A(x), 1 - \mu_A(y)\} \]
\[ \hat{\mu}_A(x \land y) \geq \min\{\hat{\mu}_A(x), \hat{\mu}_A(y)\} \]

Hence \( \square A = \langle \mu_A, \hat{\mu}_A \rangle \) is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \).

**Proposition 3.13.** Let \( A = \langle \mu_A, \lambda_A \rangle \) be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \). Then \( \Diamond A = \langle \hat{\lambda}_A, \lambda_A \rangle \) is an intuitionistic fuzzy prime ideal of \( X \), where \( \hat{\lambda}_A = 1 - \lambda_A \).

**Proof.** Let \( A = \langle \mu_A, \lambda_A \rangle \) be an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \). Then since by Proposition 3.6 \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy ideal of \( X \), so by [16, Page 843, Theorem 3.12] \( \Diamond A = \langle \hat{\lambda}_A, \lambda_A \rangle \) is an intuitionistic fuzzy ideal of \( X \). Let for any \( x, y \in X \). Then,
\[ \mu_A(x \land y) \leq \max\{\mu_A(x), \mu_A(y)\} \]
\[ \lambda_A(x \land y) \geq \min\{\lambda_A(x), \lambda_A(y)\} \]

Now
\[ 1 - \lambda_A(x \land y) \leq 1 - \min\{\lambda_A(x), \lambda_A(y)\} \]
\[ \hat{\lambda}_A(x \land y) \leq \max\{1 - \lambda_A(x), 1 - \lambda_A(y)\} \]
\[ \hat{\lambda}_A(x \land y) \leq \max\{\hat{\lambda}_A(x), \hat{\lambda}_A(y)\} \]

Hence, \( \Diamond A = \langle \hat{\lambda}_A, \lambda_A \rangle \) is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \).

**Theorem 3.14.** Let \( A = \langle \mu_A, \lambda_A \rangle \) be an IFS in a commutative BCK-algebra \( X \). Then \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \) if and only if \( \square A = \langle \mu_A, \hat{\mu}_A \rangle \) and \( \Diamond A = \langle \hat{\lambda}_A, \lambda_A \rangle \) are an intuitionistic fuzzy prime ideals of \( X \).

**Definition 3.15.** Let \( A = \langle \mu_A, \lambda_A \rangle \) be an intuitionistic fuzzy set of a BCK-algebra \( X \). Then for \( s, t \in [0, 1] \), the set \( U(\mu_A, t) = \{x \in X/\mu_A(x) \geq t\} \) is called upper \( t \)-level cut of \( \mu_A \) and the set \( L(\lambda_A, s) = \{x \in X/\lambda_A(x) \leq s\} \) is called lower \( s \)-level cut of \( \lambda_A \).

**Theorem 3.16.** An IFS \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \) if and only if for all \( s, t \in [0, 1] \), the set’s \( U(\mu_A, t) \) and \( L(\lambda_A, s) \) are prime ideals of \( X \).

**Proof.** Suppose \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \). Then since by Proposition 3.6 \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \). So by [16, Page 843, Theorem 3.13] \( U(\mu_A, t) \) and \( L(\lambda_A, t) \) are ideal of \( X \). Let \( x \land y \in U(\mu_A, t) \) this implies \( \mu_A(x \land y) \geq t \) and \( \mu_A(x \land y) \leq \max\{\mu_A(x), \mu_A(y)\} \) this implies \( \mu_A(x \land y) \geq\)
y) ≥ t implies that \( \max\{\mu_A(x), \mu_A(y)\} \ge t \); this implies \( \mu_A(x) \ge t \) or \( \mu_A(y) \ge t \). So that \( x \in U(\mu_A, t) \) and \( y \in U(\mu_A, t) \). Hence \( U(\mu_A, t) \) is a prime ideal of \( X \). Similarly \( L(\lambda_A, s) \) is a prime of \( X \).

Conversely, let \( U(\mu_A, t) \) and \( L(\lambda_A, s) \) be prime ideals of \( X \). Then by \( [7] \) \( U(\mu_A, t) \) and \( L(\lambda_A, s) \) are ideals of \( X \). Since by \( [16] \) Page 843, Theorem 3.13 \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy ideal of \( X \). On contrary \( A = \langle \mu_A, \lambda_A \rangle \) is not an intuitionistic fuzzy prime ideal of \( X \). Then there exist \( x, y \in X \) such that

\[
\mu_A(x \land y) > \max\{\mu_A(x), \mu_A(y)\}
\]

Let \( t = \frac{1}{2}(\mu_A(x \land y) + \max\{\mu_A(x), \mu_A(y)\}) \)

this implies \( \mu_A(x \land y) > t > \max\{\mu_A(x), \mu_A(y)\} \)

this implies \( x \land y \in U(\mu_A, t) \) but \( x \notin U(\mu_A, t) \) and \( y \notin U(\mu_A, t) \)

which is a contradiction. Hence, \( A = \langle \mu_A, \lambda_A \rangle \) is an intuitionistic fuzzy prime ideal of a commutative BCK-algebra \( X \).

\[\square\]

References


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