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Interval valued intuitionistic fuzzy soft set relations

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Abstract. In this paper the concept of interval valued intuitionistic fuzzy soft set relations (IVIFSS-relations for short) is proposed. Our relations on interval valued intuitionistic fuzzy soft sets is an extension of the relations on intuitionistic fuzzy soft sets, introduced by Mukherjee and Chakraborty in 2009. The basic properties of the IVIFSS-relations are also presented and discussed. It is seen that the sub collection of the family of IVIFSS-relations form a relational topology. Also various types of IVIFSS-relations are presented. Then a solution to a decision making problem using IVIFSS-relation is presented. Finally the lower and upper soft interval valued intuitionistic fuzzy rough approximations of a IVIFSS-relation with respect to a soft interval valued intuitionistic fuzzy approximation space are presented.

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1. Introduction

The vagueness or the representation of imperfect knowledge has been a problem for a long time for the mathematicians and philosophers. However, recently it became a crucial issue for computer scientists, particularly in the area of artificial intelligence. To handle situations like this, many tools have been suggested. Some of them are probability theory, fuzzy set theory, rough set theory etc. In 1999, Molodtsov [10] introduced soft set theory which is a completely new approach for modeling vagueness and uncertainties. In soft set theory, there is no limited condition to the description of objects; so researchers can choose the form of parameters they need. Research works on soft set theory are progressing rapidly. Maji et al.[6]

defined several operations on soft set theory. Based on the analysis of several operations on soft sets introduced in [6], Ali et al.[2] presented some new algebraic operations for soft sets and proved that certain De Morgan's law holds in soft set theory with respect to these new definitions. Aktas and Cagman[1] introduced the basic properties of soft sets, compared soft sets to the related concepts of fuzzy sets[12] and rough sets[11], pointed out that every fuzzy set and every rough set may be considered as a soft set. Combining soft sets with fuzzy sets[12] and intuitionistic fuzzy sets[3], Maji et al. [7, 8] defined fuzzy soft sets and intuitionistic fuzzy soft sets which are rich potentials for solving decision making problems. As a generalization of fuzzy soft set theory, intuitionistic fuzzy soft set theory makes description of the objective more realistic, more practical and accurate in some cases, making it more promising. The notion of the interval-valued intuitionistic fuzzy set was first introduced by Atanassov and Gargov [4]. It is characterized by an interval-valued membership degree and an interval-valued non-membership degree. In 2010, Y. Jiang et al.[5] introduced the concept of interval valued intuitionistic fuzzy soft sets which is a combination of an interval valued intuitionistic fuzzy set theory and a soft set theory. The concept of relations on intuitionistic fuzzy soft sets was introduced by Mukherjee and Chakraborty[9] in 2009. The concept of IVIFSS-relations together with their basic properties on interval valued intuitionistic fuzzy soft sets is introduced in this paper. It is to be shown that the sub collection of the family of IVIFSS-relations forms relational topology. Also various types of IVIFSS-relations are presented. Then a solution to a decision making problem using IVIFSS-relation is presented. Finally the lower and upper soft interval valued intuitionistic fuzzy rough approximations of a IVIFSS-relation with respect to a soft interval valued intuitionistic fuzzy approximation space are presented.

2. Preliminaries

Definition 2.1 ([12]). Let X be a non empty set. Then a fuzzy set (FS for short) A is a set having the form A=$\{(x, \mu_A(x)) \colon x \in X\}$ where the function μ_A: X→[0,1] is called the membership function and μ_A(x) is called the degree of membership of each element x∈X.

Definition 2.2 ([10]). Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A⊆ E. Then the pair (F, A) is called a soft set over U, where F is a mapping given by F: A→ P(U).

In other words, a soft set over U is a parameterized family of subsets of U. For e∈A, F(e) may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.3 ([7]). Let U be an initial universe and E be a set of parameters. Let F(U) be the set of all fuzzy subsets of U and A⊆E. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A → F(U).

Definition 2.4 ([3]). Let X be a non empty set. Then an intuitionistic fuzzy set (IFS for short) A is a set having the form A=$\{\langle x, \mu_A(x), \gamma_A(x)\rangle \colon x \in X\}$ where the functions μ_A: X→[0,1] and γ_A: X→[0,1] represents the degree of membership and the degree of non-membership respectively of each element x∈U and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each x∈X.

Definition 2.5 ([8]). Let U be an initial universe and E be a set of parameters. Let IF(U) be the set of all intuitionistic fuzzy subsets of U and A$\subseteq$ E. Then the pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by F: A $\rightarrow$ IF(U).

Definition 2.6 ([4]). An interval valued intuitionistic fuzzy set A over an universe set U is defined as the object of the form A=$\{<x, \mu_A(x), \gamma_A(x)>: x\in U\}$, where μ_A: U$\rightarrow$ Int([0,1]) and γ_A: U$\rightarrow$ Int([0, 1]) are functions such that the condition: $\forall x\in U$, $\sup\mu_A(x)+\sup\gamma_A(x)\leq 1$ is satisfied.

The class of all interval valued intuitionistic fuzzy sets on U is denoted by IVIFS(U). Let A, B$\in$IVIFS(U). Then

- the union of A and B is denoted by A$\bigvee$B where

$A\bigvee B=\{(x, [\max(\inf\mu_A(x), \inf\mu_B(x)), \max(\sup\mu_A(x), \sup\mu_B(x))], [\min(\inf\gamma_A(x), \inf\gamma_B(x)), \min(\sup\gamma_A(x), \sup\gamma_B(x))]): x\in U\}$

- the intersection of A and B is denoted by A$\bigwedge$B where

$A\bigwedge B=\{(x, [\min(\inf\mu_A(x), \inf\gamma_B(x)), \min(\sup\mu_A(x), \sup\mu_B(x))], [\max(\inf\gamma_A(x), \inf\gamma_B(x)), \max(\sup\gamma_A(x), \sup\gamma_B(x))]): x\in U\}$

Atanassov and Gargov shows in [4] that A$\bigvee$B and A$\bigwedge$B are again IVIFSs.

Definition 2.7 ([5]). Let U be an initial universe and E be a set of parameters. Let IVIFS(U) be the set of all interval valued intuitionistic fuzzy sets on U and A $\subseteq$E. Then the pair (F, A) is called an interval valued intuitionistic fuzzy soft set (IVIFSS for short) over U, where F is a mapping given by F: A $\rightarrow$ IVIFS(U).

Definition 2.8 ([11]). Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R$. Now for X$\subseteq$U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R_*X and R^*X and are defined by

$R_*X=\{x\in U: [x]_R\subseteq X\}$,

$R^*X=\{x\in U: [x]_R\cap X\neq\varnothing\}$.

Now if $R_*X=R^*X$, then X is called definable; otherwise X is called a rough set.

3. Relations on interval valued intuitionistic fuzzy soft sets

Definition 3.1. Let U be an initial universe and (F, A) and (G, B) be two interval valued intuitionistic fuzzy soft sets. Then a relation between them is defined as a pair (H, A×B), where H is mapping given by H: A×B$\rightarrow$IVIFS(U). This is called an interval valued intuitionistic fuzzy soft set relation (IVIFSS-relation for short). The collection of relations on interval valued intuitionistic fuzzy soft sets on A×B over U is denoted by ρ_U(A×B).

Remark 3.2. Let U be an initial universe and (F_1, A_1), (F_2, A_2),...,(F_n, A_n) be n numbers of interval valued intuitionistic fuzzy soft sets over U. Then a relation ρ between them is defined as a pair (H, $A_1\times A_2\times ...\times A_n$), where H is mapping given by H: $A_1\times A_2\times .... \times A_n\rightarrow$ IVIFS(U).

Example 3.3. (i) Let us consider an interval valued intuitionistic fuzzy soft set (F, A) which describes the 'attractiveness of the houses' under consideration. Let the universe set U={h_1, h_2, h_3 ,h_4, h_5} and the set of parameter A={beautiful(e_1), in the green surroundings (e_3)}.

Then the tabular representation of the interval valued intuitionistic fuzzy soft set (F, A) is given below:

U	Beautiful(e_1)	In the green surroundings(e_3)
h_1	([.1, .7], [.1, .3])	([.3, .4], [.2, .5])
h_2	([.3, .5], [.3, .4])	([.2, .3], [.4, .5])
h_3	([.2, .6], [.1, .3])	([.1, .3], [.2, .3])
h_4	([.3, .4], [.1, .5])	([.1, .6], [.2, .3])
h_5	([.1, .3], [.2, .3])	([.4, .5], [.1, .4])

(ii) Now let us consider the interval valued intuitionistic fuzzy soft set (G, B) which describes the 'cost of the houses' under consideration. Let the universe set U={h_1, h_2, h_3 ,h_4, h_5} and the set of parameter B={costly(e_2), moderate(e_4)}.

Then the tabular representation of the interval valued intuitionistic fuzzy soft set (G, B) is given below:

U	Costly(e_2)	Moderate(e_4)
h_1	([.2, .6], [.3, .4])	([.2, .4], [.3, .5])
h_2	([.1, .2], [.4, .6])	([.4, .5], [.1, .3])
h_3	([.4, .6], [.2, .3])	([.2, .5], [.1, .3])
h_4	([.1, .3], [.3, .6])	([.3, .4], [.2, .5])
h_5	([.3, .5], [.2, .4])	([.4, .5], [.3, .4])

Let us consider the two IVIFSS-relations P and Q on the two given interval valued intuitionistic fuzzy soft sets given below:

(1) P= (H, A×B):

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([.1, .6], [.1, .3])	([.2, .4], [.3, .5])	([.2, .6], [.3, .4])	([.2, .3], [.3, .6])
h_2	([.3, .4], [.3, .4])	([.4, .5], [.1, .3])	([.3,.5], [.4, .5])	([.4, .7], [.1, .3])
h_3	([.2, .3], [.5, .6])	([.2, .6],[.2, .3])	([.2, .5], [.3, .4])	([.2, .5],[.3, .4])
h_4	([.3, .4], [.3, .5])	([.3, .4], [.4, .5])	([.3, .4], [.2, .3])	([.3, .4], [.3, .5])
h_5	([.1, .3],[.2, .4])	([.3, .5], [.2, .4])	([.3, .5], [.2,.4])	([.3, .6], [.3, .4])

(2) Q= (J, A×B):

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([.5, .8], [.1, .2])	([.2, .5], [.3, .4])	([.3, .7], [.1, .2])	([.2, .4], [.2, .3])
h_2	([.4, .5], [.2, .4])	([.4, .6], [.1, .3])	([.3, .5], [.4, .5])	([.4, .8], [.1, .2])
h_3	([.2, .6],[.1, .4])	([.2, .7], [.1, .3])	([.2, .6], [.2, .3])	([.3, .5], [.2, .3])
h_4	([.3, .5], [.3, .4])	([.3,.4], [.4, .5])	([.7, .8], [.1, .2])	([.4, .6], [.2, .4])
h_5	([.1, .4], [.1, .2])	([.3, .6], [.2, .3])	([.3, .6], [.2, .3])	([.3, .7], [.1, .3])

The tabular representations of P and Q are called relational matrices for P and Q respectively. From above we have, $\mu_{H(e_1,e_2)}(h_1)$=[0.1, 0.6] and $\gamma_{J(e_1,e_2)}(h_2)$=[0.2, 0.4] etc. But this intervals lie on the 1st row-1st column and 2nd row-1st column respectively. So we denote $\mu_{H(e_1,e_2)}(h_1)|_{(1,1)}$=[0.1, 0.6] and $\gamma_{J(e_1,e_2)}(h_2)|_{(2,1)}$=[0.2, 0.4] etc to make the clear concept about what are the positions of the intervals in the relational matrices.

Definition 3.4. The order of the relational matrix is (α, β), where α=number of the universal points and β=number of pairs of parameters considered in the relational matrix. In example 3.3, both the relational matrix for P and Q are of order (5, 4). If $\alpha=\beta$, then the relation matrix is called a square matrix.

Definition 3.5. Let P, Q$\in\rho_U$(A×B), P=(H, A×B), Q=(J, A×B) and the order of their relational matrices are same. Then we define

(i) P$\bigcup$Q=(H⧫J, A×B), where H⧫J: A×B→ IVIFS(U) is defined as (H⧫J)(e_i, e_j)=H(e_i, e_j)$\bigvee$J(e_j e_j) for (e_i, e_j)∈A×B, where $\bigvee$ denotes the interval valued intuitionistic fuzzy union.

(ii) P$\bigcap$Q=(H•J, A×B), where H•J: A×B→ IVIFS(U) is defined as (H•J)(e_i, e_j)=H(e_i, e_j)$\bigwedge$J(e_j, e_j) for (e_i, e_j)∈A×B, where $\bigwedge$ denotes the interval valued intuitionistic fuzzy intersection.

(iii) P^C =(∼H, A×B), where ∼H: A×B→IVIFS(U) is defined as ∼H(e_i, e_j)=[H(e_i, e_j)]$^\sharp$ for (e_i, e_j)∈ A×B, where $\sharp$ denotes the interval valued intuitionistic fuzzy complement.

Example 3.6. Consider the interval valued intuitionistic fuzzy soft sets (F, A) and (G, B) given in example 3.3. Let us consider the two IVIFSS-relations P_1 and Q_1 given below:

(1) P_1= (H, A×B):

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([.1, .5], [.1, .3])	([.2, .4], [.3, .5])	([.2, .6], [.1, .4])	([.2, .3], [.3, .6])
h_2	([.3, .4], [.3, .4])	([.4, .5], [.1, .3])	([.3, .5], [.4, .5])	([.1, .7], [.1, .3])
h_3	([.2, .3], [.5, .7])	([.2, .6], [.2, .3])	([.2, .5], [.3, .4])	([.2, .5], [.2, .4])
h_4	([.3, .4], [.2, .5])	([.3, .4], [.4, .5])	([.3, .4], [.2, .4])	([.3, .4], [.3, .5])
h_5	([.1, .3], [.2, .4])	([.3, .5], [.2, .4])	([.3, .5], [.2, .4])	([.5, .6], [.3, .4])

(2) Q_1= (J, A×B):

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([.3, .4], [.1, .2])	([.2, .3], [.3, .4])	([.3, .5], [.1, .2])	([.2, .4], [.2, .3])
h_2	([.4, .5], [.2, .5])	([.4, .6], [.1, .3])	([.3, .5], [.4, .5])	([.4, .6], [.1, .2])
h_3	([.2, .6], [.1, .4])	([.2, .4], [.1, .3])	([.2, .6], [.2, .3])	([.3, .5], [.2, .5])
h_4	([.3, .4], [.3, .4])	([.3, .4], [.4, .5])	([.7, .8], [.1, .2])	([.4, .5], [.2, .4])
h_5	([.3, .4], [.2, .5])	([.2, .6], [.2, .3])	([.5, .6], [.2, .3])	([.3, .7], [.1, .3])

Then (i) $P_1\bigcup Q_1$:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([.3, .5], [.1, .2])	([.2, .4], [.3, .4])	([.3, .6], [.1, .2])	([.2, .4], [.2, .3])
h_2	([.4, .5], [.2, .4])	([.4, .6], [.1, .3])	([.3, .5], [.4, .5])	([.4, .7], [.1, .2])
h_3	([.2, .6], [.1, .4])	([.2, .6], [.1, .3])	([.2, .6], [.2, .3])	([.3, .5], [.2, .4])
h_4	([.3, .4], [.2, .4])	([.3, .4], [.4, .5])	([.7, .8], [.1, .2])	([.4, .5], [.2, .4])
h_5	([.3, .4], [.2, .4])	([.3, .6], [.2, .3])	([.5, .6], [.2, .3])	([.5, .7], [.1, .3])

(ii) $P_1\bigcap Q_1$:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([.1, .4], [.1, .3])	([.2, .3], [.3, .5])	([.2, .5], [.1, .4])	([.2, .3], [.3, .6])
h_2	([.3, .4], [.3, .5])	([.4, .5], [.1, .3])	([.3, .5], [.4, .5])	([.1, .6], [.1, .3])
h_3	([.2, .3], [.5, .7])	([.2, .4], [.2, .3])	([.2, .5], [.3, .4])	([.2, .5], [.2, .5])
h_4	([.3, .4], [.3, .5])	([.3, .4], [.4, .5])	([.3, .4], [.2, .4])	([.3, .4], [.3, .5])
h_5	([.1, .3], [.2, .5])	([.2, .5], [.2, .4])	([.3, .5], [.2, .4])	([.3, .6], [.3, .4])

(iii) $P_1{}^C$:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([.1, .3], [.1, .5])	([.3, .5], [.2, .4])	([.1, .4], .2, .6])	([.3, .6], [.2, .3])
h_2	([.3, .4], [.3, .4])	([.1, .3], [.4, .5])	([.4, .5], [.3, .5])	([.1, .3], [.1, .7])
h_3	([.5, .7], [.2, .3])	([.2, .3], [.2, .6])	([.3, .4], [.2, .5])	([.2, .4], [.2, .5])
h_4	([.2, .5], [.3, .4])	([.4, .5], [.3, .4])	([.2, .4], [.3, .4])	([.3, .5], [.3, .4])
h_5	([.2, .4], [.1, .3])	([.2, .4], [.3, .5])	([.2, .4], [.3, .5])	([.3, .4], [.5, .6])

Theorem 3.7. *Let P, Q, R$\in\rho_U(A\times B)$ and the order of their relational matrices are same. Then the following properties hold:*
(a) $(P\bigcup Q)^C = P^C\bigcap Q^C$.
(b) $(P\bigcap Q)^C = P^C\bigcup Q^C$.
(c) $P\bigcup(Q\bigcup R)=(P\bigcup Q)\bigcup R$
(d) $P\bigcap(Q\bigcap R)=(P\bigcap Q)\bigcap R$
(e) $P\bigcap(Q\bigcup R)=(P\bigcap Q)\bigcup(P\bigcap R)$
(f) $P\bigcup(Q\bigcap R)=(P\bigcup Q)\bigcap(P\bigcup R)$

Proof. (a) Let P=(H, A×B), Q=(J, A×B). Then $P\bigcup Q=(H\blacklozenge J, A\times B)$, where $H\blacklozenge J$: A×B→IVIFS(U) is defined as $(H\blacklozenge J)(e_i, e_j)= H(e_i, e_j)\bigvee J(e_i, e_j)$ for $(e_i, e_j)\in A\times B$.
So $(P\bigcup Q)^C =(\sim H\blacklozenge J, A\times B)$, where $\sim H\blacklozenge J$: A×B → IVIFS(U) is defined as
$(\sim H\blacklozenge J)(e_i, e_j)$
$= \sharp(H(e_i, e_j)\bigvee J(e_i, e_j))$
$= \sharp(\{\langle h_k, \mu_{H(e_i,e_j)}(h_k), \gamma_{H(e_i,e_j)}(h_k)\rangle : h_k \in U\}$
$\quad \bigvee\{\langle h_k, \mu_{J(e_i,e_j)}(h_k), \gamma_{J(e_i,e_j)}(h_k)\rangle : h_k \in U\})$
$= \sharp(\{\langle h_k, [max(inf\mu_{H(e_i,e_j)}(h_k), inf\mu_{J(e_i,e_j)}(h_k)),$
$\quad max(sup\mu_{H(e_i,e_j)}(h_k), sup\mu_{J(e_i,e_j)}(h_k))],$
$\quad [min(inf\gamma_{H(e_i,e_j)}(h_k), inf\gamma_{J(e_i,e_j)}(h_k)),$
$\quad min(sup\gamma_{H(e_i,e_j)}(h_k), sup\gamma_{J(e_i,e_j)}(h_k))]\rangle : h_k \in U\})$
$= \{\langle h_k, [min(inf\gamma_{H(e_i,e_j)}(h_k), inf\gamma_{J(e_i,e_j)}(h_k)),$
$\quad min(sup\gamma_{H(e_i,e_j)}(h_k), sup\gamma_{J(e_i,e_j)}(h_k))],$
$\quad [max(inf\mu_{H(e_i,e_j)}(h_k), inf\mu_{J(e_i,e_j)}(h_k)),$
$\quad max(sup\mu_{H(e_i,e_j)}(h_k), sup\mu_{J(e_i,e_j)}(h_k))]\rangle : h_k \in U\}$
Now $P^c\bigcap Q^C = (\sim H, A\times B)\bigcap(\sim J, A\times B)$, where $\sim H, \sim J : A\times B \to IVIFS(U)$ are defined as

$$\sim H(e_i, e_j) = [H(e_i, e_j)]^\sharp \text{ and } \sim J(e_i, e_j) = [J(e_i, e_j)]^\sharp$$

for $(e_i, e_j) \in A\times B$. We have $(\sim H, A\times B)\bigcap(\sim J, A\times B) = (\sim H\bullet \sim J, A\times B)$.
Now for $(e_i, e_j) \in A\times B$,
$(\sim H\bullet \sim J)(e_i, e_j) = \sim H(e_i, e_j) \bigwedge \sim J(e_j, e_j)$
$= \{\langle h_k, \gamma_{H(e_i,e_j)}(h_k), \mu_{H(e_i,e_j)}(h_k)\rangle : h_k \in U\}$

$$\bigwedge\{\langle h_k, \gamma_{J(e_i,e_j)}(h_k), \mu_{J(e_i,e_j)}(h_k)\rangle : h_k \in U\}$$
$$= \{\langle h_k, [min(inf\gamma_{H(e_i,e_j)}(h_k), inf\gamma_{J(e_i,e_j)}(h_k)), min(sup\gamma_{H(e_i,e_j)}(h_k), sup\gamma_{J(e_i,e_j)}(h_k))], [max(inf\mu_{H(e_i,e_j)}(h_k), inf\mu_{J(e_i,e_j)}(h_k)), max(sup\mu_{H(e_i,e_j)}(h_k), sup\mu_{J(e_i,e_j)}(h_k))]\rangle : h_k \in U\}.$$

Consequently, $(P\bigcup Q)^C = P^C \bigcap Q^C$.

(b) Proof is similar to (a).

(c) Let P=(H, A×B), Q=(J, A×B) and R=(K, A×B). Then $P\bigcup Q$=(H⧫J, A×B), where H⧫J: $A \times B \longrightarrow IVIFS(U)$ is defined as (H⧫J)(e_i, e_j)= H(e_i, e_j)$\bigvee$ J(e_i, e_j) for (e_i, e_j)∈ A×B. So $(P\bigcup Q)\bigcup R = ((H⧫J)⧫K, A\times B)$, where (H⧫J)⧫K: A×B → IVIFS(U) is defined as for (e_i, e_j)∈ A×B, ((H⧫J)⧫K)(e_i, e_j)=$(H(e_i,e_j)\bigvee J(e_i,e_j))$ $\bigvee K(e_i,e_j)$. Now as $(H(e_i,e_j)\bigvee J(e_i,e_j))\bigvee K(e_i,e_j)$=$H(e_i,e_j)\bigvee(J(e_i,e_j)$ $\bigvee K(e_i,e_j))$, therefore ((H⧫J)⧫K)(e_i, e_j)=((H⧫(J⧫K))(e_i, e_j). Also we have $P\bigcup(Q\bigcup R)$=(H⧫(J⧫K), A×B). Consequently, $P\bigcup(Q\bigcup R) = (P\bigcup Q)\bigcup R$.

(d) Proof is similar to (c).

(e) Let P=(H, A×B), Q=(J, A×B) and R=(K, A×B). Then Q⋃R=(J⧫K, A×B), where J⧫K: A×B→ IVIFS(U) is defined as (J⧫K)(e_i, e_j)= J(e_i, e_j)$\bigvee$ K(e_j e_j) for (e_i, e_j)∈ A×B. Then P⋂(Q⋃R)=(H•(J⧫K), A×B), where H•(J⧫K): A×B→ IVIFS(U) is defined as for (e_i, e_j)∈ A×B, (H•(J⧫K))(e_i, e_j)=$H(e_i,e_j)\bigwedge(J(e_i,e_j))\bigvee K(e_i,e_j))$. Since

$$H(e_i,e_j)\bigwedge(J(e_i,e_j)\bigvee K(e_i,e_j))=(H(e_i,e_j)\bigwedge J(e_i,e_j))\bigvee(H(e_i,e_j)\bigwedge K(e_i,e_j)),$$

we have $(H \bullet (J⧫K))(e_i,e_j)$=$(H(e_i,e_j)\bigwedge J(e_i,e_j))\bigvee(H(e_i,e_j)\bigwedge K(e_i,e_j))$. Also $(P\bigcap Q)\bigcup(P\bigcap R) = (H\bullet J, A\times B)\bigcup(H\bullet K, A\times B)$. $=((H\bullet J)⧫(H\bullet K), A\times B)$ Now for $(e_i,e_j) \in A\times B$, $((H\bullet J)⧫(H\bullet K))(e_i,e_j)$ $=(H\bullet J)(e_i,e_j)\bigvee(H\bullet K)(e_i,e_j)$ $=(H(e_i,e_j)\bigwedge J(e_i,e_j))\bigvee(H(e_i,e_j)\bigwedge K(e_i,e_j))$ $=(H\bullet(J⧫K))(e_i,e_j)$. Consequently, $P\cap(Q\cup R) = (P\cap Q)\cup(P\cap R)$.

(f) Proof is similar to (e). □

Definition 3.8. Let P, Q $\in\rho_U$(A×B) and the order of their relational matrices are same. Then P⊆Q iff H(e_i, e_j)⊆J(e_i, e_j) for (e_i, e_j)∈ $A\times B$ where P=(H, A×B) and Q= (J, A×B).

Example 3.9. In the example 3.3, $P \subset Q$.

Definition 3.10. Let U be an initial universe and (F, A) and (G, B) be two interval valued intuitionistic fuzzy soft sets. Then a null relation between them is denoted by O_U and is defined as $O_U = (H_O, A\times B)$ where $H_O(e_i,e_j)$=$\{< h_k, [0,0], [1,1] >: h_k \in U\}$ for (e_i, e_j)∈ A×B.

Example 3.11. Consider the interval valued intuitionistic fuzzy soft sets (F, A) and (G, B) given in example 3.3. Then a null relation between them is given by:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])
h_2	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])
h_3	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])
h_4	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])
h_5	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])

Remark 3.12. It can be easily seen that $P \bigcup O_U = P$ and $P \bigcap O_U = O_U$ for any P∈ρ_U(A×B).

Definition 3.13. Let U be an initial universe and (F, A) and (G, B) be two interval valued intuitionistic fuzzy soft sets. Then an absolute relation between them is denoted by I_U and is defined as $I_U = (H_I, A\times B)$, where $H_I(e_i, e_j)$=$\{\langle h_k, [1, 1], [0, 0]\rangle : h_k \in U\}$ for $(e_i, e_j) \in A \times B$.

Example 3.14. Consider the interval valued intuitionistic fuzzy soft sets (F, A) and (G, B) given in example 3.3. Then an absolute relation between them is given by:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])
h_2	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])
h_3	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])
h_4	([1, 1], [0, 0])	([1, 1], [0, 0]])	([1, 1], [0, 0])	([1, 1], [0, 0])
h_5	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])

Remark 3.15. It can be easily seen that $P \bigcup I_U = I_U$ and $P \bigcap I_U = P$ for any P∈ρ_U(A×B).

Definition 3.16. Let τ be a sub-collection of interval valued intuitionistic fuzzy soft set relations of the same order belonging to ρ_U(A×B). Then τ is said to form a relational topology over ρ_U(A×B) if the following conditions are satisfied:
(i) $O_U, I_U \in \tau$
(ii) If $P_\lambda \in \tau$ for $\lambda \in \Lambda$, then $\bigcup_\lambda P_\lambda \in \tau$
(iii) If $P_1, P_2 \in \tau$, then $P_1 \bigcap P_2 \in \tau$.
Then we say that (ρ_U(A×B),τ) is a conditional relational topological space.

Example 3.17. Consider example 3.3. Then the collection $\tau = \{O_U, I_U, P, Q\}$ forms a relational topology on ρ_U(A×B).

4. Various types of interval valued intuitionistic fuzzy soft set relations

Definition 4.1. Let P∈ρ_U(A×B) and P=(H, A×B) whose relational matrix is a square matrix. Then P is called a reflexive IVIFSS-relation if for (e_i, e_j)∈ $A \times B$ and $h_k \in U$, we have, $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)}$=[1, 1] and $\gamma_{H(e_i,e_j)}(h_k)|_{(m,n)}$=[0, 0] for m=n=k.

Example 4.2. Let U={h_1, h_2, h_3, h_4}. Let us consider the interval valued intuitionistic fuzzy soft sets (F, A) and (G, B) where $A = \{e_1, e_3\}$ and $B = \{e_2, e_4\}$ Then a reflexive IVIFSS-relation between them is

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([1, 1], [0, 0])	([.2, .4], [.3, .5])	([.2, .6], [.3, .4])	([.2, .3], [.3, .6])
h_2	([.3, .4], [.3, .4])	([1, 1], [0, 0])	([.4, .5], [.1, .3])	([.4, .7], [.1, .3])
h_3	([.2, .6],[.1, .4])	([.2, .6], [.1, .3])	([1, 1], [0, 0])	([.2, .5], [.2, .3])
h_4	([.3, .4], [.3, .5])	([.3, .4], [.4, .5])	([.3, .4], [.2, .3])	([1, 1], [0, 0])

Definition 4.3. Let P$\in\rho_U$(A×B) and P=(H, A×B) whose relational matrix is a square matrix. Then P is called an anti-reflexive IVIFSS-relation if for (e_i, e_j)∈A×B and h_k∈ U,

we have, $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)}$=[0, 0] and $\gamma_{H(e_i,e_j)}(h_k)|_{(m,n)}$=[1, 1] for m=n=k.

Example 4.4. Let U={h_1, h_2, h_3, h_4}. Let us consider the interval valued intuitionistic fuzzy soft sets (F, A) and (G, B) where A={e_1, e_3} and B={e_2, e_4} Then an anti-reflexive IVIFSS-relation between them is

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0, 0], [1, 1])	([.2, .4], [.3, .5])	([.2, .6], [.3, .4])	([.2, .3], [.3, .6])
h_2	([.3, .4], [.3, .4])	([0, 0], [1, 1])	([.4,.5], [.1, .3])	([.3, .4], [.4, .5])
h_3	([.2, .3],[.1, .4])	([.4, .5], [.1, .3])	([0, 0], [1, 1])	([.2, .5], [.2, .3])
h_4	([.3, .6], [.3, .4])	([.3, .4], [.4, .5])	([.2, .5], [.2, .5])	([0, 0], [1, 1])

Definition 4.5. Let P$\in\rho_U$(A×B) and P=(H, A×B) whose relational matrix is a square matrix. Then P is called a symmetric IVIFSS-relation if for each $(e_i, e_j) \in A \times B$ and $h_k \in U$, $\exists(e_p, e_q) \in A \times B$ and $h_l \in U$ such that $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)} = \mu_{H(e_p,e_q)}(h_l)|_{(n,m)}$ and $\gamma_{H(e_i,e_j)}(h_k)|_{(m,n)} = \gamma_{H(e_p,e_q)}(h_l)|_{(n,m)}$.

Example 4.6. Let U={h_1, h_2, h_3, h_4}. Let us consider the interval valued intuitionistic fuzzy soft sets (F, A) and (G, B) where A={e_1, e_3} and B={e_2, e_4}. Then a symmetric IVIFSS-relation between them is

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([.1, .2], [.2, .6])	([.2, .4], [.3, .5])	([.2, .6], [.3, .4])	([.2, .3], [.3, .6])
h_2	([.2, .4], [.3, .5])	([0, 0], [1, 1])	([.4, .5], [.1, .3])	([.3, .4], [.4, .5])
h_3	([.2, .6],[.1, .4])	([.4, .5], [.1, .3])	([.3, .4], [.2, .5])	([.2, .5], [.2, .3])
h_4	([.2, .3], [.3, .6])	([.3, .4], [.4, .5])	([.2, .5], [.2, .3])	([.5, .6], [.1, .3])

Definition 4.7. Let P$\in\rho_U$(A×B) and P=(H, A×B) whose relational matrix is a square matrix. Then P is called an anti symmetric IVIFSS-relation if for each $(e_i, e_j) \in A \times B$ and $h_k \in U$, $\exists(e_p, e_q) \in A \times B$ and $h_l \in U$ such that either $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)} \neq \mu_{H(e_p,e_q)}(h_l)|_{(n,m)}$ and $\gamma_{H(e_i,e_j)}(h_k)|_{(m,n)} \neq \gamma_{H(e_p,e_q)}(h_l)|_{(n,m)}$ or $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)} = \mu_{H(e_p,e_q)}(h_l)|_{(n,m)} = [0,0]$ and $\gamma_{H(e_i,e_j)}(h_k)|_{(m,n)} = \gamma_{H(e_p,e_q)}(h_l)|_{(n,m)} = [1,1]$

Example 4.8. Let U={h_1, h_2, h_3, h_4}. Let us consider the interval valued intuitionistic fuzzy soft sets (F, A) and (G, B) where A={e_1, e_3} and B={e_2, e_4}. Then an anti-symmetric IVIFSS-relation between them is

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([.1, .2], [.2, .6])	([.2, .4], [.3, .5])	([.2, .6], [.3, .4])	([0, 0], [1, 1])
h_2	([.3, .5], [.2, .4])	([.1, .4], [.1, .4])	([.4, .5], [.1, .3])	([.3, .4], [.4, .5])
h_3	([.4, .5], [.1, .3])	([.5, .6], [.2, .4])	([.3, .4], [.2, .5])	([0, 0], [1, 1])
h_4	([0, 0], [1, 1])	([.2, .3], [.5, .6])	([0, 0], [1, 1])	([.5, .6], [.1, .3])

Definition 4.9. Let P$\in\rho_U$(A×B) and P=(H, A×B) whose relational matrix is a square matrix. Then P is called a perfectly anti symmetric IVIFSS-relation if for each $(e_i, e_j) \in A \times B$ and $h_k \in U$, $\exists(e_p, e_q) \in A \times B$ and $h_l \in U$ such that whenever $inf\mu_{H(e_i,e_j)}(h_k)|_{(m,n)} > 0$ and $inf\gamma_{H(e_i,e_j)}(h_k)|_{(m,n)} > 0$, $\mu_{H(e_p,e_q)}(h_l)|_{(n,m)} = [0,0]$ and $\gamma_{H(e_p,e_q)}(h_l)|_{(n,m)} = [1,1]$.

Example 4.10. Let U={h_1, h_2, h_3, h_4}. Let us consider the interval valued intuitionistic fuzzy soft sets (F, A) and (G, B) where A={e_1, e_3} and B={e_2, e_4}. Then a perfectly anti-symmetric IVIFSS-relation between them is

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([.1, .2], [.2, .6])	([.2, .4], [.3, .5])	([0, 0], [1, 1])	([0, 0], [1, 1])
h_2	([0, 0], [1, 1])	([.1, .4], [.1, .4])	([.4, .5], [.1, .3])	([0, 0], [1, 1])
h_3	([.2, .5], [.1, .4])	([0, 0], [1, 1])	([.3, .4], [.2, .5])	([0, .5], [0, .2])
h_4	([.2, .4], [.5, .6])	([.2, .3], [.5, .6])	([0, .3], [0, .5])	([.5, .6], [.1, .3])

Definition 4.11. Let P, Q$\in\rho_U$(A×A) and P=(H, A×A), Q= (J, A×A) and the order of their relational matrices are same. Then the composition of P and Q, denoted by P*Q, is defined by P$*$Q=(H$\circ$ J, A×A) where H$\circ$J: A×A$\longrightarrow$ IVIFS(U) is defined as $(H \circ J)(e_i, e_j) = \{\langle h_k, \mu_{(H\circ J)(e_i,e_j)}(h_k), \gamma_{(H\circ J)(e_i,e_j)}(h_k)\rangle : h_k \in U\}$, where $\mu_{(H\circ J)(e_i,e_j)}(h_k) = [max_l(min(inf\mu_{H(e_i,e_l)}(h_k), inf\mu_{J(e_l,e_j)}(h_k))),$
$max_l(min(sup\mu_{H(e_i,e_l)}(h_k), sup\mu_{J(e_l,e_j)}(h_k)))]$ and
$\gamma_{(H\circ J)(e_i,e_j)}(h_k) = [min_l(max(inf\gamma_{H(e_i,e_l)}(h_k), inf\gamma_{J(e_l,e_j)}(h_k))),$
$min_l(max(sup\gamma_{H(e_i,e_l)}(h_k), sup\gamma_{J(e_l,e_j)}(h_k)))]$ for $(e_i, e_j) \in A \times A$.

Example 4.12. Let U={h_1, h_2, h_3, h_4}. Let us consider the interval valued intuitionistic fuzzy soft sets (F, A) and (G, A) where A={e_1, e_2}. Let P, Q $\in\rho_U$(A×A) and P=(H, A×A), Q= (J, A×A) where P:

U	(e_1, e_1)	(e_1, e_2)	(e_2, e_1)	(e_2, e_2)
h_1	([.3, .4], [.3, .4])	([.2, .4], [.3, .5])	([.2, .5], [.3, .4])	([.2, .3], [.3, .6])
h_2	([1, 1], [0, 0])	([.1, .2], [0, 0])	([.4, .5], [.1, .3])	([.4, .7], [.1, .3])
h_3	([.2, .6], [.1, .4])	([.2, .6], [.1, .3])	([.2, .3], [.4, .6])	([.2, .5], [.2, .3])
h_4	([.2, .4], [.3, .5])	([.3, .4], [.4, .5])	([.3, .4], [.2, .3])	([0, .2], [.4, .5])

Q:

U	(e_1, e_1)	(e_1, e_2)	(e_2, e_1)	(e_2, e_2)
h_1	([.5, .8], [.1, .2])	([.2, .3], [.3, .6])	([.1, .4], [.3, .5])	([.2, 0.4], [.2, .3])
h_2	([.4, .5], [.2, .4])	([.4, .6], [.2, .3])	([.1, .5], [.4, .5])	([.4, .5], [.1, .2])
h_3	([.2, .3], [.5, .6])	([.3, .4], [.4, .5])	([.7, .8], [.1, .2])	([.3, .5], [.3, .4])
h_4	([.3, .5], [.3, .4])	([.3, .5], [.2, .4])	([.2, .4], [.2, .3])	([.3, .7], [.1, .3])

Then, P$*$Q:

U	(e_1, e_1)	(e_1, e_2)	(e_2, e_1)	(e_2, e_2)
h_1	([.3, .4], [.3, .4])	([.2, .4], [.3, .5])	([.2, .5], [.3, .4])	([.2, .3], [.3, .6])
h_2	([.4, .5], [.2, .4])	([.1, .6], [.1, .2])	([.4, .5], [.2, .4])	([.4, .5], [.1, .3])
h_3	([.2, .6], [.1, .3])	([.2, .5], [.3, .4])	([.2, .5], [.2, .3])	([.2, .5], [.3, .4])
h_4	([.2, .4], [.3, .5])	([.3, .4], [.3, .5])	([.3, .4], [.3, .4])	([.3, .4], [.2, .4])

Definition 4.13. Let P $\in\rho_U$(A×A) and P=(H, A×A). Then P is called a transitive IVIFSS- relation if P*P $\subseteq$P.

Example 4.14. Let U={h_1, h_2, h_3, h_4}. Let us consider the interval valued intuitionistic fuzzy soft sets (F, A) where $A = \{e_1, e_2\}$. Let P $\in\rho_U$(A×A) and P=(H, A× A) where P:

U	(e_1, e_1)	(e_1, e_2)	(e_2, e_1)	(e_2, e_2)
h_1	([.3, .4], [.3, .4])	([.2, .4], [.3, .6])	([.2, .5], [.3, .4])	([.2, .4], [.3, .6])
h_2	([1, 1], [0, 0])	([.1, .2], [0, 0])	([.4, .5], [.1, .3])	([.4, .7], [.1, .3])
h_3	([.2, .6], [.1, .4])	([.2, .6], [.1, .3])	([.2, .3], [.4, .6])	([.2, .5], [.2, .3])
h_4	([.3, .4], [.3, .4])	([.2, .4], [.3, .5])	([.2, .5], [.3, .4])	([.2, .4], [.3, .5])

Then P∗P:

U	(e_1, e_1)	(e_1, e_2)	(e_2, e_1)	(e_2, e_2)
h_1	([.3, .4], [.3, .4])	([.2, .4], [.3, .6])	([.2, .4], [.3, .4])	([.2, .4], [.3, .6])
h_2	([1, 1], [0, 0])	([.1, .2], [0, 0])	([.4, .5], [.1, .3])	([.4, .7], [.1, .3])
h_3	([.2, .6], [.1, .4])	([.2, .6], [.1, .3])	([.2, .3], [.4, .6])	([.2, .5], [.2, .3])
h_4	([.3, .4], [.3, .4])	([.2, .4], [.3, .5])	([.2, .4], [.3, .4])	([.2, .4], [.3, .5])

Thus, P∗P⊂P and so P is a transitive IVIFSS- relation.

5. Solution of a decision making problem

The concept of interval valued intuitionistic fuzzy soft relations can be used effectively for solving a wide range of decision making problems. Using the interval valued intuitionistic fuzzy soft relations there is an inherent reduction in the computational effect. This fact is illustrated by the following real life problem:

Let U={h_1, h_2, h_3, h_4, h_5} be the set of five houses and E={e_1(expensive), e_2(wooden), e_3(beautiful), e_4(cheap), e_5(in the green surroundings), e_6(concrete), e_7(in the main town), e_8(moderate)} be the set of parameters. Let us consider the four soft sets (F_1, A_1), (F_2, A_2), (F_3, A_3), (F_4, A_4) which describes the 'cost of the houses', 'attractiveness of the houses', 'physical trait of the houses', 'characteristic of the place where the houses are located' respectively. Now suppose that Mr. X is interested in buying a house on the basis of his choice of parameters 'beautiful', 'wooden', 'cheap', 'in the green surroundings'. This implies that from the available houses in U, he will select the house which satisfies with all the parameters of his choice.

Step-1: Let the tabular representations of the above soft sets are given by respectively: (F_1, A_1):

U	e_1	e_4
h_1	([0.2, 0.6], [0.3, 0.4])	([0.2, 0.4], [0.3, 0.5])
h_2	([0.1, 0.2], [0.4, 0.6])	([0.4, 0.5], [0.1, 0.3])
h_3	([0.3, 0.6], [0.2 , 0.3])	([0.2, 0.5], [0.1, 0.2])
h_4	([0.1, 0.3], [0.3, 0.5])	([0.3, 0.4], [0.3, 0.5])
h_5	([0.5,0.7], [0.1, 0.2])	([0.6, 0.8], [0.1, 0.2])

(F_2, A_2):

U	e_3	e_8
h_1	([0.5, 0.6], [0.1, 0.2])	([0.2, 0.3], [0.4, 0.6])
h_2	([0.2, 0.4], [0.3, 0.4])	([0.6, 0.7], [0.1, 0.2])
h_3	([0.3, 0.5], [0.3 , 0.4])	([0.4, 0.6], [0.3, 0.4])
h_4	([0.1, 0.3], [0.4, 0.5])	([0.5, 0.6], [0.1, 0.3])
h_5	([0.6,0.8], [0.1, 0.2])	([0.1, 0.2], [0.5, 0.6])

(F_3, A_3):

U	e_2	e_6
h_1	([0.6, 0.7], [0.1, 0.2])	([0.2, 0.5], [0.3, 0.4])
h_2	([0.2, 0.5], [0.2, 0.4])	([0.6, 0.8], [0.1, 0.2])
h_3	([0.4, 0.6], [0.1 , 0.4])	([0.3, 0.6], [0.3, 0.4])
h_4	([0.3, 0.4], [0.4, 0.5])	([0.1, 0.2], [0.5, 0.7])
h_5	([0.2,0.3], [0.3, 0.7])	([0.5, 0.6], [0.2, 0.3])

(F_4, A_4):

U	e_5	e_7
h_1	([0.4, 0.5], [0.3, 0.4])	([0.5, 0.6], [0.2, 0.3])
h_2	([0.2, 0.4], [0.3, 0.4])	([0.1, 0.4], [0.2, 0.5])
h_3	([0.1, 0.3], [0.5 , 0.6])	([0.4, 0.6], [0.1, 0.2])
h_4	([0.3, 0.5], [0.2, 0.4])	([0.1, 0.3], [0.4, 0.6])
h_5	([0.5, 0.7], [0.1, 0.2])	([0.2, 0.3], [0.5, 0.7])

Step-2: To solve this problem let us consider a soft set relation ρ=(F, $A_1 \times A_2 \times A_3 \times A_4$), where F: $A_1 \times A_2 \times A_3 \times A_4 \to$ IVIFS(U) is defined by

$F((e_i, e_j), (e_k, e_l)) = \{\langle h_\lambda, [min(inf\mu_{F_1(e_i)}(h_\lambda), inf\mu_{F_2(e_j)}(h_\lambda), inf\mu_{F_3(e_k)}(h_\lambda), inf\mu_{F_4(e_l)}(h_\lambda)), min(sup\mu_{F_1(e_i)}(h_\lambda), sup\mu_{F_2(e_j)}(h_\lambda), sup\mu_{F_3(e_k)}(h_\lambda), sup\mu_{F_4(e_l)}(h_\lambda))], [min(inf\gamma_{F_1(e_i)}(h_\lambda), inf\gamma_{F_2(e_j)}(h_\lambda), inf\gamma_{F_3(e_k)}(h_\lambda), inf\gamma_{F_4(e_l)}(h_\lambda)), min(sup\gamma_{F_1(e_i)}(h_\lambda), sup\gamma_{F_2(e_j)}(h_\lambda), sup\gamma_{F_3(e_k)}(h_\lambda), sup\gamma_{F_4(e_l)}(h_\lambda))]\rangle : h_\lambda \in U\}$

where $e_i \in A_1$, $e_j \in A_2$, $e_k \in A_3$, $e_l \in A_4$. Then we have,

$F((e_4, e_3), (e_2, e_5)) = \{< h_1, [min(0.2, 0.5, 0.6, 0.4), min(0.4, 0.6, 0.7, 0.5)], [min(0.3, 0.1, 0.1, 0.3), min(0.5, 0.2, 0.2, 0.4)] >, < h_2, [min(0.4, 0.2, 0.2, 0.2), min(0.5, 0.4, 0.5, 0.4)], [min(0.1, 0.3, 0.2, 0.3), min(0.3, 0.4, 0.4, 0.4)] >, < h_3, [min(0.2, 0.3, 0.4, 0.1), min(0.5, 0.5, 0.6, 0.3)], [min(0.1, 0.3, 0.1, 0.5), min(0.2, 0.4, 0.4, 0.6)] >, < h_4, [min(0.3, 0.1, 0.3, 0.3), min(0.4, 0.3, 0.4, 0.5)], [min(0.3, 0.4, 0.4, 0.2), min(0.5, 0.5, 0.5, 0.4)] >, < h_5, [min(0.6, 0.6, 0.2, 0.5), min(0.8, 0.8, 0.3, 0.7)], [min(0.1, 0.1, 0.3, 0.1), min(0.2, 0.2, 0.7, 0.2)] >\}$

i.e.,

$F((e_4, e_3), (e_2, e_5)) = \{< h_1, [0.2, 0.4], [0.1, 0.2] >, < h_2, [0.2, 0.4], [0.1, 0.3] >, < h_3, [0.1, 0.3], [0.1, 0.2] >, < h_4, [0.1, 0.3], [0.2, 0.4] >, < h_5, [0.2, 0.3], [0.1, 0.2] >\}$.

Step-3: We define the score of h_i (denoted by S(h_i)) by: $\forall h_j \in U$, S(h_i)=the total number of times so that supμ(h_i))$\geq$supμ(h_j)) for j$\neq$i, $h_i \in U$.

Then we have, S(h_1)=4, S(h_2)=4, S(h_3)=2, S(h_4)=2, S(h_5)=2.

Step-4: Now we define the decision set as D={$h_i \in U$: S(h_i) is maximum}.

Then we have D={h_1, h_2}. So Mr. X will buy any one house between h_1 and h_2.

6. Lower and upper soft interval valued intuitionistic fuzzy rough approximations of an IVIFSS-relation

Definition 6.1. Let R$\in\rho_U$(A×A) and R=(H, A× A). Let Θ=(f, B) be an interval valued intuitionistic fuzzy soft set over U and S=(U, Θ) be the soft interval valued intuitionistic fuzzy approximation space. Then the the lower and upper soft interval valued intuitionistic fuzzy rough approximations of R with respect to S are denoted by Lwr_S(R) and Upr_S(R) respectively, which are IVIFSS- relations over A×B in U given by:

Lwr_S(R)=(J, A× B) and Upr_S(R)=(K, A×B) where

J(e_i, e_k)={<x, [$\bigwedge_{e_j\in A}$(inf$\mu_{H(e_i,e_j)}$(x)$\bigwedge$inf$\mu_{f(e_k)}$(x)), $\bigwedge_{e_j\in A}$(sup$\mu_{H(e_i,e_j)}$(x)$\bigwedge$ sup$\mu_{f(e_k)}$(x))], [$\bigwedge_{e_j\in A}$(inf$\gamma_{H(e_i,e_j)}$(x)$\bigvee$inf$\gamma_{f(e_k)}$(x)), $\bigwedge_{e_j\in A}$(sup$\gamma_{H(e_i,e_j)}$(x) $\bigvee$ sup$\gamma_{f(e_k)}$(x))]>: x∈U},

K(e_i , e_k) ={<x, [$\bigwedge_{e_j\in A}$(inf$\mu_{H(e_i,e_j)}$(x)$\bigvee$inf$\mu_{f(e_k)}$(x)), $\bigwedge_{e_j\in A}$(sup$\mu_{H(e_i,e_j)}$(x) $\bigvee$sup$\mu_{f(e_k)}$(x))], [$\bigwedge_{e_j\in A}$(inf$\gamma_{H(e_i,e_j)}$(x)$\bigwedge$inf$\gamma_{f(e_k)}$(x)), $\bigwedge_{e_j\in A}$(sup$\gamma_{H(e_i,e_j)}$(x) $\bigwedge$ sup$\gamma_{f(e_k)}$(x))]>: x∈U}, for e_i ∈A, e_k∈B.

Theorem 6.2. *Let Θ=(f, B) be an interval valued intuitionistic fuzzy soft set over U and S=(U, Θ) be the soft approximation space. Let R_1, $R_2\in\rho_U(A\times A)$ and R_1=(G, A ×A) and R_2=(H, A ×A). Then*

(i) $Lwr_S(O_U)=O_U$
(ii) $Upr_S(I_U)=I_U$
(iii) $R_1\subseteq R_2\Rightarrow Lwr_S(R_1)\subseteq Lwr_S(R_2)$
(iv) $R_1\subseteq R_2\Rightarrow Upr_S(R_1)\subseteq Upr_S(R_2)$
(v) $Lwr_S(R_1\bigcap R_2)\subseteq Lwr_S(R_1)\bigcap Lwr_S(R_2)$
(vi) $Upr_S(R_1\bigcap R_2)\subseteq Upr_S(R_1)\bigcap Upr_S(R_2)$
(vii) $Lwr_S(R_1)\bigcup Lwr_S(R_2)\subseteq Lwr_S(R_1\bigcup R_2)$
(viii) $Upr_S(R_1)\bigcup Upr_S(R_2)\subseteq Upr_S(R_1\bigcup R_2)$

Proof. (i)-(iv) are straight forward.

(v) Let Lwr$_S$($R_1\bigcap R_2$)=(S, A ×B). Then for (e_i , e_k)∈ A× B, we have

S(e_i, e_k)={<x,[$\bigwedge_{e_j\in A}$(inf$\mu_{(G\bullet H))(e_i,e_j)}(x)\bigwedge$inf$\mu_{f(e_k)}$(x)), $\bigwedge_{e_j\in A}$($sup\mu_{(G\bullet H)(e_i,e_j)}(x)$$\bigwedge$sup$\mu_{f(e_k)}$(x))],[$\bigwedge_{e_j\in A}$(inf$\gamma_{(G\bullet H))(e_i,e_j)}(x)\bigvee$inf$\gamma_{f(e_k)}$(x)), $\bigwedge_{e_j\in A}$($sup\gamma_{(G\bullet H)(e_i,e_j)}(x)$$\bigvee$ sup$\gamma_{f(e_k)}$(x))]>: x∈U}

$=\{<x,[\bigwedge_{e_j\in A}(min(inf\mu_{G(e_i,e_j)}(x),inf\mu_{H(e_i,e_j)}(x))\bigwedge inf\mu_{f(e_k)}(x)),$
$\bigwedge_{e_j\in A}(min(sup\mu_{G(e_i,e_j)}(x),sup\mu_{H(e_i,e_j)}(x))\bigwedge sup\mu_{f(e_k)}(x))],$
$[\bigwedge_{e_j\in A}(max(inf\gamma_{G(e_i,e_j)}(x),inf\gamma_{H(e_i,e_j)}(x))\bigvee inf\gamma_{f(e_k)}(x)),$
$\bigwedge_{e_j\in A}(max(sup\gamma_{G(e_i,e_j)}(x),sup\gamma_{H(e_i,e_j)}(x))\bigvee sup\gamma_{f(e_k)}(x))]>: x\in U\}.$

Also for $Lwr_S(R_1)\bigcap Lwr_S(R_2)=(Z,A\times B)$ and $(e_i,e_k)\in A\times B$, we have,
$Z(e_i,e_k)=\{<x,[min(\bigwedge_{e_j\in A}(inf\mu_{G(e_i,e_j)}(x)\bigwedge inf\mu_{f(e_k)}(x)),\bigwedge_{e_j\in A}(inf\mu_{H(e_i,e_j)}(x)\bigwedge inf\mu_{f(e_k)}(x))),min(\bigwedge_{e_j\in A}(sup\mu_{G(e_i,e_j)}(x)\bigwedge sup\mu_{f(e_k)}(x)),\bigwedge_{e_j\in A}(sup\mu_{H(e_i,e_j)}(x)\bigwedge sup\mu_{f(e_k)}(x)))],[max(\bigwedge_{e_j\in A}(inf\gamma_{G(e_i,e_j)}(x)\bigvee inf\gamma_{f(e_k)}(x)),\bigwedge_{e_j\in A}(inf\gamma_{H(e_i,e_j)}(x)\bigvee inf\gamma_{f(e_k)}(x))),max(\bigwedge_{e_j\in A}(sup\gamma_{G(e_i,e_j)}(x)\bigvee sup\gamma_{f(e_k)}(x)),\bigwedge_{e_j\in A}(sup\gamma_{H(e_i,e_j)}(x)\bigvee sup\gamma_{f(e_k)}(x)))]>: x\in U\}$

Now since $min(inf\mu_{G(e_i,e_j)}(x),inf\mu_{H(e_i,e_j)}(x))\leq inf\mu_{G(e_i,e_j)}(x)$ and $min(inf\mu_{G(e_i,e_j)}(x),inf\mu_{H(e_i,e_j)}(x))\leq inf\mu_{H(e_i,e_j)}(x))$,we have
$\bigwedge_{e_j\in A}(min(inf\mu_{G(e_i,e_j)}(x),inf\mu_{H(e_i,e_j)}(x))\bigwedge inf\mu_{f(e_k)}(x))\leq$
$min(\bigwedge_{e_j\in A}(inf\mu_{G(e_i,e_j)}(x)\bigwedge inf\mu_{f(e_k)}(x)),\bigwedge_{e_j\in A}(inf\mu_{H(e_i,e_j)}(x)$
$\bigwedge inf\mu_{f(e_k)}(x))).$

Similarly we can get $\bigwedge_{e_j\in A}(min(sup\mu_{G(e_i,e_j)}(x),sup\mu_{H(e_i,e_j)}(x))\bigwedge sup\mu_{f(e_k)}(x))\leq$

$min(\bigwedge_{e_j\in A}(sup\mu_{G(e_i,e_j)}(x)\bigwedge sup\mu_{f(e_k)}(x)),\bigwedge_{e_j\in A}(sup\mu_{H(e_i,e_j)}(x)\bigwedge sup\mu_{f(e_k)}(x))).$
Again as $max(inf\gamma_{G(e_i,e_j)}(x),inf\gamma_{H(e_i,e_j)}(x))\geq inf\gamma_{G(e_i,e_j)}(x)$ and
$max(inf\gamma_{G(e_i,e_j)}(x),inf\gamma_{H(e_i,e_j)}(x))\geq inf\gamma_{H(e_i,e_j)}(x))$,we have
$\bigwedge_{e_j\in A}(max(inf\gamma_{G(e_i,e_j)}(x),inf\gamma_{H(e_i,e_j)}(x))\bigvee inf\gamma_{f(e_k)}(x))\geq$

$max(\bigwedge_{e_j \in A}(inf\gamma_{G(e_i,e_j)}(x) \bigvee inf\gamma_{f(e_k)}(x)), \bigwedge_{e_j \in A}(inf\gamma_{H(e_i,e_j)}(x)$
$\bigvee inf\gamma_{f(e_k)}(x)))$.
Similarly we can get $\bigwedge_{e_j \in A}(max(sup\gamma_{G(e_i,e_j)}(x), sup\gamma_{H(e_i,e_j)}(x)) \bigvee sup\gamma_{f(e_k)}(x)) \geq$
$max(\bigwedge_{e_j \in A}(sup\gamma_{G(e_i,e_j)}(x) \bigvee sup\gamma_{f(e_k)}(x)), \bigwedge_{e_j \in A}(sup\gamma_{H(e_i,e_j)}(x)$
$\bigvee sup\gamma_{f(e_k)}(x)))$.

Consequently, $Lwr_S(R_1 \bigcap R_2) \subseteq Lwr_S(R_1) \bigcap Lwr_S(R_2)$.
(vi) Proof is similar to (v).
(vii) Let $Lwr_S(R_1 \bigcup R_2)$=(S, A ×B). Then for (e_i , e_k)∈ A× B, we have
$S(e_i, e_k)=\{<x,[\bigwedge_{e_j \in A}(\inf\mu_{(G\blacklozenge H))(e_i,e_j)}(x) \bigwedge \inf\mu_{f(e_k)}(x)),$
$\bigwedge_{e_j \in A}(\sup\mu_{(G\blacklozenge H))(e_i,e_j)}(x) \bigwedge \sup\mu_{f(e_k)}(x))],$
$[\bigwedge_{e_j \in A}(\inf\gamma_{(G\blacklozenge H))(e_i,e_j)}(x) \bigvee \inf\gamma_{f(e_k)}(x)),$
$\bigwedge_{e_j \in A}(\sup\gamma_{(G\blacklozenge H))(e_i,e_j)}(x) \bigvee \sup\gamma_{f(e_k)}(x))]>: x \in U\}$
$=\{< x, [\bigwedge_{e_j \in A}(max(inf\mu_{G(e_i,e_j)}(x), inf\mu_{H(e_i,e_j)}(x)) \bigwedge inf\mu_{f(e_k)}(x)),$
$\bigwedge_{e_j \in A}(max(sup\mu_{G(e_i,e_j)}(x), sup\mu_{H(e_i,e_j)}(x)) \bigwedge sup\mu_{f(e_k)}(x))],$
$[\bigwedge_{e_j \in A}(min(inf\gamma_{G(e_i,e_j)}(x), inf\gamma_{H(e_i,e_j)}(x)) \bigvee inf\gamma_{f(e_k)}(x)),$
$\bigwedge_{e_j \in A}(min(sup\gamma_{G(e_i,e_j)}(x), sup\gamma_{H(e_i,e_j)}(x)) \bigvee sup\gamma_{f(e_k)}(x))] >: x \in U\}$.
Also for $Lwr_S(R_1) \bigcup Lwr_S(R_2) = (Z, A \times B)$ and $(e_i, e_k) \in A \times B$, we have,
$Z(e_i, e_k) = \{< x, [max(\bigwedge_{e_j \in A}(inf\mu_{G(e_i,e_j)}(x) \bigwedge inf\mu_{f(e_k)}(x)), \bigwedge_{e_j \in A}(inf\mu_{H(e_i,e_j)}(x)$
$\bigwedge inf\mu_{f(e_k)}(x))), max(\bigwedge_{e_j \in A}(sup\mu_{G(e_i,e_j)}(x) \bigwedge sup\mu_{f(e_k)}(x)), \bigwedge_{e_j \in A}(sup\mu_{H(e_i,e_j)}(x)$
$\bigwedge sup\mu_{f(e_k)}(x)))], [min(\bigwedge_{e_j \in A}(inf\gamma_{G(e_i,e_j)}(x) \bigvee inf\gamma_{f(e_k)}(x)), \bigwedge_{e_j \in A}(inf\gamma_{H(e_i,e_j)}(x)$
$\bigvee inf\gamma_{f(e_k)}(x))), min(\bigwedge_{e_j \in A}(sup\gamma_{G(e_i,e_j)}(x) \bigvee sup\gamma_{f(e_k)}(x)), \bigwedge_{e_j \in A}(sup\gamma_{H(e_i,e_j)}(x)$
$\bigvee sup\gamma_{f(e_k)}(x)))] >: x \in U\}$
Now since $max(inf\mu_{G(e_i,e_j)}(x), inf\mu_{H(e_i,e_j)}(x)) \geq inf\mu_{G(e_i,e_j)}(x)$ and
$max(inf\mu_{G(e_i,e_j)}(x), inf\mu_{H(e_i,e_j)}(x)) \geq inf\mu_{H(e_i,e_j)}(x))$,we have
$\bigwedge_{e_j \in A}(max(inf\mu_{G(e_i,e_j)}(x), inf\mu_{H(e_i,e_j)}(x)) \bigwedge inf\mu_{f(e_k)}(x)) \geq$
$max(\bigwedge_{e_j \in A}(inf\mu_{G(e_i,e_j)}(x) \bigwedge inf\mu_{f(e_k)}(x)), \bigwedge_{e_j \in A}(inf\mu_{H(e_i,e_j)}(x)$
$\bigwedge inf\mu_{f(e_k)}(x)))$.

Similarly we can get
$\bigwedge_{e_j \in A}(max(sup\mu_{G(e_i,e_j)}(x), sup\mu_{H(e_i,e_j)}(x)) \bigwedge sup\mu_{f(e_k)}(x)) \geq$
$max(\bigwedge_{e_j \in A}(sup\mu_{G(e_i,e_j)}(x) \bigwedge sup\mu_{f(e_k)}(x)), \bigwedge_{e_j \in A}(sup\mu_{H(e_i,e_j)}(x) \bigwedge sup\mu_{f(e_k)}(x)))$.
Again as $min(inf\gamma_{G(e_i,e_j)}(x), inf\gamma_{H(e_i,e_j)}(x)) \leq inf\gamma_{G(e_i,e_j)}(x)$ and
$min(inf\gamma_{G(e_i,e_j)}(x), inf\gamma_{H(e_i,e_j)}(x)) \leq inf\gamma_{H(e_i,e_j)}(x))$,we have
$\bigwedge_{e_j \in A}(min(inf\gamma_{G(e_i,e_j)}(x), inf\gamma_{H(e_i,e_j)}(x)) \bigvee inf\gamma_{f(e_k)}(x)) \leq$
$min(\bigwedge_{e_j \in A}(inf\gamma_{G(e_i,e_j)}(x) \bigvee inf\gamma_{f(e_k)}(x)), \bigwedge_{e_j \in A}(inf\gamma_{H(e_i,e_j)}(x)$
$\bigvee inf\gamma_{f(e_k)}(x)))$.
Similarly we can get $\bigwedge_{e_j \in A}(min(sup\gamma_{G(e_i,e_j)}(x), sup\gamma_{H(e_i,e_j)}(x)) \bigvee sup\gamma_{f(e_k)}(x)) \leq$
$min(\bigwedge_{e_j \in A}(sup\gamma_{G(e_i,e_j)}(x) \bigvee sup\gamma_{f(e_k)}(x)), \bigwedge_{e_j \in A}(sup\gamma_{H(e_i,e_j)}(x)$
$\bigvee sup\gamma_{f(e_k)}(x)))$.

Consequently, $Lwr_S(R_1) \bigcup Lwr_S(R_2) \subseteq Lwr_S(R_1 \bigcup R_2)$.
(viii) Proof is similar to (vii). □

7. Conclusion

In the present paper we extend the concepts of relations on intuitionistic fuzzy soft sets to interval valued intuitionistic fuzzy soft set relations (IVIFSS-relations for short). We also made an attempt to form the topological structure based on IVIFSS-relations. We have discussed various types of IVIFSS-relations with suitable examples and presented a solution to a decision making problem. It is expected that the approach will be useful to handle several realistic uncertain problems. Lastly we have studied the lower and upper soft interval valued intuitionistic fuzzy rough approximations of an IVIFSS-relation.

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