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Fantastic filters and their fuzzification in BE-algebras

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Abstract. The concept of fantastic filters is introduced in BE-algebras. A necessary and sufficient condition is established for every filter of a BE-algebra to become a fantastic filter. Fuzzification of fantastic filters is also considered in BE-algebras. Some properties of fuzzy fantastic filters are studied with respect to fuzzy relations and cartesian products.

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1. Introduction

The notion of BE-algebras was introduced and extensively studied by H.S. Kim and Y.H. Kim in [6]. Some properties of filters of BE-algebras were studied by S.S. Ahn and K.S. So in [1] and then by H.S. Kim and Y.H. Kim in [6]. The concepts of a fuzzy set and a fuzzy relation on a set was initially defined by L.A. Zadeh [11]. Fuzzy relations on a group have been studied by Bhattacharya and Mukherjee [2]. In 1996, Y.B. Jun and S.M. Hong [5] discussed the fuzzy deductive systems of Hilbert algebras. Later, W.A. Dudek and Y.B. Jun [3] considered the fuzzification of ideals in Hilbert algebras and discussed the relation between fuzzy ideals and fuzzy deductive systems. In [8], the author introduced the notion of fuzzy filters in BE-algebras and discussed some related properties. Recently, the concept of fuzzy implicative filters [9] is introduced in BE-algebra and studied the properties of these implicative filters. Later in [10], the author studied some more properties of fuzzy filters and also normal fuzzy filters of BE-algebras.

In this paper, the notion of fantastic filters is introduced in BE-algebras analogous to that in a lattice implication algebra. A necessary and sufficient condition is established for every filter of a BE-algebra to become a fantastic filter. The concept of fuzzy fantastic filters is also introduced in a BE-algebra. Some properties of fuzzy fantastic filters are studied. A necessary and sufficient condition is derived for every fuzzy filter of a BE-algebra to become a fuzzy fantastic filter. Properties of fuzzy fantastic filters are studied in terms of fuzzy relations and cartesian products.

2. Preliminaries

In this section, we present certain definitions and results which are taken mostly from the papers [6], [8], [9] and [11] for the ready reference of the reader.

Definition 2.1 ([6]). An algebra $(X, *, 1)$ of type $(2, 0)$ is called a BE-algebra if it satisfies the following properties:

(1) $x * x = 1$
(2) $x * 1 = 1$
(3) $1 * x = x$
(4) $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$

Theorem 2.2 ([6]). *Let $(X, *, 1)$ be a BE-algebra. Then we have the following:*

(1) $x * (y * x) = 1$
(2) $x * ((x * y) * y)) = 1$

We introduce a relation $\leq$ on a BE-algebra X by $x \leq y$ implies $x * y = 1$. A BE-algebra X is called self-distributive if $x*(y*z) = (x*y)*(x*z)$ for all $x, y, z \in X$.

Definition 2.3 ([6]). Let $(X, *, 1)$ be a BE-algebra. A non-empty subset F of X is called a filter of X if, for all $x, y \in X$, it satisfies the following properties:

(a) $1 \in F$
(b) $x \in F$ and $x * y \in F$ imply that $y \in F$

Definition 2.4 ([11]). Let X be a set. Then a fuzzy set in X is a function $\mu : X \longrightarrow [0, 1]$.

Definition 2.5 ([8]). Let X be a BE-algebra. A fuzzy set μ of X is called a fuzzy filter if it satisfies the following properties, for all $x, y \in X$:

(F_1) $\mu(1) \geq \mu(x)$
(F_2) $\mu(y) \geq min\{\mu(x), \mu(x * y)\}$

Definition 2.6 ([8]). Let μ be a fuzzy set in a BE-algebra X. For any $\alpha \in [0, 1]$, the set $\mu_\alpha = \{x \in X \mid \mu(x) \geq \alpha\}$ is called a level subset of X.

Definition 2.7 ([9]). A fuzzy relation on a set S is a fuzzy set $\mu : S \times S \longrightarrow [0, 1]$.

Definition 2.8 ([9]). Let μ be a fuzzy relation on a set S and ν a fuzzy set in S. Then μ is a fuzzy relation on ν if for all $x, y \in S$, it satisfies

$$\mu(x, y) \leq min\{\nu(x), \nu(y)\}$$

Definition 2.9 ([9]). Let μ and ν be two fuzzy sets in a BE-algebra X. Then the cartesian product of μ and ν is defined by

$$(\mu \times \nu)(x, y) = min\{\mu(x), \nu(y)\}$$

for all $x, y \in X$.

3. Fantastic filters

In this section, the notion of fantastic filters is introduced in BE-algebras. Some properties of fantastic filters of BE-algebras are studied. A necessary and sufficient conditions is derived for any filter of a BE-algebra to become a fantastic filter.

Definition 3.1. A subset F of a BE-algebra X is called a fantastic filter if, for all $x, y, z \in X$, it satisfies the following properties.

$(F1)$ $1 \in F$

$(F2)$ $z * (y * x) \in F$ and $z \in F$ imply that $((x * y) * y) * x \in F$

Example 3.2. Let $X = \{1, a, b, c\}$ be a non-empty set. Define a binary operation $*$ on X as follows:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	b
b	1	a	1	b	a
c	1	a	1	1	a
d	1	1	1	b	1

Then it can be easily verified that $(X, *, 1)$ is a BE-algebra. It is easy to check that $F = \{b, c, 1\}$ is a fantastic filter in X.

Proposition 3.3. *Every fantastic filter of a BE-algebra is a filter.*

Proof. Let F be a fantastic filter of a BE-algebra X. Let $x, y \in X$ be such that $x \in F$ and $x * y \in F$. Then we get $x * (1 * y) = x * y \in F$ and $x \in F$. Hence by condition $(F2)$, we get $y = ((y * 1) * 1) * y \in F$. Therefore F is a filter in X. □

Since every fantastic filter of a BE-algebra is a filter, the following corollary is a direct consequence of the above proposition.

Corollary 3.4. *Let F be a fantastic filter of X. Then the following hold.*

(1) *$x \in F$ and $x \leq y$ imply that $y \in F$*

(2) *$x \leq y * z$ implies $z \in F$*

The converse of the above Proposition 3.3 is not true in general. It can be seen in the following example.

Example 3.5. Let $X = \{1, a, b, c\}$ be a non-empty set. Define a binary operation $*$ on X as follows:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	a	1	c	c
c	1	1	b	1	b
d	1	1	1	1	1

Then $(X, *, 1)$ is a BE-algebra. It is clear that $F = \{1\}$ is a filter in X but it is not a fantastic filter because of $1 * ((c * a) * c) = 1 \in F$ and $1 \in F$. We can see that $((a * c) * c) * a = (c * c) * a = 1 * a = a \notin F$.

However, we now derive a set of equivalent conditions for every filter of a BE-algebra to become a fantastic filter.

Theorem 3.6. *Let F be a filter of a BE-algebra X. Then F is a fantastic filter if and only if it satisfies the following condition.*

$$y * x \in F \textit{ implies } ((x * y) * y) * x \in F \quad \textit{for all } x, y \in X$$

Proof. Assume that F is a fantastic filter of X. Let $x, y \in X$ and $y * x \in F$. Then clearly $1 * (y * x) = y * x \in F$ and $1 \in F$. Since F is a fantastic filter, we get that $((x * y) * y) * x \in F$. Conversely, assume the condition. Let $x, y, z \in X$. Suppose $z * (y * x) \in F$ and $z \in F$. Since F is a filter, we get that $y * x \in F$. Hence by the condition, we get $((x * y) * y) * x \in F$. Therefore F is a fantastic filter in X. □

We now discuss the fuzzification of fantastic filters in BE-algebras.

Definition 3.7. A fuzzy set μ of a BE-algebra X is called a fuzzy fantastic filter of X if it satisfies the following properties.

(1) $\mu(1) \geq \mu(x)$

(2) $\mu(((x * y) * y) * x) \geq min\{\mu(z), \mu(z * (y * x))\}$ for all $x, y, z \in X$

Proposition 3.8. *Every fuzzy fantastic filter of a BE-algebra is a fuzzy filter.*

Proof. Let μ be a fuzzy fantastic filter of a BE-algebra X. Hence we get the following.

$$\begin{aligned} \mu(y) &= \mu((((y * 1) * 1) * y)) \\ &\geq min\{\mu(z), \mu(z * (1 * y))\} \\ &= min\{\mu(z), \mu(z * y)\} \end{aligned}$$

Therefore μ is a fuzzy filter in X. □

The converse of the above proposition is not true. That is a fuzzy filter of a BE-algebra is not a fuzzy fantastic filter as shown in the following example.

Example 3.9. Let $X = \{1, a, b, c, d\}$ be a non-empty set. Define a binary operation $*$ on X as follows:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	a	1	c	c
c	1	1	b	1	b
d	1	1	1	1	1

Then it can be easily verified that $(X, *, 1)$ is a BE-algebra. Define a fuzzy set μ on X as follows:

$$\mu(x) = \begin{cases} 0.8 & \text{if } x = 1 \\ 0.3 & \text{otherwise} \end{cases}$$

for all $x \in X$. Then clearly μ is a fuzzy filter of X, but μ is not a fuzzy fantastic filter of X since $\mu(c) \ngeq min\{\mu(a), \mu(a * ((b * c) * b))\}$.

We now derive a necessary and sufficient condition for every fuzzy filter of a BE-algebra to become a fuzzy fantastic filter.

Theorem 3.10. *A fuzzy filter μ of a BE-algebra X is a fuzzy fantastic filter if and only if it satisfies the following condition.*

$$\mu(((x*y)*y)*x) \geq \mu(y*x) \text{ for all } x,y \in X$$

Proof. Let μ be a fuzzy filter of X. Assume that μ is a fuzzy fantastic filter in X. Let $x, y \in X$. Since μ is a fuzzy fantastic filter, we get

$$\begin{aligned} \mu(((x*y)*y)*x) &\geq min\{\mu(1), \mu(1*(y*x))\} \\ &= \mu(1*(y*x)) \\ &= \mu(y*x) \end{aligned}$$

Conversely, assume that the condition holds in X. Clearly $\mu(1) \geq \mu(x)$ for all $x \in X$. Let $x, y \in X$. Since μ is a fuzzy filter, by the assumed condition, we get

$$\begin{aligned} \mu(((x*y)*y)*x) &\geq \mu(y*x) \\ &\geq min\{\mu(z), \mu(z*(y*x))\} \end{aligned}$$

Therefore μ is a fuzzy fantastic filter in X. □

Theorem 3.11. *Let F be a non-empty subset of a BE-algebra X. Define a fuzzy set $\mu_F : X \longrightarrow [0,1]$ as follows:*

$$\mu_F(x) = \begin{cases} \alpha & \text{if } x \in F \\ 0 & \text{if } x \notin F \end{cases}$$

where $0 < \alpha < 1$ is fixed. Then μ_F is a fuzzy fantastic filter in X if and only if F is a fantastic filter in X.

Proof. Assume that μ_F is a fuzzy fantastic filter in X. Since $\mu_F(1) \geq \mu_F(x)$ for all $x \in X$, we get $\mu_F(1) = \alpha$ and hence $1 \in F$. Let $x, y, z \in X$ be such that $z, z*(y*x) \in F$. Then $\mu_F(z) = \mu_F(z*(y*x)) = \alpha$. Since μ_F is a fuzzy fantastic filter, we get the following:

$$\mu_F(((x*y)*y)*x) \geq min\{\mu_F(z), \mu_F(z*(y*x))\} = min\{\alpha, \alpha\} = \alpha$$

Hence we get that $\mu_F(((x*y)*y)*x) = \alpha$ and so $((x*y)*y)*x \in F$. Therefore it concludes that F is a fantastic filter in X.

Conversely, assume that F is a fantastic filter of X. Since $1 \in F$, we get that $\mu_F(1) = \alpha$. Hence $\mu_F(1) \geq \mu_F(x)$ for all $x \in X$. Let $x, y, z \in X$. Suppose $z, z*(y*x) \in F$. Since F is a fantastic filter, we get $((x*y)*y)*x \in F$. Then $\mu_F(z) = \mu_F(z*(y*x)) = \mu_F(((x*y)*y)*x) = \alpha$. Hence $\mu_F(((x*y)*y)*x) \geq min\{\mu_F(z), \mu_F(z*(y*x))\}$. Suppose $z*(y*x) \notin F$ and $z \notin F$. Then $\mu_F(z) = \mu_F(z*(y*x)) = 0$. Hence $\mu_F(((x*y)*y)*x) \geq min\{\mu_F(z), \mu_F(z*(y*x))\}$. If exactly one of z and $z*(y*x)$ is in F, then exactly one of $\mu_F(z)$ and $\mu_F(z*(y*x))$ is equal to 0. Hence $\mu_F(((x*y)*y)*x) \geq min\{\mu_F(z), \mu_F(z*(y*x))\}$. By summarizing the above results, we get $\mu_F(((x*y)*y)*x) \geq min\{\mu_F(z), \mu_F(z*(y*x))\}$ for all $x, y, z \in X$. Therefore μ_F is a fuzzy fantastic filter of X. □

Proposition 3.12. *Let μ be a fuzzy set in a BE-algebra X. Then μ is a fuzzy fantastic filter in X if and only if for each $\alpha \in [0,1]$, the level subset μ_α is a fantastic filter in X, when $\mu_\alpha \neq \varnothing$.*

Proof. Assume that μ is a fuzzy fantastic filter of X. Then $\mu(1) \geq \mu(x)$ for all $x \in X$. In particular, $\mu(1) \geq \mu(x) \geq \alpha$ for all $x \in \mu_\alpha$. Hence $1 \in \mu_\alpha$. Let $z, z*(y*x) \in \mu_\alpha$. Then $\mu(z) \geq \alpha$ and $\mu(z*(y*x)) \geq \alpha$. Since μ is a fuzzy fantastic filter, we get $\mu(((x*y)*y)*x) \geq min\{\mu(z), \mu(z*(y*x))\} \geq \alpha$. Thus it concludes that $((x*y)*y)*x \in \mu_\alpha$. Therefore μ_α is a fantastic filter in X.

Conversely, assume that μ_α is a fantastic filter of X for each $\alpha \in [0,1]$ with $\mu_\alpha \neq \varnothing$. Suppose there exists $x_0 \in X$ such that $\mu(1) < \mu(x_0)$. Let $\alpha_0 = \frac{1}{2}(\mu(1) + \mu(x_0))$. Then $\mu(1) < \alpha_0$ and $0 \leq \alpha_0 < \mu(x_0) \leq 1$. Hence $x_0 \in \mu_{\alpha_0}$ and $\mu_{\alpha_0} \neq \varnothing$. Since μ_{α_0} is a fantastic filter in X, we get $1 \in \mu_{\alpha_0}$ and hence $\mu(1) \geq \alpha_0$, which is a contradiction. Therefore $\mu(1) \geq \mu(x)$ for all $x \in X$. Let $x, y, z \in X$ be such that $\mu(z*(y*x)) = \alpha_1$ and $\mu(z) = \alpha_2$. Then $z*(y*x) \in \mu_{\alpha_1}$ and $z \in \mu_{\alpha_2}$. Without loss of generality, assume that $\alpha_1 \leq \alpha_2$. Then clearly $\mu_{\alpha_2} \subseteq \mu_{\alpha_1}$. Hence $z \in \mu_{\alpha_1}$. Since μ_{α_1} is a fantastic filter in X, we get $((x*y)*y)*x \in \mu_{\alpha_1}$. Thus it concludes that $\mu(((x*y)*y)*x) \geq \alpha_1 = min\{\alpha_1, \alpha_2\} = \min\{\mu(z*(y*x)), \mu(z)\}$. Therefore μ is a fuzzy fantastic filter of X. □

For any $\alpha \in [0,1]$, the above fantastic filter μ_α of a BE-algebra is called a level fantastic filter. Now, in the following, we derive a necessary and sufficient condition for two level fantastic filters of a BE-algebra to become equal.

Theorem 3.13. *Let μ be a fuzzy fantastic filter of a BE-algebra X. Then two level fantastic filters μ_{α_1} and μ_{α_2}(with $\alpha_1 < \alpha_2$) of μ are equal if and only if there is no $x \in X$ such that $\alpha_1 \leq \mu(x) < \alpha_2$.*

Proof. Assume that $\mu_{\alpha_1} = \mu_{\alpha_2}$ for $\alpha_1 < \alpha_2$. Suppose there exists some $x \in X$ such that $\alpha_1 \leq \mu(x) < \alpha_2$. Then μ_{α_2} is a proper subset of μ_{α_1}, which is impossible. Conversely, assume that there is no $x \in X$ such that $\alpha_1 \leq \mu(x) < \alpha_2$. Since $\alpha_1 < \alpha_2$, we get that $\mu_{\alpha_2} \subseteq \mu_{\alpha_1}$. If $x \in \mu_{\alpha_1}$, then $\mu(x) \geq \alpha_1$. Hence by the assume condition, we get $\mu(x) \geq \alpha_2$. Hence $x \in \mu_{\alpha_2}$ and so $\mu_{\alpha_1} \subseteq \mu_{\alpha_2}$. Therefore $\mu_{\alpha_1} = \mu_{\alpha_2}$. □

Definition 3.14. Let $f : X \longrightarrow Y$ be a homomorphism of BE-algebras and μ is a fuzzy set in Y. Then define a mapping $\mu^f : X \longrightarrow [0,1]$ such that $\mu^f(x) = \mu(f(x))$ for all $x \in X$.

Clearly the above mapping μ^f is well-defined and a fuzzy set in X.

Theorem 3.15. *Let $f : X \longrightarrow Y$ be an onto homomorphism of BE-algebras and μ is a fuzzy set in Y. Then μ is a fuzzy fantastic filter in Y if and only if μ^f is a fuzzy fantastic filter in X.*

Proof. Assume that μ is a fuzzy fantastic of Y. For any $x \in X$, we have $\mu^f(1) = \mu(f(1)) = \mu(1') \geq \mu(f(x)) = \mu^f(x)$. Let $x, y, z \in X$. Then

$$\begin{aligned}
\mu^f(((x*y)*y)*x) &= \mu(f(((x*y)*y)*x)) \\
&= \mu(((f(x)*f(y))*f(y))*f(x)) \\
&\geq min\{\mu(f(z)), \mu(f(z)*(f(y)*f(x)))\} \\
&= min\{\mu(f(z)), \mu(f(z*(y*x)))\} \\
&= min\{\mu^f(z), \mu^f(z*(y*x))\}
\end{aligned}$$

Hence μ^f is a fuzzy fantastic filter of X. Conversely, assume that μ^f is a fuzzy fantastic filter of X. Let $x \in Y$. Since f is onto, there exists $y \in X$ such that $f(y) = x$. Then $\mu(1') = \mu(f(1)) = \mu^f(1) \geq \mu^f(y) = \mu(f(y)) = \mu(x)$. Let $x, y, z \in Y$. Then there exist $a, b, c \in X$ such that $f(a) = x, f(b) = y$ and $f(c) = z$. Hence we get

$$\begin{aligned} \mu(((x*y)*y)*x) &= \mu(((f(a)*f(b))*f(b))*f(a)) \\ &= \mu(f(((a*b)*b)*a)) \\ &= \mu^f(((a*b)*b)*a) \\ &\geq min\{\mu^f(c), \mu^f(c*(b*a))\} \\ &= min\{\mu(f(c)), \mu(f(c)*(f(b)*f(a)))\} \\ &= min\{\mu(z), \mu(z*(y*x))\} \end{aligned}$$

Therefore μ is a fuzzy fantastic filter in Y. □

The following Lemma is a direct consequence.

Lemma 3.16. *Let μ and ν be two fuzzy sets in a BE-algebra X. Then the following conditions hold.*

(1) *$\mu \times \nu$ is a fuzzy relation on X*
(2) *$(\mu \times \nu)_\alpha = \mu_\alpha \times \nu_\alpha$ for all $\alpha \in [0, 1]$*

For any two BE-algebras X and Y, define an operation $*$ on $X \times Y$ as follows:

$(x, y) * (x', y') = (x * x', y * y')$ for all $x, x' \in X$ and $y, y' \in Y$

Then it can be easily observed that $(X \times Y, *, (1, 1))$ is a BE-algebra.

Theorem 3.17. *Let μ and ν be two fuzzy fantastic filters of a BE-algebra X. Then $\mu \times \nu$ is a fuzzy fantastic filter in $X \times X$.*

Proof. Let $(x, y) \in X \times X$. Since μ, ν are fuzzy fantastic filters in X, we get

$$\begin{aligned} (\mu \times \nu)(1, 1) &= min\{\mu(1), \nu(1)\} \\ &\geq min\{\mu(x), \nu(y)\} \qquad \text{for all } x, y \in X \\ &= (\mu \times \nu)(x, y) \end{aligned}$$

Let $(x, x'), (y, y'), (z, z') \in X \times X$ be such that $x_0 = (x, x'), y_0 = (y, y')$ and $z_0 = (z, z')$. Put $t = z * (y * x)$ and $t' = z' * (y' * x')$. Clearly $(t, t') = z_0 * (y_0 * x_0)$. Since μ and ν are fuzzy fantastic filters in X, we can obtain the following consequence.

$$\begin{aligned} (\mu \times \nu)(((x_0 * y_0) * y_0) * x_0) &= (\mu \times \nu)(((x*y)*y)*x, ((x'*y')*y')*x') \\ &= min\{\mu(((x*y)*y)*x), \nu(((x'*y')*y')*x')\} \\ &\geq min\{min\{\mu(z), \mu(t)\}, min\{\nu(z'), \nu(t')\}\} \\ &= min\{min\{\mu(z), \nu(z')\}, min\{\mu(t), \nu(t')\}\} \\ &= min\{(\mu \times \nu)(z, z'), (\mu \times \nu)(t, t')\} \\ &= min\{(\mu \times \nu)(z_0), (\mu \times \nu)(z_0 * (y_0 * x_0))\} \end{aligned}$$

Therefore $\mu \times \nu$ is a fuzzy fantastic filter in $X \times X$. □

Definition 3.18. Let ν be a fuzzy set in a BE-algebra X. Then the strongest fuzzy relation μ_ν is a fuzzy relation on X defined by

$$\mu_\nu(x,y) = min\{\nu(x),\nu(y)\}$$

for all $x, y \in X$.

Theorem 3.19. *Let ν be a fuzzy set in a BE-algebra X and μ_ν the strongest fuzzy relation on X. Then ν is a fuzzy fantastic filter in X if and only if μ_ν is a fuzzy fantastic filter of $X \times X$.*

Proof. Assume that ν is a fuzzy fantastic of X. Then for any $(x,y) \in X \times X$, we have the following:

$$\mu_\nu(x,y) = min\{\nu(x),\nu(y)\} \leq min\{\nu(1),\nu(1)\} = \mu_\nu(1,1)$$

Let $(x,x'),(y,y'),(z,z') \in X \times X$ be such that $x_0 = (x,x'), y_0 = (y,y')$ and $z_0 = (z,z')$. Put $t = z*(y*x)$ and $t' = z'*(y'*x')$. Clearly $(t,t') = z_0*(y_0*x_0)$.

$$\begin{aligned}
\mu_\nu(((x_0*y_0)*y_0)*x_0) &= \mu_\nu(((x*y)*y)*x,((x'*y')*y')*x') \\
&= min\{\nu(((x*y)*y)*x),\nu((x'*y')*y')*x')\} \\
&\geq min\{min\{\nu(z),\nu(t)\},min\{\nu(z'),\nu(t')\}\} \\
&= min\{min\{\nu(z),\nu(z')\},min\{\nu(t),\nu(t')\}\} \\
&= min\{\mu_\nu(z,z'),\mu_\nu(t,t')\} \\
&= min\{\mu_\nu(z,z'),\mu_\nu(z*(y*x),z'*(y'*x'))\} \\
&= min\{\mu_\nu(z_0),\mu_\nu(z_0*(y_0*x_0))\}
\end{aligned}$$

Therefore μ_ν is a fuzzy fantastic filter in $X \times X$. Conversely, assume that μ_ν is a fuzzy fantastic filter in $X \times X$. Then

$$\nu(1) = min\{\nu(1),\nu(1)\} = \mu_\nu(1,1) \geq \mu_\nu(x,x) = min\{\nu(x),\nu(x)\} = \nu(x)$$

for all $x \in X$. Hence it yields that $\nu(x) \leq \nu(1)$ for all $x \in X$. Let $x, y, z \in X$. Then we have the following consequence.

$$\begin{aligned}
\nu(((x*y)*y)*x) &= min\{\nu((x*y)*y)*x),\nu(1)\} \\
&= \mu_\nu(((x*y)*y)*x,1) \\
&\geq min\{\mu_\nu(z,1),\mu_\nu((z,1)*((y,1)*(x,1)))\} \\
&= min\{\mu_\nu(z,1),\mu_\nu(z*(y*x),1*(1*1))\} \\
&= min\{\mu_\nu(z,1),\mu_\nu(z*(y*x),1)\} \\
&= min\{min\{\nu(z),\nu(1)\},min\{\nu(z*(y*x)),\nu(1)\}\} \\
&= min\{\nu(z),\nu(z*(y*x))\}
\end{aligned}$$

Therefore ν is a fuzzy fantastic filter in X. □

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