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Chromatic number of fuzzy graphs

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Abstract. Coloring of fuzzy graphs plays a vital role in theory and practical applications. The concept of chromatic number of fuzzy graphs was introduced by Munoz[7] et.al. Later Eslahchi and Onagh [2]defined fuzzy coloring of fuzzy graphs and defined fuzzy chromatic number $\chi^f(G)$. Incorporating the features of these two definitions, the definition of chromatic number of a fuzzy graph $\chi(G)$, is modified in terms of chromatic number of threshold graph G_α and established that $\chi^f(G) = \chi(G)$. The advantage of this definition is that the chromatic number of a fuzzy graph can be obtained directly from the chromatic number of threshold graph G_α, which is a crisp graph. Also algorithms are proposed to find chromatic number of crisp graph and fuzzy graph. The solution for the banquet problem is found using the chromatic number of the corresponding fuzzy graph.

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1. Introduction

Many practical problems such as scheduling, allocation, network problems etc can be modeled as coloring problems and hence coloring is one of the most studied areas in the research of graph theory. A large number of variations in coloring of graphs is available in literature. With the emergence of fuzzy set theory and fuzzy graph theory, most of the real situations are modeled with more precision and flexibility than their classical counterparts. More research is done on fuzzy graphs [1],[5],[8],interval valued fuzzy graphs[10] and the applications of coloring of fuzzy graphs since in many practical situations, the relation between nodes will show various degrees of incompatibility [7],[9].

The paper is organized as follows:
The existing definitions of coloring of fuzzy graphs are studied and incorporating the features of both definitions, the concept of chromatic number of a fuzzy graph in terms of threshold graph is defined and it is established that the chromatic number based on this concept is same as the fuzzy chromatic number by Eslahchi and Onagh [2]. Algorithms to find the chromatic number of a crisp graph and a similar algorithm to find the chromatic number of a fuzzy graph is proposed. Finally the banquet problem is solved using chromatic number of fuzzy graph.

2. Preliminaries

The following basic definitions are taken from [6]. A fuzzy graph is an ordered triple $G : (V, \sigma, \mu)$ where V is a set of vertices $\{u_1, u_2...u_n\}$, σ is a fuzzy subset[11] of V i.e, $\sigma : V \to [0, 1]$ and is denoted by $\sigma = \{(u_1, \sigma(u_1)), (u_2, \sigma(u_2))...(u_n, \sigma(u_n))\}$ and μ is a fuzzy relation on σ, i.e $\mu(u, v) \leq \sigma(u) \wedge \sigma(v) \forall u, v \in V$. We consider fuzzy graph G with no loops and assume that V is finite and nonempty, μ is reflexive (i.e.,$\mu(u, u) = \sigma(u), \forall u \in V$) and symmetric(i.e., $\mu(u, v) = \mu(v, u), \forall(u, v) \in V \times V$). In all the examples σ is chosen suitably. Also, we denote the underlying crisp graph of G by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V : \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V : \mu(u, v) > 0\}$. Through out we assume that $\sigma^* = V$. The level set of fuzzy set σ is defined as $L_\sigma = \{\alpha/\sigma(u) = \alpha$ for some $u \in \sigma^*\}$ and the level set of μ is defined as $L_\mu = \{\alpha/\mu(u, v) = \alpha$ for some $(u, v) \in \mu^*\}$. The fundamental set of the fuzzy graph $G = (V, \sigma, \mu)$ is defined as $L = L_\sigma \bigcup L_\mu$. For each $\alpha \in L$, G_α denotes the crisp graph $G_\alpha = (\sigma_\alpha, \mu_\alpha)$ where $\sigma_\alpha = \{u \in V/\sigma(u) \geq \alpha\}$, $\mu_\alpha = \{(u, v) \in V \times V/\mu(u, v) \geq \alpha\}$.

A path P of length n is a sequence of distinct nodes
$u_0, u_1, ...u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, ..., n$ and the degree of membership of a weakest arc in P is defined as the strength of P. If $u_0 = u_n$ and $n \geq 3$ then P is called a cycle and P is called a fuzzy cycle, if it contains more than one weakest arc. The strength of a cycle is the strength of the weakest arc in it. A fuzzy cycle of length n is denoted by C_n. The maximum strength among all paths from u to v is denoted by $CONN_G(u, v)$. A fuzzy graph $G : (\sigma, \mu)$ is connected if for every u, v in $\sigma^*, CONN_G(u, v) > 0$.

The fuzzy graph $H = (S, \nu, \tau)$ is called a fuzzy subgraph of $G : (V, \sigma, \mu)$ induced by S, if $S \subset V$, $\nu(u) = \sigma(u)$ for all $u \in S$ and $\tau(u, v) = \mu(u, v)$ for all $u, v \in S$. The notation $\langle S \rangle$ is used to denote the fuzzy subgraph of G induced by S.

Let X be a universal set. The standard fuzzy union [3]of two fuzzy sets A and B over X with membership functions σ_A and σ_B respectively is the fuzzy set $A \bigcup B$ with membership function $\sigma_{A \bigcup B} : X \to [0, 1]$ defined as $\sigma_{A \bigcup B}(x) = \max \{\sigma_A(x), \sigma_B(x)\} \ \forall x \in X$.

3. Major section : coloring of fuzzy graphs

A coloring of a crisp graph $G = (V, E)$ [4] is an assignment of colors to its vertices so that no two adjacent vertices have the same color(also called proper coloring). The set of all vertices with any one color is independent and is called a color class. A k coloring of a graph G uses k colors, and thereby partitions V into k color classes. The chromatic number $\chi(G)$ is defined as the minimum number k for which G has a proper k coloring. Also note that G_α is a crisp graph and $\chi_\alpha = \chi(G_\alpha)$ denote the chromatic number of G_α.

The concept of chromatic number of fuzzy graph was introduced by Munoz et al.[7]. The authors considered fuzzy graphs with crisp vertex set i.e fuzzy graphs for which $\sigma(x) = 1 \forall x \epsilon V$ and edges with membership degree in $[0, 1]$.

Definition 3.1 ([7])**.** If $G : (V, \mu)$ is such a fuzzy graph where $V = \{1, 2, 3...n\}$ and μ is a fuzzy number on the set of all subsets of $V \times V$. Assume $I = A \cup \{0\}$ where $A = \{\alpha_1 < \alpha_2 < ... < \alpha_k\}$ is the fundamental set (level set) of G. For each $\alpha \in I, G_\alpha$ denote the crisp graph $G_\alpha = (V, E_\alpha)$ where $E_\alpha = \{ij/1 \le i < j \le n, \mu(i, j) \ge \alpha\}$ and $\chi_\alpha = \chi(G_\alpha)$ denote the chromatic number of crisp graph G_α. By this definition the chromatic number of fuzzy graph G is the fuzzy number $\chi(G) = \{(i, \nu(i))/i \in X\}$ where $\nu(i) = \max \{\alpha \in I/i \in A_\alpha\}$ and $A_\alpha = \{1, 2, 3..., \chi_\alpha\}$.

Definition 3.2 ([2])**.** Later Eslahchi and Onagh introduced fuzzy coloring of fuzzy graphs as follows. A family $\Gamma = \{\gamma_1, \gamma_2, ...\gamma_k\}$ of fuzzy sets on a set V is called a k-fuzzy coloring of $G = (V, \sigma, \mu)$ if

(i) $\vee\Gamma = \sigma$,

(ii) $\gamma_i \wedge \gamma_j = 0$,

(iii) for every strong edge (x, y) (i.e $\mu(x, y) > 0$) of G, min $\{\gamma_i(x), \gamma_i(y)\} = 0 (1 \le i \le k)$.

The minimum number k for which there exists a k-fuzzy coloring is called the fuzzy chromatic number of G, denoted as $\chi^f(G)$.

Remark 3.3. In fuzzy coloring given in [2], (i) $\vee\Gamma = \sigma$, means no vertex belongs to two different color classes. (iii) for every strong edge (x, y) of G, min $\{\gamma_i(x), \gamma_i(y)\} = 0, (1 \le i \le k)$, means that one of the vertices is having color 0 in a particular color class. This is exactly the same as crisp coloring in which no vertex is given two different colors and the membership value of one of the adjacent vertices in a particular color class is zero. This means that fuzzy coloring partitions the vertex set into different color classes and the number of distinct color classes is the fuzzy chromatic number.

Incorporating the features of the above two definitions , the chromatic number $\chi(G)$ of fuzzy graph is modified. It is established that $\chi^f(G) = \chi(G)$

Definition 3.4. The chromatic number of fuzzy graph $G : (V, \sigma, \mu)$ is defined as $\chi(G) = \max \{(\chi_\alpha/\alpha \in L\}$ where $\chi_\alpha = \chi(G_\alpha)$.

Remark 3.5. If $\alpha_i < \alpha_j$, $\chi_{\alpha_i} \ge \chi_{\alpha_j}$.

Theorem 3.6. *For a fuzzy graph $G : (V, \sigma, \mu)$, $\chi(G) = \chi^f(G)$.*

Proof. Let $G = (V, \sigma, \mu)$ be a fuzzy graph on n vertices, $\{u_1, u_2...u_n\}$. Let $\chi^f(G) = k$.
$\Leftrightarrow \Gamma = \{\gamma_1, \gamma_2, ...\gamma_k\}$ is a k-fuzzy coloring and let C_j be the color assigned to vertices in $\gamma_j^*, j = 1, 2, ...k$.
$\Leftrightarrow \{\gamma_1, \gamma_2, ...\gamma_k\}$ is a family of fuzzy sets where
$\gamma_j(u_i) = \{(u_j, \sigma(u_j))\} \bigcup \{(u_i, \sigma(u_i))/\mu(u_i, u_j) = 0, i \ne j\}$ which follows from (i) and (iii) of Definition 3.2. Also $\bigcup_{j=1}^{k} \gamma_j^* = V$ and $\gamma_i^* \cap \gamma_j^* = \phi, i \ne j$ which follows from

(ii) of Definition 3.2.
$\Leftrightarrow \gamma_j^*$ is an independent set of vertices (i.e no two vertices in γ_j^* are adjacent) for each $j = 1, 2, ...k$. $\Leftrightarrow \chi(G^*) = k$, G^* is the underlying crisp graph of G.
Now, $\chi(G^*) = \chi(G_t) = k$ where $t = \min\ \{\alpha/\alpha \in L\} = \max\ \{\chi_\alpha/\alpha\epsilon L\}$ (by remark 3.5) and hence the proof. □

Example 3.7. Consider the following fuzzy graph G and the crisp graphs $G_{.5}$, $G_{.4}$, $G_{.3}$, $G_{.2}$ and $G_{.1}$ given in Fig I corresponding to the values in the level set of the edges of the fuzzy graph, $L = \{.5, .4, .3, .2, .1\}$

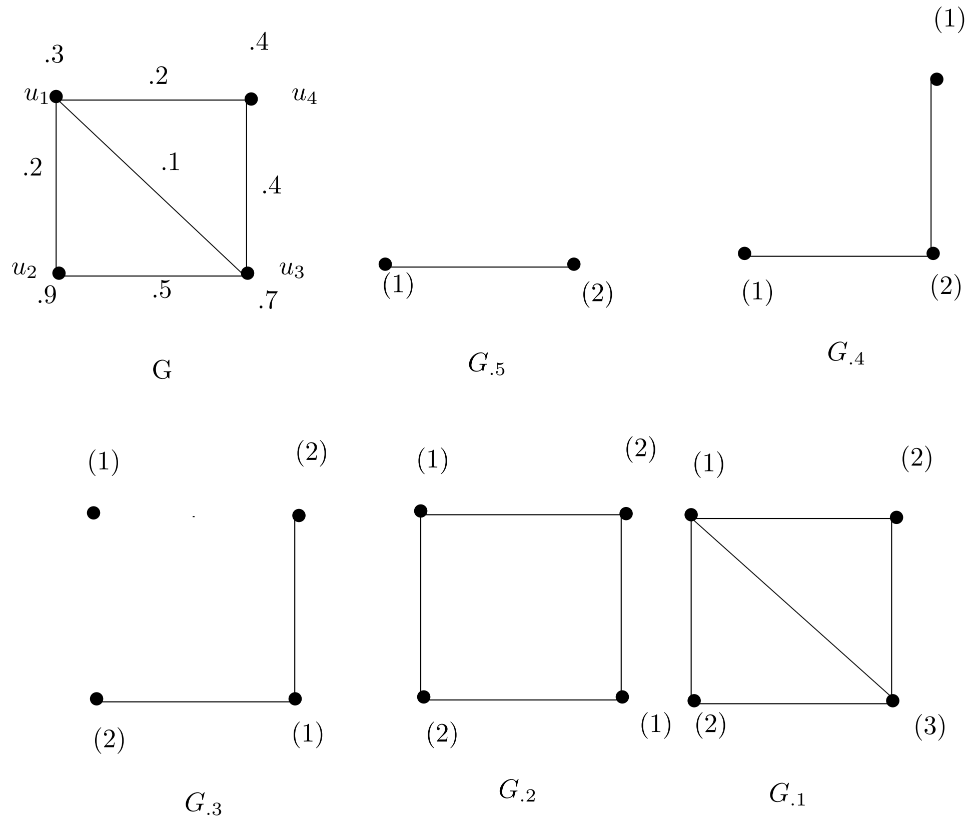

Fig I

The fuzzy coloring is $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$,

$$\gamma_1(u_i) = \begin{cases} .3 & i = 1 \\ 0 & otherwise. \end{cases}$$

$$\gamma_2(u_i) = \begin{cases} .9 & i = 2 \\ .4 & i = 4 \\ 0 & otherwise. \end{cases}$$

$$\gamma_3(u_i) = \begin{cases} .7 & i = 3 \\ 0 & otherwise. \end{cases}$$

Hence $\chi^f(G) = 3$ and the crisp graph coloring yields $\chi_{.5} = \chi_{.4} = \chi_{.3} = \chi(G_{.2}) = 2$

and $\chi_{.1} = 3$ (The integers at the vertices denotes colors).
Here $\chi(G^*) = \chi_{.1} = 3 = \max\ \{\chi_\alpha/\alpha \epsilon L\}$.

4. Algorithm (chromatic number of a crisp graph)

An algorithm based on fuzzy chromatic number defined by Eslahchi and Onagh [2] is developed for crisp graph as follows.
Let $G = (V, E)$ be a crisp graph with vertex set $V = \{u_1, u_2...u_n\}$ and edge set E.
Step I: Initialize $k' = 0$.
Step II: If $|V| = 0$, return $\chi(G) = 0$.
Step III: Choose a family of color classes $\Gamma = \{\gamma_1, \gamma_2, ...\gamma_n\}$
where $\gamma_j(u_i) = \{u_j\} \bigcup \{u_i/(u_i, u_j) \notin E, i \neq j\}$ such that $(u_i, u_m) \notin E$, $u_i, u_m \in \gamma_j$, $i \neq m$
Step IV: If $\exists$ a subfamily $\Gamma' = \{\gamma_1, \gamma_2, ...\gamma_k\}$ such that
$\gamma_i \bigcap \gamma_j = \phi$, $i \neq j$, $i, j = 1, 2...k$ and $\sum_{j=1}^{k} |\gamma_j|$ is maximum, let $S = V - \bigcup_{j=1}^{k} \gamma_j$.
Step V: If$|S| = 0$,
{
If $k = 1$, then $\chi(G) = 1$ and Goto Step VII
else if $1 < k \leq n$, assign color C_j to γ_j and then $\chi(G) = k$ and Go to Step VII.
}
Step VI: While $|S| \neq 0$
Assign color C_j to γ_j, $j = 1, 2...k$ and then $\chi(G) = k$.
Let $k' = \chi(G) + k'$.
Put $G = \langle S \rangle$ and Go to Step III.
Step VII: $\chi(G) = k + k'$
Step VIII: End

Example 4.1. Consider Fig II where the graph $G = (V, E)$

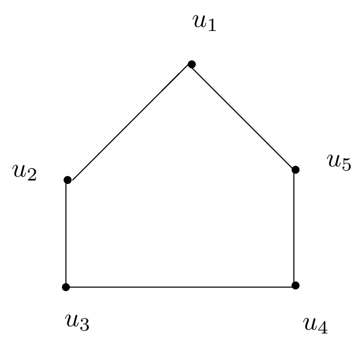

Fig II

Here $\Gamma = \{\gamma_1, \gamma_2, ...\gamma_5\}$, $\gamma_1 = \{u_1, u_3\}$, $\gamma_2 = \{u_2, u_4\}$, $\gamma_3 = \{u_3, u_5\}$, $\gamma_4 = \{u_4, u_1\}$, $\gamma_5 = \{u_5, u_2\}$.
One of the subfamilies of Γ satisfying the condition $\gamma_i \bigcap \gamma_j = \phi$, $i \neq j$ is $\Gamma' = \{\gamma_1, \gamma_2\}$.
Hence $S = \{u_5\}$

Assigning colors C_1 and C_2 to γ_1 and γ_2 respectively, $k = 2$ and $k' = 2$. Put $G = \langle S \rangle$. But since $|S| = 1$, assign a different color C_3 to $\gamma = \{u_5\}$ and $\chi(G) = 1$. Hence new $k' = 2 + 1 = 3$. i.e the chromatic number $\chi(G) = 3$.

A similar algorithm to find the chromatic number of a fuzzy graph is given as the following:

5. Algorithm (chromatic number of a fuzzy graph)

Let $G = (V, \sigma, \mu)$ be a fuzzy graph with n vertices $V = \{u_1, u_2 ... u_n\}$.
Step I: Initialize $k' = 0$.
Step II: If $|V| = 0$, return $\chi(G) = 0$.
Step III: Choose a collection of fuzzy sets $\{\gamma_1, \gamma_2, ... \gamma_n\}$ where
$\gamma_j(u_i) = \{(u_j, \sigma(u_j))\} \bigcup \{(u_i, \sigma(u_i))/\mu(u_i, u_j) = 0, i \neq j\}$(where $\bigcup$ stands for standard fuzzy union), such that $\mu(u_i, u_m) = 0$, $u_i, u_m \in \gamma_j^*$, $i \neq m$.
Step IV: If $\exists$ a subfamily $\Gamma' = \{\gamma_1, \gamma_2, ... \gamma_k\}$ such that
$\gamma_i \wedge \gamma_j = 0$, $i \neq j$, $i, j = 1, 2...k$ and $\sum_{j=1}^{k} |\gamma_j^*|$ is maximum, let $S = V - \bigcup_{j=1}^{k} \gamma_j^*$.
Step V: If$|S| = 0$,
{
If $k = 1$, then $\chi(G) = 1$ and Goto Step VII
else if $1 < k \leq n$, assign color C_j to γ_j^* and then $\chi(G) = k$ and Go to Step VII.
}
Step VI: While $|S| \neq 0$
Assign color C_j to $\gamma_j^*, j = 1, 2...k$ and then $\chi(G) = k$.
Let $k' = \chi(G) + k'$.
Put $G = \langle S \rangle$ and Go to Step III.
Step VII: $\chi(G) = k + k'$
Step VIII: End

Example 5.1. Consider the following fuzzy graph $G = (V, \sigma, \mu)$, given in Fig III.

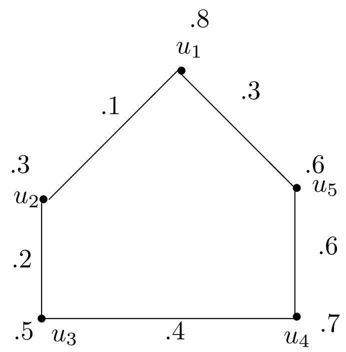

Fig III

Here $\Gamma = \{\gamma_1, \gamma_2, ...\gamma_5\}$ where $\gamma_1(u_i) = \{(u_1, .8)\} \bigcup \{(u_4, .7)\} = \{(u_1, .8), (u_4, .7)\}$. Hence $\gamma_1^* = \{u_1, u_4\}$,
$\gamma_2(u_i) = \{(u_2, .3)\} \bigcup \{(u_5, .6)\} = \{(u_2, .3)\}, \{(u_5, .6)\}$ and $\gamma_2^* = \{u_2, u_5\}$,
$\gamma_3(u_i) = \{(u_3, .5)\} \bigcup \{(u_5, .6)\} = \{(u_3, .5)\}, \{(u_5, .6)\}$ and $\gamma_3^* = \{u_3, u_5\}$,
$\gamma_4(u_i) = \{(u_4, .7)\} \bigcup \{(u_1, .8)\} = \{(u_4, .7)\}, \{(u_1, .8)\}$ $\gamma_4^* = \{u_4, u_1\}$,
$\gamma_5(u_i) = \{(u_5, .6)\} \bigcup \{(u_2, .3)\}$ and $\gamma_5^* = \{u_5, u_2\}$. One of the subfamilies of Γ satisfying $\gamma_i \wedge \gamma_j = 0$ is $\Gamma' = \{\gamma_1, \gamma_2\}$. Hence $S = \{u_3\}$. Assigning colors C_1 and C_2 to γ_1^* and γ_2^* respectively, $k = 2$ and $k' = 2$. Next put $G = \langle S \rangle$. But since $|S| = 1$ only one color is required to color $G = \langle S \rangle$. Hence $\Gamma' = \{\gamma\}$ where $\gamma = \{(u_3, .5)\}$ and $\gamma^* = \{u_3\}$. Assigning color C_3 to γ^*, $\chi(G) = 1$. Hence new $k' = 2 + 1 = 3$ and the chromatic number $\chi(G) = 3$.

6. Application

An approach to solve the banquet problem by using chromatic number of fuzzy graphs is given below:

Banquet Problem

The Problem:

The menu for a banquet includes a number of cooked dishes which must be prepared in advance. A dish d must be cooked for $m(d)$ minutes at a temperature interval (not necessarily a constant i.e it varies as high, medium and low). If there is only one oven, to determine the optimal scheduling of dishes so that total cooking time is minimized.

Solution:

Consider a fuzzy graph model whose vertices represent dishes and two vertices are adjacent if their corresponding dishes have no common temperature levels (and hence cannot be in the oven at the same time). The temperature intervals insures that there is always a common acceptable temperature for all the dishes being simultaneously cooked.

Consider the fuzzy graph $G = (V, \sigma, \mu)$ with V as the set of all dishes, $\sigma(u_i) = 1 \forall u_i \in V$, $\mu(u_i, u_j)$ is assigned membership values as follows .(i) If u_i is having temperature value high and u_j is having low temperature, $\mu(u_i, u_j)$ is assigned a high value to indicate the degree of incompatibility between the temperature levels. (ii)If u_i is having temperature value medium and u_j is having low temperature, $\mu(u_i, u_j)$ is assigned a medium value since the degree of incompatibility is not as high as the previous case and (iii)if u_i is having temperature value high and u_j is having medium temperature, $\mu(u_i, u_j)$ is assigned a value comparatively lesser than both the other cases . Thus $\mu(u_i, u_j)$ indicates the degree of incompatibility between the dishes u_i and u_j, that cannot be cooked at the same time. Consider the threshold graph G_α for different values of $\alpha = \mu(u_i, u_j)$ and determine the chromatic number $\chi(G_\alpha)$ which represents the number of times the oven works. The objective is to minimize the number of times the oven works which is same as the chromatic number of the fuzzy graph $\chi(G) = max_\alpha \ \{\chi_\alpha\}$.

Example 6.1. Consider an example of a banquet problem, Fig IV where six dishes are to be cooked, say $V = \{u_1, u_2, u_3, u_4, u_5, u_6\}$.

The temperature levels are given as follows:

u_1 and $u_4 \rightarrow$ high temperature.
u_2 and $u_5 \rightarrow$ medium temperature.
u_3 and $u_6 \rightarrow$ low temperature.
Assigning membership values to edges(between those dishes having varying temperature levels) say as given below.
$\mu(u_1, u_2) = 0.5$, $\mu(u_1, u_5) = 0.4$, $\mu(u_2, u_4) = 0.3$, $\mu(u_4, u_5) = 0.2$,$\mu(u_1, u_3) = 0.9$, $\mu(u_1, u_6) = 0.8$, $\mu(u_4, u_3) = 0.85$, $\mu(u_4, u_6) = 0.95$, $\mu(u_2, u_3) = 0.7$, $\mu(u_2, u_6) = 0.6$, $\mu(u_3, u_5) = 0.65$ and $\mu(u_6, u_5) = 0.75$.

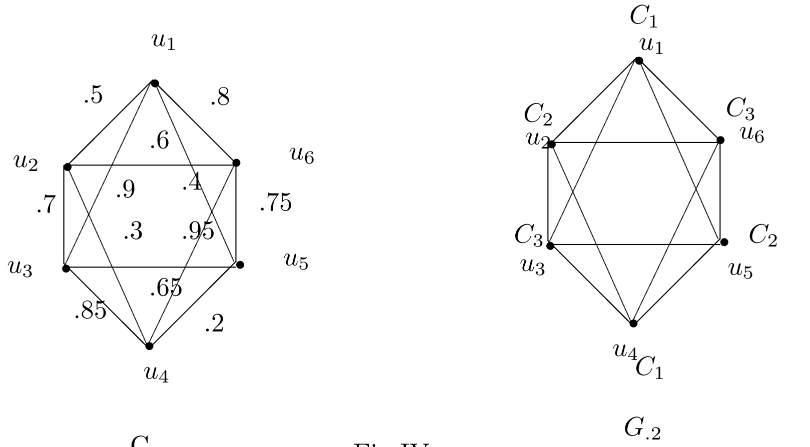

Fig IV

Note that $\chi_{.2} = \chi_{.3} = \chi_{.4} = \chi_{.5} = 3$, $\chi_{.6} = \chi_{.65} ... \chi_{.95} = 2$. Hence $\chi(G) = 3$ which gives an optimal working time for the banquet problem. The oven has to work three times to cook all the dishes. The three color classes are $\gamma_1 = \{u_1, u_4\}$, $\gamma_2 = \{u_2, u_5\}$ and $\gamma_3 = \{u_3, u_6\}$ gives the optimal scheduling of dishes.

7. Conclusion

In this paper, we propose algorithms to determine the chromatic number of crisp and fuzzy graphs. We defined the chromatic number of fuzzy graphs in terms of chromatic number of threshold graph G_α which is a crisp graph. This makes coloring of fuzzy graphs easy since the coloring of crisp graphs is a widely studied area in graph theory. The optimal scheduling of banquet problem using chromatic number is also obtained.

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