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A realization of intuitionistic fuzzy ideals of semirings

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Abstract. In this paper a correspondence between the set of all intuitionistic fuzzy ideals of a Γ-semiring and the set of all intuitionistic fuzzy ideals of its operator semirings is established and used them to study some properties of the semiring $\begin{pmatrix} R & \Gamma \\ S & L \end{pmatrix}$.

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1. Introduction

There are many concepts of universal algebras generalizing an associative ring $(R, +, \cdot)$. Some of them have been found to be very useful for solving problems in different areas of applied mathematics and information sciences. Semiring is one such concept. The structure of a semiring, in particular ideals of semiring, provides an algebraic framework for modeling and studying the key factors in these applied areas.

The notion of Γ-semiring was introduced by Rao[6] as a generalization of semiring. The theory of Γ-semirings has been enriched by the introduction of operator semirings of a Γ-semiring by Dutta and Sardar[2]. To make operator semirings effective in the study of Γ-semirings Dutta et al [2] established a correspondence between the ideals of a Γ-semiring S and the ideals of the operator semirings. Saha and Sardar[12] generalized the notion of Γ-semiring to Nobusawa Γ-semiring. A Nobusawa Γ-semiring S is simply Γ-semiring where Γ is also an S-semiring. To each Nobusawa Γ-semiring S, Saha and Sardar [7] associate a semiring called matrix semiring and it was denoted by S_2 or $\begin{pmatrix} R & \Gamma \\ S & L \end{pmatrix}$ where R, L are respectively the right and left operator semirings of the Γ-semiring S. In [11] Sardar, Mandal and Davvaz studied $\begin{pmatrix} R & \Gamma \\ S & L \end{pmatrix}$ via fuzzy subsets and fuzzy h-ideals.

The main aim of this paper is to study the semiring $\begin{pmatrix} R & \Gamma \\ S & L \end{pmatrix}$ via intuitionistic fuzzy ideals.

2. Preliminaries

A *hemiring* (respectively *semiring*)[3] is a nonempty set S on which operations addition and multiplication have been defined such that $(S,+)$ is a commutative monoid with identity 0_S, $(S,\cdot)$ is a semigroup (respectively monoid with identity 1_S), multiplication distributes over addition from either side, $1_S \neq 0_S$ and $0_S s = 0_S = s0_S$ for all $s \in S$.

Let S and Γ be two additive commutative semigroups with zero. Then S is called a Γ-*semiring* if there exists a mapping $S \times \Gamma \times S \longrightarrow S$, $(a,\alpha,b) \mapsto a\alpha b$, satisfying the following conditions:
(1) $(a+b)\alpha c = a\alpha c + b\alpha c$,
(2) $a\alpha(b+c) = a\alpha b + a\alpha c$,
(3) $a(\alpha+\beta)b = a\alpha b + a\beta b$,
(4) $a\alpha(b\beta c) = (a\alpha b)\beta c$,
(5) $0_S\alpha a = 0_S = a\alpha 0_S$,
(6) $a0_\Gamma b = 0_S = b0_\Gamma a$,
for all $a,b,c \in S$ and for all $\alpha,\beta \in \Gamma$. In addition, if there exists a mapping $\Gamma \times S \times \Gamma \longrightarrow \Gamma$, $(\alpha,s,\beta) \mapsto \alpha s\beta$, satisfying the similar conditions as above together with $a\alpha(b\beta c) = (a\alpha b)\beta c = a(\alpha b\beta)c$, then S is called a *Nobusawa* Γ-*semiring*.

For simplification we write 0 instead of 0_S and 0_Γ.

Example 2.1. Let S be the set of all $m \times n$ matrices over $\mathbb{Z}_{\mathbf{0}}^-$ (the set of all non-positive integers) and Γ be the set of all $n \times m$ matrices over $\mathbb{Z}_{\mathbf{0}}^-$. Then S forms a Γ-semiring with usual addition and multiplication of matrices.

Now, we recall the following definitions from [2].

Let S be a Γ-semiring and F be the free additive commutative semigroup generated by $S \times \Gamma$. We define a relation ρ on F as follows:

$$\sum_{i=1}^{m}(x_i,\alpha_i)\ \rho \sum_{j=1}^{n}(y_j,\beta_j) \text{ if and only if } \sum_{i=1}^{m}x_i\alpha_i a = \sum_{j=1}^{n}y_j\beta_j a,$$

for all $a \in S$ $(m,n \in \mathbb{Z}^+)$. Then ρ is a congruence on F. We denote the congruence class containing $\sum_{i=1}^{m}(x_i,\alpha_i)$ by $\sum_{i=1}^{m}[x_i,\alpha_i]$. Then F/ρ is an additive commutative semigroup. Now, F/ρ forms a semiring with the multiplication defined by

$$\left(\sum_{i=1}^{m}[x_i,\alpha_i]\right)\left(\sum_{j=1}^{n}[y_j,\beta_j]\right) = \sum_{i,j}[x_i\alpha_i y_j,\beta_j].$$

We denote this semiring by L and call it the *left operator semiring* of the Γ-semiring S. Dually we define the *right operator semiring* R of the Γ-semiring S.

Let S be a Γ-semiring and L be the left operator semiring of S and R be the right one. If there exists an element $\sum_{i=1}^{m}[e_i,\delta_i]\in L$(resp. $\sum_{j=1}^{n}[\gamma_j,f_j]\in R$) such that $\sum_{i=1}^{m}e_i\delta_i a=a$ (respectively, $\sum_{j=1}^{n}a\gamma_j f_j=a$) for all $a\in S$, then S is said to have the *left unity* $\sum_{i=1}^{m}[e_i,\delta_i]$ (respectively, the *right unity* $\sum_{j=1}^{n}[\gamma_j,f_j]$). Also, if there exists an element $[e,\delta]\in L$ (respectively, $[\gamma,f]\in R$) such that $e\delta a=a$ (respectively, $a\gamma f=a$) for all $a\in S$, then S is said to have the *strong left unity* $[e,\delta]$ (respectively, strong right unity $[\gamma,f]$) [6].

Let S be a Γ-semiring, L be the left operator semiring of S and R be the right one. Let $P\subseteq L$ ($\subseteq R$). According to [2], we define $P^+=\{a\in S:[a,\Gamma]\subseteq P\}$ (respectively, $P^*=\{a\in S:[\Gamma,a]\subseteq P\}$) and for $Q\subseteq S$,

$$Q^{+'}=\left\{\sum_{i=1}^{m}[x_i,\alpha_i]\in L:\ \left(\sum_{i=1}^{m}([x_i,\alpha_i]\right)S\subseteq Q\right\},$$

where $\left(\sum_{i=1}^{m}[x_i,\alpha_i]\right)S$ denotes the set of all finite sums $\sum_{i,k}x_i\alpha_i s_k,\quad s_k\in S$ and

$$Q^{*'}=\left\{\sum_{i=1}^{m}[\alpha_i,x_i]\in R:\ S\left(\sum_{i=1}^{m}([\alpha_i,x_i]\right)\subseteq Q\right\},$$

where $S\left(\sum_{i=1}^{m}[x_i,\alpha_i]\right)$ denotes the set of all finite sums $\sum_{i,k}s_k\alpha_i x_i,\quad s_k\in S$.

For more preliminaries of semirings and Γ-semirings we refer to [3] and[2], respectively.

For some recent works on Γ structures in fuzzy setting we refer to [4], [5], [8], [9], [10], [13].

Throughout this paper unless otherwise mentioned for different elements of L (respectively, R) we take the same index say i whose range is finite.

3. Intuitionistic fuzzy ideal and its correspondence

According to [14] a *fuzzy subset* μ of a non-empty set S is a function $\mu:S\to[0,1]$.

Let μ be a non-empty fuzzy subset of a Γ-semiring S (i.e., $\mu(x)\neq 0$ for some $x\in S$). Then μ is called a *fuzzy left ideal* (respectively, *fuzzy right ideal*) of S if for all $x,y\in S$ and for all $\gamma\in\Gamma$,

(1) $\mu(x+y)\geq\min[\mu(x),\mu(y)]$,

(2) $\mu(x\gamma y)\geq\mu(y)$ (respectively, $\mu(x\gamma y)\geq\mu(x)$).

A *fuzzy ideal* of a Γ-semiring S is a non-empty fuzzy subset of S which is a fuzzy left ideal as well as a fuzzy right ideal of S. Note that if μ is a fuzzy left or right ideal of a Γ-semiring S, then $\mu(0)\geq\mu(x)$ for all $x\in S$.

As an important generalization of the notion of fuzzy sets, Atanassov [1] introduced the concept of an intuitionistic fuzzy set defined on a non-empty set R, denoted by $IFS(R)$. An $IFS(R)$ is an object having the form

$$A = (\mu_A, \lambda_A) = \{x, \mu_A(x), \lambda_A(x) : x \in R\}$$

where the fuzzy sets μ_A and λ_A denote the degree of membership(namely $\mu_A(x)$) and the degree of non-membership(namely $\lambda_A(x)$) of each element $x \in R$ to the set A respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in R$.
According to [1], for every two intuitionistic fuzzy sets $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ in R,we define $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$ for all $x \in R$. Obviously $A = B$ means that $A \subseteq B$ and $B \subseteq A$. Also $A \cap B = \{< x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\} >: x \in R\} = (\mu_A \cap \mu_B, \lambda_A \cup \lambda_B)$, and $A \cup B = \{< x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\} >: x \in R\} = (\mu_A \cup \mu_B, \lambda_A \cap \lambda_B)$,

An intuitionistic fuzzy subset $A = (\mu_A, \lambda_A)$ of a Γ-semiring R is called an *intuitionistic fuzzy left ideal* if

(1) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all $x, y \in R$.
(2) $\lambda_A(x + y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$, for all $x, y \in R$.
(3) $\mu_A(x\gamma y) \geq \mu_A(y)$, for all $x, y \in R$ and for all $\gamma \in \Gamma$.
(4) $\lambda_A(x\gamma y) \leq \lambda_A(y)$, for all $x, y \in R$ and for all $\gamma \in \Gamma$.

Example 3.1. let us consider $R = N, \Gamma = N$ with usual addition(+) and multiplication (.) defined on natural numbers. Then $(R, +, \Gamma)$ is a Γ-semiring.
Consider $A = (\mu_A, \lambda_A)$ where

$$\mu_A(x) = \begin{cases} 1 \text{ for } x = 0 \\ 0.8 \text{ for } x \in pN \sim \{0\} \\ 0.6 \text{ for } x \notin pN \text{ and } x \neq 0 \end{cases}$$

$$\lambda_A(x) = \begin{cases} 0 \text{ for } x = 0 \\ 0.2 \text{ for } x \in pN \sim \{0\} \\ 0.4 \text{ for } x \notin pN \text{ and } x \neq 0 \end{cases}$$

where p is a prime number.Then it is easy to check that A is an *Intuitionistic fuzzy left ideal* of R.

Throughout this paper unless otherwise mentioned S denotes a Nobusawa Γ-semiring with left unity and right unity, R denotes the right operator semiring and L denotes the left operator semiring of the Nobusawa Γ-semiring S and $IFL\text{-}I(S)$, $IFR\text{-}I(S)$ and $IF\text{-}I(S)$ denote respectively the set of all intuitionistic fuzzy left ideals, the set of all intuitionistic fuzzy right ideals and the set of all intuitionistic fuzzy ideals of the Γ-semiring S. The meanings of $IFL\text{-}I(L)$, $IFL\text{-}I(R)$, $IFR\text{-}I(L)$, $IFR\text{-}I(R)$, $IF\text{-}I(L)$, $IF\text{-}I(R)$ are similar, where L and R are respectively the left operator and right operator semirings of the Γ-semiring S. Also, in this section we assume that $\mu(0) = 1$ for a fuzzy left ideal (respectively, fuzzy right ideal, fuzzy ideal) μ of a Γ-semiring S. Similarly, we assume that $\mu(0_L) = 1$ (respectively, $\mu(0_R) = 1$) for a fuzzy left ideal (respectively, fuzzy right ideal, fuzzy ideal) μ of the left operator semiring (respectively, right operator semiring R) of a Γ-semiring S.

Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy subset of L, we define an intuitionistic fuzzy subset $A^+ = (\mu_A^+, \lambda_A^+)$ of S by

$$\mu_A^+(x) = \inf_{\gamma\in\Gamma}\mu_A\left([x,\gamma]\right), \lambda_A^+(x) = \sup_{\gamma\in\Gamma}\lambda_A\left([x,\gamma]\right),$$

for all $x \in S$.
If $B = (\sigma_B, \tau_B)$ is an intuitionistic fuzzy subset of S, we define an intuitionistic fuzzy subset $B^{+'} = (\sigma_B^{+'}, \tau_B^{+'})$ of L by

$$\sigma_B^{+'}\left(\sum_i [x_i,\alpha_i]\right) = \inf_{s\in S}\sigma_B\left(\sum_i x_i\alpha_i s\right), \tau_B^{+'}\left(\sum_i [x_i,\alpha_i]\right) = \sup_{s\in S}\tau_B\left(\sum_i x_i\alpha_i s\right),$$

for all $\sum_i [x_i, \alpha_i] \in L$.
If $C = (\delta_C, \upsilon_C)$ is an intuitionistic fuzzy subset of R, we define an intuitionistic fuzzy subset $C^* = (\delta_C^*, \upsilon_C^*)$ of S by

$$\delta_C^*(x) = \inf_{\gamma\in\Gamma}\delta_C\left([\gamma,x]\right), \upsilon_C^*(x) = \sup_{\gamma\in\Gamma}\upsilon_C\left([\gamma,x]\right),$$

for all $x \in S$.
If $D = (\eta_D, \nu_D)$ is an intuitionistic fuzzy subset of S, we define an intuitionistic fuzzy subset $D^{*'} = (\eta_D^{*'}, \nu_D^{*'})$ of R by

$$\eta_D^{*'}\left(\sum_i [\alpha_i,x_i]\right) = \inf_{s\in S}\eta_D\left(\sum_i s\alpha_i x_i\right), \nu_D^{*'}\left(\sum_i [\alpha_i,x_i]\right) = \sup_{s\in S}\nu_D\left(\sum_i s\alpha_i x_i\right),$$

for all $\sum_i [\alpha_i, x_i] \in R$.

Theorem 3.2.
(1) *If* $A = (\mu_A, \lambda_A) \in IF\text{-}I(L)$, *then* $A^+ = (\mu_A^+, \lambda_A^+) \in IF\text{-}I(S)$.
(2) *If* $B = (\sigma_B, \tau_B) \in IF\text{-}I(S)$, *then* $B^{+'} \in IF\text{-}I(L)$.
(3) *If* $C = (\delta_C, \upsilon_C) \in IF\text{-}I(R)$, *then* $C^* = (\delta_C^*, \upsilon_C^*) \in IF\text{-}I(S)$.
(4) *If* $D = (\eta_D, \nu_D) \in IF\text{-}I(S)$, *then* $D^{*'} = (\eta_D^{*'}, \nu_D^{*'}) \in IF\text{-}I(R)$.

Proof. (1) Let $A = (\mu_A, \lambda_A) \in IF\text{-}I(L)$. Let $x, y \in S$ and $\alpha \in \Gamma$. Then

$$\begin{aligned} \mu_A^+(x+y) &= \inf_{\gamma\in\Gamma}\mu_A([x+y,\gamma]) \\ &= \inf_{\gamma\in\Gamma}\mu_A([x,\gamma]+[y,\gamma]) \\ &\geq \inf_{\gamma\in\Gamma}\min\{\mu_A([x,\gamma]),\mu_A([y,\gamma])\} \\ &= \min\left\{\inf_{\gamma\in\Gamma}\mu_A([x,\gamma]), \inf_{\gamma\in\Gamma}\mu_A([y,\gamma])\right\} \\ &= \min\{\mu_A^+(x),\mu_A^+(y)\}, \end{aligned}$$

$$\begin{aligned}\lambda_A^+(x+y) &= \sup_{\gamma\in\Gamma}\lambda_A([x+y,\gamma]) \\ &= \sup_{\gamma\in\Gamma}\lambda_A([x,\gamma]+[y,\gamma]) \\ &\le \sup_{\gamma\in\Gamma}\max\{\lambda_A([x,\gamma]),\lambda_A([y,\gamma])\} \\ &= \max\left\{\sup_{\gamma\in\Gamma}\lambda_A([x,\gamma]),\sup_{\gamma\in\Gamma}\lambda_A([y,\gamma])\right\} \\ &= \max\{\lambda_A^+(x),\lambda_A^+(y)\},\end{aligned}$$

$$\begin{aligned}\mu_A^+(x\alpha y) &= \inf_{\gamma\in\Gamma}\mu_A([x\alpha y,\gamma]) \\ &= \inf_{\gamma\in\Gamma}\mu_A([x,\alpha][y,\gamma]) \\ &\ge \inf_{\gamma\in\Gamma}\mu_A([y,\gamma]) \\ &= \mu_A^+(y),\end{aligned}$$

and similarly $\mu_A^+(x\alpha y)\ge\mu_A^+(x)$. Also

$$\begin{aligned}\lambda_A^+(x\alpha y) &= \sup_{\gamma\in\Gamma}\lambda_A([x\alpha y,\gamma]) \\ &= \sup_{\gamma\in\Gamma}\lambda_A([x,\alpha][y,\gamma]) \\ &\le \sup_{\gamma\in\Gamma}\lambda_A([y,\gamma]) \\ &= \lambda_A^+(y),\end{aligned}$$

and similarly $\lambda_A^+(x\alpha y)\le\lambda_A^+(x)$. Hence $A^+=(\mu_A^+,\lambda_A^+)\in IF\text{-}I(S)$.

(2) Let $B=(\sigma_B,\tau_B)\in IF\text{-}I(S)$. Then $\sigma_B(0_S)=1$ and $\tau_B(0_S)=0$. Now we have $\sigma_B^{+'}([0_S,\gamma])=\inf_{s\in S}\{\sigma_B(0_S\gamma s)\}=\sigma_B(0_S)=1$ and hence $\tau_B^{+'}([0_S,\gamma])=0$ for all $\gamma\in\Gamma$. Therefore $B^{+'}=(\sigma_B^{+'},\tau_B^{+'})$ is non-empty and $\sigma_B^{+'}(0_L)=1,\tau_B^{+'}(0_L)=0$ where $0_L=[0_S,\gamma]$ which is the zero element of L. Suppose $\sum_i[x_i,\alpha_i],\sum_j[y_j,\beta_j]\in L$. Then

$$\begin{aligned}\sigma_B^{+'}(\sum_i[x_i,\alpha_i]+\sum_j[y_j,\beta_j]) &= \inf_{s\in S}\{\sigma_B(\sum_i x_i\alpha_i s+\sum_j y_j\beta_j s)\} \\ &\ge \inf_{s\in S}\{\min\{\sigma_B(\sum_i x_i\alpha_i s),\sigma_B(\sum_j y_j\beta_j s)\}\} \\ &= \min\{\inf_{s\in S}\{\sigma_B(\sum_i x_i\alpha_i s)\},\inf_{s\in S}\{\sigma_B(\sum_j y_j\beta_j s)\}\} \\ &= \min\{\sigma_B^{+'}(\sum_i[x_i,\alpha_i]),\sigma_B^{+'}(\sum_j[y_j,\beta_j])\},\end{aligned}$$

$$\begin{aligned}
\tau_B^{+'}(\sum_i[x_i,\alpha_i]+\sum_j[y_j,\beta_j]) &= \sup_{s\in S}\{\tau_B(\sum_i x_i\alpha_i s+\sum_j y_j\beta_j s)\}\\
&\leq \sup_{s\in S}\{\max\{\tau_B(\sum_i x_i\alpha_i s),\tau_B(\sum_j y_j\beta_j s)\}\}\\
&= \max\{\sup_{s\in S}\{\tau_B(\sum_i x_i\alpha_i s)\},\sup_{s\in S}\{\tau_B(\sum_j y_j\beta_j s)\}\}\\
&= \max\{\tau_B^{+'}(\sum_i[x_i,\alpha_i]),\tau_B^{+'}(\sum_j[y_j,\beta_j])\},
\end{aligned}$$

$$\begin{aligned}
\sigma_B^{+'}(\sum_i[x_i,\alpha_i]\sum_j[y_j,\beta_j]) &= \sigma_B^{+'}(\sum_{i,j}[x_i\alpha_i y_j,\beta_j])\\
&= \inf_{s\in S}\{\sigma_B(\sum_{i,j}x_i\alpha_i y_j\beta_j s)\}\\
&\geq \inf_{s\in S}\{\min\{\sigma_B(\sum_i x_i\alpha_i y_1\beta_1 s),\sigma_B(\sum_i x_i\alpha_i y_2\beta_2 s),...\}\}\\
&\geq \inf_{s\in S}\{\min\{\sigma_B(\sum_i x_i\alpha_i y_1),\sigma_B(\sum_i x_i\alpha_i y_2),...\}\}\\
&= \min\{\sigma_B(\sum_i x_i\alpha_i y_1),\sigma_B(\sum_i x_i\alpha_i y_2),...\}\\
&\geq \inf_{s\in S}\{\sigma_B(\sum_i x_i\alpha_i s)\}\\
&= \sigma_B^{+'}(\sum_i[x_i,\alpha_i]),
\end{aligned}$$

$$\begin{aligned}
\tau_B^{+'}(\sum_i[x_i,\alpha_i]\sum_j[y_j,\beta_j]) &= \tau_B^{+'}(\sum_{i,j}[x_i\alpha_i y_j,\beta_j])\\
&= \sup_{s\in S}\{\tau_B(\sum_{i,j}x_i\alpha_i y_j\beta_j s)\}\\
&\leq \sup_{s\in S}\{\max\{\tau_B(\sum_i x_i\alpha_i y_1\beta_1 s),\tau_B(\sum_i x_i\alpha_i y_2\beta_2 s),...\}\}\\
&\leq \sup_{s\in S}\{\max\{\tau_B(\sum_i x_i\alpha_i y_1),\tau_B(\sum_i x_i\alpha_i y_2),...\}\}\\
&= \max\{\tau_B(\sum_i x_i\alpha_i y_1),\tau_B(\sum_i x_i\alpha_i y_2),...\}\\
&\leq \sup_{s\in S}\{\tau_B(\sum_i x_i\alpha_i s)\}\\
&= \tau_B^{+'}(\sum_i[x_i,\alpha_i]).
\end{aligned}$$

Hence $B^{+'}=(\sigma_B^{+'},\tau_B^{+'})\in\ IF\text{-}I(L)$.

We can prove (3) and (4) in a similar fashion. □

4. Intuitionistic fuzzy ideals of $\begin{pmatrix} R & \Gamma \\ S & L \end{pmatrix}$

Definition 4.1. Let S be a Nobusawa Γ-semiring and $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy subset of S. Then we define an intuitionistic fuzzy subset $\Gamma(A) = (\Gamma(\mu_A), \Gamma(\lambda_A))$ of Γ by

$$\Gamma(\mu_A)(\gamma) = \inf_{s,s' \in S} \mu_A(s\gamma s'),$$

$$\Gamma(\lambda_A)(\gamma) = \sup_{s,s' \in S} \lambda_A(s\gamma s'),$$

for all $\gamma \in \Gamma$.

Here a point to be noted is that when $B = (\nu_B, \eta_B)$ is an intuitionistic fuzzy subset of Γ, we can find an intuitionistic fuzzy subset $S(B) = (S(\nu_B), S(\eta_B))$ of S corresponding to B.

Throughout this section, unless otherwise mentioned, we consider S to be Nobusawa Γ-semiring with unities.

Theorem 4.2. *If $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of S, then $\Gamma(A)$ is also an intuitionistic fuzzy ideal of Γ.*

Proof. Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of S and $\gamma_1, \gamma_2 \in \Gamma$. Then

$$\begin{aligned} \Gamma(\mu_A)(\gamma_1 + \gamma_2) &= \inf_{s,s' \in S} \mu_A(s(\gamma_1 + \gamma_2)s') \\ &= \inf_{s,s' \in S} \mu_A((s\gamma_1 s') + (s\gamma_2 s')) \\ &\geq \inf_{s,s' \in S} \min\{\mu_A(s\gamma_1 s'), \mu_A(s\gamma_2 s')\} \\ &\geq \min\{\inf_{s,s' \in S} \mu_A(s\gamma_1 s'), \inf_{s,s' \in S} \mu_A(s\gamma_2 s')\} \\ &= \min\{\Gamma(\mu_A)(\gamma_1), \Gamma(\mu_A)(\gamma_2)\} \end{aligned}$$

and for all $s \in S$,

$$\begin{aligned} \Gamma(\mu_A)(\gamma_1 s \gamma_2) &= \inf_{s',s'' \in S} \mu_A(s'(\gamma_1 s \gamma_2)s'') \\ &\geq \inf_{s',s \in S} \mu_A(s'\gamma_1 s) \\ &= \Gamma(\mu_A)(\gamma_1). \end{aligned}$$

Similarly, we have for all $s \in S$,

$$\begin{aligned} \Gamma(\mu_A)(\gamma_1 s \gamma_2) &= \inf_{s',s'' \in S} \mu_A(s'(\gamma_1 s \gamma_2)s'') \\ &\geq \inf_{s,s'' \in S} \mu_A(s\gamma_2 s'') \\ &= \Gamma(\mu_A)(\gamma_2). \end{aligned}$$

Again,

$$\begin{aligned}\Gamma(\lambda_A)(\gamma_1+\gamma_2) &= \sup_{s,s^{'}\in S}\lambda_A(s(\gamma_1+\gamma_2)s^{'})\\ &= \sup_{s,s^{'}\in S}\lambda_A((s\gamma_1 s^{'})+(s\gamma_2 s^{'}))\\ &\leq \sup_{s,s^{'}\in S}\max\{\lambda_A(s\gamma_1 s^{'}),\lambda_A(s\gamma_2 s^{'})\}\\ &\leq \max\{\sup_{s,s^{'}\in S}\lambda_A(s\gamma_1 s^{'}),\sup_{s,s^{'}\in S}\lambda_A(s\gamma_2 s^{'})\}\\ &= \max\{\Gamma(\lambda_A)(\gamma_1),\Gamma(\lambda_A)(\gamma_2)\}\end{aligned}$$

and for all $s \in S$,

$$\begin{aligned}\Gamma(\lambda_A)(\gamma_1 s\gamma_2) &= \sup_{s^{'},s^{''}\in S}\lambda_A(s^{'}(\gamma_1 s\gamma_2)s^{''})\\ &\leq \sup_{s^{'},s\in S}\lambda_A(s^{'}\gamma_1 s)\\ &= \Gamma(\lambda_A)(\gamma_1).\end{aligned}$$

Similarly, we have for all $s \in S$,

$$\begin{aligned}\Gamma(\lambda_A)(\gamma_1 s\gamma_2) &= \sup_{s^{'},s^{''}\in S}\lambda_A(s^{'}(\gamma_1 s\gamma_2)s^{''})\\ &\leq \sup_{s,s^{''}\in S}\lambda_A(s\gamma_2 s^{''})\\ &= \Gamma(\lambda_A)(\gamma_2).\end{aligned}$$

Hence, $\Gamma(\lambda_A)$ is an intuitionistic fuzzy ideal of Γ. Therefore $\Gamma(\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of Γ. □

Similarly, we can prove that when $B = (\nu_B, \eta_B)$ is an intuitionistic fuzzy ideal of Γ, $S(B)$ defined as

$$S(B) = (S(\nu_B), S(\eta_B)),$$

where

$$S(\nu_B)(s) = \inf_{\gamma,\gamma^{'}\in\Gamma}\nu_B(\gamma s\gamma^{'}),$$

$$S(\eta_B)(s) = \sup_{\gamma,\gamma^{'}\in\Gamma}\eta_B(\gamma s\gamma^{'}),$$

for all $s \in S$, is also a fuzzy h-ideal of S corresponding to $B = (\nu_B, \eta_B)$. Now, we observe that if S is a Nobusawa Γ-semiring with strong unities then for $A = (\mu_A, \lambda_A) \in IF\text{-}I(S)$ and $x \in S$

$$\begin{aligned}S(\Gamma(\mu_A))(x) &= \inf_{\gamma_1,\gamma_2\in\Gamma}\Gamma(\mu_A)(\gamma_1 x\gamma_2)\\ &= \inf_{\gamma_1,\gamma_2\in\Gamma}\inf_{s_1,s_2\in S}\mu_A(s_1\gamma_1 x\gamma_2 s_2) \geq \mu_A(x).\end{aligned}$$

and

$$\begin{aligned} S(\Gamma(\lambda_A))(x) &= \sup_{\gamma_1,\gamma_2\in\Gamma} \Gamma(\lambda_A)(\gamma_1 x\gamma_2) \\ &= \sup_{\gamma_1,\gamma_2\in\Gamma} \sup_{s_1,s_2\in S} \lambda_A(s_1\gamma_1 x\gamma_2 s_2) \leq \lambda_A(x). \end{aligned}$$

Also we see that

$$\begin{aligned} \mu_A(x) &= \mu_A(e\delta x) \ (\textit{since } [e,\delta] \textit{ is strong left unity of } S) \\ &= \mu_A(e\delta x\gamma f) \ (\textit{since } [\gamma,f] \textit{ is strong right unity of } S) \\ &\geq \inf_{s_1,s_2\in S} \mu_A(s_1\delta x\gamma s_2) \\ &\geq \inf_{s_1,s_2\in S} \inf_{\gamma_1,\gamma_2\in\Gamma} \mu_A(s_1\gamma_1 x\gamma_2 s_2) \\ &= \inf_{\gamma_1,\gamma_2\in\Gamma} \inf_{s_1,s_2\in S} \mu_A(s_1\gamma_1 x\gamma_2 s_2) \\ &= S(\Gamma(\mu_A))(x). \end{aligned}$$

and

$$\begin{aligned} \lambda_A(x) &= \lambda_A(e\delta x) \ (\textit{since } [e,\delta] \textit{ is strong left unity of } S) \\ &= \lambda_A(e\delta x\gamma f) \ (\textit{since } [\gamma,f] \textit{ is strong right unity of } S) \\ &\leq \sup_{s_1,s_2\in S} \lambda_A(s_1\delta x\gamma s_2) \\ &\leq \sup_{s_1,s_2\in S} \sup_{\gamma_1,\gamma_2\in\Gamma} \lambda_A(s_1\gamma_1 x\gamma_2 s_2) \\ &= \sup_{\gamma_1,\gamma_2\in\Gamma} \sup_{s_1,s_2\in S} \lambda_A(s_1\gamma_1 x\gamma_2 s_2) \\ &= S(\Gamma(\lambda_A))(x). \end{aligned}$$

Thus $(\mu_A, \lambda_A) = (S(\Gamma(\mu_A)), S(\Gamma(\lambda_A)))$. Similarly, for $(\nu_B, \eta_B) \in IF\text{-}I(\Gamma)$, (ν_B, η_B) $= (\Gamma(S(\nu_B)), \Gamma(S(\eta_B)))$. Consequently, we obtain the following theorem.

Theorem 4.3. *Let S be a Nobusawa Γ-semiring with strong unities. Then there exists a bijection between the set of all intuitionistic fuzzy ideals of S and the set of all intuitionistic fuzzy ideals of Γ.*

Theorem 4.4 ([7])**.** *Let S be a Nobusawa Γ-semiring; L and R be its left and right operator semiring, respectively. Then $S_2 = \begin{pmatrix} R & \Gamma \\ S & L \end{pmatrix}$ forms a semiring with respect to the addition and multiplication defined by*

$$\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} + \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} = \begin{pmatrix} r_1+r_2 & \gamma_1+\gamma_2 \\ s_1+s_2 & l_1+l_2 \end{pmatrix}$$

and

$$\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} \cdot \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} = \begin{pmatrix} r_1r_2+[\gamma_1,s_2] & r_1\gamma_2+\gamma_1 l_2 \\ s_1r_2+l_1s_2 & [s_1,\gamma_2]+l_1l_2 \end{pmatrix}.$$

Theorem 4.5 ([7])**.** *Let S be a Nobusawa Γ-semiring and I be an ideal of S. Then $I_2 = \begin{pmatrix} I^{*'} & \Gamma(I) \\ I & I^{+'} \end{pmatrix}$ is an ideal of S_2.*

Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy subset of S. If we define $\begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix}$ by

$$\begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} = \min\{\mu_A^{*'}(r_1), \Gamma(\mu_A)(\gamma_1), \mu_A(s_1), \mu_A^{+'}(l_1)\}$$

and if we define $\begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix}$ by

$$\begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} = \max\{\lambda_A^{*'}(r_1), \Gamma(\lambda_A)(\gamma_1), \lambda_A(s_1), \lambda_A^{+'}(l_1)\},$$

where $\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} \in S_2$, then $\left(\begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix}, \begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \right)$ is an intuitionistic fuzzy subset of S_2.

Definition 4.6. Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy subset of S. Then the intuitionistic fuzzy subset $\left(\begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix}, \begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \right)$ of S_2 is called the *intuitionistic fuzzy subset of* S_2 *associated with* A.

Theorem 4.7. *Let* $A = (\mu_A, \lambda_A)$ *be an intuitionistic fuzzy ideal of* S*. Then* $\left(\begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix}, \begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \right)$ *is an intuitionistic fuzzy ideal of* S_2*.*

Proof. Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of S and $\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix}, \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \in S_2$. Then

$$\begin{aligned}
&\begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \left(\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} + \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \right) \\
&= \begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \begin{pmatrix} r_1 + r_2 & \gamma_1 + \gamma_2 \\ s_1 + s_2 & l_1 + l_2 \end{pmatrix} \\
&= \min\{\mu_A^{*'}(r_1 + r_2), \Gamma(\mu_A)(\gamma_1 + \gamma_2), \mu_A(s_1 + s_2), \mu^{+'}(l_1 + l_2)\} \\
&\geq \min\{\min\{\mu_A^{*'}(r_1), \Gamma(\mu_A)(\gamma_1), \mu_A(s_1), \mu_A^{+'}(l_1)\}, \\
&\qquad \min\{\mu_A^{*'}(r_2), \Gamma(\mu_A)(\gamma_2), \mu_A(s_2), \mu_A^{+'}(l_2)\}\} \\
&= \min\left\{ \begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix}, \begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \right\}
\end{aligned}$$

and

$$\begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \left(\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} \cdot \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \right)$$
$$= \begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \begin{pmatrix} r_1r_2 + [\gamma_1, s_2] & r_1\gamma_2 + \gamma_1 l_2 \\ s_1r_2 + l_1s_2 & [s_1, \gamma_2] + l_1l_2 \end{pmatrix}$$
$$= \min\{\mu_A^{*'}(r_1r_2 + [\gamma_1, s_2]), \Gamma(\mu_A)(r_1\gamma_2 + \gamma_1l_2), \mu_A(s_1r_2 + l_1s_2), \mu_A^{+'}([s_1, \gamma_2] + l_1l_2)\}$$
$$\geq \min\{\mu_A^{*'}(r_1r_2), \mu_A^{*'}([\gamma_1, s_2]), \Gamma(\mu_A)(r_1\gamma_2), \Gamma(\mu_A)(\gamma_1l_2),$$
$$\mu_A(s_1r_2), \mu_A(l_1s_2), \mu_A^{+'}(l_1l_2), \mu_A^{+'}([s_1, \gamma_2])\} \qquad (1)$$

Now, we have the following observations:

- $\mu_A^{*'}([\gamma_1, s_2]) = \inf_{s\in S} \mu_A(s\gamma_1 s_2) \geq \mu_A(s_2)$,
- $\Gamma(\mu_A)(\gamma_1 l_2) = \inf_{s,s' \in S} \mu_A(s\gamma_1 l_2 s') \geq \inf_{s'\in S} \mu_A(l_2 s') = \mu_A^{+'}(l_2)$,
- $\mu_A^{*'}(r_2) = \inf_{s\in S} \mu_A(sr_2) \leq \mu_A(s_1 r_2)$,
- $\mu_A^{+'}([s_1, \gamma_2]) = \inf_{s\in S} \mu_A(s_1\gamma_2 s) = \inf_{s\in S} \inf_{\gamma,\gamma' \in \Gamma} \Gamma(\mu_A)(\gamma s_1 \gamma_2 s \gamma') \geq \Gamma(\mu_A)(\gamma_2)$ (since we have already shown that $S(\Gamma(\mu_A)) = \mu_A$).

Therefore, from (1) we deduce that

$$\begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \left(\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} \cdot \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \right) \geq \min\{\mu_A^{*'}(r_2), \mu_A(s_2), \Gamma(\mu_A)(\gamma_2), \mu_A^{+'}(l_2)\}$$
$$= \begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix}$$

In a similar way we can prove that
$\begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \left(\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} \cdot \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \right) \geq \begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix}$.

Again

$$\begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \left(\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} + \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \right)$$
$$= \begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \begin{pmatrix} r_1 + r_2 & \gamma_1 + \gamma_2 \\ s_1 + s_2 & l_1 + l_2 \end{pmatrix}$$
$$= \max\{\lambda_A^{*'}(r_1 + r_2), \Gamma(\lambda_A)(\gamma_1 + \gamma_2), \lambda_A(s_1 + s_2), \lambda^{+'}(l_1 + l_2)\}$$
$$\leq \max\{\max\{\lambda_A^{*'}(r_1), \Gamma(\lambda_A)(\gamma_1), \lambda_A(s_1), \lambda_A^{+'}(l_1)\},$$
$$\max\{\lambda_A^{*'}(r_2), \Gamma(\lambda_A)(\gamma_2), \lambda_A(s_2), \lambda_A^{+'}(l_2)\}\}$$
$$= \max\left\{ \begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix}, \begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \right\}$$

and

$\begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \left(\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} \cdot \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \right)$

$= \begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \begin{pmatrix} r_1 r_2 + [\gamma_1, s_2] & r_1\gamma_2 + \gamma_1 l_2 \\ s_1 r_2 + l_1 s_2 & [s_1, \gamma_2] + l_1 l_2 \end{pmatrix}$

$= \max\{\lambda_A^{*'}(r_1 r_2 + [\gamma_1, s_2]), \Gamma(\lambda_A)(r_1\gamma_2 + \gamma_1 l_2), \lambda_A(s_1 r_2 + l_1 s_2), \lambda_A^{+'}([s_1, \gamma_2] + l_1 l_2)\}$

$\leq \max\{\max\{\lambda_A^{*'}(r_1 r_2), \lambda_A^{*'}([\gamma_1, s_2])\}, \max\{\Gamma(\lambda_A)(r_1\gamma_2), \Gamma(\lambda_A)(\gamma_1 l_2)\},$

$\max\{\lambda_A(s_1 r_2), \lambda_A(l_1 s_2)\}, \max\{\lambda_A^{+'}(l_1 l_2), \lambda_A^{+'}([s_1, \gamma_2])\}\}$

$= \max\{\lambda_A^{*'}(r_1 r_2), \lambda_A^{*'}([\gamma_1, s_2]), \Gamma(\lambda_A)(r_1\gamma_2), \Gamma(\lambda_A)(\gamma_1 l_2), \lambda_A(s_1 r_2), \lambda_A(l_1 s_2),$

$\lambda_A^{+'}(l_1 l_2), \lambda_A^{+'}([s_1, \gamma_2])\}$ (2)

Now, we have the following observations:

- $\lambda_A^{*'}([\gamma_1, s_2]) = \sup\limits_{s\in S}\{\lambda_A(s\gamma_1 s_2)\} \leq \lambda_A(s_2),$
- $\Gamma(\lambda_A)(\gamma_1 l_2) = \sup\limits_{s,s'\in S}\{\lambda_A(s\gamma_1 l_2 s')\} \leq \sup\limits_{s'\in S}\{\lambda_A(l_2 s')\} = \lambda_A^{+'}(l_2),$
- $\lambda_A^{*'}(r_2) = \sup\limits_{s\in S}\{\lambda_A(s r_2)\} \geq \lambda_A(s_1 r_2),$
- $\lambda_A^{+'}([s_1, \gamma_2]) = \sup\limits_{s\in S}\{\lambda_A(s_1\gamma_2 s)\} = \sup\limits_{s\in S}\sup\limits_{\gamma,\gamma'\in\Gamma}\{\Gamma(\lambda_A)(\gamma s_1\gamma_2 s\gamma')\} \leq \Gamma(\lambda_A)(\gamma_2).$

Therefore, from (2) we deduce that

$$\begin{aligned} &\begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \left(\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} \cdot \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \right) \\ &\leq \max\{\lambda_A^{*'}(r_2), \Gamma(\lambda_A)(\gamma_2), \lambda_A(s_2), \lambda_A^{+'}(l_2)\} \\ &= \begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2. \end{pmatrix} \end{aligned}$$

In a similar way we can prove that

$\begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \left(\begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1 \end{pmatrix} \cdot \begin{pmatrix} r_2 & \gamma_2 \\ s_2 & l_2 \end{pmatrix} \right) \leq \begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \begin{pmatrix} r_1 & \gamma_1 \\ s_1 & l_1. \end{pmatrix}.$

Hence $\left(\begin{pmatrix} \mu_A^{*'} & \Gamma(\mu_A) \\ \mu_A & \mu_A^{+'} \end{pmatrix} \begin{pmatrix} \lambda_A^{*'} & \Gamma(\lambda_A) \\ \lambda_A & \lambda_A^{+'} \end{pmatrix} \right)$ is an intuitionistic fuzzy ideal of S_2. □

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