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Overdetermined linear system of equations with trapezoidal fuzzy numbers

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Abstract. Recently, linear system of equations whose parameters are Trapezoidal fuzzy numbers is introduced (see in [8]). In this paper, the main aim is to develop a method for solving a $m \times n$ linear system of equations with trapezoidal fuzzy numbers for $m > n$. Conditions for the existence of a fuzzy solution are derived and a numerical procedure for calculating the solution is illustrated with some examples.

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1. Introduction

A general model for solving a fuzzy linear system whose coefficients matrix is crisp and the right-hand side column is an arbitrary fuzzy vector, was first proposed by Friedman et al. [6]. Friedman and its colleagues used the embedding method and replaced the original fuzzy linear system by a crisp linear system and then they solved it. after that Several methods for solving fuzzy linear systems have been introduced by many authors [1, 2, 3, 4, 5, 7]. In all cases, the authors just assumed that parameters are fuzzy numbers in which the core of fuzzy number is including just one point. Recently, we generalized those systems and assumed the parameters be trapezoidal fuzzy numbers (see in [8]). In Continution to our previous work, in this paper we focus on liner system of equations with trapezoidal fuzzy numbers and will develop similar approach to solve the $m \times n$ linear system of equations with trapezoidal fuzzy numbers by a $2m \times 2n$ crisp. In Section 2, we discuss some basic definitions and reselts on fuzzy numbers. then, in Section 3, we define an $m \times n$ fuzzy linear system of equations and a new method for solving these systems. In particular, we give an alternative method to find an orthogonal matrix which helps us to solve overdetermied linear system of equations with trapezoidal fuzzy numbers. The algorithms are illustrated with two examples in Section 4 and finally the conclusions are drawn in Section 5.

2. Preliminaries

We represent an arbitrary fuzzy number by an ordered pair of functions $(\underline{u}(r), \overline{u}(r))$, $0 \leq r \leq 1$ which satisfy the following requirements (as given in [6]):

1. $\underline{u}(r)$ is a bounded left continuous nondecreasing function over $[0,1]$.
2. $\overline{u}(r)$ is a bounded left continuous nonincreasing function over $[0,1]$.
3. $\underline{u}(r) \leq \overline{u}(r)$, $0 \leq r \leq 1$.

For every fuzzy number $x = (\underline{x}(r), \overline{x}(r))$, $y = (\underline{y}(r), \overline{y}(r))$ and real number k,

(a) $x = y$ if and only if $\underline{x}(r) = \underline{y}(r)$ and $\overline{x}(r) = \overline{y}(r)$.

(b) $x + y = (\underline{x}(r) + \underline{y}(r), \overline{x}(r) + \overline{y}(r))$.

(c) $kx = \begin{cases} (k\underline{x}, k\overline{x}), & k \geq 0, \\ (k\overline{x}, k\underline{x}), & k < 0. \end{cases}$

Definition 2.1. The linear system

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= y_1, \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= y_2, \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= y_m,
\end{aligned}
\tag{2.1}
$$

where the coefficient matrix $A = (a_{ij})$, $1 \leq i \leq m$, $1 \leq j \leq n$ is an $m \times n$ crisp matrix and y_i, $1 \leq i \leq m$ are fuzzy numbers, is called a Fuzzy Linear System (FLS). In this paper, we assume that rank (A)=n for $(n < m)$.

Definition 2.2. A fuzzy number vector $x = (x_1, x_2, \cdots, x_n)^T$, where

$$x_i = (\underline{x}_i(r), \overline{x}_i(r)), \quad 1 \leq i \leq n, 0 \leq r \leq 1,$$

is called a solution of fuzzy linear system if:

$$
\begin{aligned}
\underline{\sum_{j=1}^{n} a_{ij}x_j} &= \sum_{j=1}^{n} \underline{a_{ij}x_j} = \underline{y}_i, \\
\overline{\sum_{j=1}^{n} a_{ij}x_j} &= \sum_{j=1}^{n} \overline{a_{ij}x_j} = \overline{y}_i, \quad i = 1, ..., m.
\end{aligned}
\tag{2.2}
$$

Now, let s_{ij}, $1 \leq i \leq m$, $1 \leq j \leq n$ is defined as follows:

$$
\begin{aligned}
a_{ij} \geq 0 &\Rightarrow s_{ij} = a_{ij},\ s_{i+m,j+n} = a_{ij}, \\
a_{ij} < 0 &\Rightarrow s_{i,j+n} = -a_{ij},\ s_{i+m,j} = -a_{ij},
\end{aligned}
\tag{2.3}
$$

and any s_{ij} that is not determined by (2.3) is zero. Then (2.2) is equivalent to the system

$$SX = Y \tag{2.4}$$

where $S = (s_{ij})$, $s_{ij} \geq 0$, $1 \leq i \leq 2m$, $1 \leq j \leq 2n$ and

$$X = \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \vdots \\ \underline{x}_n \\ -\overline{x}_1 \\ -\overline{x}_2 \\ \vdots \\ -\overline{x}_n \end{bmatrix} = \begin{bmatrix} \underline{x} \\ -\overline{x} \end{bmatrix}, \quad Y = \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \vdots \\ \underline{y}_m \\ -\overline{y}_1 \\ -\overline{y}_2 \\ \vdots \\ -\overline{y}_m \end{bmatrix} = \begin{bmatrix} \underline{y} \\ -\overline{y} \end{bmatrix}.$$

In fact, the structure of S implies that

$$S = \begin{bmatrix} B & C \\ C & B \end{bmatrix},$$

where B contains the positive entries of A, C the absolute values of the negative entries of A and $A = B - C$. In the following we only consider triangular fuzzy numbers, i.e. $\underline{y}_i(r)$, $\overline{y}_i(r)$ and consequently $\underline{x}_i(r)$, $\overline{x}_i(r)$ are all linear functions of r. Let X satisfies (2.4), then the fuzzy solution of (3.3) is defined in the below.

Definition 2.3. Let $X = \{(\underline{x}_i(r), \overline{x}_i(r)),\ 1 \leq i \leq n\}$ denotes the unique solution of (3.3). The fuzzy number vector $u = \{(\underline{u}_i(r), \overline{u}_i(r)),\ 1 \leq i \leq n\}$ defined by

$$\begin{aligned} \underline{u}_i(r) &= min\{\underline{x}_i(r), \overline{x}_i(r), x_i(1)\} \\ \overline{u}_i(r) &= max\{\underline{x}_i(r), \overline{x}_i(r), x_i(1)\} \end{aligned} \tag{2.5}$$

is called the fuzzy solution of $SX = Y$. Moreover if $(\underline{x}_i(r), \overline{x}_i(r))$, $1 \leq i \leq n$ are all fuzzy numbers then we have $\underline{x}_i(r) = \underline{u}_i(r)$, $\overline{x}_i(r) = \overline{u}_i(r)$, $1 \leq i \leq n$, and u is called a strong fuzzy solution. Otherwise, u is a week fuzzy solution.

Theorem 2.4. *The rank of matrix S is $2n$ if and only if the rank of matrices $A = B - C$ and $B + C$ are both n.*

Proof. See in [6] □

2.1. **Overdetermined fuzzy system of linear equation.** To solve the fuzzy system of linear equations (2.4), where the coefficient matrix $S = s_{ij}$, $1 \leq i \leq 2m$, $1 \leq j \leq 2n$, is a crisp $2m \times 2n$ matrix where $m > n$ is named overdetermined. Such system cannot be solved.
Instead of solving the original system, we attempt to minimize the norm $\|r\|$ of the residual vector

$$r = Y - SX, \tag{2.6}$$

we discuss the least-squares norm and obtain the normal equation, which, in our formulation, have the form

$$S^t SX = S^t Y. \tag{2.7}$$

Theorem 2.5. *The solution X of Eq.* (2.4) *is a fuzzy vector for an arbitrary Y if and only if $((S^tS)^{-1}S^t)$ is non-negative, i.e.*
$((S^tS)^{-1}S^t) \geq 0, \quad 1 \leq i \leq 2m, \quad 1 \leq j \leq 2n.$

Proof. See in [2] □

It can be shown that the matrix S^tS is positive definite and hence non-singular if the columns of S are linearly independent. Nevertheless, the matrix S^tS may be ill conditioned and for moderate values of m, the solution Y will not be accurate. Hence, an alternative method must be used. Some of such methods are used of orthogonal polynomials, use of elementary Hermitian matrices or Householder transformation, and Givens transformations.
With the aid of this result, we can find an orthogonal $(2m) \times (2m)$ matrix, Q, which when applied to S yields a matrix of the form $(R, 0)^t$, where $(2n) \times (2n)$ matrix R is an upper triangular matrix where $r_{ii} > 0$, $1 \leq i \leq 2n$. Thus

$$QS = \begin{bmatrix} R \\ 0 \end{bmatrix}, \quad QY = \begin{bmatrix} c \\ d \end{bmatrix} \tag{2.8}$$

Corollary 2.6. *Matrix R in Eq. (2.8) must have the special structure, i.e.,*

$$R = \begin{bmatrix} R_1 & 0 \\ 0 & R_1 \end{bmatrix}. \tag{2.9}$$

By Eq. (2.6) we have

$$Qr = \begin{bmatrix} c \\ d \end{bmatrix} - QSX = \begin{bmatrix} c \\ d \end{bmatrix} - \begin{bmatrix} R \\ 0 \end{bmatrix} X = \begin{bmatrix} c - Rx \\ d \end{bmatrix}. \tag{2.10}$$

Since Q is orthogonal,

$$\|r\|_2^2 = \|Qr\|_2^2 = \|c - RX\|_2^2 + \|d\|_2^2 \tag{2.11}$$

and $\|r\|_2^2$ is minimized if $c - RX = 0$; thus, our least-squares solution is

$$\begin{bmatrix} \underline{x} \\ -\overline{x} \end{bmatrix} = \begin{bmatrix} R_1^{-1} & 0 \\ 0 & R_1^{-1} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}, \tag{2.12}$$

where

$$c = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{2.13}$$

and c_1, c_2 are $n \times 1$ matrices and hence

$$\underline{x} = R_1^{-1} c_1, \quad \overline{x} = -R_1^{-1} c_1 \tag{2.14}$$

and the residual vector r is given by

$$r = Q^t \,[0 \;\; d]^t\,. \tag{2.15}$$

3. Linear system of equations with trapezoidal fuzzy numbers

The following definition is concerning to a new achievement to the fuzzy linear systems.

Definition 3.1. The linear system

$$(3.1)\qquad \begin{aligned} a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2 + \cdots + a_{1n}\tilde{x}_n &= \tilde{y}_1 \\ a_{21}\tilde{x}_1 + a_{22}\tilde{x}_2 + \cdots + a_{2n}\tilde{x}_n &= \tilde{y}_2 \\ &\vdots \\ a_{m1}\tilde{x}_1 + a_{m2}\tilde{x}_2 + \cdots + a_{mn}\tilde{x}_n &= \tilde{y}_m \end{aligned}$$

where the coefficient matrix $A = (a_{ij})$, $1 \leq i \leq m$, $1 \leq j \leq n$ is an $m \times n$ crisp matrix and $\tilde{y}_i = (y_i^L, y_i^U, y_i^\alpha, y_i^\beta)$, $1 \leq i \leq m$ are trapezoidal fuzzy numbers, is called a Linear System of Equations with Trapezoidal Fuzzy Numbers (LSETFN).

Definition 3.2. A trapezoidal fuzzy number vector $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)^T$, where

$$\tilde{x}_i = (x_i^L, x_i^U, x_i^\alpha, x_i^\beta), \quad 1 \leq i \leq m,$$

is called a solution of LSETFN if:

$$(3.2)\qquad \sum_{j=1}^{n} a_{ij}\tilde{x}_j = \tilde{y}_i.$$

In the following definition we give an associated triangular fuzzy number for each trapezoidal fuzzy number which will be used in throughout the paper.

Definition 3.3. For every trapezoidal fuzzy number $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$, we define an associated triangular fuzzy number $\hat{a} = (a^m, \hat{a}^\alpha, \hat{a}^\beta)$ such that $a^m = \frac{a^L + a^U}{2}$, $\hat{a}^\alpha = a^\alpha + (a^m - a^L)$ and $\hat{a}^\beta = a^\beta + (a^U - a^m)$.

In fact, the support and the core of the associated triangular fuzzy number are $(a^m - \hat{a}^\alpha, a^m + \hat{a}^\beta)$ and the single point (real number) a^m, respectively. Note that the support of the trapezoidal fuzzy number $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ and its associated triangular fuzzy number is same.

According to the above definition we may define an associated fuzzy linear system of equations for every LSETFN which is defined in (6) as follows:

$$(3.3)\qquad \begin{aligned} a_{11}\hat{x}_1 + a_{12}\hat{x}_2 + \cdots + a_{1n}\hat{x}_n &= \hat{y}_1 \\ a_{21}\hat{x}_1 + a_{22}\hat{x}_2 + \cdots + a_{2n}\hat{x}_n &= \hat{y}_2 \\ &\vdots \\ a_{m1}\hat{x}_1 + a_{m2}\hat{x}_2 + \cdots + a_{mn}\hat{x}_n &= \hat{y}_m \end{aligned}$$

where $\hat{y}_i = (y_i^m, \hat{y_i}^\alpha, \hat{y_i}^\beta)$ such that $y_i^m = \frac{y_i^L + y_i^U}{2}$, $\hat{y_i}^\alpha = y_i^\alpha + (y_i^m - y_i^L)$ and $\hat{y_i}^\beta = y_i^\beta + (y_i^U - y_i^m)$. We name the above fuzzy linear system as "auxiliary fuzzy linear system" (AFLS). Also according to Definition 6, it is clear that there exist a unique auxiliary fuzzy linear system for every TFLS.

Now we are in place that can describe how to solve the linear system of equations with trapezoidal fuzzy numbers.

For achieving to this objective, we first solve the auxiliary fuzzy linear system (18) by one of the various methods which have proposed for solving these systems (see in [1, 2, 3, 4, 5]). So if we assume that $\hat{x} = (\hat{x}_1, ..., \hat{x}_j, ..., \hat{x}_n)^T$ be a weak or strong solution for system (18) such that $\hat{x}_j = (\hat{x}_j^m, \hat{x}_j^\alpha, \hat{x}_j^\beta)$, for all $j = 1, ..., n$, then we must obtain a trapezoidal fuzzy solution. This aim is also complied by the following process.

For the *i–th* right-hand-side $\tilde{y}_i = (y_i^L, y_i^U, y_i^\alpha, y_i^\beta)$, we first define

$$R_i = \frac{y_i^U - y_i^L}{(y_i^U + y_i^\beta) - (y_i^L - y_i^\alpha)}, \tag{3.4}$$

and then set

$$\gamma = \max_{1 \leq i \leq n} \{R_i\}. \tag{3.5}$$

Now for all $j = 1, ..., n$ set

$$\gamma_j = \gamma * \min\{\hat{x}_j^\alpha, \hat{x}_j^\beta\}. \tag{3.6}$$

Then the *j–th* component of the approximated trapezoidal fuzzy solution will be defined as follows:

$$\bar{x}_j = (x_j^L, x_j^U, x_j^\alpha, x_j^\beta),$$

where $x_j^L = \hat{x}_j^m - \gamma_j$, $x_j^U = \hat{x}_j^m + \gamma_j$, $x_j^\alpha = \hat{x}_j^\alpha - \gamma_j$ and $x_j^\beta = \hat{x}_j^\beta - \gamma_j$.

Thus, the support and the core of the mentioned trapezoidal fuzzy numbers are respectively $(x_j^L - x_j^\alpha, x_j^U + x_j^\beta)$ and $[x_j^L, x_j^U]$.

Now based on the above discussion we can propose the following algorithm for solving TFLSs.

Algorithm 3.1 (TFLS Solver):
Assumption The *i–th* component of the right-hand-side of ISETFN is given by $\tilde{y}_i = (y_i^L, y_i^U, y_i^\alpha, y_i^\beta)$, for all $i = 1, .., n$.

Step 1: For every $i = 1, ..., n$, compute the parameters y_i^m, $\hat{y_i}^\alpha$, $\hat{y_i}^\beta$ and R_i by $y_i^m = \frac{y_i^L + y_i^U}{2}$, $\hat{y_i}^\alpha = y_i^\alpha + (y_i^m - y_i^L)$, $\hat{y_i}^\beta = y_i^\beta + (y_i^U - y_i^m)$, and also $R_i = \frac{y_i^U - y_i^L}{(y_i^U + y_i^\beta) - (y_i^L - y_i^\alpha)}$.

Step 2: Solve the fuzzy linear system (18) by each algorithm which is proposed for solving fuzzy linear system solver and assume that the *i–th* component of the triangular fuzzy solution is $\hat{x}_i = (\hat{x}_i^m, \hat{x}_i^\alpha, \hat{x}_i^\beta)$.

Step 3: Compute the parameter γ_i, for all $i = 1, ..., n$ by

$$\gamma_i = \gamma * \min\{\hat{x}_i^\alpha, \hat{x}_i^\beta\},$$

where

$$\gamma = \max_{1 \leq i \leq n} \{R_i\}.$$

Step 4: Compute the *i–th* component of the approximated trapezoidal fuzzy solution by

$$\bar{x}_i = (x_i^L, x_i^U, x_i^\alpha, x_i^\beta),$$

where $x_i^L = \hat{x}_i^m - \gamma_i$, $x_i^U = \hat{x}_i^m + \gamma_i$, $x_i^\alpha = \hat{x}_i^\alpha - \gamma_i$ and $x_i^\beta = \hat{x}_i^\beta - \gamma_i$, and then **stop**.

4. Numerical examples

Here, we give some illustrative examples to show how Algorithm 3.1 solves the *ISETFNs*.

Example 4.1. Consider the following *ISETFN*

$$2\tilde{x}_1 = \tilde{y}_1 = (-8, -2, 6, 2)$$
$$-\tilde{x}_1 = \tilde{y}_2 = (1, 4, 1, 3)$$

The associated fuzzy linear system of equations for the above ISETFN is as follows:

$$2\hat{x}_1 = \hat{y}_1 = (\underline{y}_1(r), \bar{y}_1(r)) = (-14 + 9r, -5r),$$

$$-\hat{x}_1 = \hat{y}_2 = (\underline{y}_2(r), \bar{y}_2(r)) = (\frac{5}{2}r, 7 - \frac{9}{2}r).$$

1. Using pseudo inverse of S, the strong fuzzy solution of the current fuzzy linear system is as follows:

$$\hat{x}_1 = (\underline{x}_1(r), \bar{x}_1(r)) = (-7 + \frac{9}{2}r, -\frac{5}{2}r),$$

Thus the approximated trapezoidal fuzzy solution is as follows:

$$\bar{x}_1 = (-4.43, -0.57, 2.57, 0.57).$$

Note that the trapezoidal fuzzy solution of the mentioned ISETFN is as follows:

$$\tilde{x}_1 = (-4, -1, 3, 1).$$

2. Using orthogonal matrix

$$Q = \begin{bmatrix} 2/\sqrt{5} & 0 & 0 & 1/\sqrt{5} \\ 0 & 1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & -2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -1/\sqrt{5} & 0 & 0 & 2/\sqrt{5} \end{bmatrix}$$

thus

$$QS = \begin{bmatrix} 5/\sqrt{5} & 0 \\ 0 & 5/\sqrt{5} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad QY = \begin{bmatrix} 22.5/\sqrt{5}r - 35/\sqrt{5} \\ 12.5/\sqrt{5} \\ 0 \\ 0 \end{bmatrix}$$

and hence

$$\begin{bmatrix} \underline{x}_1(r) \\ -\overline{x}_1(r) \end{bmatrix} = \begin{bmatrix} 1/\sqrt{5} & 0 \\ 0 & 1/\sqrt{5} \end{bmatrix} \begin{bmatrix} 22.5/\sqrt{5}r - 35/\sqrt{5} \\ 12.5/\sqrt{5} \end{bmatrix} = \begin{bmatrix} 4.5r - 7 \\ 2.5r \end{bmatrix}$$

and therefore least-square solution is

$$\underline{x}_1(r) = 4.5r - 7, \quad \overline{x}_1(r) = -2.5r.$$

Thus the approximated trapezoidal fuzzy solution is as follows:

$$\bar{x}_1 = (-4.43, -0.57, 2.57, 0.57).$$

Example 4.2. Consider the following *ISETFN*

$$2\tilde{x}_1 = \tilde{y}_1 = (-32, 40, 8, 20)$$
$$-\tilde{x}_1 = \tilde{y}_2 = (-20, 16, 10, 4)$$
$$-2\tilde{x}_1 = \tilde{y}_3 = (-40, 32, 20, 8)$$

The associated fuzzy linear system of equations for the above ISETFN is as follows:

$$2\hat{x}_1 = \hat{y}_1 = (\underline{y}_1(r), \bar{y}_1(r)) = (-40 + 44r, 60 - 56r),$$

$$-\hat{x}_1 = \hat{y}_2 = (\underline{y}_2(r), \bar{y}_2(r)) = (-30 + 28r, 20 - 22r),$$

$$-2\hat{x}_1 = \hat{y}_3 = (\underline{y}_3(r), \bar{y}_3(r)) = (-60 + 56r, 40 - 44r).$$

1. Using pseudo inverse of S, the strong fuzzy solution of the current fuzzy linear system is as follows:

$$\hat{x}_1 = (\underline{x}_1(r), \bar{x}_1(r)) = (22r - 20, -28r + 30),$$

Thus the approximated trapezoidal fuzzy solution by the 4-th step of Algorithm 3.1 is as follows:

$$\bar{x}_1 = (\underline{x}_1(r), \bar{x}_1(r)) = (-18.16, 22.16, 1.84, 7.84),$$

Note that the exact trapezoidal fuzzy solution of the mentioned ISETFN is as follows:

$$\tilde{x}_1 = (-16, 20, 4, 10).$$

2. Using orthogonal matrix

$$Q = \begin{bmatrix} 2/3 & 0 & 0 & 0 & 1/3 & 2/3 \\ 0 & 1/3 & 2/3 & 2/3 & 0 & 0 \\ 0 & -2/\sqrt{5} & 1/\sqrt{5} & 0 & 0 & 0 \\ 0 & -2/3\sqrt{5} & -4/3\sqrt{5} & \sqrt{5}/3 & 0 & 0 \\ -1/\sqrt{5} & 0 & 0 & 02/\sqrt{5} & 0 & \\ -4/3\sqrt{5} & 0 & 0 & 0 & -2/3\sqrt{5} & \sqrt{5}/3 \end{bmatrix}$$

thus

$$QS = \begin{bmatrix} 3 & 0 \\ 0 & 3 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad QY = \begin{bmatrix} 66r - 60 \\ 84r - 90 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and hence

$$\begin{bmatrix} \underline{x}_1(r) \\ -\overline{x}_1(r) \end{bmatrix} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} 66r - 60 \\ 84r - 90 \end{bmatrix} = \begin{bmatrix} 22r - 20 \\ 28r - 30 \end{bmatrix}$$

and therefore least-square solution is

$$\underline{x}_1(r) = 22r - 20, \quad \overline{x}_1(r) = -28r + 30.$$

Thus the approximated trapezoidal fuzzy solution is as follows:

$$\bar{x}_1 = (-18.16, 22.16, 1.84, 7.84).$$

5. Conclusion

In this paper, we introduced a general model for solving the trapezoidal fuzzy linear system of equations with $n(m > n)$ variables. The trapezoidal origignal system is replaced by a triangel crisp linear system and we find least-square solution by using pseudo inverse and an orthogonal matrix. For finding the trapezoidal solution, we proposed a new algorithm. Here according to our suggestion we may obtain a unique trapezoidal fuzzy solution for the given ISETFN. We examined some numerical examples to show how solve these systems by Algorithm 3.1.

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