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On somewhat pairwise soft fuzzy quasi uniform almost C-open function

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Abstract. In this paper the concepts of soft fuzzy quasi uniform space and soft fuzzy quasi uniform topological space are introduced. Somewhat pairwise soft fuzzy quasi uniform C-continuous function, somewhat pairwise soft fuzzy quasi uniform almost C-open function, pairwise soft fuzzy quasi uniform $\mathcal{D}^*$-space, pairwise soft fuzzy quasi uniform $\mathcal{D}_C$-space are introduced. In this connection, several properties are discussed. Interrelations among the continuity are established with counter examples.
Moreover, constructing the continuous function by several methods is used in Analysis. Among which some generalize to arbitrary topological space and a few others do not. In particular, for any continuous function, expanding the domain in general topology as well as fuzzy topology is invalid whereas it is true in this paper due to the nature of the peculiar set, "C-open set", which was introduced by E.Hatir, T. Noiri and S. Yuksel.

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1. Introduction

Zheadeh [12] introduced the fundamental concepts of fuzzy sets in his classical paper. Fuzzy sets have applications in many fields such as information [8] and control [9]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Chang [2] introduced and developed the concept of fuzzy topological spaces. The notion of C-set in general topology was introduced by E.Hatir, T.Noiri and S.Yuksel [4]. E.Roja, M.K.Uma, G.Balasubramanian [10] introduced the concept of fuzzy C-set. Bruce Hutton [5] introduced the new structure, called fuzzy quasi uniformity. The concept of soft fuzzy topological space is introduced by Yogalakshmi.T., Roja.E., Uma.M.K. [11]. The idea of somewhat continuity was introduced and studied by Karl.R.Gentry, Hughes.B.Hoyle [3] in classical sense. In 1989, Kandil [6] introduced the concept of fuzzy bitopological spaces. Azad. K.K. [1] introduced and discussed about the fuzzy almost continuous function.

In this paper, soft fuzzy quasi uniform space and the soft fuzzy quasi uniform topology $\tau_{\mathcal{U}}$, generated by the uniformity is introduced. Somewhat pairwise soft fuzzy quasi uniform C-continuous function, somewhat pairwise soft fuzzy quasi uniform almost C-open function, pairwise soft fuzzy quasi uniform $\mathcal{D}^*$-space, pairwise soft fuzzy quasi uniform $\mathcal{D}_C$-space are introduced. Several characterization are discussed. Interrelations among the continuity are studied with counter examples.

2. Preliminaries

Definition 2.1 ([11])**.** Let X be a set, μ be a fuzzy subset of X and $M \subseteq X$. Then, the pair (μ, M) is called a **soft fuzzy set** of X. The set of all soft fuzzy subsets of X is denoted by **SF(X)**.

Definition 2.2 ([11])**.** The relation $\sqsubseteq$ on $SF(X)$ is given by $(\mu, M) \sqsubseteq (\lambda, N) \Leftrightarrow \mu(x) \leq \lambda(x), \forall x \in X$ and $M \subseteq N$.

Definition 2.3 ([11])**.** For $(\mu, M) \in SF(X)$, the soft fuzzy set $(\mu, M)^{'} = (1 - \mu, X|M)$ is called the **complement** of (μ, M).

Definition 2.4 ([11])**.** A subset $\tau \subseteq SF(X)$ is called a **soft fuzzy topology** on X if

(1) $(0, \phi)$ and $(1, X) \in \tau$.
(2) $(\mu_j, M_j) \in \tau$, j=1,2,3,...n $\Rightarrow \sqcap_{j=1}^{n}(\mu_j, M_j) \in \tau$.
(3) $(\mu_j, M_j) \in \tau, j \in J \Rightarrow \sqcup_{j \in J}(\mu_j, M_j) \in \tau$.

Then, the elements of τ are called **soft fuzzy open sets**, and those of $\tau^{'} = \{(\mu, M) : (\mu, M)^{'} \in \tau\}$ are the **soft fuzzy closed sets**.

If τ is a SF-topology on X, then, the ordered pair (X, τ) is called as a **soft fuzzy topological space**(in short, **SFTS**).

Definition 2.5 ([11])**.** Let $\psi : X \to Y$ be a function. If (λ, N) is a soft fuzzy set in Y, then its **pre-image** under ψ , denoted $\psi^{\leftharpoonup}(\lambda, N)$ is defined as,

$$\psi^{\leftharpoonup}(\lambda, N) = (\lambda \circ \psi, \psi^{\leftharpoonup}(N))$$

where, $\psi^{\leftharpoonup}(N) = \{x \in X : \psi(x) = y, for\, y \in N\}$.

Definition 2.6 ([11])**.** Let $\psi : X \to Y$ be a function. If (μ, M) is a soft fuzzy set in X, then its **image** under ψ , denoted $\psi^{\rightharpoonup}(\mu, M)$ is defined as,

$$\psi^{\rightharpoonup}(\mu, M) = (\gamma, K)$$

where, $\gamma(y) = \psi^{\rightharpoonup}(\mu)(y) = sup\{\mu(x) : x \in \psi^{\leftharpoonup}(y)\}$
$K = \{\psi^{\rightharpoonup}(x) : x \in M\}$.

Definition 2.7 ([11])**.** Let (X, τ) be a SFTS. A soft fuzzy set (λ, N) is said to be **soft fuzzy α^*-open** , if $int(\lambda, N) = int(cl(int(\lambda, N)))$.

Definition 2.8 ([11])**.** Let (X, τ) be a SFTS. A soft fuzzy set (λ, N) is said to be **soft fuzzy C-open** , if

$$(\lambda, N) = (\mu, M) \sqcap (\gamma, K)$$

where,(μ, M) is a soft fuzzy open set and (γ, K) is a soft fuzzy α^*-open set.

The complement of soft fuzzy C-open set (in short. **SFcOS**) is called as a **soft fuzzy C-closed set**. (in short.**SFcCS**)

Definition 2.9 ([1]). Let (X, τ) and (Y, σ) be any two fuzzy topological spaces. A function $f : X \to Y$ is a **fuzzy almost continuous function** if the inverse image of each fuzzy regular open set in Y is a fuzzy open set in X.

3. Soft fuzzy quasi uniform topological space

Let D denote the family of all functions f: $SF(X) \to SF(X)$ with the following properties

(1) $f(0, \phi) = (0, \phi)$.
(2) $(\mu, M) \sqsubseteq f(\mu, M)$, for every $(\mu, M) \in SF(X)$.

Definition 3.1. A soft fuzzy quasi uniformity on X is a subcollection $\mathcal{U} \subset D$ satisfying the following axioms

(i) If $f \in \mathcal{U}$, $f \sqsubseteq g$ and $g \in D$, then $g \in \mathcal{U}$.
(ii) If $f_1, f_2 \in \mathcal{U}$, then there exists $g \in \mathcal{U}$ such that $g \sqsubseteq f_1 \sqcap f_2$.
(iii) For every $f \in \mathcal{U}$, there exists $g \in \mathcal{U}$ such that $g \circ g \sqsubseteq f$.

Then, the pair $(X, \mathcal{U})$ is called **soft fuzzy quasi uniform space**.

Definition 3.2. Let $(X, \mathcal{U})$ be a soft fuzzy quasi uniform space. The **operator** $Int : SF(X) \to SF(X)$ defined by

$$Int(\mu, M) = \sqcup\{(\lambda, N) \in SF(X) : f(\lambda, N) \sqsubseteq (\mu, M) \text{for some} f \in \mathcal{U}\}.$$

Definition 3.3. Let $(X, \mathcal{U})$ be a soft fuzzy quasi uniform space. Then,

$$\tau_{\mathcal{U}} = \{(\mu, M) \in SF(X) : Int(\mu, M) = (\mu, M)\}$$

is said to be the **soft fuzzy quasi uniform topology**, which is generated by $\mathcal{U}$. The ordered pair $(X, \tau_{\mathcal{U}})$ is called as a **soft fuzzy quasi uniform topological space**. The members of $\tau_{\mathcal{U}}$ are called **soft fuzzy quasi $\mathcal{U}$-open set**. The complement of a soft fuzzy quasi $\mathcal{U}$-open set is **soft fuzzy quasi $\mathcal{U}$-closed**.

Definition 3.4. Let $(X, \tau_{\mathcal{U}})$ be a soft fuzzy quasi uniform topological space. Then, the **soft fuzzy quasi $\mathcal{U}$-interior** of a soft fuzzy set (λ, N) in $(X, \tau_{\mathcal{U}})$ is defined as follows

$int_{\mathcal{U}}(\lambda, N) = \sqcup\{(\mu, M) : (\mu, M) \sqsubseteq (\lambda, N)$ and (μ, M) is soft fuzzy quasi $\mathcal{U}$-open set $\}$

The **soft fuzzy quasi $\mathcal{U}$-closure** of a soft fuzzy set (λ, N) in $(X, \tau_{\mathcal{U}})$ is defined as follows

$cl_{\mathcal{U}}(\lambda, N) = \sqcap\{(\mu, M) : (\mu, M) \sqsupseteq (\lambda, N)$ and (μ, M) is soft fuzzy quasi $\mathcal{U}$-closed set $\}$.

Definition 3.5. A soft fuzzy set (λ, N) in $(X, \tau_{\mathcal{U}})$ is said to be a **soft fuzzy quasi regular $\mathcal{U}$-open set**, if $int_{\mathcal{U}}(cl_{\mathcal{U}}(\lambda, N)) = (\lambda, N)$.

The complement of soft fuzzy quasi regular $\mathcal{U}$-open set is **soft fuzzy quasi regular $\mathcal{U}$-closed set**.

Definition 3.6. A soft fuzzy set (λ, N) in $(X, \tau_{\mathcal{U}})$ is called a **soft fuzzy quasi uniform dense set** if there exists no soft fuzzy quasi $\mathcal{U}$-closed set, (μ, M) such that $(\lambda, N) \sqsubseteq (\mu, M) \sqsubseteq (1, X)$.

Definition 3.7. Let $(X, \tau_{\mathcal{U}})$ be a soft fuzzy quasi uniform topological space. A soft fuzzy set (λ, N) is said to be **soft fuzzy quasi** α^* **$\mathcal{U}$-open set** , if $int_{\mathcal{U}}(\lambda, N) = int_{\mathcal{U}}(cl_{\mathcal{U}}(int_{\mathcal{U}}(\lambda, N)))$.

Definition 3.8. Let $(X, \tau_{\mathcal{U}})$ be a soft fuzzy quasi uniform topological space. A soft fuzzy set (λ, N) is said to be **soft fuzzy C-quasi $\mathcal{U}$-open set** , if

$$(\lambda, N)=(\mu, M) \sqcap (\gamma, K)$$

where, (μ, M) is a soft fuzzy quasi $\mathcal{U}$-open set and (γ, K) is a soft fuzzy quasi α^* $\mathcal{U}$-open set.

The complement of soft fuzzy C-quasi $\mathcal{U}$-open set is called as a **soft fuzzy C-quasi $\mathcal{U}$-closed set**.

Definition 3.9. Let $(X, \tau_{\mathcal{U}})$ be a soft fuzzy quasi uniform topological space. Let (λ, N) be a soft fuzzy set in $(X, \tau_{\mathcal{U}})$. Then, the **soft fuzzy C-quasi $\mathcal{U}$-interior** and **soft fuzzy C-quasi $\mathcal{U}$-closure** of (λ, N) are defined respectively as

$$\begin{aligned} C\text{-}int_{\mathcal{U}}(\lambda, N) &= \sqcup\{(\mu, M) : (\mu, M) \text{ is soft fuzzy C-quasi } \mathcal{U}\text{-open set and} \\ &\qquad (\mu, M) \sqsubseteq (\lambda, N)\} \\ C\text{-}cl_{\mathcal{U}}(\lambda, N) &= \sqcap\{(\gamma, K) : (\gamma, K) \text{ is soft fuzzy C-quasi } \mathcal{U}\text{-closed set and} \\ &\qquad (\gamma, K) \sqsupseteq (\lambda, N)\} \end{aligned}$$

Definition 3.10. A soft fuzzy set (λ, N) in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ is said to be a **soft fuzzy C-quasi regular $\mathcal{U}$-open set**, if $C\text{-}int_{\mathcal{U}}(C\text{-}cl_{\mathcal{U}}(\lambda, N)) = (\lambda, N)$.

The complement of soft fuzzy C-quasi regular $\mathcal{U}$-open set is **soft fuzzy C-quasi regular $\mathcal{U}$-closed set**.

4. Somewhat pairwise soft fuzzy quasi uniform C-continuous function

Definition 4.1. A **soft fuzzy quasi uniform bitopological space** is a 3-tuple $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$, where X is a set , $\tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2}$ are any two soft fuzzy quasi uniform topologies on X.

Definition 4.2. Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. A function $\psi : (X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2}) \to (Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ is **pairwise soft fuzzy quasi uniform continuous (almost continuous) function**, if for each soft fuzzy quasi $\mathcal{V}_1$-open (regular $\mathcal{V}_1$-open) set or soft fuzzy quasi $\mathcal{V}_2$-open (regular $\mathcal{V}_2$-open) set (λ, N) in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$, the inverse image $\psi^{\leftharpoonup}(\lambda, N)$ is a soft fuzzy quasi $\mathcal{U}_1$-open set or soft fuzzy quasi $\mathcal{U}_2$-open set in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$.

Definition 4.3. Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. Then, $\psi : X \to Y$ is said to be a **pairwise soft fuzzy quasi uniform almost open function**, if (λ, N) is soft fuzzy quasi regular $\mathcal{U}_1$-open set or soft fuzzy quasi regular $\mathcal{U}_2$-open set in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$, $\psi^{\rightharpoonup}(\lambda, N)$ is a soft fuzzy quasi $\mathcal{V}_1$-open set or soft fuzzy quasi $\mathcal{V}_2$-open set.

Remark 4.4. If a function $\psi : X \to Y$ is both pairwise soft fuzzy quasi uniform almost open function and pairwise soft fuzzy quasi uniform almost continuous function, then the inverse image $\psi^{\leftharpoonup}(\lambda, N)$ of each soft fuzzy quasi regular $\mathcal{V}_1/\mathcal{V}_2$-open set (λ, N) of Y is a soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-open set in X.

Proof. Proof is analogous to Theorem. 3.5 in [7]. □

Proposition 4.5. *Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ be any soft fuzzy quasi uniform bitopological space. For each soft fuzzy set (λ, N),*

(1) $C\text{-}int_{\mathcal{U}_1/\mathcal{U}_2}((1, X) - (\lambda, N)) = (1, X) - C\text{-}cl_{\mathcal{U}_1/\mathcal{U}_2}(\lambda, N)$
(2) $C\text{-}cl_{\mathcal{U}_1/\mathcal{U}_2}((1, X) - (\lambda, N)) = (1, X) - C\text{-}int_{\mathcal{U}_1/\mathcal{U}_2}(\lambda, N)$

Proof. Proof is obvious. □

Definition 4.6. Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. A function $\psi : (X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2}) \to (Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ is **pairwise soft fuzzy quasi uniform C-continuous function**, if for each soft fuzzy quasi $\mathcal{V}_1$-open set or soft fuzzy quasi $\mathcal{V}_2$-open set (λ, N) in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$, the inverse image $\psi^{\leftharpoonup}(\lambda, N)$ is a soft fuzzy C-quasi $\mathcal{U}_1$-open set or soft fuzzy C-quasi $\mathcal{U}_2$-open set in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$.

Definition 4.7. Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. A function $\psi : (X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2}) \to (Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ is **somewhat pairwise soft fuzzy quasi uniform C-continuous function**, if (λ, N) is soft fuzzy quasi $\mathcal{V}_1$-open set or soft fuzzy quasi $\mathcal{V}_2$-open set and $\psi^{\leftharpoonup}(\lambda, N) \neq (0, \phi)$, then there exists a soft fuzzy C-quasi $\mathcal{U}_1$-open set or soft fuzzy C-quasi $\mathcal{U}_2$-open set, (μ, M) such that $(\mu, M) \neq (0, \phi)$ and $(\mu, M) \sqsubseteq \psi^{\leftharpoonup}(\lambda, N)$.

It is clear from the definition, that every pairwise soft fuzzy quasi uniform C-continuous function is somewhat pairwise soft fuzzy quasi uniform C-continuous, but the converse need not be true as shown in the following example.

Example 4.8. Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$. Let N, M_1, M_2 be the subsets of X. Let D_1 denote the family of functions $f_1, f_2, f_3 : SF(X) \to SF(X)$ be defined as follows

$$f_1(\lambda, N) = \begin{cases} (0, \phi), & \text{if } (\lambda, N) = (0, \phi) \\ (1, X), & \text{otherwise} \end{cases}$$

$$f_2(\lambda, N) = \begin{cases} (0, \phi), & \text{if } (\lambda, N) = (0, \phi) \\ (\mu_1, M_1), & \text{if} (0, \phi) \neq (\lambda, N) \sqsubseteq (\mu_1, M_1) \\ (1, X), & \text{otherwise} \end{cases}$$

$$f_3(\lambda, N) = \begin{cases} (0, \phi), & \text{if } (\lambda, N) = (0, \phi) \\ (\mu_2, M_2), & \text{if} (0, \phi) \neq (\lambda, N) \sqsubseteq (\mu_2, M_2) \\ (1, X), & \text{otherwise} \end{cases}$$

where (μ_1, M_1) and (μ_2, M_2) are defined as follows

$$\begin{array}{llll} \mu_1(a) = 0 & \mu_1(b) = 0 & \mu_1(c) = 1 & M_1 = \{a, b\} \\ \mu_2(a) = 0 & \mu_2(b) = 0 & \mu_2(c) = 1 & M_2 = \{b\} \end{array}$$

Let $\mathcal{U}_1, \mathcal{U}_2$ be the subcollection of D_1. Now, $(X,\mathcal{U}_1)$ and $(X,\mathcal{U}_2)$ form the soft fuzzy quasi uniform space. Then,

$$\tau_{\mathcal{U}_1} = \{(0,\phi),(1,X),(\mu_1,M_1)\}$$
$$\tau_{\mathcal{U}_2} = \{(0,\phi),(1,X),(\mu_2,M_2)\}$$

are the soft fuzzy quasi uniform topologies, which are generated by $\mathcal{U}_1$ and $\mathcal{U}_2$. Thus, $(X,\tau_{\mathcal{U}_1},\tau_{\mathcal{U}_2})$ is a soft fuzzy quasi uniform bitopological space.
Let M, K be the subset of Y. Let D_2 denote the family of functions $g_1, g_2 : SF(Y) \to SF(Y)$ be defined as follows

$$g_1(\mu,M) = \begin{cases} (0,\phi), & \text{if } (\mu,M) = (0,\phi) \\ (1,X), & \text{otherwise} \end{cases}$$

$$g_2(\mu,M) = \begin{cases} (0,\phi), & \text{if } (\mu,M) = (0,\phi) \\ (\gamma,K), & \text{if} (0,\phi) \neq (\mu,M) \sqsubseteq (\gamma,K) \\ (1,X), & \text{otherwise} \end{cases}$$

where (γ,K) is defined as follows

$$\gamma(a) = 0 \qquad \gamma(b) = 1 \qquad \gamma(c) = 1 \qquad K = \{p\}$$

Let $\mathcal{V}_1$, $\mathcal{V}_2$ be the subcollection of D_2. Now, $(X,\mathcal{V}_1)$ and $(X,\mathcal{V}_2)$ form the soft fuzzy quasi uniform space. Then,

$$\tau_{\mathcal{V}_1} = \{(0,\phi),(1,X)\}$$
$$\tau_{\mathcal{V}_2} = \{(0,\phi),(1,X),(\gamma,K)\}$$

are the soft fuzzy quasi uniform topologies, which are generated by $\mathcal{V}_1$ and $\mathcal{V}_2$. Thus, $(Y,\tau_{\mathcal{V}_1},\tau_{\mathcal{V}_2})$ is a soft fuzzy quasi uniform bitopological space.
Let $\psi : (X,\tau_{\mathcal{U}_1},\tau_{\mathcal{U}_2}) \to (Y,\tau_{\mathcal{V}_1},\tau_{\mathcal{V}_2})$ be defined as

$$\psi(a) = p \qquad \psi(b) = p \qquad \psi(c) = r$$

Let $L \subseteq X$ and $\delta : X \to [0,1]$ be defined as

$$\delta(a) = 0 \qquad \delta(b) = 0 \qquad \delta(c) = 0.5 \qquad L = \{a\}$$

Now, (δ,L) is a soft fuzzy C-quasi $\mathcal{U}_1/\mathcal{U}_2$-open set, which is coarser than $\psi^{\leftarrow}(\gamma,K)$. Thus, ψ **is somewhat pairwise soft fuzzy quasi uniform C-continuous function but not pairwise soft fuzzy quasi uniform C-continuous function**.

Definition 4.9. A soft fuzzy set (λ,N) in $(X,\tau_{\mathcal{U}_1},\tau_{\mathcal{U}_2})$ is called a **pairwise soft fuzzy quasi uniform C-dense set** if there exists no soft fuzzy C-quasi $\mathcal{U}_1$-closed set and soft fuzzy C-quasi $\mathcal{U}_2$-closed set, (μ,M) such that $(\lambda,N) \sqsubseteq (\mu,M) \sqsubseteq (1,X)$.

Proposition 4.10. *Let $(X,\tau_{\mathcal{U}_1},\tau_{\mathcal{U}_2})$ and $(Y,\tau_{\mathcal{V}_1},\tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. Let $\psi : X \to Y$ be a surjective function. Then, the following are equivalent:*
(i) ψ is somewhat pairwise soft fuzzy quasi uniform C-continuous function.
(ii) If (λ,N) is soft fuzzy quasi $\mathcal{V}_1$-closed set or soft fuzzy quasi $\mathcal{V}_2$-closed set such that $\psi^{\leftarrow}(\lambda,N) \neq (1,X)$, then there exists a proper soft fuzzy C-quasi $\mathcal{U}_1$-closed set or a soft fuzzy C-quasi $\mathcal{U}_2$-closed set (μ,M) such that $(\mu,M) \sqsupset \psi^{\leftarrow}(\lambda,N)$.
(iii) If (λ,N) is a pairwise soft fuzzy quasi uniform C-dense set, then $\psi^{\rightarrow}(\lambda,N)$ is a pairwise soft fuzzy quasi uniform dense set in $(Y,\tau_{\mathcal{V}_1},\tau_{\mathcal{V}_2})$.

Proof. Proof is obvious □

Notation: Let $A \subseteq X$ and (λ, N) be any soft fuzzy set of X. Then, $(\lambda, N)|A = (\lambda|A, N \cap A)$.

The following Proposition exhibits how the nature of the peculiar set, "C-set" plays a vital role in it.

Proposition 4.11. *Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. Let $\psi : X \to Y$ be a somewhat pairwise soft fuzzy quasi uniform C-continuous function. Let $A \subseteq X$ be such that $(1_A, A) \sqcap (\mu, M) \neq (0, \phi)$, for all non-zero soft fuzzy C-quasi $\mathcal{U}_1$-open set or soft fuzzy C-quasi $\mathcal{U}_2$-open set $(\mu, M) \neq (0, \phi)$. Let $\tau_{\mathcal{U}_1}|A$ and $\tau_{\mathcal{U}_2}|A$ be the induced soft fuzzy quasi uniform topological spaces on A. Then $\psi|A : (A, \tau_{\mathcal{U}_1}|A, \tau_{\mathcal{U}_2}|A) \to (Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ is somewhat pairwise soft fuzzy quasi uniform C-continuous.*

Proof. Let (λ, N) be a soft fuzzy quasi $\mathcal{V}_1$-open set or soft fuzzy quasi $\mathcal{V}_2$-open set such that $\psi^{\leftharpoonup}(\lambda, N) \neq (0, \phi)$. Since ψ is somewhat pairwise soft fuzzy quasi uniform C-continuous, there exists a soft fuzzy C-quasi $\mathcal{U}_1$-open set or soft fuzzy C-quasi $\mathcal{U}_2$-open set $(\mu, M) \neq (0, \phi)$ and $(\mu, M) \sqsubseteq \psi^{\leftharpoonup}(\lambda, N)$. Now, clearly $(\mu, M)|A$ is soft fuzzy C-quasi $\mathcal{U}_1$-open set or soft fuzzy C-quasi $\mathcal{U}_2$-open set on A and $(\mu, M)|A \neq (0, \phi)$. Also,

$$\begin{aligned}(\psi|A)^{\leftharpoonup}(\lambda, N) &= (\lambda \circ (\psi|A), (\psi|A)^{\leftharpoonup}(N)) \\ &= (\lambda \circ \psi, \psi^{\leftharpoonup}(N)) \\ &= \psi^{\leftharpoonup}(\lambda, N)\end{aligned}$$

Hence, $\psi^{\leftharpoonup}(\lambda, N) \neq (0, \phi)$ and $\psi^{\leftharpoonup}(\lambda, N) \sqsupseteq (\mu, M) = (\mu, M)|A$.

Assume that $\psi|A$ is a somewhat pairwise soft fuzzy quasi uniform C-continuous function. It must be shown that, ψ is a somewhat pairwise soft fuzzy C-continuous function. Let (λ, N) be a soft fuzzy quasi $\mathcal{V}_1/\mathcal{V}_2$-open set such that $(\psi|A)^{\leftharpoonup}(\lambda, N) \neq (0, \phi)$ in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$. From the hypothesis, there exists a soft fuzzy C-quasi $\mathcal{U}_1/\mathcal{U}_2$-open set, $(\mu, M) \neq (0, \phi)$ on A in X such that $(\mu, M) \sqsubseteq (\psi|A)^{\leftharpoonup}(\lambda, N)$.
Define $\gamma : X \to I$ as follows:

$$\gamma(x) = \begin{cases} \mu(x), & x \in A \\ 0, & otherwise \end{cases}$$

and K be any subset of X. Clearly, $(\gamma, K)|A = (\mu, M)$ is a soft fuzzy C-quasi $\mathcal{U}_1/\mathcal{U}_2$-open set. Now, (γ, K) is also a soft fuzzy C-quasi $\mathcal{U}_1/\mathcal{U}_2$-open set such that $(\gamma, K) \sqsubseteq \psi^{\leftharpoonup}(\lambda, N)$. This implies that, ψ is a somewhat pairwise soft fuzzy quasi uniform C-continuous function. □

5. Somewhat pairwise soft fuzzy quasi uniform almost C-continuous function

Definition 5.1. Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. A function $\psi : X \to Y$ is **pairwise soft fuzzy quasi uniform almost C-continuous function** if the inverse image of each soft fuzzy quasi regular $\mathcal{V}_1/\mathcal{V}_2$-open set in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ is a soft fuzzy C-quasi $\mathcal{U}_1/\mathcal{U}_2$-open set in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$.

Proposition 5.2. *Every pairwise soft fuzzy quasi uniform C-continuous function is pairwise soft fuzzy quasi uniform almost C-continuous function.*

Proof. Proof is obvious. □

The converse of the above property need not be true which is shown in the following example.

Example 5.3. Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$. Let N, M_1, M_2 be the subset of X. Let D_1 denote the family of functions $f_1, f_2, f_3 : SF(X) \to SF(X)$ be defined as follows

$$f_1(\lambda, N) = \begin{cases} (0,\phi), & \text{if}(\lambda, N) = (0,\phi) \\ (1,X), & \text{otherwise} \end{cases}$$

$$f_2(\lambda, N) = \begin{cases} (0,\phi), & \text{if } (\lambda, N) = (0,\phi) \\ (\mu_1, M_1), & \text{if}(0,\phi) \neq (\lambda, N) \sqsubseteq (\mu_1, M_1) \\ (1,X), & \text{otherwise} \end{cases}$$

$$f_3(\lambda, N) = \begin{cases} (0,\phi), & \text{if } (\lambda, N) = (0,\phi) \\ (\mu_2, M_2), & \text{if}(0,\phi) \neq (\lambda, N) \sqsubseteq (\mu_2, M_2) \\ (1,X), & \text{otherwise} \end{cases}$$

where (μ_1, M_1) and (μ_2, M_2) are defined as follows

$$\begin{array}{llll} \mu_1(a) = 1 & \mu_1(b) = 0 & \mu_1(c) = 1 & M_1 = \{a, b\} \\ \mu_2(a) = 1 & \mu_2(b) = 1 & \mu_2(c) = 0 & M_2 = \{a\} \end{array}$$

Let $\mathcal{U}_1, \mathcal{U}_2$ be the subcollection of D_1. Now, $(X,\mathcal{U}_1)$ and $(X,\mathcal{U}_2)$ form the soft fuzzy quasi uniform space. Then,

$$\tau_{\mathcal{U}_1} = \{(0,\phi), (1,X), (\mu_1, M_1)\}$$
$$\tau_{\mathcal{U}_2} = \{(0,\phi), (1,X), (\mu_2, M_2)\}$$

are the soft fuzzy quasi uniform topologies, which are generated by $\mathcal{U}_1$ and $\mathcal{U}_2$. Thus, $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ is a soft fuzzy quasi uniform bitopological space.
Let M, K_1, K_2, K_3 be the subset of Y. Let D_2 denote the family of functions $g_1, g_2, g_3, g_4 : SF(Y) \to SF(Y)$ be defined as follows

$$g_1(\mu, M) = \begin{cases} (0,\phi), & \text{if } (\mu, M) = (0,\phi) \\ (1,X), & \text{otherwise} \end{cases}$$

$$g_2(\mu, M) = \begin{cases} (0,\phi), & \text{if } (\mu, M) = (0,\phi) \\ (\gamma_1, K_1), & \text{if}(0,\phi) \neq (\mu, M) \sqsubseteq (\gamma_1, K_1) \\ (1,X), & \text{otherwise} \end{cases}$$

$$g_3(\mu, M) = \begin{cases} (0,\phi), & \text{if } (\mu, M) = (0,\phi) \\ (\gamma_2, K_2), & \text{if}(0,\phi) \neq (\mu, M) \sqsubseteq (\gamma_2, K_2) \\ (1,X), & \text{otherwise} \end{cases}$$

$$g_4(\mu, M) = \begin{cases} (0,\phi), & \text{if } (\mu, M) = (0,\phi) \\ (\gamma_3, K_3), & \text{if}(0,\phi) \neq (\mu, M) \sqsubseteq (\gamma_3, K_3) \\ (1,X), & \text{otherwise} \end{cases}$$

where (γ_i, K_i) for i=1,2,3 are defined as follows:

$$\begin{array}{llll} \gamma_1(a) = 1 & \gamma_1(b) = 0 & \gamma_1(c) = 0 & K_1 = \{p\} \\ \gamma_2(a) = 1 & \gamma_2(b) = 0 & \gamma_2(c) = 1 & K_2 = \{p\} \\ \gamma_3(a) = 0 & \gamma_3(b) = 1 & \gamma_3(c) = 1 & K_3 = \{q, r\} \end{array}$$

Let $\mathcal{V}_1$, $\mathcal{V}_2$ be the subcollection of D_2. Now, $(X, \mathcal{V}_1)$ and $(X, \mathcal{V}_2)$ form the soft fuzzy quasi uniform space. Then,

$$\tau_{\mathcal{V}_1} = \{(0, \phi), (1, X)\}$$
$$\tau_{\mathcal{V}_2} = \{(0, \phi), (1, X), (\gamma_1, K_1), (\gamma_2, K_2), (\gamma_3, K_3)\}$$

are the soft fuzzy quasi uniform topologies, which are generated by $\mathcal{V}_1$ and $\mathcal{V}_2$. Thus, $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ is a soft fuzzy quasi uniform bitopological space.
Let $\psi : (X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2}) \to (Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be defined as

$$\psi(a) = p \qquad \psi(b) = p \qquad \psi(c) = r$$

Thus, ψ **is pairwise soft fuzzy quasi uniform almost C-continuous function but not pairwise soft fuzzy quasi uniform C-continuous function.**

Proposition 5.4. *Suppose $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. Let $\psi : X \to Y$ be a function. Then, the following conditions are equivalent.*

(i) *ψ is pairwise soft fuzzy quasi uniform almost C-continuous function.*
(ii) *$\psi^{\leftharpoonup}(\lambda, N) \sqsubseteq C\text{–}int_{\mathcal{U}_1/\mathcal{U}_2}(\psi^{\leftharpoonup}(int_{\mathcal{V}_1}(cl_{\mathcal{V}_1}(\lambda, N))))$ or $\psi^{\leftharpoonup}(\lambda, N) \sqsubseteq C\text{–}int_{\mathcal{U}_1/\mathcal{U}_2}(\psi^{\leftharpoonup}(int_{\mathcal{V}_2}(cl_{\mathcal{V}_2}(\lambda, N))))$ for each soft fuzzy quasi $\mathcal{V}_1$-open set or soft fuzzy quasi $\mathcal{V}_2$-open set (λ, N) in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$.*
iii) *$C\text{-}cl_{\mathcal{U}_1/\mathcal{U}_2}(\psi^{\leftharpoonup}(cl_{\mathcal{V}_1}(int_{\mathcal{V}_1}(\mu, M)))) \sqsubseteq \psi^{\leftharpoonup}(\mu, M)$ or $C\text{-}cl_{\mathcal{U}_1/\mathcal{U}_2}(\psi^{\leftharpoonup}(cl_{\mathcal{V}_2}(int_{\mathcal{V}_2}(\mu, M)))) \sqsubseteq \psi^{\leftharpoonup}(\mu, M)$, for each soft fuzzy quasi $\mathcal{V}_1$-closed set or soft fuzzy quasi $\mathcal{V}_2$-closed set (μ, M) in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$.*
(iv) *$\psi^{\leftharpoonup}(\lambda, N)$ is a soft fuzzy C-quasi $\mathcal{U}_1$-closed set or C-quasi $\mathcal{U}_2$-closed set, for each soft fuzzy quasi regular $\mathcal{V}_1/\mathcal{V}_2$-closed set (λ, N) in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$.*

Proof. Proof is obvious. □

Definition 5.5. A soft fuzzy set (λ, N) in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ is called a **pairwise soft fuzzy quasi uniform dense* set** if there exists no soft fuzzy quasi $\mathcal{U}_1$-clopen set and soft fuzzy quasi $\mathcal{U}_2$-clopen set, (μ, M) such that $(\lambda, N) \sqsubset (\mu, M) \sqsubset (1, X)$.

Definition 5.6. Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. A function $\psi : (X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2}) \to (Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ is **somewhat pairwise soft fuzzy quasi uniform almost C-continuous function**, if (λ, N) is soft fuzzy quasi regular $\mathcal{V}_1$-open set or soft fuzzy quasi regular $\mathcal{V}_2$-open set and $\psi^{\leftharpoonup}(\lambda, N) \neq (0, \phi)$, then there exists a soft fuzzy C-quasi $\mathcal{U}_1$-open set or soft fuzzy C-quasi $\mathcal{U}_2$-open set (μ, M) such that $(\mu, M) \neq (0, \phi)$ and $(\mu, M) \sqsubseteq \psi^{\leftharpoonup}(\lambda, N)$.

It is clear from the definition, that every pairwise soft fuzzy quasi uniform almost C-continuous function is somewhat pairwise soft fuzzy quasi uniform almost C-continuous , but the converse need not be true as shown in the following example.

Example 5.7. Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$. Let N, M_1, M_2 be the subset of X. Let D_1 denote the family of functions $f_1, f_2, f_3 : SF(X) \to SF(X)$ be defined as follows

$$f_1(\lambda, N) = \begin{cases} (0, \phi), & \text{if } (\lambda, N) = (0, \phi) \\ (1, X), & \text{otherwise} \end{cases}$$

$$f_2(\lambda, N) = \begin{cases} (0, \phi), & \text{if } (\lambda, N) = (0, \phi) \\ (\mu_1, M_1), & \text{if} (0, \phi) \neq (\lambda, N) \sqsubseteq (\mu_1, M_1) \\ (1, X), & \text{otherwise} \end{cases}$$

$$f_3(\lambda, N) = \begin{cases} (0, \phi), & \text{if } (\lambda, N) = (0, \phi) \\ (\mu_2, M_2), & \text{if} (0, \phi) \neq (\lambda, N) \sqsubseteq (\mu_2, M_2) \\ (1, X), & \text{otherwise} \end{cases}$$

where (μ_1, M_1) and (μ_2, M_2) are defined as follows

$$\begin{array}{llll} \mu_1(a) = 1 & \mu_1(b) = 0 & \mu_1(c) = 1 & M_1 = \{a, b\} \\ \mu_2(a) = 1 & \mu_2(b) = 0 & \mu_2(c) = 1 & M_2 = \{a\} \end{array}$$

Let $\mathcal{U}_1, \mathcal{U}_2$ be the subcollection of D_1. Now, $(X,\mathcal{U}_1)$ and $(X,\mathcal{U}_2)$ form the soft fuzzy quasi uniform space. Then,

$$\tau_{\mathcal{U}_1} = \{(0, \phi), (1, X), (\mu_1, M_1)\}$$
$$\tau_{\mathcal{U}_2} = \{(0, \phi), (1, X), (\mu_2, M_2)\}$$

are the soft fuzzy quasi uniform topologies, which are generated by $\mathcal{U}_1$ and $\mathcal{U}_2$. Thus, $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ is a soft fuzzy quasi uniform bitopological space.
Let M, K_1, K_2 be the subsets of Y. Let D_2 denote the family of functions $g_1, g_2, g_3 : SF(Y) \to SF(Y)$ be defined as follows

$$g_1(\mu, M) = \begin{cases} (0, \phi), & \text{if } (\mu, M) = (0, \phi) \\ (1, X), & \text{otherwise} \end{cases}$$

$$g_2(\mu, M) = \begin{cases} (0, \phi), & \text{if } (\mu, M) = (0, \phi) \\ (\gamma_1, K_1), & \text{if} (0, \phi) \neq (\mu, M) \sqsubseteq (\gamma_1, K_1) \\ (1, X), & \text{otherwise} \end{cases}$$

$$g_3(\mu, M) = \begin{cases} (0, \phi), & \text{if } (\mu, M) = (0, \phi) \\ (\gamma_2, K_2), & \text{if} (0, \phi) \neq (\mu, M) \sqsubseteq (\gamma_2, K_2) \\ (1, X), & \text{otherwise} \end{cases}$$

where (γ_1, K_1) and (γ_2, K_2) are defined as follows

$$\begin{array}{llll} \gamma_1(a) = 1 & \gamma_1(b) = 0 & \gamma_1(c) = 0 & K_1 = \{p\} \\ \gamma_2(a) = 1 & \gamma_2(b) = 0 & \gamma_2(c) = 0 & K_2 = \{q, r\} \end{array}$$

Let $\mathcal{V}_1$, $\mathcal{V}_2$ be the subcollection of D_2. Now, $(X, \mathcal{V}_1)$ and $(X, \mathcal{V}_2)$ form the soft fuzzy quasi uniform space. Then,

$$\tau_{\mathcal{V}_1} = \{(0, \phi), (1, X)\}$$
$$\tau_{\mathcal{V}_2} = \{(0, \phi), (1, X), (\gamma_1, K_1), (\gamma_2, K_2)\}$$

are the soft fuzzy quasi uniform topologies, which are generated by $\mathcal{V}_1$ and $\mathcal{V}_2$. Thus, $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ is a soft fuzzy quasi uniform bitopological space.
Let $\psi : (X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2}) \to (Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be defined as

$$\psi(a) = p \qquad \psi(b) = q \qquad \psi(c) = p$$

Thus, ψ **is somewhat pairwise soft fuzzy quasi uniform almost C-continuous function but not pairwise soft fuzzy quasi uniform almost C-continuous function.**

Proposition 5.8. *Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$, $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ and $(Z, \tau_{\mathcal{W}_1}, \tau_{\mathcal{W}_2})$ be the soft fuzzy quasi uniform bitopological spaces. Let $\psi : X \to Y$ be a somewhat pairwise soft fuzzy quasi uniform almost C-continuous and let $\phi : Y \to Z$ be a pairwise soft fuzzy quasi uniform almost continuous and pairwise soft fuzzy quasi uniform almost open function. Then, $\phi \circ \psi$ is a somewhat pairwise soft fuzzy almost C-continuous function.*

Proof. Proof is clear from the Remark: 4.4. □

Proposition 5.9. *Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. Let $\psi : X \to Y$ be any function. Then, the following are equivalent:*
(i) ψ is somewhat pairwise soft fuzzy quasi uniform almost C-continuous function.
(ii) If (λ, N) is a pairwise soft fuzzy quasi uniform C-dense set, then $\psi^{\rightharpoonup}(\lambda, N)$ is a pairwise soft fuzzy quasi uniform dense* set in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$.

Proof. Proof is obvious. □

From the results proved so far, the following diagram of implications is shown below :

$$\begin{array}{c}
\text{Somewhat pairwise soft fuzzy quasi uniform C} - \text{continuous function} \\
\uparrow \\
\text{Pairwise soft fuzzy quasi uniform C} - \text{continuous function} \\
\downarrow \\
\text{Pairwise soft fuzzy quasi uniform almost C} - \text{continuous function} \\
\downarrow \\
\text{Somewhat pairwise soft fuzzy quasi uniform almost C} - \text{continuous function}
\end{array}$$

6. Somewhat pairwise soft fuzzy quasi uniform almost C-open function

Definition 6.1. Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. Then, $\psi : X \to Y$ is said to be a **pairwise soft fuzzy quasi uniform almost C-open function**, if (λ, N) is soft fuzzy quasi regular $\mathcal{U}_1$-open set or soft fuzzy quasi regular $\mathcal{U}_2$-open set in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$, $\psi^{\rightharpoonup}(\lambda, N)$ is a soft fuzzy C-quasi $\mathcal{V}_1$-open set or soft fuzzy C-quasi $\mathcal{V}_2$-open set.

Definition 6.2. Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. Then, $\psi : X \to Y$ is said to be a **somewhat pairwise soft fuzzy quasi uniform almost C-open function** iff (λ, N) is a soft fuzzy quasi regular $\mathcal{U}_1$-open set or soft fuzzy quasi regular $\mathcal{U}_2$-open set and $(\lambda, N) \neq (0, \phi)$, then there exists a soft fuzzy C-quasi $\mathcal{V}_1$-open set or a soft fuzzy C-quasi $\mathcal{V}_2$-open set, (μ, M) such that $(\mu, M) \neq (0, \phi)$ and $(\mu, M) \sqsubseteq \psi^{\rightharpoonup}(\lambda, N)$.

Proposition 6.3. *Suppose $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. Let $\psi : X \to Y$ be any function. Then the following conditions are equivalent.*
(i) ψ is somewhat pairwise soft fuzzy quasi uniform almost C-open function.
(ii) If (λ, N) is a soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-closed set such that $\psi^{\rightharpoonup}(\lambda, N) \neq (1, Y)$, then there exists a soft fuzzy C-quasi $\mathcal{V}_1/\mathcal{V}_2$-closed set (μ, M) such that $(\mu, M) \neq (1, Y)$ and $(\mu, M) \sqsupset \psi^{\rightharpoonup}(\lambda, N)$.

Proof. $(i) \Rightarrow (ii)$ Let (λ, N) be a soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-closed set in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ such that $\psi^{\rightharpoonup}(\lambda, N) \neq (1, Y)$. Then, $(1, X) - (\lambda, N)$ is a soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-open set such that $\psi^{\rightharpoonup}((1, X) - (\lambda, N)) = (1, Y) - \psi^{\rightharpoonup}(\lambda, N) \neq (0, \phi)$. As ψ is somewhat pairwise soft fuzzy quasi uniform almost C-open function, there exists a soft fuzzy C-quasi $\mathcal{V}_1/\mathcal{V}_2$-open set (μ, M) such that $(\mu, M) \neq (0, \phi)$ and $(\mu, M) \sqsubseteq \psi^{\rightharpoonup}[(1, X) - (\lambda, N)]$. Since (μ, M) is soft fuzzy C-quasi $\mathcal{V}_1/\mathcal{V}_2$-open set such that $(\mu, M) \neq (0, \phi)$, $(1, Y) - (\mu, M)$ is soft fuzzy C-quasi $\mathcal{V}_1/\mathcal{V}_2$-closed set (μ, M) in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ such that $(1, Y) - (\mu, M) \neq (1, Y)$ and $(1, Y) - (\mu, M) \sqsupset \psi^{\rightharpoonup}(\lambda, N)$.

$(ii) \Rightarrow (i)$ Let (λ, N) be a soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-open set such that $(\lambda, N) \neq (0, \phi)$. Then $(1, X) - (\lambda, N)$ is a soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-closed set and $(1, X) - (\lambda, N) \neq (1, X)$. Now $\psi^{\rightharpoonup}((1, X) - (\lambda, N)) = (1, Y) - \psi^{\rightharpoonup}(\lambda, N) \neq (1, Y)$. Hence by hypothesis, there exists soft fuzzy C-quasi $\mathcal{V}_1/\mathcal{V}_2$-closed set (μ, M) in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ such that $(\mu, M) \neq (1, Y)$ and $(\mu, M) \sqsupset \psi^{\rightharpoonup}((1, X) - (\lambda, N)) = (1, Y) - \psi^{\rightharpoonup}(\lambda, N)$. This implies $\psi^{\rightharpoonup}(\lambda, N) \sqsupseteq (1, Y) - (\mu, M)$. Clearly $(1, Y) - (\mu, M)$ is soft fuzzy C-quasi $\mathcal{V}_1/\mathcal{V}_2$-open set in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ such that $(1, Y) - (\mu, M) \sqsubseteq \psi^{\rightharpoonup}(\lambda, N)$ and $(1, Y) - (\mu, M) \neq (0, \phi)$. This shows that ψ is somewhat pairwise soft fuzzy quasi uniform almost C-open function. □

Proposition 6.4. *Suppose $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be any two soft fuzzy quasi uniform bitopological spaces. Let $\psi : X \to Y$ be a surjective function. Then, the following conditions are equivalent.*
(i) ψ is somewhat pairwise soft fuzzy quasi uniform almost C-open function.
(ii) If (λ, N) is a pairwise soft fuzzy quasi uniform C-dense set in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$, then $\psi^{\leftharpoonup}(\lambda, N)$ is a pairwise soft fuzzy quasi uniform dense* set in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$.

Proof. $(i) \Rightarrow (ii)$ Assume that ψ is somewhat pairwise soft fuzzy quasi uniform almost C-open function. Suppose that (λ, N) is a pairwise soft fuzzy quasi uniform C-dense set in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$. It must be shown that, $\psi^{\leftharpoonup}(\lambda, N)$ is a pairwise soft fuzzy quasi uniform *dense** set in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$. Suppose if it is not, then there exists a soft fuzzy quasi $\mathcal{U}_1/\mathcal{U}_2$-clopen set, (μ, M) such that $\psi^{\leftharpoonup}(\lambda, N) \sqsubset (\mu, M) \sqsubset (1, X)$. Now, $(\lambda, N) = \psi^{\rightharpoonup}(\psi^{\leftharpoonup}(\lambda, N)) \sqsubset \psi^{\rightharpoonup}(\mu, M) \sqsubset \psi^{\rightharpoonup}(1, X) = (1, Y)$. Since ψ is somewhat pairwise soft fuzzy quasi uniform almost C-open function and every soft fuzzy quasi $\mathcal{U}_1/\mathcal{U}_2$-clopen set is soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-clopen set, there exists a soft fuzzy C-quasi $\mathcal{V}_1$-closed set or soft fuzzy C-quasi $\mathcal{V}_2$-closed set (δ, L) such that $(\delta, L) \neq (1, Y)$ and $(\delta, L) \sqsupset \psi^{\rightharpoonup}(\mu, M)$. Thus, $(\lambda, N) \sqsubset \psi^{\rightharpoonup}(\mu, M) \sqsubset (\delta, L) \neq (1, Y)$. This leads a contradiction that (λ, N) being a pairwise soft fuzzy quasi uniform C-dense set in $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$. Hence $\psi^{\leftharpoonup}(\lambda, N)$ is a pairwise soft fuzzy quasi *dense** set in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$.

$(ii) \Rightarrow (i)$ Suppose $(\lambda, N) \neq (0, \phi)$ is a soft fuzzy quasi regular $\mathcal{U}_1$-open set or a soft fuzzy quasi regular $\mathcal{U}_2$-open set. It must be shown that $C\text{-}int_{\mathcal{V}_1}(\psi^{\rightharpoonup}(\lambda, N)) \neq (0, \phi)$ or $C\text{-}int_{\mathcal{V}_2}(\psi^{\rightharpoonup}(\lambda, N)) \neq (0, \phi)$. Suppose that $C\text{-}int_{\mathcal{V}_1}(\psi^{\rightharpoonup}(\lambda, N)) = (0, \phi)$ and $C\text{-}int_{\mathcal{V}_2}(\psi^{\rightharpoonup}(\lambda, N)) = (0, \phi)$. Now

$$C\text{-}cl_{\mathcal{V}_1}((1, Y) - \psi^{\rightharpoonup}(\lambda, N)) = (1, Y) - C\text{-}int_{\mathcal{V}_1}(\psi^{\rightharpoonup}(\lambda, N)) = (1, Y)$$

and

$$C\text{-}cl_{\mathcal{V}_2}((1, Y) - \psi^{\rightharpoonup}(\lambda, N)) = (1, Y) - C\text{-}int_{\mathcal{V}_2}(\psi^{\rightharpoonup}(\lambda, N)) = (1, Y).$$

This implies, $(1, Y) - (\psi^{\rightharpoonup}(\lambda, N))$ is a pairwise soft fuzzy quasi uniform C-dense set. Therefore by (ii), $\psi^{\leftharpoonup}((1, Y) - \psi^{\rightharpoonup}(\lambda, N))$ is a pairwise soft fuzzy quasi uniform *dense** set in $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$. Therefore, $(1, X) = cl_{\mathcal{U}_1}(\psi^{\leftharpoonup}((1, Y) - \psi^{\rightharpoonup}(\lambda, N))) \sqsubseteq cl_{\mathcal{U}_1}((1, X) - (\lambda, N)) = (1, X) - (\lambda, N)$ and $(1, X) = cl_{\mathcal{U}_2}(\psi^{\leftharpoonup}((1, Y) - \psi^{\rightharpoonup}(\lambda, N))) \sqsubseteq cl_{\mathcal{U}_2}((1, X) - (\lambda, N)) = (1, X) - (\lambda, N)$. This implies, $(1, X) \sqsubseteq (1, X) - (\lambda, N)$. That is, $(\lambda, N) \sqsubseteq (0, \phi)$. This leads a contradiction that (λ, N). Therefore, $C\text{-}int_{\mathcal{V}_1}(\psi(\lambda, N)) \neq (0, \phi)$ or $C\text{-}int_{\mathcal{V}_2}(\psi(\lambda, N)) \neq (0, \phi)$. Thus, ψ is somewhat pairwise soft fuzzy quasi uniform almost C-open function. □

Proposition 6.5. *Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$, $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ and $(Z, \tau_{\mathcal{W}_1}, \tau_{\mathcal{W}_2})$ be the soft fuzzy quasi uniform bitopological spaces. Let $\psi : X \to Y$ be a pairwise soft fuzzy quasi uniform almost C-open function of a space X onto a space Y and let $\phi : Y \to Z$. If $\phi \circ \psi$ is a pairwise soft fuzzy quasi uniform almost continuous and pairwise soft fuzzy quasi uniform almost open functions, then, ϕ is pairwise soft fuzzy quasi uniform almost C-continuous function.*

Proof. Let (λ, N) be a soft fuzzy quasi regular $\mathcal{W}_1/\mathcal{W}_2$-open set in Z. By the Remark:4.1, $(\phi \circ \psi)^{\leftharpoonup}(\lambda, N) = \psi^{\leftharpoonup}(\phi^{\leftharpoonup}(\lambda, N))$ is soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-open set in X. Since ψ is a pairwise soft fuzzy quasi uniform almost C-open function and ψ is surjective, $\psi^{\rightharpoonup}(\psi^{\leftharpoonup}(\phi^{\leftharpoonup}(\lambda, N))) = \phi^{\leftharpoonup}(\lambda, N)$ is soft fuzzy C-quasi $\mathcal{V}_1/\mathcal{V}_2$-open set in Y. Thus, ϕ is pairwise soft fuzzy quasi uniform almost C-continuous function. □

Proposition 6.6. *Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$, $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ and $(Z, \tau_{\mathcal{W}_1}, \tau_{\mathcal{W}_2})$ be the soft fuzzy quasi uniform bitopological spaces. Let $\psi : X \to Y$, $\phi : Y \to Z$ and suppose that $\phi \circ \psi$ is a somewhat pairwise soft fuzzy quasi uniform almost C-open function. If ψ is pairwise soft fuzzy quasi uniform almost continuous and pairwise soft fuzzy quasi uniform almost open surjection, then ϕ is somewhat pairwise soft fuzzy quasi uniform almost C-open function.*

Proof. Proof is obvious by using Remark : 4.4. □

Proposition 6.7. *Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be the soft fuzzy quasi uniform bitopological spaces. Let $\psi : X \to Y$ be a pairwise soft fuzzy quasi uniform almost C-open function and a bijective function. For any $(\lambda, N) \in SF(Y)$ and any soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-closed set $(\mu, M) \sqsupseteq \psi^{\leftharpoonup}(\lambda, N)$, there exists a soft fuzzy C-quasi $\mathcal{V}_1/\mathcal{V}_2$-closed set $(\delta, L) \sqsupseteq (\lambda, N)$ such that $\psi^{\leftharpoonup}(\delta, L) \sqsubseteq (\mu, M)$.*

Proof. Let $(\lambda, N) \in SF(Y)$ and (μ, M) be any soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-closed set such that $(\mu, M) \sqsupseteq \psi^{\leftharpoonup}(\lambda, N)$. Since ψ is a pairwise soft fuzzy quasi uniform almost C-open function and $(1, X) - (\mu, M)$ is a soft fuzzy quasi regular $\mathcal{U}_1/\mathcal{U}_2$-open set, it follows that, $(\delta, L) = (1, Y) - \psi^{\rightharpoonup}((1, X) - (\mu, M))$ is a soft fuzzy C-quasi $\mathcal{V}_1/\mathcal{V}_2$-closed set in Y and $(\delta, L) \sqsupseteq (1, Y) - \psi^{\rightharpoonup}((1, X) - \psi^{\leftharpoonup}(\lambda, N)) = \psi^{\rightharpoonup}(\psi^{\leftharpoonup}(\lambda, N)) = (\lambda, N)$. Now, $\psi^{\leftharpoonup}(\delta, L) = \psi^{\leftharpoonup}((1, Y) - \psi^{\rightharpoonup}((1, X) - (\mu, M))) = (1, X) - \psi^{\leftharpoonup}(\psi^{\rightharpoonup}((1, X) - (\mu, M))) \sqsubseteq (\mu, M)$. Hence, it is proved. □

Proposition 6.8. *Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be the soft fuzzy quasi uniform bitopological spaces. If $\psi : X \to Y$ is a pairwise soft fuzzy quasi uniform continuous and pairwise soft fuzzy quasi uniform almost C-open bijection function, then $\psi^{\leftharpoonup}(C\text{-}cl_{\mathcal{V}_1/\mathcal{V}_2}(int_{\mathcal{V}_1/\mathcal{V}_2}(\lambda, N))) \sqsubseteq cl_{\mathcal{U}_1/\mathcal{U}_2}(\psi^{\leftharpoonup}(int_{\mathcal{V}_1/\mathcal{V}_2}(\lambda, N)))$, for each soft fuzzy set (λ, N) in Y.*

Proof. Proof is clear. □

Definition 6.9. A soft fuzzy quasi uniform bitopological space $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ is called as **a pairwise soft fuzzy quasi uniform $\mathcal{D}^*$-space ($\mathcal{D}_C$-space)**, if every non-zero soft fuzzy quasi $\mathcal{U}_1/\mathcal{U}_2$-open set (λ, N) of X is soft fuzzy quasi uniform dense* (soft fuzzy quasi C-dense) set in X.

Proposition 6.10. *Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be the soft fuzzy quasi uniform bitopological spaces. Let ψ be a pairwise soft fuzzy quasi uniform almost C-open function from X onto the pairwise soft fuzzy quasi uniform $\mathcal{D}_C$-space, Y. Then X is pairwise soft fuzzy quasi uniform $\mathcal{D}^*$-space.*

Proof. Proof is obvious. □

Proposition 6.11. *Let $(X, \tau_{\mathcal{U}_1}, \tau_{\mathcal{U}_2})$ and $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ be the soft fuzzy quasi uniform bitopological spaces. Let ψ be a pairwise soft fuzzy quasi uniform almost C-open and surjective function. If Y is the pairwise soft fuzzy quasi uniform $\mathcal{D}_C$-space, then $cl_{\mathcal{U}_1/\mathcal{U}_2}(\psi^{\leftharpoonup}(int_{\mathcal{V}_1/\mathcal{V}_2}(\lambda, N))) = (1, X)$, for any soft fuzzy set (λ, N) in Y.*

Proof. Let (λ, N) be any soft fuzzy set in Y. Let $int_{\mathcal{V}_1/\mathcal{V}_2}(\lambda, N) \neq (0, \phi)$ be a soft fuzzy quasi $\mathcal{V}_1/\mathcal{V}_2$-open set in Y. Since $(Y, \tau_{\mathcal{V}_1}, \tau_{\mathcal{V}_2})$ is the pairwise soft fuzzy quasi uniform $\mathcal{D}_C$-space and ψ is the soft fuzzy quasi uniform almost C-open function, $int_{\mathcal{V}_1/\mathcal{V}_2}(\lambda, N)$ is a soft fuzzy quasi uniform C-dense set in Y and $\psi^{\leftharpoonup}(int_{\mathcal{V}_1/\mathcal{V}_2}(\lambda, N))$ is the soft fuzzy quasi uniform dense* in X. This implies that,$cl_{\mathcal{U}_1/\mathcal{U}_2}(\psi^{\leftharpoonup}\ (int_{\mathcal{V}_1/\mathcal{V}_2}(\lambda, N))) = (1, X)$. □

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