Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 7, No. 3, (March 2014), pp. 355–368
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

@FMI
© Kyung Moon Sa Co.
http://www.kyungmoon.com

A novel approach for solving fully fuzzy linear programming problems using membership function concepts

Seyed Hadi Nasseri, Farzaneh Khalili , Nemat Allah Taghi-Nezhad, Seyedeh Marziyeh Mortezania

Received 21 April 2013; Accepted 8 July 2013

Abstract. Modeling and solving optimization problems is one of the most important issues in our real world problems. This paper presents a new method for solving Fully Fuzzy Linear Programming (FFLP) problems. Although, it has been considered and expanded from many various points of view in more than a decade, but still it is useful to develop new procedures to present better fit into real world problems as much as possible. After introducing FFLP, we propose a new method to solve these kinds of problems. The method is based on the definition of membership function and using the convenient techniques for solving the classical multi-objective programming. Furthermore, for describing the solution process, we have gave two examples. The computational results show that this method has a good performance in compare with others in the literature.

2010 AMS Classification: 90C70, 90C05

Keywords: Fully fuzzy linear programming, Triangular fuzzy number, Multi objective problem, Membership function.

Corresponding Author: Farzaneh Khalili (`farzaneh.khalili@stu.umz.ac.ir`)

1. Introduction

Linear programming is one of the most important applied operations research techniques. Due to adapt the real world situations, various attempts were proposed by researchers. Although the concept of decision making in fuzzy environment was first proposed by Belman and Zadeh [3], however the concept of fuzzy linear programming on general level was first proposed by Tanaka et al. [15] in the framework of the fuzzy decision of Bellman and Zadeh [3]. Many researchers adopted this concept for solving fuzzy linear programming problems, but less attention has focused on formulation of fully fuzzy linear programming. Zimmermann [17] proposed the first formulation of fuzzy linear programming in different objective. Campos and Verdegay [4] studied linear programming problem with fuzzy constraints and coefficients in both matrix and right hand side of the constraints sets. Kumar [10] proposed a new method to solve Fully Fuzzy Linear Programming (FFLP) where the constraints are all inequality. Mariano Jimenez et al. [9] presented a method for solving linear programming problems where all the coefficients are, in general, fuzzy numbers. They used a fuzzy ranking method to rank the fuzzy objective values and to deal with the inequality relation on constraints. Nasseri and Ebrahimnejad [14] proposed a fuzzy primal simplex algorithm for solving the flexible linear programming problem and then suggested the fuzzy primal simplex method to solve the flexible linear programming problems directly without solving any auxiliary problem. Ebrahimnejad and Nasseri [7] used the complementary slackness theorem to solve fuzzy linear programming problem with fuzzy parameters without the need of a simplex tableau. Allahviranloo et al. [1] solved the fuzzy integer linear programming problem by reducing it into a crisp integer linear programming problem. After that Allahviranloo et al. [2] proposed a new method for solving fully fuzzy linear programming problems by use of a ranking function. Lotfi et al. [8] discussed FFLP problems by representing all parameters and variables as triangular fuzzy numbers. Dehghan et al. [6] proposed a fuzzy linear programming approach for finding the exact solution of Fully Fuzzy Linear System (FFLS) of equations. Kumar et al. [11] presented a new method for solving FFLP and showed the deficiency of the methods which were given in [6, 8]. Mishmast Nehi and Hajmohamadi [13] presented a new method in which the comparison of fuzzy numbers was used by a linear ordered function for solving multi-objective linear Programming (MOLP) where the parameters and objectives were fuzzy. Kumar and Singh [12] proposed a new method for finding the optimal solution of FLP problems. The main advantage of the proposed method over existing methods was that the fuzzy linear programming problems which can be solved by the existing methods can also be solved by the proposed method but there exist several fuzzy linear programming problems which can be solved only by using the proposed method. In this study we proposed a new method for solving Fully Fuzzy Linear Programming (FFLP) where all constraints, coefficients, parameters and variables are fuzzy numbers. This method is based on membership function and multi objective concepts. This paper is organized in 5 sections. In Section 2, we present some basic notations and definitions of fuzzy set theory. In Section 3 we introduce the FFLP program and then propose a new method for solving these programs. The mentioned algorithm is presented in 6 Steps. Illustrative examples in Section 4 demonstrate the efficiency of our proposed method. Finally conclusions are presented in Section 5.

2. Preliminaries

In this section we review some basic backgrounds and notions of fuzzy set theory, which is taken from [5] and[14].

2.1. **Terminology and notation.**

Definition 2.1. Let X be a collection of objects denoted generically by x, a fuzzy set $\tilde{A}$ in X is defined to be a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$, where

$\mu_{\tilde{A}}(x)$ is called membership function for fuzzy set $\tilde{A}$. The membership function maps each element to a membership degree.

Definition 2.2. The support of a fuzzy set $\tilde{A}$ is the set of points x in X with $\mu_{\tilde{A}}(x) > 0$.

Definition 2.3. A fuzzy subset A of universe set X is normal, iff there is at least one $x \in X$, that $\mu_{\tilde{A}}(x) = 1$.

Definition 2.4. The α–cut or α–level set of a fuzzy set is a certain set defined as follow:

$$\tilde{A}_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) > \alpha\}.$$

Definition 2.5. A fuzzy set $\tilde{A}$ of universe set X is convex if and only if for any $x, y \in X$ and $\lambda \in [0, 1]$, we have

$$\mu_{\tilde{A}}(\lambda x + (1 - \lambda)y) \geq min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}.$$

Definition 2.6. A fuzzy set $\tilde{A}$ is a fuzzy number if and only if $\tilde{A}$ satisfies in normality and convexity conditions on the real line.

Definition 2.7. A triangular fuzzy number $\tilde{A} = (a, b, c)$ is a fuzzy number on $\mathbb{R}$. with a membership function $\mu_{\tilde{A}}$ defined by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b] \\ \dfrac{x-c}{b-c}, & x \in [b,c] \\ 0, & o.w. \end{cases}$$

We also denote the set of all triangular fuzzy numbers with $\mathbf{F}(\mathbf{R})$.

Definition 2.8. A triangular fuzzy number $\tilde{A} = (a, b, c)$ is said to be nonnegative triangular fuzzy number, iff $a \geq 0$.

2.2. **Arithmetic on triangular fuzzy numbers.** Let $\tilde{A} = (a, b, c)$ and $\tilde{B} = (d, e, f)$ be two triangular fuzzy numbers. Define:

1) $\tilde{A} \oplus \tilde{B} = (a + d, b + e, c + f)$.

2) $-\tilde{A} = (-c, -b, -a)$,

3) $\tilde{A} \ominus \tilde{B} = (a - f, b - e, c - d)$,

4) $\tilde{A} = \tilde{B}$ if and only if $a = d, b = e, c = f$,

5) Suppose $\tilde{A}$ be any triangular fuzzy number and $\tilde{B}$ be a non-negative triangular fuzzy number according to Definition 2.8, then we define:

$$\tilde{A} \otimes \tilde{B} \simeq \begin{cases} (ad, be, cf), & a \geq 0 \\ (af, be, cf), & a < 0,\ c \geq 0 \\ (af, be, cd), & c < 0 \end{cases}$$

3. FFLP problem and a new solving method

In this section, we first define FFLP problem, and after that give a new method to solve the mentioned problem.

Definition 3.1. FFLP problems with μ fuzzy constraints and n variables may be formulated as follows:

$$maximize\ \ (or\ minimize)\ \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j$$

$$\text{(3.1)} \qquad s.t. \begin{cases} \sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \preceq, =, \succeq \tilde{b}_i,\ \forall i = 1, \dots, m \\ \tilde{x}_j \ \ is\ nonnegative\ fuzzy\ number \end{cases}$$

where $\tilde{C} = [\tilde{c}_j]_{1\times n}$, $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{b} = \left[\tilde{b}_i\right]_{m\times 1}$ $\tilde{X} = [\tilde{x}_j]_{n\times 1}$.

3.1. **Main steps of the proposed algorithm.** In this section we propose a novel method for solving Fully Fuzzy Linear Programming (FFLP) problems in 7 steps. The steps of this method are as follows:

Step 1: Suppose our FFLP problem is similar to (3.1). If all $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i$ and $\tilde{x}_j$, are represented by triangular fuzzy numbers (p_j, q_j, r_j), (a_{ij}, b_{ij}, c_{ij}), (b_i, g_i, h_i) and (x_j, y_j, z_j) respectively, then the FFLP problem, may be written as:

$$max\ (min)\ \sum_{j=1}^{n} (p_j, q_j, r_j) \otimes (x_j, y_j, z_j)$$

$$\text{(3.2)} \quad s.t. \begin{cases} \sum_{j=1}^{n} (a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) \preceq, =, \succeq (b_i, g_i, h_i),\ \forall i = 1, \dots, m \\ (x_j, y_j, z_j) \ \ is\ nonnegative\ fuzzy\ number \end{cases}$$

Step 2: By using the arithmetic operations defined in subsection 2.2, the fuzzy linear programming problem obtained in Step 1, is converted into the following equivalent problem:

$$max\ (min) \sum_{j=1}^{n} (p_j, q_j, r_j) \otimes (x_j, y_j, z_j) = \sum_{j=1}^{n} (\alpha_j, \beta_j, \gamma_j)$$

$$(3.3) \qquad s.t. \begin{cases} \sum_{j=1}^{n} m_{ij} \leq, =, \geq b_i, \ \forall i = 1, \ldots, m \\ \sum_{j=1}^{n} n_{ij} \leq, =, \geq g_i, \ \forall i = 1, \ldots, m \\ \sum_{j=1}^{n} o_{ij} \leq, =, \geq h_i, \ \forall i = 1, \ldots, m \\ y_j - x_j \geq 0, z_j - y_j \geq 0 \end{cases}$$

where

$$(a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) = (m_{ij}, n_{ij}, o_{ij})$$

$$(p_j, q_j, r_j) \otimes (x_j, y_j, z_j) = (\alpha_j, \beta_j, \gamma_j)$$

Step 3: Suppose the problem is in form of maximizing, (we can easily expand the problem to the minimizing form), then we convert the objective function into three objectives as follows:

- Minimize $\sum_{j=1}^{n} \beta_j - \alpha_j$
- Maximize $\sum_{j=1}^{n} \beta_j$
- Maximize $\sum_{j=1}^{n} \gamma_j - \beta_j$

Therefore our problem would change as follows:

$$Z_1 = Minimize \sum_{j=1}^{n} \beta_j - \alpha_j$$

$$Z_2 = Maximize \sum_{j=1}^{n} \beta_j$$

$$Z_3 = Maximize \sum_{j=1}^{n} \gamma_j - \beta_j$$

$$(3.4) \qquad s.t. \begin{cases} \sum_{j=1}^{n} m_{ij} \leq, =, \geq b_i, \ \forall i = 1, \ldots, m \\ \sum_{j=1}^{n} n_{ij} \leq, =, \geq g_i, \ \forall i = 1, \ldots, m \\ \sum_{j=1}^{n} o_{ij} \leq, =, \geq h_i, \ \forall i = 1, \ldots, m \\ y_j - x_j \geq 0, z_j - y_j \geq 0 \end{cases}$$

Step 4: Determine the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for each objective function by solving the corresponding model as follows:

$$Z_1{}^{PIS} = Minimize \sum_{j=1}^{n} \beta_j - \alpha_j \tag{3.5}$$
$$x \in \mathbf{F}$$

$$Z_1{}^{NIS} = Maximize \sum_{j=1}^{n} \beta_j - \alpha_j \tag{3.6}$$
$$x \in \mathbf{F}$$

$$Z_2{}^{PIS} = Maximize \sum_{j=1}^{n} \beta_j \tag{3.7}$$
$$x \in \mathbf{F}$$

$$Z_2{}^{NIS} = Minimize \sum_{j=1}^{n} \beta_j \tag{3.8}$$
$$x \in \mathbf{F}$$

$$Z_3{}^{PIS} = Maximize \sum_{j=1}^{n} \gamma_j - \beta_j \tag{3.9}$$
$$x \in \mathbf{F}$$

$$Z_3{}^{NIS} = Minimize \sum_{j=1}^{n} \gamma_j - \beta_j \tag{3.10}$$
$$x \in \mathbf{F}$$

Assume that, $\boldsymbol{F}$ be the set of all constraints. Obtaining the above ideal solutions requires solving six linear programming problems. To reduce the computational time, the negative ideal solutions can be determined by using the positive ideal solutions. Let v_h^* and $Z_h(v_h^*)$ denote the decision vector associated with the PIS of hth objective function and the corresponding value of hth objective function, respectively. So, the related NIS could be estimated as follows:

$$Z_1^{NIS} = max_{k=1,2,3}\{Z_1(v_k^*)\} \qquad Z_h^{NIS} = min_{k=1,2,3}\left\{Z_h\left(v_k^*\right)\right\}; h = 2, 3$$

Step 5: Determine a linear membership function for each objective function according to positive and negative ideal points as follows:

$$\mu_1(v) = \begin{cases} 1, & \text{if } Z_1 < Z_1^{PIS} \\ \dfrac{Z_1^{NIS} - Z_1}{Z_1^{NIS} - Z_1^{PIS}}, & \text{if } Z_1^{NIS} \geq Z_1 \geq Z_1^{PIS} \\ 0, & \text{if } Z_1 > Z_1^{NIS} \end{cases} \tag{3.11}$$

$$\mu_2(v) = \begin{cases} 1, & \text{if } Z_2 > Z_2^{PIS} \\ \dfrac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}}, & \text{if } Z_2^{NIS} \leq Z_2 \leq Z_2^{PIS} \\ 0, & \text{if } Z_2 < Z_2^{NIS} \end{cases} \tag{3.12}$$

$$\mu_3(v) = \begin{cases} 1, & \text{if } Z_3 > Z_3^{PIS} \\ \dfrac{Z_3^{NIS} - Z_3}{Z_3^{NIS} - Z_3^{PIS}}, & \text{if } Z_3^{NIS} \leq Z_3 \leq Z_3^{PIS} \\ 0, & \text{if } Z_3 < Z_3^{NIS} \end{cases} \tag{3.13}$$

In practice, $\mu_i(v)\,; i = 1,2,3$ presents the satisfaction level of ith objective function for the given solution vector v. The graphs of these membership functions were represented in Figures 1 and 2 see also in [16]

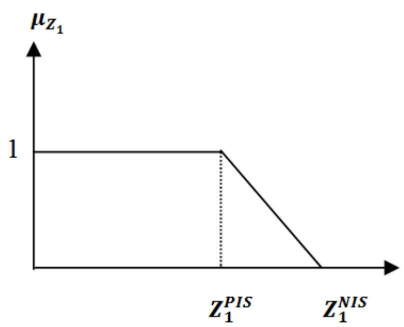

Fig.1. Linear membership function for $\mathbf{Z}_1$

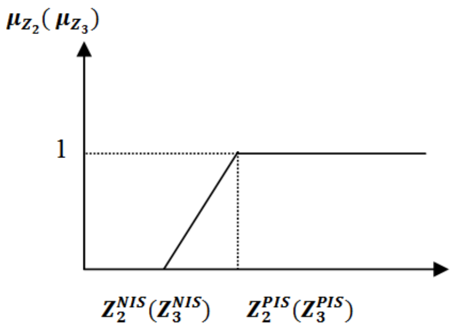

Fig.2. Linear membership function for $\mathbf{Z}_2(\mathbf{Z}_3)$

Step 6: Convert the auxiliary LP model into an equivalent single-objective LP by using the following auxiliary crisp formulation:

$$\max \gamma\lambda \ + (1-\gamma)\sum_{i=1}^{3} \theta_i Z_i(v)$$

$$s.t.\begin{cases} 0\le\lambda,\gamma\le 1 \\ \lambda\le\mu_i\left(v\right);i=1,2,3 \\ v\in F\left(v\right)\end{cases} \tag{3.14}$$

where $\mu_i\left(v\right);i=1,2,3$ presents the satisfaction level of ith objective function for the given solution vector v and λ denote the minimum satisfaction degree of all objectives. This formulation has a new achievement function defined as a convex combination of the lower bound for satisfaction degree of objectives (λ), and the weighted sum of the objective functions to ensure yielding an adjustably balanced compromise solution. Moreover, θ_i and γ indicate the relative importance of the ith objective function and the coefficient of compensation, respectively. The selection of ,θ_i depends to the aims and opinion of decision maker. The main aim in this problem is to find the maximum of minimum satisfaction degree of all objectives in order to find a better solution for the primal FFLP problem.

Step 7: After solving, the solutions must be put into the objective function of primal FFLP problem in order to find the fuzzy objective value of problem. In this section, a numerical example is considered to indicate the performance of proposed method. To illustrate our method, we will solve the following FFLP and then we compare our answer with the answer solving by A. Kumar et. al method in [5].

4. Numerical examples

Example 4.1. Consider the following FFLP problem. By solving this problem, we are going to explain the main steps of our proposed algorithm. Due to explaining our method in maximum form, here we solve a problem in minimum form to expand it:

$$\min Z=((1,6,9)\otimes\tilde{x}_1\oplus(2,2,8)\otimes\tilde{x}_2)$$

$$s.t.\begin{cases}(0,1,1)\otimes\tilde{x}_1\oplus(2,2,3)\otimes\tilde{x}_2\succcurlyeq(4,7,14)\\(2,2,3)\otimes\tilde{x}_1\oplus(-1,4,4)\otimes\tilde{x}_2\preccurlyeq(-4,14,22)\\(2,3,4)\otimes\tilde{x}_1\ominus(1,2,3)\otimes\tilde{x}_2\preccurlyeq(-12,-3,6)\end{cases} \tag{4.1}$$

$\tilde{x}_1$ and $\tilde{x}_2$ are non-negative triangular fuzzy numbers

According to Step 1, let $\tilde{x}_1=(x_1,y_1,z_1)$ and $\tilde{x}_2=(x_2,y_2,z_2)$. Then the original FFLP problem changes as follows:

$$\min Z=((1,6,9)\otimes(x_1,y_1,z_1)\oplus(2,2,8)\otimes(x_2,y_2,z_2))$$

$$s.t.\begin{cases}(0,1,1)\otimes(x_1,y_1,z_1)\oplus(2,2,3)\otimes(x_2,y_2,z_2)\succcurlyeq(4,7,14)\\(2,2,3)\otimes(x_1,y_1,z_1)\oplus(-1,4,4)\otimes(x_2,y_2,z_2)\preccurlyeq(-4,14,22)\\(2,3,4)\otimes(x_1,y_1,z_1)\ominus(1,2,3)\otimes(x_2,y_2,z_2)\preccurlyeq(-12,-3,6)\end{cases} \tag{4.2}$$

By using Step 2 and arithmetic operations defined in subsection 2.2, the mentioned FFLP problem changes as following problem:

$$\min Z = ((x_1{+}2x_2, 6y_1 + 2y_2, 9z_1 + 8z_2))$$
$$s.t. \begin{cases} 2x_2 \geq 4 \\ y_1 + 2y_2 \geq 7 \\ z_1 + 3z_2 \geq 14 \\ 2x_1 - z_2 \leq -4 \\ 2y_1 + 4y_2 \leq 14 \\ 3x_1 + 4z_2 \leq 22 \\ 2x_1 - 3z_2 \leq -12 \\ 3y_1 - 2y_2 \leq -3 \\ 4z_1 - x_2 \leq 6 \\ y_1 - x_1 \geq 0, y_2 - x_2 \geq 0 \\ z_1 - y_1 \geq 0, z_2 - y_2 \geq 0 \end{cases}$$

By using Step 3, the objective function is converted into three objective functions as follows:

$Z_1 = max\{6y_1 + 2y_2 - x_1 - 2x_2 \}$

$Z_2 = min\{6y_1 + 2y_2 \}$

$Z_3 = min\{9z_1 + 8z_2 - 6y_1 - 2y_2\}$

the problem changes as follow:

$$Z_1 = max\{6y_1 + 2y_2 - x_1 - 2x_2\}$$
$$Z_2 = min\{6y_1 + 2y_2\}$$
$$Z_3 = min\{9z_1 + 8z_2 - 6y_1 - 2y_2\}$$

$$(4.3) \qquad s.t. \begin{cases} 2x_2 \geq 4 \\ y_1 + 2y_2 \geq 7 \\ z_1 + 3z_2 \geq 14 \\ 2x_1 - z_2 \leq -4 \\ 2y_1 + 4y_2 \leq 14 \\ 3x_1 + 4z_2 \leq 22 \\ 2x_1 - 3z_2 \leq -12 \\ 3y_1 - 2y_2 \leq -3 \\ 4z_1 - x_2 \leq 6 \\ y_1 - x_1 \geq 0, y_2 - x_2 \geq 0 \\ z_1 - y_1 \geq 0, z_2 - y_2 \geq 0 \end{cases}$$

In Step 4, we determine the positive and negative ideal points for each objective. The result is obtained as follows:

$$v_1^* = \begin{cases} (x_1, y_1, z_1)_{z_2} = (0, 1, 0) \\ (x_2, y_2, z_2)_{z_2} = (2, 3, 5.5) \end{cases}$$
$$v_2^* = \begin{cases} (x_1, y_1, z_1)_{z_1} = (0, 0, 2) \\ (x_2, y_2, z_2)_{z_1} = (2, 3.5, 4) \end{cases}$$
$$v_2^* = \begin{cases} (x_1, y_1, z_1)_{z_3} = (0, 1, 0) \\ (x_2, y_2, z_2)_{z_3} = (2, 3.4.6667) \end{cases}$$

According to Step 4, we have:

$Z_1^{PIS} = 8,$ $\qquad Z_1^{NIS} = 3;;$

$Z_2^{PIS} = 7$, $\qquad Z_2^{NIS} = 12;$

$Z_3^{PIS} = 76/3 \approx 25.3334,$ $\qquad Z_3^{NIS} = 43.$

Therefore, according to (4.4), and Step 5, the membership functions of these objective functions are as below:

$$\mu_1(v) = \begin{cases} 1, & \text{if } Z_1 > 8 \\ \dfrac{Z_1 - 3}{5}, & \text{if } 3 \leq Z_1 \leq 8 \\ 0, & \text{if } Z_1 < 0 \end{cases}$$

$$\mu_2(v) = \begin{cases} 1, & \text{if } Z_2 < 7 \\ \dfrac{12 - Z_2}{5}, & \text{if } 7 \leq Z_2 \leq 12 \\ 0, & \text{if } Z_2 > 12 \end{cases}$$

$$\mu_3(v) = \begin{cases} 1, & \text{if } 3 > 38 \\ \dfrac{43 - Z_3}{43 - 25.3334}, & \text{if } 0 \leq Z_3 \leq 38 \\ 0, & \text{if } Z_3 < 0 \end{cases}$$

Finally converting the auxiliary LP model into an equivalent single-objective LP by using the following auxiliary crisp formulation is obtained as follows:

$$\max \gamma\lambda + (1-\gamma)\sum_{i=1}^{3} \theta_i Z_i = max 0.5\gamma + 0.5(Z_1 + 4Z_2 + Z_3)$$

$$(4.4)\qquad s.t.\begin{cases} 5\lambda - Z_1 \leq -3 \\ 5\lambda + Z_2 \leq 12 \\ 17.6667\lambda + Z_3 \leq 43 \\ 2x_2 \geq 4 \\ y_1 + 2y_2 \geq 7 \\ z_1 + 3z_2 \geq 14 \\ 2x_1 - z_2 \leq -4 \\ 2y_1 + 4y_2 \leq 14 \\ 3x_1 + 4z_2 \leq 22 \\ 2x_1 - 3z_2 \leq -12 \\ 3y_1 - 2y_2 \leq -3 \\ 4z_1 - x_2 \leq 6 \\ y_1 - x_1 \geq 0, y_2 - x_2 \geq 0 \\ z_1 - y_1 \geq 0, z_2 - y_2 \geq 0 \end{cases}$$

This problem is a conventional linear programming problem. The optimal solution of this problem for $\gamma = 0.5$, $\theta_1 = 1$, $\theta_2 = 4$ and $\theta_3 = 1$ is obtained as follows:

$$\tilde{x}_1 = (x_1, y_1, z_1) = (0, 0, 0), \quad \tilde{x}_2 = (x_2, y_2, z_2) = (2, 3.5, 4, 6667)$$

By these values, the objective value of the (4.1) is as follows:

$$Z^* = (1, 6, 9) \otimes (0, 0, 0) \oplus (2, 2, 8) \otimes (2, 3.5, 4, 6667) = (4, 7, 37.333)$$

The optimal solution of the proposed method is $Z^* = (4, 7, 37.333)$. By solving this problem with the proposed method by Kumar et. al in [10] the optimal solution is $Z^0 = (4, 7, 37.333)$. In comparison with Z^0, our solution for this problem is equal with the solution achieving in[10].

Example 4.2. Consider the following FFLP problem. By solving this problem, which is the same as Kumar et. al example in[10]. we solve the following FFLP and then compare our solution with the solution which was obtained in [10].

$$\max Z = ((1, 2, 3) \otimes \tilde{x}_1 \oplus (2, 3, 4) \otimes \tilde{x}_2)$$
$$s.t.\begin{cases} (0, 1, 2) \otimes \tilde{x}_1 \oplus (1, 2, 3) \otimes \tilde{x}_2 \succcurlyeq (1, 10, 27) \\ (1, 2, 3) \otimes \tilde{x}_1 \oplus (0, 1, 2) \otimes \tilde{x}_2 \preccurlyeq (2, 11, 28) \end{cases}$$
$$\tilde{x}_1 \text{ and } \tilde{x}_2 \text{ are non-negative triangular fuzzy numbers}$$

According to Step 1, let $\tilde{x}_1 = (x_1, y_1, z_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2)$. Then the original FFLP problem changes as follows:

$$\max Z = ((1, 2, 3) \otimes (x_1, y_1, z_1) \oplus (2, 3, 4) \otimes (x_2, y_2, z_2))$$
$$s.t.\begin{cases} (0, 1, 2) \otimes (x_1, y_1, z_1) \oplus (1, 2, 3) \otimes (x_2, y_2, z_2) \succcurlyeq (1, 10, 27) \\ \\ (1, 2, 3) \otimes (x_1, y_1, z_1) \ominus (0, 1, 2) \otimes (x_2, y_2, z_2) \preccurlyeq (2, 11, 28) \end{cases}$$

By using Step 2 and arithmetic operations defined in subsection 2.2, the mentioned FFLP problem changes as following problem:

$$\max Z = ((x_1 + 2x_2, 2y_1 + 3y_2, 3z_1 + 4z_2))$$

$$s.t. \begin{cases} x_2 \le 1 \\ y_1 + 2y_2 \le 10 \\ 2z_1 + 3z_2 \le 27 \\ x_1 \le 2 \\ 2y_1 + y_2 \le 11 \\ 3z_1 + 2z_2 \le 28 \\ y_1 - x_1 \ge 0, y_2 - x_2 \ge 0 \\ z_1 - y_1 \ge 0, z_2 - y_2 \ge 0 \end{cases}$$

By using Step 3, the objective function is converted into three objective functions as follows:

$$Z_1 = min\{2y_1 + 3y_2 - x_1 - 2x_2\}$$

$$Z_2 = max\{2y_1 + 3y_2\}$$

$$Z_3 = max\{3z_1 + 4z_2 - 2y_1 - 3y_2\}$$

$$s.t. \begin{cases} x_2 \le 1 \\ y_1 + 2y_2 \le 10 \\ 2z_1 + 3z_2 \le 27 \\ x_1 \le 2 \\ 2y_1 + y_2 \le 11 \\ 3z_1 + 2z_2 \le 28 \\ y_1 - x_1 \ge 0, y_2 - x_2 \ge 0 \\ z_1 - y_1 \ge 0, z_2 - y_2 \ge 0 \end{cases}$$

Then, we determine the PIS and NIS for each objective. The result is as below:

$Z_1^{PIS} = 0, \qquad Z_1^{NIS} = 17;$

$Z_2^{PIS} = 17\ , \qquad Z_2^{NIS} = 0;$

$Z_3^{PIS} = 38, \qquad Z_3^{NIS} = 0.$

Therefore, the membership functions of these objective functions are as below:

$$\mu_1(v) = \begin{cases} 1, & \text{if } Z_1 < 0 \\ \dfrac{Z_1 - 17}{17}, & \text{if } 0 \le Z_1 \le 17 \\ 0, & \text{if } Z_1 > 17 \end{cases}$$

$$\mu_2(v) = \begin{cases} 1, & \text{if } Z_2 > 17 \\ \dfrac{0 - Z_2}{-17}, & \text{if } 0 \le Z_2 \le 17 \\ 0, & \text{if } Z_2 < 0 \end{cases}$$

$$\mu_3(v) = \begin{cases} 1, & \text{if } Z_3 > 38 \\ \dfrac{0 - Z_3}{-38}, & \text{if } 0 \le Z_3 \le 38 \\ 0, & \text{if } Z_3 < 0 \end{cases}$$

Finally converting the auxiliary LP model into an equivalent single-objective LP by using the following auxiliary crisp formulation is obtained as follows:

$$Z = \max\{\gamma\lambda + (1-\gamma)\sum_{i=1}^{3}\theta_i Z_i\} = max\{0.5\ \gamma + 0.5(-Z_1 + 4Z_2 + Z_3)\}$$

$$s.t. \begin{cases} 0 \le \lambda \le 1 \\ 17\lambda + Z_1 \le 17 \\ 17\lambda - Z_2 \le 0 \\ 38\lambda - Z_3 \le 0 \\ 0 \le Z_1 \le 17 \\ 0 \le Z_2 \le 17 \\ 0 \le Z_3 \le 38 \\ x_2 \le 1 \\ y_1 + 2y_2 \le 10 \\ 2z_1 + 3z_2 \le 27 \\ x_1 \le 2 \\ 2y_1 + y_2 \le 11 \\ 3z_1 + 2z_2 \le 28 \\ y_1 - x_1 \ge 0, y_2 - x_2 \ge 0 \\ z_1 - y_1 \ge 0, z_2 - y_2 \ge 0 \\ 0 \le \gamma \le 1 \end{cases}$$

This problem is a conventional linear programming problem. The optimal solution of this problem for $\gamma = 0.5,\ \theta_1 = 1, \theta_2 = 4$ and $\theta_3 = 1$ is obtained as follows:

$$\tilde{x}_1 = (x_1, y_1, z_1) = (2, 4, 6)\,,\ \tilde{x}_2 = (x_2, y_2, z_2) = (1, 3, 5)$$

By these values, the objective value of the (4.2) is as follows:

$$Z^* = (1, 2, 3) \otimes (2, 4, 6) \oplus (2, 3, 4) \otimes (1, 3, 5) = (4, 17, 38)$$

The optimal solution of the proposed method is $Z^* = (4, 17, 38)$. In comparison with Kumar et. al [10], both methods have the same solutions.

5. Conclusion remarks

This paper proposed a novel method for solving Fully Fuzzy Linear Programming problem by representing all parameters as triangular fuzzy numbers. After introducing FFLP problem, we express a new algorithm for solve FFLP problem, which is based on membership function definition and interactive fuzzy programming solution approach. Computational results and illustrative numerical examples show that this method has good performance in comparing with others in literature.

References

[1] T. Allahviranloo, KH. Shamsolkotabi, N. A. Kiani and L. Alizadeh, Fuzzy integer linear programming problems, Int. J. Contemp. Math. Sci. 2(1-4) (2007), no. 1-4, 167–181.

[2] T. Allahviranloo, F. H. Lotfi, M. K. Kiasary, N. A. Kiani and L. Alizadeh, Solving full fuzzy linear programming problem by the ranking function, Appl. Math. Sci. 2 (2008) 19–32.

[3] R. E. Bellman and L. A. Zadeh, Decision making in a fuzzy environment, Management Sci. 17 (1970/71), B141–B164.

[4] L. Campos and J. L. Verdegay, Linear programming problems and ranking of fuzzy numbers, Fuzzy Sets and Systems 32 (1989) 1–11.

[5] B. Y. Cao, Optimal Models and Methods with Fuzzy Quantities, springer, Studies in Fuzziness and Soft Computing, 248, 2010.

[6] M. Dehghan, B. Hashemi and M. Ghatee, Computational methods for solving fully fuzzy linear systems, Appl. Math. Comput. 179 (2006) 328–343

[7] A. Ebrahimnejad and S. H. Nasseri, Using complementary slackness property to solve linear programming with fuzzy parameters, Fuzzy Information and Engineering 3 (2009) 233–245.

[8] F. Hosseinzadeh Lotfi, T. Allahviranloo, M. Alimardani Jondabeh and L. Alizadeh, Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution, Appl. Math. Model. 33 (2009) 3151–3156.

[9] M. Jimenez, M. Arenas, A. Bilbao, M. Victoria Rodrguez, Linear programming with fuzzy parameters: An interactive method resolution, European J. Oper. Res. 177(3) (2007) 1599–1609.

[10] A. Kumar, J. Kaur and P. Singh, Fuzzy optimal solution of fully fuzzy linear programming problems with inequality constraints, International Journal of Mathematical and Computer Sciences 6(1) (2010) 37–41.

[11] A. Kumar, J. Kaur and P. Singh, A new method for solving fully fuzzy linear programming problems, Appl. Math. Model. 35 (2011) 817–823.

[12] A. Kumar and P. Singh, , A new method for solving fully fuzzy linear programming problems, Ann. Fuzzy Math. Inform. 3(1) (2012) 103–118.

[13] H. Mishmast Nehi and H. Hajmohamadi, A ranking function method for solving fuzzy multi-objective linear programming problem, Ann. Fuzzy Math. Inform. 3(1) (2012) 31–38.

[14] S.H. Nasseri and A. Ebrahimnejad, A fuzzy primal simplex algorithm and its application for solving flexible linear programming problems, Eur. J. Ind. Eng. 4(3) (2010) 372–389.

[15] H. Tanaka, T. Okuda and K. Asai, On fuzzy mathematical programming, J. Cybernet. 3(4) (1973) 37–46.

[16] S. T. Torabi, E. Hassini, An interactive possibilistic programming approach for multiple objective supply chain master planning, Fuzzy Sets and Systems 159 (2008) 193–214.

[17] H. J. Zimmermann, Fuzzy programming and linear programming with several objective functions, Fuzzy Sets and Systems 1 (1978) 45–55.

S. H. Nasseri (nasseri@umz.ac.ir)
Department of Mathematical Sciences, University of Mazandaran, Babolsar, Iran

F. Khalili (farzaneh.khalili@stu.umz.ac.ir)
Department of Mathematical Sciences, University of Mazandaran, Babolsar, Iran

N. A. TaghiNezhad (nemat.taghi.nezhad@gmail.com)
Department of Mathematical Sciences, University of Mazandaran, Babolsar, Iran

S.M. Mortezania (m_mortezania@yahoo.com)
Department of Mathematical Sciences, University of Mazandaran, Babolsar, Iran