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Pairwise T_i (i=1,2) ordered space and pairwise normally ordered space in an intuitionistic fuzzy topological space

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Abstract. In this paper we introduce the concept of a new class of an ordered intuitionistic fuzzy bitopological spaces. Besides giving some interesting properties of these spaces. We also prove analogues of Uryshon's lemma and Tietze extension theorem in an ordered intuitionistic fuzzy bitopological spaces.

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh[12]. Fuzzy sets have applications in many fields such as information[9] and control[10]. The theory of fuzzy topological spaces was introduced and developed by Chang[5]. The concept of fuzzy normal space was introduced by Bruce Hutton[4]. Atanassov[1] introduced and studied intuitionistic fuzzy sets. On the otherhand, Coker[6] introduced the notions of an intuitionistic fuzzy topological space and some other concepts. The concept of an ordered fuzzy topological spaces was introduced and developed by A.K.Katsaras[8]. Later G.Balasubmanian[3] was introduced and studied the concepts of an ordered L-fuzzy bitopological spaces. Ganster and Relly used locally closed sets[7] to define LC-continuity and LC-irresoluteness. G.Balasubramanian[2] introduced and studied the concept of fuzzy β-open set in a fuzzy topological space. The concept of an π-open set in a topological space was introduced by V.Zaitsev[13]. In this paper we introduced the concepts of pairwise intuitionistic fuzzy π-β-locally T_1-ordered space, pairwise intuitionistic fuzzy π-β-locally T_2-ordered space, weakly pairwise intuitionistic fuzzy π-β-locally T_2-ordered space, almost pairwise intuitionistic fuzzy π-β-locally T_2-ordered space and strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space are introduced. Some interesting propositions are discussed. Urysohn's lemma and Tietze extension theorem of an strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space are studied and estabilished.

2. Preliminaries

Definition 2.1 ([1]). Let X be a nonempty fixed set and I is the closed interval [0,1]. An intuitionistic fuzzy set(IFS) A is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, where the mapping $\mu_A : X \longrightarrow I$ and $\gamma_A : X \longrightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form, $A = \{\langle x, \mu_A(x), 1-\mu_A(x)\rangle : x \in X\}$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.2 ([1]). Let X be a nonempty set and the IFSs A and B in the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Then
(i) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;
(ii) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$.

Definition 2.3 ([1]). The IFSs $0_\sim$ and $1_\sim$ are defined by $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$.

Definition 2.4 ([6]). An intuitionistic fuzzy topology (IFT) in Coker's sense on a non empty set X is a family τ of IFSs in X satisfying the following axioms.
(T_1) $0_\sim$,$1_\sim \in \tau$
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(T_3) $\cup G_i \in \tau$ for arbitrary family $\{G_i / i \in I\} \subseteq \tau$
In this paper by (X, τ) or simply by X we will denote the Cocker's intuitionistic fuzzy topological space (IFTS). Each IFSs in τ is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.5 ([6]). Let A be an IFS in IFTS X. Then
$int(A) = \bigcup\{G \mid G$ is an IFOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy interior of A;
$clA = \bigcap\{G \mid G$ is an IFCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy closure of A.

Definition 2.6 ([5]). Let (X, τ) be an IFTS on X. If $A = int(cl(A))$, then A is called an intuitionistic fuzzy regular open set in X.

Definition 2.7 ([6]). Let (X, τ) and (Y, σ) be two IFTSs and let $f : X \to Y$ be a function. Then f is said to be fuzzy continuous iff the preimage of each IFS in σ is an IFS in τ.

Definition 2.8 ([3]). A L-fuzzy set μ in a fuzzy topological space X is called a neighbourhood of a point $x \in X$, if there exists an L-fuzzy set μ_1 with $\mu_1 \leq \mu$ and $\mu_1(x) > 0$. It can be shown that a L-fuzzy set μ is open $\Longleftrightarrow$ μ is a neighbourhood of each $x \in X$ for which $\mu(x) > 0$.

Definition 2.9 ([3]). The L-fuzzy real line $R(L)$ is the set of all monotone decreasing elements $\lambda \in L^R$ satisfying $\vee\{\lambda(t) \mid t \in R\} = 1$ and $\wedge\{\lambda(t) \mid t \in R\} = 0$, after the identification of $\lambda, \mu \in L^R$ iff $\lambda(t-) = \mu(t-)$ and $\lambda(t+) = \mu(t+)$ for all $t \in R$ where $\lambda(t-) = \wedge\{\lambda(s) \mid s < t\}$ and $\lambda(t+) = \vee\{\lambda(s) \mid s > t\}$.

Definition 2.10 ([3]). The natural L-fuzzy topology on $R(L)$ is generated from the subbasis $\{L_t, R_t \mid t \in R\}$, where $L_t[\lambda]{=}\lambda(t-)'$ and $R_t[\lambda]{=}\lambda(t+)'$.

Definition 2.11 ([3]). A partial order on $R(L)$ is defined by $[\lambda] \leq [\mu] \Leftrightarrow \lambda(t-) \leq \mu(t-)$ and $\lambda(t+) \leq \mu(t+)$ for all $t \in R$.

Definition 2.12 ([3]). The L-fuzzy unit interval $I(L)$ is a subset of $R(L)$ such that $[\lambda] \in I(L)$ if $\lambda(t) = 1$ for $0 < t$ and $\lambda(t) = 0$ for $t > 1$. It is equipped with the subspace L-fuzzy topology.

Definition 2.13 ([3]). Let (X, τ) be an L-fuzzy topological space. A function $f : X \to R(L)$ is called lower (upper) semicontinuous if $f^{-1}(R_t)(f^{-1}(L_t))$ is open for each $t \in R$. Equivalently f is lower (upper) semicontinuous $\Leftrightarrow$ it is continuous w.r.t the right hand (left hand) L-fuzzy topology on $R(L)$ where the right hand (left hand) topology is generated from the basis $\{R_t \mid t \in R\}(\{L_t \mid t \in R\})$. Lower and upper semi continuous with values in $I(L)$ are defined in the analogous way.

Definition 2.14 ([3]). A L-fuzzy set λ in a partially ordered set X is called
(i) Increasing if $x \leq y \Longrightarrow \lambda(x) \leq \lambda(y)$
(ii) Decreasing if $x \leq y \Longrightarrow \lambda(x) \geq \lambda(y)$.

Definition 2.15 ([2]). Let λ be any fuzzy set of the fuzzy topological space (X, T). Then λ is called fuzzy β-open set if $\lambda \leq cl(int(cl(A)))$.

The complement of fuzzy β-open set is called fuzzy β-closed set.

Definition 2.16 ([12]). The finite union of regular open sets is said to be π-open set. The complement of π-open set is said to be π-closed set.

Definition 2.17 ([7]). A subset A of a space (X, τ) is called locally closed (briefly lc) if $A = C \cap D$, where C is open and D is closed in (X, τ).

Definition 2.18 ([11]). Let (X, T) be a normal L-space, $A \subset X$ suitable closed and $f : (A, T_A) \to I(L)$ continuous. Then there exists a continuous function $F : X \to I(L)$ which extends f over X.

Corollary 2.19 ([11]). *(Urysohn's type lemma). An L-space (X, T) is normal iff for each K', $U \in T$ such that $K \leq U$ there exists a continuous function $f : X \to I(L)$ such that $K \leq {L_1}'f \leq R_0 f \leq U$.*

3. Ordered intuitionistic fuzzy π-β-locally bitopological spaces

In this section, the concepts of an intuitionistic fuzzy π-open set, intuitionistic fuzzy β-closed set,intuitionistic fuzzy π-β-locally closed set, upper pairwise intuitionistic fuzzy π-β-locally T_1-ordered space, lower pairwise intuitionistic fuzzy

π-β-locally T_1-ordered space, pairwise intuitionistic fuzzy π-β-locally T_1-ordered space, pairwise intuitionistic fuzzy π-β-locally T_2-ordered space, weakly pairwise intuitionistic fuzzy π-β-locally T_2-ordered space, almost pairwise intuitionistic fuzzy π-β-locally T_2-ordered space and strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space are introduced. Some interesting propositions and characterizations are discussed. Urysohn's lemma and Tietze extension theorem of an strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space are studied and estabilished.

Definition 3.1. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). Then A is said be an intuitionistic fuzzy π-open set if $A = \bigcup_{i=1}^{n} A_i$, where $A_i = \langle x, \mu_{A_i}, \gamma_{A_i} \rangle$ is an intuitionistic fuzzy regular open set in an intuitionistic fuzzy topological space (X,T).

The complement of an intuitionistic fuzzy π-open set is said to be an intuitionistic fuzzy π-closed set.

Definition 3.2. Let A be any intuitionistic fuzzy set of an intuitionistic fuzzy topological space (X, T). Then A is called an intuitionistic fuzzy β-open set ($IF\beta OS$) if $A \subseteq cl(int(cl(A)))$.

The complement of an intuitionistic fuzzy β-open set is said to be an intuitionistic fuzzy β-closed set.

Definition 3.3. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). Then A is said be an intuitionistic fuzzy π-β-locally closed set (in short, $IF\pi$-β-lcs) if $A = B \cap C$, where B is an intuitionistic fuzzy π-open set and C is an intuitionistic fuzzy β-closed set.

The complement of an intuitionistic fuzzy π-β-locally closed set is said to be an intuitionistic fuzzy π-β-locally open set (in short, $IF\pi$-β-los).

Definition 3.4. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, T). The intuitionistic fuzzy π-β-locally closure of A is denoted and defined by

$IF\pi$-β-$lcl(A)$=$\bigcap\{B$:$B = \langle x, \mu_B, \gamma_B \rangle$ is an intuitionistic fuzzy π-β-locally closed set in X and $A \subseteq B\}$.

Definition 3.5. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, T). The intuitionistic fuzzy π-β-locally interior of A is denoted and defined by

$IF\pi$-β-$lint(A)$=$\bigcup\{B$:$B = \langle x, \mu_B, \gamma_B \rangle$ is an intuitionistic fuzzy π-β-locally open set in X and $B \subseteq A\}$.

Definition 3.6. An intuitionistic fuzzy set $A = \langle x, \mu_A, \gamma_A \rangle$ in an intuitionistic fuzzy topological space (X, T) is said to be an intuitionistic fuzzy neighbourhood of a point $x \in X$, if there exists an intuitionistic fuzzy open set $B = \langle x, \mu_B, \gamma_B \rangle$ with $B \subseteq A$ and $B(x) \supseteq 0_\sim$.

Definition 3.7. An intuitionistic fuzzy set $A = \langle x, \mu_A, \gamma_A \rangle$ in an intuitionistic fuzzy topological space (X, T) is said to be an intuitionistic fuzzy π-β-locally neighbourhood of a point $x \in X$, if there exists an intuitionistic fuzzy π-β-locally open set $B = \langle x, \mu_B, \gamma_B \rangle$ with $B \subseteq A$ and $B(x) \supseteq 0_\sim$.

Definition 3.8. An intuitionistic fuzzy set $A = \langle x, \mu_A, \gamma_A \rangle$ in a partially ordered set $(X, \leq)$ is said to be an

(i) increasing intuitionistic fuzzy set if $x \leq y$ implies $A(x) \subseteq A(y)$. That is $\mu_A(x) \leq \mu_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$.

(ii) decreasing intuitionistic fuzzy set if $x \leq y$ implies $A(x) \supseteq A(y)$. That is $\mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$.

Definition 3.9. An ordered intuitionistic fuzzy bitopological space is an intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ (where τ_1 and τ_2 are intuitionistic fuzzy topologies on X) equipped with a partial order $\leq$.

Definition 3.10. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an upper pairwise intuitionistic fuzzy T_1-ordered space if $a, b \in X$ such that $a \nleq b$, there exists an decreasing τ_1 intuitionistic fuzzy neighbourhood (or) an decreasing τ_2 intuitionistic fuzzy neighbourhood A of b such that $A = \langle x, \mu_A, \gamma_A \rangle$ is not an intuitionistic fuzzy neighbourhood of a.

Definition 3.11. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an lower pairwise intuitionistic fuzzy T_1-ordered space if $a, b \in X$ such that $a \nleq b$, there exists an increasing τ_1 intuitionistic fuzzy neighbourhood (or) an increasing τ_2 intuitionistic fuzzy neighbourhood A of a such that $A = \langle x, \mu_A, \gamma_A \rangle$ is not an intuitionistic fuzzy neighbourhood of b.

Definition 3.12. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an pairwise intuitionistic fuzzy T_1-ordered space if and only if it is both upper and lower pairwise intuitionistic fuzzy T_1-ordered space.

Definition 3.13. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an upper pairwise intuitionistic fuzzy π-β-locally T_1-ordered space if $a, b \in X$ such that $a \nleq b$, there exists an decreasing τ_1 intuitionistic fuzzy π-β-locally neighbourhood (or) an decreasing τ_2 intuitionistic fuzzy π-β-locally neighbourhood $A = \langle x, \mu_A, \gamma_A \rangle$ of b such that A is not an intuitionistic fuzzy π-β-locally neighbourhood of a.

Definition 3.14. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an lower pairwise intuitionistic fuzzy π-β-locally T_1-ordered space if $a, b \in X$ such that $a \nleq b$, there exists an increasing τ_1 intuitionistic fuzzy π-β-locally neighbourhood (or) an increasing τ_2 intuitionistic fuzzy π-β-locally neighbourhood $A = \langle x, \mu_A, \gamma_A \rangle$ of a such that A is not an intuitionistic fuzzy π-β-locally neighbourhood of b.

Definition 3.15. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an pairwise intuitionistic fuzzy π-β-locally T_1-ordered space if and only if it is both upper and lower pairwise intuitionistic fuzzy π-β-locally T_1-ordered space.

Remark 3.16. Russian translation of A. Kaufmann's book [301] and decided to add to the definition, a second degree (degree of nonmembership) and studied the properties of a set with both degrees. Of course, observed that the new set is an extension of the ordinary fuzzy set, but did not immediately notice that it has essentially different properties. So the first research works of mine in this area followed, step-by-step, the existing results in fuzzy sets theory. Of course, some concepts are not so difficult to extend formally. It is interesting to show that the respective extension has specific properties, absent in the basic concept. An intuitionistic fuzzy set $A = \langle x, \mu_A(x), \gamma_A(x)\rangle$, when $\gamma_A(x) = 1 - \mu_A(x)$. Then A is also called fuzzy set.

Proposition 3.17. *For an ordered intuitionistic fuzzy bitopological space* $(X, \tau_1, \tau_2, \leq)$ *the following are equivalent*

(i) *X is an lower (resp.upper) pairwise intuitionistic fuzzy π-β-locally T_1-ordered space.*

(ii) *For each $a, b \in X$ such that $a \nleq b$, there exists an increasing (resp.decreasing) τ_1 intuitionistic fuzzy π-β-locally open set (or) an increasing (resp.decreasing) τ_2 intuitionistic fuzzy π-β-locally open set $A = \langle x, \mu_A, \gamma_A\rangle$ such that $A(a) > 0$ (resp.$A(b) > 0$) and A is not an intuitionistic fuzzy π-β-locally neighbourhood of b (resp.a).*

Proof. **(i)⇒(ii)** Let X be an lower pairwise intuitionistic fuzzy π-β-locally T_1-ordered space. Let $a, b \in X$ such that $a \nleq b$. There exists an increasing τ_1 intuitionistic fuzzy π-β-locally neighbourhood (or) an increasing τ_2 intuitionistic fuzzy π-β-locally neighbourhood A of a such that A is not an intuitionistic fuzzy π-β-locally neighbourhood of b.It follows that there exists an τ_i intuitionistic fuzzy π-β-locally open set$(i = 1(or)2)$, $A_i = \langle x, \mu_{A_i}, \gamma_{A_i}\rangle$ with $A_i \subseteq A$ and $A_i(a) = A(a) > 0$. As A is an increasing intuitionistic fuzzy set, $A(a) > A(b)$ and since A is not an intuitionistic fuzzy π-β-locally neighbourhood of b, $A_i(b) < A(b)$ implies $A_i(a) = A(a) > A(b) \geq A_i(b)$. This shows that A_i is an increasing intuitionistic fuzzy set and A_i is not an intuitionistic fuzzy π-β-locally neighbourhood of b, since A is not an intuitionistic fuzzy π-β-locally neighbourhood of b.

(ii)⇒(i) Since A_1 is an increasing τ_1 intuitionistic fuzzy π-β-locally open set (or) increasing τ_2 intuitionistic fuzzy π-β-locally open set. Now, A_1 is an intuitionistic fuzzy π-β-locally neighbourhood of a with $A_1(a) > 0$. By (ii), A_1 is not an intuitionistic fuzzy π-β-locally neighbourhood of b. This implies, X is an lower pairwise intuitionistic fuzzy π-β-locally T_1-ordered space. □

Remark 3.18. Similar proof holds for upper pairwise intuitionistic fuzzy π-β-locally T_1-ordered space.

Proposition 3.19. *If $(X, \tau_1, \tau_2, \leq)$ is an lower (resp.upper) pairwise intuitionistic fuzzy π-β-locally T_1-ordered space and $\tau_1 \subseteq \tau_1^*, \tau_2 \subseteq \tau_2^*$, then $(X, {\tau_1}^*, {\tau_2}^*, \leq)$ is an lower (resp. upper) pairwise intuitionistic fuzzy π-β-locally T_1-ordered space.*

Proof. Let $(X, \tau_1, \tau_2, \leq)$ be an lower pairwise intuitionistic fuzzy π-β-locally T_1-ordered space. Then if $a, b \in X$ such that $a \nleq b$, there exists an increasing τ_1 intuitionistic fuzzy π-β-locally neighbourhood (or) an increasing τ_2 intuitionistic fuzzy π-β-locally neighbourhood $A = \langle x, \mu_A, \gamma_A\rangle$ of a such that A is not an intuitionistic fuzzy π-β-locally neighbourhood of b. Since $\tau_1 \subseteq \tau_1^*$ and $\tau_2 \subseteq \tau_2^*$. Therefore, if $a, b \in X$ such that $a \nleq b$, there exists an increasing ${\tau_1}^*$ intuitionistic fuzzy π-β-locally neighbourhood (or) an increasing $\tau_2{}^*$ intuitionistic fuzzy π-β-locally neighbourhood $A = \langle x, \mu_A, \gamma_A \rangle$ of a such that A is not an intuitionistic fuzzy π-β-locally neighbourhood of b. Thus $(X, \tau_1{}^*, \tau_2{}^*, \leq)$ is an lower pairwise intuitionistic fuzzy π-β-locally T_1-ordered space. □

Remark 3.20. Similar proof holds for upper pairwise intuitionistic fuzzy π-β-locally T_1-ordered space.

Definition 3.21. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an pairwise intuitionistic fuzzy T_2-ordered space if for $a, b \in X$ with $a \nleq b$, there exist an intuitionistic fuzzy open sets $A = \langle x, \mu_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \gamma_B \rangle$ such that A is an increasing τ_i intuitionistic fuzzy neighbourhood of a,B is an decreasing τ_j intuitionistic fuzzy neighbourhood of b $(i, j = 1, 2$ and $i \neq j)$ and $A \cap B = 0_\sim$.

Definition 3.22. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an pairwise intuitionistic fuzzy π-β-locally T_2-ordered space if for $a, b \in X$ with $a \nleq b$, there exist an intuitionistic fuzzy π-β-locally open sets $A = \langle x, \mu_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \gamma_B \rangle$ such that A is an increasing τ_i intuitionistic fuzzy π-β-locally neighbourhood of a,B is an decreasing τ_j intuitionistic fuzzy π-β-locally neighbourhood of b $(i, j = 1, 2$ and $i \neq j)$ and $A \cap B = 0_\sim$.

Definition 3.23. Let $(X, \leq)$ be a partially ordered set. Let $G = \{(x, y) \in X \times X \mid x \leq y, y = f(x)\}$. Then G is called an intuitionistic fuzzy graph of the partially ordered $\leq$.

Definition 3.24. Let (X, T) be an intuitionistic fuzzy topological space and $A \subset X$ be a subset of X. An intuitionistic fuzzy characteristic function of $A = \langle x, \mu_A, \gamma_A \rangle$ is defined as $\chi_A(x) = \begin{cases} 1_\sim & if \quad x \in A \\ 0_\sim & if \quad x \notin A \end{cases}$

Definition 3.25. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set in an ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$. Then for $i = 1(or)2$, we define

I_{τ_i}-π-β-$li(A)$ = increasing τ_i intuitionistic fuzzy π-β-locally interior of A
= the greatest increasing τ_i intuitionistic fuzzy π-β-locally open set contained in A

D_{τ_i}-π-β-$li(A)$ = decreasing τ_i intuitionistic fuzzy π-β-locally interior of A
= the greatest decreasing τ_i intuitionistic fuzzy π-β-locally open set contained in A

I_{τ_i}-π-β-$lc(A)$ = increasing τ_i intuitionistic fuzzy π-β-locally closure of A
= the smallest increasing τ_i intuitionistic fuzzy π-β-locally closed set containing in A

D_{τ_i}-π-β-$lc(A)$ = decreasing τ_i intuitionistic fuzzy π-β-locally closure of A
= the smallest decreasing τ_i intuitionistic fuzzy π-β-locally closed set containing in A.

Notation 3.26.

(i) *The complement of the characteristic function* χ_G,*where* G *is the intuitionistic fuzzy graph of the partial order of* X *is denoted by* $\chi_{\overline{G}}$.

(ii) I_{τ_i}-π-β-$lc(A)$ *is denoted by* $I_i(A)$ *and* D_{τ_j}-π-β-$lc(A)$ *is denoted by* $D_j(A)$ *where*

$A = \langle x, \mu_A, \gamma_A \rangle$ *is an intuitionistic fuzzy set in an ordered intuitionistic fuzzy bitopological space* $(X, \tau_1, \tau_2, \leq)$, *for* $i, j = 1, 2$ *and* $i \neq j$.

(iii) I_{τ_i}-π-β-$li(A)$ *is denoted by* $I_i{}^{\circ}(A)$ *and* D_{τ_j}-π-β-$li(A)$ *is denoted by* $D_j{}^{\circ}(A)$ *where*
$A = \langle x, \mu_A, \gamma_A \rangle$ *is an intuitionistic fuzzy set in an ordered intuitionistic fuzzy bitopological space* $(X, \tau_1, \tau_2, \leq)$, *for* $i, j = 1, 2$ *and* $i \neq j$.

Proposition 3.27. *For an ordered intuitionistic fuzzy bitopological space* $(X, \tau_1, \tau_2, \leq)$ *the following are equivalent*

(i) X *is a pairwise intuitionistic fuzzy* π-β-*locally* T_2-*ordered space.*

(ii) *For each pair* $a, b \in X$ *such that* $a \nleq b$, *there exists an* τ_i *intuitionistic fuzzy* π-β-*locally open set* $A = \langle x, \mu_A, \gamma_A \rangle$ *and* τ_j *intuitionistic fuzzy* π-β-*locally open set* $B = \langle x, \mu_B, \gamma_B \rangle$ *such that* $A(a) > 0, B(b) > 0$ *and* $A(x) > 0, B(y) > 0$ *together imply that* $x \nleq y$.

(iii) *The characteristic function* χ_G,*where* G *is the intuitionistic fuzzy graph of the partial order of* X *is a* τ^*-*intuitionistic fuzzy* π-β-*locally closed set,where* τ^* *is either* $\tau_1 \times \tau_2$ *or* $\tau_2 \times \tau_1$ *in* $X \times X$.

Proof. **(i)⇒(ii)** Let X be a pairwise intuitionistic fuzzy π-β-locally T_2-ordered space. Assume that suppose $A(x) > 0$,$B(y) > 0$ and suppose $x \leq y$. Since A is an increasing τ_i intuitionistic fuzzy π-β-locally open set and B is an decreasing τ_j intuitionistic fuzzy π-β-locally open set, $A(x) \leq A(y)$ and $B(y) \leq B(x)$. Therefore $0 < A(x) \cap B(y) \leq A(y) \cap B(x)$, which is a contradiction to the fact that $A \cap B = 0_{\sim}$. Therefore $x \nleq y$.

(ii)⇒(i) Let $a, b \in X$ with $a \nleq b$, there exists an intuitionistic fuzzy sets A and B satisfying the properties in (ii). Since $I_i{}^{\circ}(A)$ is an increasing τ_i intuitionistic fuzzy π-β-locally open set and $D_j{}^{\circ}(B)$is an decreasing τ_j intuitionistic fuzzy π-β-locally open set, we have $I_i{}^{\circ}(A) \cap D_j{}^{\circ}(B)$=$0_{\sim}$. Suppose $z \in X$ is such that $I_i{}^{\circ}(A)(z) \cap D_j{}^{\circ}(B)(z) > 0$.Then $I_i{}^{\circ}(A) > 0$ and $D_j{}^{\circ}(B)(z) > 0$. If $x \leq z \leq y$, then $x \leq z$ implies that $D_j{}^{\circ}(B)(x) \geq D_j{}^{\circ}(B)(z) > 0$ and $z \leq y$ implies that $I_i{}^{\circ}(A)(y) \geq I_i{}^{\circ}(A)(z) > 0$ then $D_j{}^{\circ}(B)(x) > 0$ and $I_i{}^{\circ}(A)(y) > 0$. Hence by (ii), $x \nleq y$ but then $x \leq y$. This is a contradiction. This implies that X is pairwise intuitionistic fuzzy π-β-locally T_2-ordered space.

(i)⇒ (iii) We want to show that χ_G is an τ^* intuitionistionic fuzzy π-β-locally closed set. That is to show that $\chi_{\overline{G}}$ is an τ^* intuitionistionic fuzzy π-β-locally open set. It is sufficient to prove that $\chi_{\overline{G}}$ is an intuitionistionic fuzzy π-β-locally neighbourhood of a point $(x, y) \in X \times X$ such that $\chi_{\overline{G}}\ (x, y) > 0$. Suppose $(x, y) \in X \times X$ is such that $\chi_{\overline{G}}\ (x, y) > 0$. That is $\chi_G(x, y) < 1$. This means $\chi_G(x, y) = 0$.That is $(x, y) \notin G$. That is $x \nleq y$. Therefore by assumption (i) there exist intuitionistic fuzzy π-β-locally open sets A and B such that A is an increasing τ_i intuitionistic fuzzy π-β-locally neighbourhood of a, B is an decreasing τ_j intuitionistic fuzzy π-β-locally neighbourhood of $b(i, j = 1, 2$ and $i \neq j)$and $A \cap B = 0_{\sim}$. Clearly $A \times B$ is an $IF\tau^*$ π-β-locally neighbourhood of (x, y). It is easy to verify that $A \times B \subseteq \chi_{\overline{G}}$. Thus we find that $\chi_{\overline{G}}$ is an τ^* $IF\pi$-β-locally open set. Hence (iii) is established.

(iii)⇒(i) Suppose $x \nleq y$. Then $(x, y) \notin G$, where G is an intuitionistic fuzzy graph of the partial order. Given that χ_G is an τ^* intuitionistic fuzzy π-β-locally closed set. That is $\chi_{\overline{G}}$ is an τ^* intuitionistic fuzzy π-β-locally open set. Now,

$(x, y) \notin G$ implies that $\chi_{\overline{G}}(x, y) > 0$. Therefore $\chi_{\overline{G}}$ is an τ^* intuitionistic fuzzy π-β-locally neighbourhood of $(x, y) \in X \times X$. Hence we can find that τ^* intuitionistic fuzzy π-β-locally open set $A \times B$ such that $A \times B \subseteq \chi_{\overline{G}}$ and A is an τ_i intuitionistic fuzzy π-β-locally open set such that $A(x) > 0$ and B is an τ_j intuitionistic fuzzy π-β-locally open set such that $B(y) > 0$. We now claim that $I_i{}^{\circ}(A) \cap D_j{}^{\circ}(B)$=$0_{\sim}$. For if $z \in X$ is such that $(I_i{}^{\circ}(A) \cap D_j{}^{\circ}(B))(z)> 0$, then $I_i{}^{\circ}(A)(z) \cap D_j{}^{\circ}(B)(z) > 0$. This means $I_i{}^{\circ}(A)(z)$$> 0$ and $D_j{}^{\circ}(B)(z)$$> 0$. And if $a \leq z \leq b$, then $z \leq b$ implies that $I_i{}^{\circ}(A)(b)$$\geq$ $I_i{}^{\circ}(A)(z)$$> 0$ and $a \leq z$ implies that $D_j{}^{\circ}(B)(a)$$\geq$ $D_j{}^{\circ}(B)(z)$$> 0$. Then $D_j{}^{\circ}(B)(a)$$> 0$ and $I_i{}^{\circ}(A)(b)$$> 0$ implies that $a \nleq b$ but then $a \leq b$. This is a contradiction. Hence (i) is established. □

Definition 3.28. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a weakly pairwise intuitionistic fuzzy T_2-ordered space if given $b < a$ (that is $b \leq a$ and $b \neq a$),there exist an τ_i intuitionistic fuzzy open set $A = \langle x, \mu_A, \gamma_A \rangle$ such that $A(a) > 0$ and τ_j intuitionistic fuzzy open set $B = \langle x, \mu_B, \gamma_B \rangle$ such that $B(b) > 0$ ($i, j = 1, 2$ and $i \neq j$) such that if $x, y \in X$, $A(x) > 0$, $B(y) > 0$ together imply that $y < x$.

Definition 3.29. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a weakly pairwise intuitionistic fuzzy π-β-locally T_2-ordered space if given $b < a$ (that is $b \leq a$ and $b \neq a$), there exist an τ_i intuitionistic fuzzy π-β-locally open set $A = \langle x, \mu_A, \gamma_A \rangle$ such that $A(a) > 0$ and τ_j intuitionistic fuzzy π-β-locally open set $B = \langle x, \mu_B, \gamma_B \rangle$ such that $B(b) > 0$ ($i, j = 1, 2$ and $i \neq j$) such that if $x, y \in X$, $A(x) > 0$, $B(y) > 0$ together imply that $y < x$.

Definition 3.30. The symbol $x \parallel y$ means that $x \nleq y$ and $y \nleq x$.

Example 3.31. Let $x, y \in X$ and $A = \langle a, (0.2, 0.3), (0.7, 0.5) \rangle$ be an intuitionistic fuzzy set. This implies that $x \nleq y$ and $y \nleq x$. Therefore $x||y$ if and only if $x \leq y$ and $y \leq x$.

Definition 3.32. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a almost pairwise intuitionistic fuzzy T_2-ordered space if given $a \parallel b$, there exist an τ_i intuitionistic fuzzy open set $A = \langle x, \mu_A, \gamma_A \rangle$ such that $A(a) > 0$ and τ_j intuitionistic fuzzy open set $B = \langle x, \mu_B, \gamma_B \rangle$ such that $B(b) > 0$ ($i, j = 1, 2$ and $i \neq j$) such that if $x, y \in X$, $A(x) > 0$ and $B(y) > 0$ together imply that $x \parallel y$.

Definition 3.33. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a almost pairwise intuitionistic fuzzy π-β-locally T_2-ordered space if given $a \parallel b$, there exist an τ_i intuitionistic fuzzy π-β-locally open set $A = \langle x, \mu_A, \gamma_A \rangle$ such that $A(a) > 0$ and τ_j intuitionistic fuzzy π-β-locally open set $B = \langle x, \mu_B, \gamma_B \rangle$ such that $B(b) > 0$ ($i, j = 1, 2$ and $i \neq j$) such that if $x, y \in X$, $A(x) > 0$ and $B(y) > 0$ together imply that $x \parallel y$.

Proposition 3.34. *An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is a pairwise intuitionistic fuzzy π-β-locally T_2-ordered space if and only if it is a weakly pairwise intuitionistic fuzzy π-β-locally T_2-ordered and almost pairwise intuitionistic fuzzy π-β-locally T_2-ordered space.*

Proof. Let $(X, \tau_1, \tau_2, \leq)$ be a pairwise intuitionistic fuzzy π-β-locally T_2-ordered space. Then by Proposition(3.3) and Definition(3.20) it is a weakly pairwise intuitionistic fuzzy π-β-locally T_2-ordered space. Let $a \parallel b$. Then $a \nleq b$ and $b \nleq a$. Since $a \nleq b$ and X is a pairwise intuitionistic fuzzy π-β-locally T_2-ordered space. We have τ_i intuitionistic fuzzy π-β-locally open set $A = \langle x, \mu_A, \gamma_A \rangle$ and τ_j intuitionistic fuzzy π-β-locally open set $B = \langle x, \mu_B, \gamma_B \rangle$ such that $A(a) > 0$, $B(b) > 0$ and $A(x) > 0$,$B(y) > 0$ together imply that $x \nleq y$. Also since $b \nleq a$,there exist τ_i intuitionistic fuzzy π-β-locally open set A^*=$\langle x, \mu_{A^*}, \gamma_{A^*} \rangle$ and τ_j intuitionistic fuzzy π-β-locally open set B^*=$\langle x, \mu_{B^*}, \gamma_{B^*} \rangle$ such that $A^*(a) > 0$,$B^*(b) > 0$ and $A^*(x) > 0$,$B^*(y) > 0$ together imply that $y \nleq x$. Thus ${I_i}^\circ(A \cap A^*)$ is an τ_i intuitionistic fuzzy π-β-locally open set such that ${I_i}^\circ(A \cap A^*)(a) > 0$ and ${I_j}^\circ(B \cap B^*)$ is an τ_j intuitionistic fuzzy π-β-locally open set such that ${I_j}^\circ(B \cap B^*)(b) > 0$. Also ${I_i}^\circ(A \cap A^*)(x) > 0$ and ${I_j}^\circ(B \cap B^*)(y) > 0$ togetherimply that $x \parallel y$. Hence X is a almost pairwise intuitionistic fuzzy π-β-locally T_2-ordered space.

Conservely, let X be a weakly pairwise intuitionistic fuzzy π-β-locally T_2-ordered and almost pairwise intuitionistic fuzzy π-β-locally T_2-ordered space. We want to show that X is a pairwise intuitionistic fuzzy π-β-locally T_2-ordered space. Let $a \nleq b$.Then either $b < a$ (or) $b \nleq a$. If $b < a$ then X being weakly pairwise intuitionistic fuzzy π-β-locally T_2-ordered space, there exist τ_i intuitionistic fuzzy π-β-locally open set A and τ_j intuitionistic fuzzy π-β-locally open set B such that $A(a) > 0$, $B(b) > 0$ and such that $A(x) > 0$, $B(y) > 0$ together imply that $y < x$. That is $x \nleq y$.If $b \nleq a$, then $a \parallel b$ and the result follows easily since X is a almost pairwise intuitionistic fuzzy π-β-locally T_2-ordered space. Hence X is a pairwise intuitionistic fuzzy π-β-locally T_2-ordered space. □

Definition 3.35. Let $A = \langle x, \mu_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \gamma_B \rangle$ be intuitionistic fuzzy sets in an ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$. Then A is said to be an τ_i intuitionistic fuzzy neighbourhood of B if $B \subseteq A$ and there exists τ_i intuitionistic fuzzy open set $C = \langle x, \mu_C, \gamma_C \rangle$ such that $B \subseteq C \subseteq A$, $(i = 1(or)2)$.

Definition 3.36. Let $A = \langle x, \mu_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \gamma_B \rangle$ be intuitionistic fuzzy sets in an ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$. Then A is said to be an τ_i intuitionistic fuzzy π-β-locally neighbourhood of B if $B \subseteq A$ and there exists τ_i intuitionistic fuzzy π-β-locally open set $C = \langle x, \mu_C, \gamma_C \rangle$ such that $B \subseteq C \subseteq A$,$(i = 1(or)2)$.

Definition 3.37. An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space if for every pair $A = \langle x, \mu_A, \gamma_A \rangle$ is an decreasing τ_i intuitionistic fuzzy π-β-locally closed set and $B = \langle x, \mu_B, \gamma_B \rangle$ is an decreasing τ_j intuitionistic fuzzy π-β-locally open set such that $A \subseteq B$ then there exist decreasing τ_j intuitionistic fuzzy π-β-locally open set A_1=$\langle x, \mu_{A_1}, \gamma_{A_1} \rangle$ such that $A \subseteq A_1 \subseteq D_i(A_1) \subseteq B$, $(i, j = 1, 2$ and $i \neq j)$.

Proposition 3.38. *An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ the following are equivalent*

(i) $(X, \tau_1, \tau_2, \leq)$ *is a strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space.*

(ii) *For each increasing τ_i intuitionistic fuzzy π-β-locally open set A=$\langle x, \mu_A, \gamma_A\rangle$ and decreasing τ_j intuitionistic fuzzy π-β-locally open set B=$\langle x, \mu_B, \gamma_B\rangle$ with $A \subseteq B$ there exists an decreasing τ_j intuitionistic fuzzy π-β-locally open set A_1 such that $A \subseteq A_1 \subseteq IF\pi\text{-}\beta\text{-}lcl_{\tau_i}(A_1) \subseteq B$, $(i, j = 1, 2$ and $i \neq j)$.*

Proof. The Proof is simple. □

Definition 3.39. Let $(X, \tau_1, \tau_2, \leq)$ be an ordered intuitionistic fuzzy bitopological space. A function $f : X \to R(I)$ is said to be an τ_i *lower**(*resp.upper**) intuitionistic fuzzy π-β-locally continuous function if $f^{-1}(R_t)$ $(resp.f^{-1}(L_t))$ is an increasing (or) an decreasing τ_i (resp.τ_j) intuitionistic fuzzy π-β-locally open set,for each $t \in R$ $(i, j = 1, 2$ and $i \neq j)$.

Proposition 3.40. *Let $(X, \tau_1, \tau_2, \leq)$ be an ordered intuitionistic fuzzy bitopological space. Let $A = \langle x, \mu_A, \gamma_A\rangle$ be an intuitionistic fuzzy set in X and let $f : X \to R(I)$ be such that*

$$f(x)(t) = \begin{cases} 1 & if \quad t < 0 \\ A(x) & if \quad 0 \leq t \leq 1 \\ 0 & if \quad t > 1 \end{cases}$$

for all $x \in X$. Then f is an τ_i lower*(resp.$\tau_j$upper*) intuitionistic fuzzy π-β-locally continuous function if and only if A is an increasing (or) an decreasing τ_i (resp.τ_j) intuitionistic fuzzy π-β-locally open(resp.closed) set $(i, j = 1, 2$ and $i \neq j)$.

Proof. $$f^{-1}(R_t) = \begin{cases} 1 & if \quad t < 0 \\ A(x) & if \quad 0 \leq t \leq 1 \\ 0 & if \quad t > 1 \end{cases}$$
implies that f is an τ_i *lower** intuitionistic fuzzy π-β-locally continuous function if and only if A is an increasing (or) an decreasing τ_i intuitionistic fuzzy π-β-locally open set in X.

$$f^{-1}(L_t) = \begin{cases} 1 & if \quad t < 0 \\ A(x) & if \quad 0 \leq t \leq 1 \\ 0 & if \quad t > 1 \end{cases}$$
implies that f is an τ_j *upper** intuitionistic fuzzy π-β-locally continuous function if and only if A is an increasing (or) an decreasing τ_j intuitionistic fuzzy π-β-locally closed set in X $(i, j = 1, 2$ and $i \neq j)$. □

Uryshon's Lemma

Proposition 3.41. *An ordered intuitionistic fuzzy bitopological space $(X, \tau_1, \tau_2, \leq)$ is a strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space if and only if for every $A = \langle x, \mu_A, \gamma_A\rangle$ is an decreasing τ_i intuitionistic fuzzy closed set and $B = \langle x, \mu_B, \gamma_B\rangle$ is an increasing τ_j intuitionistic fuzzy closed set with $A \subseteq \overline{B}$, there exists increasing intuitionistic fuzzy function $f : X \to I$ such that $A \subseteq f^{-1}(\overline{L_1}) \subseteq f^{-1}(R_0) \subseteq B$ and f is an τ_i upper* intuitionistic fuzzy π-β-locally continuous function and τ_j lower* intuitionistic fuzzy π-β-locally continuous function$(i, j = 1, 2$ and $i \neq j)$.*

Proof. Suppose that there exists a function f satisfying the given conditions. Let $C = \langle x, \mu_C, \gamma_C\rangle$=$f^{-1}(\overline{L_t})$ and $D = \langle x, \mu_D, \gamma_D\rangle$=$f^{-1}(R_t)$ for some $0 \leq t \leq 1$. Then $\overline{C} \in \tau_i$ and $D \in \tau_j$ and such that $A \subseteq C \subseteq D \subseteq \overline{B}$. It is easy to verify that D is an decreasing τ_j intuitionistic fuzzy π-β-locally open set and C is an increasing τ_i

intuitionistic fuzzy π-β-locally closed set. Then there exists decreasing τ_j intuitionistic fuzzy π-β-locally open set C_1 such that $C \subseteq C_1 \subseteq D_i(C_1) \subseteq D$, $(i, j = 1, 2$ and $i \neq j)$. This proves that X is a strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space.

Conversely, let X be a strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space. Let A be an decreasing τ_i intuitionistic fuzzy π-β-locally closed set and B be an increasing τ_j intuitionistic fuzzy π-β-locally closed set.By the Proposition(3.6),we can construct a collection $\{C_t \mid t \in I\} \subseteq \tau_j$,where $C = \langle x, \mu_{C_t}, \gamma_{C_t}\rangle$,$t \in I$ such that $A \subseteq C_t \subseteq B$, $IF\pi$-β-$lcl_{\tau_i}(C_s) \subseteq C_t$ whenever $s < t$, $A \subseteq C_0$, $C_1 = B$ and $C_t = 0_\sim$ for $t < 0$, $C_t = 1_\sim$ for $t > 1$. We define a function $f : X \to I$ by $f(x)(t) = C_{1-t}(x)$. Clearly f is well defined. Since $A \subseteq C_{1-t} \subseteq B$, for $t \in I$. We have $A \subseteq f^{-1}(\overline{L_1}) \subseteq f^{-1}(R_0) \subseteq B$. Furthermore $f^{-1}(R_t) = \bigcup_{s<1-t} C_s$ is an τ_j intuitionistic fuzzy π-β-locally open set and $f^{-1}(\overline{L_t}) = \bigcap_{s>1-t} C_s = \bigcap_{s>1-t} IF\pi$-$\beta$-$lcl_{\tau_i}(C_s)$ is an τ_i intuitionistic fuzzy π-β-locally closed set. Thus f is an τ_j *lower** intuitionistic fuzzy π-β-locally continuous function and τ_i *upper** intuitionistic fuzzy π-β-locally continuous function and is an increasing intuitionistic fuzzy function. □

Tietze Extension Theorem

Proposition 3.42. *Let $(X, \tau_1, \tau_2, \leq)$ be an ordered intuitionistic fuzzy bitopological space the following statements are equivalent.*

(i) *$(X, \tau_1, \tau_2, \leq)$ is a strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space.*

(ii) *If $g, h : X \to R(I)$,g is an τ_i upper* intuitionistic fuzzy π-β-locally continuous function,h is an τ_j lower* intuitionistic fuzzy π-β-locally continuous function and $g \subseteq h$,then there exists $f : X \to R(I)$ such that $g \subseteq f \subseteq h$ and f is an τ_i upper* intuitionistic fuzzy π-β-locally continuous function and τ_j lower* intuitionistic fuzzy π-β-locally continuous function$(i, j = 1, 2$ and $i \neq j)$.*

Proof. **(ii)⇒(i)** Let $A = \langle x, \mu_A, \gamma_A\rangle$ and $B = \langle x, \mu_B, \gamma_B\rangle$ be an intuitionistic fuzzy π-β-locally open sets such that $A \subseteq B$. Define $g, h : X \to R(I)$ by

$$g(x)(t) = \begin{cases} 1 & if \quad t < 0 \\ A(x) & if \quad 0 \leq t \leq 1 \\ 0 & if \quad t > 1 \end{cases} \quad \text{and}$$

$$h(x)(t) = \begin{cases} 1 & if \quad t < 0 \\ B(x) & if \quad 0 \leq t \leq 1 \\ 0 & if \quad t > 1 \end{cases}$$

for each $x \in X$. By Proposition(3.6), g is an τ_i *upper** intuitionistic fuzzy π-β-locally continuous function and h is an τ_j *lower** intuitionistic fuzzy π-β-locally continuous function. Clearly, $g \subseteq h$ holds,so that there exists $f : X \to R(L)$ such that $g \subseteq f \subseteq h$. Suppose $t \in (0, 1)$. Then $A = g^{-1}(R_t) \subseteq f^{-1}(R_t) \subseteq f^{-1}(\overline{L_t}) \subseteq h^{-1}(\overline{L_t}) = B$. By Proposition(3.7),$X$ is a strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space.

(i)⇒(ii) Define two mappings $A, B : Q \to I$ by $A(r) = A_r = h^{-1}(\overline{R_r})$ and $B(r) = B_r = g^{-1}(L_r)$,for all $r \in Q$ (Q is the set of all rationals). Clearly, A and B are monotone increasing families of an decreasing τ_i intuitionistic fuzzy π-β-locally closed sets and decreasing τ_j intuitionistic fuzzy π-β-locally open sets of

X. Moreover $A_r \subset B_{r'}$ if $r < r'$. By Proposition(3.5) there exists an decreasing τ_j intuitionistic fuzzy π-β-locally open set $C = \langle x, \mu_C, \gamma_C \rangle$ such that $A_r \subseteq IF\pi$-β-$lint_{\tau_i}(C_r)$, $IF\pi$-β-$lcl_{\tau_i}(C_r) \subseteq IF\pi$-$\beta$-$lint_{\tau_i}(C_{r'})$, $IF\pi$-β-$lcl_{\tau_i}(C_r) \subseteq B_{r'}$ whenever $r < r'$ $(r, r' \in Q)$. Letting $V_t = \bigcap_{r<t} \overline{C_r}$ for $t \in R$, we define a monotone decreasing family $\{V_t \mid t \in R\} \subseteq I$. Moreover we have $IF\pi$-β-$lcl_{\tau_i}(V_t) \subseteq IF\pi$-$\beta$-$lint_{\tau_i}(V_s)$ whenever $s < t$. We have,

$$\bigcup_{t\in R} V_t = \bigcup_{t\in R}\bigcap_{r<t} \overline{C_r} \supseteq \bigcup_{t\in R}\bigcap_{r<t} \overline{B_r} = \bigcup_{t\in R}\bigcap_{r<t} g^{-1}(\overline{L_r})$$
$$= \bigcup_{t\in R} g^{-1}(\overline{L_t}) = g^{-1}(\bigcup_{t\in R} \overline{L_t}) = 1_\sim$$

Similarly, $\bigcap_{t\in R} V_t = 0_\sim$. Now define a function $f : (X, \tau_1, \tau_2, \leq) \to R(L)$ satisfying the required conditions. Let $f(x)(t) = V_t(x)$, for all $x \in X$ and $t \in R$. By the above discussion, it follows that f is well defined. To prove f is an τ_i *upper** intuitionistic fuzzy π-β-locally continuous function and τ_j *lower** intuitionistic fuzzy π-β-locally continuous function($i, j = 1, 2$ and $i \neq j$). Observe that $\bigcup_{s>t} V_s = \bigcup_{s>t} IF\pi$-$\beta$-$lint_{\tau_i}(V_s)$ and $\bigcap_{s>t} V_s = \bigcap_{s>t} IF\pi$-$\beta$-$lcl_{\tau_i}(V_s)$. Then $f^{-1}(R_t) = \bigcup_{s>t} V_s = \bigcup_{s>t} IF\pi$-$\beta$-$lint_{\tau_i}(V_s)$ is an increasing τ_i intuitionistic fuzzy π-β-locally open set. Now, $f^{-1}(\overline{L_t}) = \bigcap_{s>t} V_s = \bigcap_{s>t} IF\pi$-$\beta$-$lcl_{\tau_i}(V_s)$ is an decreasing τ_j intuitionistic fuzzy π-β-locally closed set. So that f is an τ_i *upper** intuitionistic fuzzy π-β-locally continuous function and τ_j *lower** intuitionistic fuzzy π-β-locally continuous function. To conclude the proof it remains to show that $g \subseteq f \subseteq h$. That is $g^{-1}(\overline{L_t}) \subseteq f^{-1}(\overline{L_t}) \subseteq h^{-1}(\overline{L_t})$ and $g^{-1}(R_t) \subseteq f^{-1}(R_t) \subseteq h^{-1}(R_t)$ for each $t \in R$. We have,

$$g^{-1}(\overline{L_t}) = \bigcap_{s<t} g^{-1}(\overline{L_s}) = \bigcap_{s<t}\bigcap_{r<s} g^{-1}(\overline{L_r}) = \bigcap_{s<t}\bigcap_{r<s} \overline{B_r}$$
$$\subseteq \bigcap_{s<t}\bigcap_{r<s} \overline{C_r} = \bigcap_{s<t} V_s = f^{-1}(\overline{L_t})$$

and

$$f^{-1}(\overline{L_t}) = \bigcap_{s<t} V_s = \bigcap_{s<t}\bigcap_{r<s} \overline{C_r} \subseteq \bigcap_{s<t}\bigcap_{r<s} \overline{A_r}$$
$$= \bigcap_{s<t}\bigcap_{r<s} h^{-1}(\overline{R_r}) = \bigcap_{s<t} h^{-1}(\overline{L_s}) = h^{-1}(\overline{L_t})$$

Similarly, we obtain

$$g^{-1}(R_t) = \bigcup_{s>t} g^{-1}(R_s) = \bigcup_{s>t}\bigcup_{r>s} g^{-1}(\overline{L_r}) = \bigcup_{s>t}\bigcup_{r>s} \overline{B_r}$$
$$\subseteq \bigcup_{s>t}\bigcup_{r>s} \overline{C_r} = \bigcup_{s>t} V_s = f^{-1}(\overline{R_t})$$

and

$$f^{-1}(\overline{R_t}) = \bigcup_{s>t} V_s = \bigcup_{s>t}\bigcup_{r>s} \overline{C_r} \subseteq \bigcup_{s>t}\bigcup_{r>s} \overline{A_r}$$
$$= \bigcup_{s>t}\bigcup_{r>s} h^{-1}(\overline{R_r}) = \bigcup_{s>t} h^{-1}(\overline{R_s}) = h^{-1}(\overline{R_t})$$

This completes the proof. □

Proposition 3.43. *Let $(X, \tau_1, \tau_2, \leq)$ be a strongly pairwise intuitionistic fuzzy π-β-locally normally ordered space. Let $\overline{A} \in \tau_1$ and $\overline{A} \in \tau_2$ be crisp and let $f : (A, \tau_1/A, \tau_2/A) \to I$ be an τ_i upper* intuitionistic fuzzy π-β-locally continuous function and τ_j lower* intuitionistic fuzzy π-β-locally continuous function $(i, j = 1, 2$ and $i \neq j)$. Then f has an intuitionistic fuzzy extension over $(X, \tau_1, \tau_2, \leq)$ (that is,$F : (X, \tau_1, \tau_2, \leq) \to I$).*

Proof. Define $g : X \to I$ by

$$\begin{aligned} g(x) &= f(x) \quad if \quad x \in A \\ &= [A_0] \quad if \quad x \notin A \end{aligned}$$

and also define $h : X \to I$ by

$$\begin{aligned} h(x) &= f(x) \quad if \quad x \in A \\ &= [A_1] \quad if \quad x \notin A \end{aligned}$$

where $[A_0]$ is the equivalence class determined by $A_0 : R \to I$ such that

$$\begin{aligned} A_0(t) &= 1_\sim \quad if \quad t < 0 \\ &= 0_\sim \quad if \quad t > 0 \end{aligned}$$

and $[A_1]$ is the equivalence class determined by $A_1 : R \to I$ such that

$$\begin{aligned} A_1(t) &= 1_\sim \quad if \quad t < 1 \\ &= 0_\sim \quad if \quad t > 1 \end{aligned}$$

g is an τ_i *upper** intuitionistic fuzzy π-β-locally continuous function and h is an τ_j *lower** intuitionistic fuzzy π-β-locally continuous function and $g \subseteq h$.Hence by the Proposition(3.8),there exists a function $F : X \to I$ such that F is an τ_i *upper** intuitionistic fuzzy π-β-locally continuous function and τ_j *lower** intuitionistic fuzzy π-β-locally continuous function and $g(x) \subseteq h(x) \subseteq f(x)$ for all $x \in X$. Hence for all $x \in A$, $f(x) \subseteq F(x) \subseteq f(x)$. So that F is an required extension of f over X. □

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