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On fuzzy m_X^*- oscillation and it's application in image processing

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Abstract. The aim of this paper is to introduce the concept of fuzzy m_X^*-oscillation. The new concept of fuzzy m_X^*-oscillation has been introduced which helps us in face recognition process.In this paper we have taken an grey scale image with different pixel values lying between [0,1] using MATLAB 7.0 software.Then we compare an image of unknown object with the original image by measuring the height of oscillation. It is also shown that the open oscillation of an image is same as a closed oscillation of a negative image, which helps us in comparing an image with negative image at a time. Also it helps diagnosing the image due to various effects of light.

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1. Introduction

The concept of minimal structure in general topological space has been introduced by Popa and Noiri in 2006 [10]. Later on M. Alimohammady and M. Roohi in 2006 [1] introduced the concept of fuzzy minimal structure. After this lots of research work has been done in this field [2, 3, 4]. Minimal structure or m_X-structure consists of whole set X along with other elements.Relaxing this condition we may introduce a new concept as m_X^*-structure which may consist of atleast one of X and ϕ. In case of fuzzy set fuzzy m_X^*-structure contains atleast an element with membership value 1 and/or 0. Throughout this paper the concept of m_X^*-structure is studied in the field of fuzzy oscillation introduced in 2007 [9]. The concept of fuzzy oscillation in [5, 6, 7] has been introduced by the authors and applied in the various fields of data mining and decision making process.

In this paper fuzzy m_X^*-oscillation is introduced and it is applied in the field of image analysis(gray scale image). Gray scale images consists of pixels with atleast one pixels membership value 1 or/and 0. i.e. these pixels forms a fuzzy m_X^*-structure .So the aim of this paper is to introduce fuzzy m_X^*-oscillation and to show its application in the field of image comparison. In section two the preliminaries required to go further through this paper is cited. In section three the concept of m_X^*-oscillation is introduced and its application in the field of image analysis is shown. Lastly conclusion is given.

2. Preliminaries

In this section we shall give those notions, which are necessary to define concepts used in the paper.

Definition 2.1 ([8])**.** H. Maki called a subset A of an ordinary topological space (X,T) a Λ- set if it is the intersection of open sets containing the set i.e.

$\Lambda(A) = \cap\{ G : G \supseteq A, G$ is an open set $\} = A$ and
a V - set if $V(A) = \cup\{G : G \subseteq A, G$ is a closed set $\} = A$.
Also $\Lambda(A^C) =$ 1- V(A) , $Cl(A^C) =$ 1- Int(A) , $V(A) \subseteq A \subseteq (A)$, for any subset A of X.

Definition 2.2 ([10])**.** A subfamily m_X of P(X),power set of X, is called a minimal structure on X if $\phi \in m_X$ and $X \in m_X$. Each member of m_X is said to be a m_X-open set and the complement of a m_X-open set is said to be a m_X-closed set. By (X, m_X) we denote a non-empty set X with a minimal structure m_X on X and we say that (X, m_X) is a space with minimal structure. Let (X, m_X) be a space with minimal structure and $A \subset X$. The m_X-interior of A, denoted by m_X -Int(A), is the union of all m_X -open subsets of A. The m_X -closure of A,denoted by m_X -Cl(A), is the intersection of all m_X -closed supersets of A.

Definition 2.3 ([1])**.** A family M of fuzzy sets in P(X) is said to be a fuzzy minimal structure on X if $\alpha 1_X \in M$ for any $\alpha \in I$.In this case (X, M) is called a fuzzy minimal space.

Definition 2.4. A digital image a[m,n] described in a 2D discrete space is derived from an analog image a(x,y) in a 2D continuous space through a sampling process that is frequently referred to as digitization. The 2D continuous image a(x,y) is divided into N rows and M columns. The intersection of a row and a column is termed a pixel. The value assigned to the integer coordinates [m,n] with {m=0,1,2,...,M-1} and {n=0,1,2,...,N-1} is a[m,n]. The figure is given below:

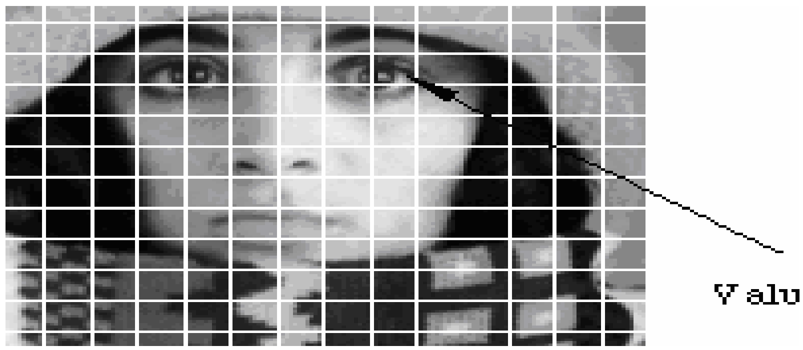

Figure: The pixel at coordinates [m=10, n=3] has the integer brightness value 110.

The image shown in Figure has been divided into N = 16 rows and M = 16 columns. The value assigned to every pixel is the average brightness in the pixel rounded to the nearest integer value.

An image is stored as a matrix using standard Matlab matrix conventions. There are five basic types of images supported by Matlab: Indexed images, Intensity images, Binary images, True color or RGB images,8-bit images etc.We are interested to work only with Intensity image or grey scale image.In a binary image there are just two gray levels which can be referred to, for example, as 'black' and 'white' or '0' and '1'. There are 256 grey levels in an 8-bit grey-scale image and the intensity of each pixel can have a value from 0 to 255, with 0 being black and 255 being white.

3. On fuzzy m_X^*-Oscillation and its application on image analysis

In this section the concept of fuzzy m_X^*-oscillation is introduced and its application is shown in the field of image comparison.

Definition 3.1. A subfamily m_X^* of fuzzy sets in P(X) is said to be fuzzy m_X^*-structure on X if at least μ_X and/or μ_ϕ belongs to m_X^* with $\alpha \in m_X^*$, where $\alpha \in I^X$. Members of m_X^*-structure are m_X*-open set. Let us now introduce the following operators from $I^x \to I^x$ which are defined on fuzzy m_X^* space.

Definition 3.2. The operator $\Lambda : I^x \to I^x$ is defined as

(i) $\Lambda_{a_j}(x) = \inf\{\ \mu_{a_j}(x_i) : \mu_{a_j}(x_i) \geq \mu_{a_j}(x), x_i \in G$,G is an open set , j=1,2,$\cdots$,n$\}$
$= \hat{I}$,if no such open set exists.

(ii) $Int_{a_j}(x) = \sup\{\mu_{a_j}(x_i) : \mu_{a_j}(x_i) \leq \mu_{a_j}(x), x_i \in G$,G is an open set , j=1,2,$\cdots$,n$\}$
$=\phi$,if no such open set exists.

(iii)$Cl_{a_j}(x) = \inf\{\ \mu_{a_j}(x_i) : \mu_{a_j}(x_i) \geq \mu_{a_j}(x), x_i \in G$,G is an closed set , j=1,2,$\cdots$,n$\}$
$= \hat{I}$,if no such closed set exists.

(iv)$V_{a_j}(x) = \sup\{\mu_{a_j}(x_i) : \mu_{a_j}(x_i) \leq \mu_{a_j}(x), x_i \in G$,G is an closed set , j=1,2,$\cdots$,n$\}$
$=\phi$,if no such closed set exists.

where $\mu_{a_j}(x_i)$is the membership value of any particular attribute a_j of any object x_i

Definition 3.3. An operator O^o: $I^X \to I^X$ such that $O^o a_j(x) = \Lambda_{a_j}(x)$- $Int_{a_j}(x)$, is said to be fuzzy m^*_X-open oscillatory operator and an operator O^c:$I^X \to I^X$ such that $O^c a_j(x) = Cl_{a_j}(x)$- $V_{a_j}(x)$,is said to be fuzzy m^*_X-closed oscillatory operator.

Theorem 3.4. *For any object y in fuzzy m^*_X- structure, $O^o a_j(y) = \alpha$ implies $O^o a_j(y^c) = -\alpha$ and viceversa where $\alpha \in I^X$.*

Proof. Let if possible $O^o a_j(y) = \alpha$, for any object y with attribute a_j.

$\iff \Lambda_{a_j}(y)$- $Int_{a_j}(y) = \alpha$.

$\iff \{1- \Lambda_{a_j}(y)\}$-$\{1- Int_{a_j}(y)\} = -\alpha$.

$\iff Cl_{a_j}(y^c)$- $V_{a_j}(y^c) = -\alpha$, [since $1- \Lambda_{a_j}(y) = Cl_{a_j}(y^c)$ and $1- Int_{a_j}(y) = V_{a_j}(y^c)$ are obvious from definitions].

$\iff O^{cl} a_j(y^c) = -\alpha$. □

Remark 3.5. Oscillation can never be negative, so $-\alpha$ may indicate oscillation in the opposite direction.

Remark 3.6. (i) If $O^o a_j(y) = O^{cl} a_j(y^c) = 0$ then $\Lambda_{a_j}(y) = Int_{a_j}(y)$ and $Cl_{a_j}(y^c) = V_{a_j}(y^c)$.

(ii) If $O^o a_j(y) = O^{cl} a_j(y^c) = 1$ then $\Lambda_{a_j}(y) = Cl_{a_j}(y^c) = 1$ and $Int_{a_j}(y) = V_{a_j}(y^c) = 0$.

4. Application of fuzzy m^*_X- oscillation on image comparison

In this subsection we apply the concept of fuzzy m^*_X oscillatory region in the field of image comparison. Here we are only considering the gray scale images. We know that there are only 256 grey level in an 8 bit grey scale image and the intensity of each pixel can have a value from 0 to 255 with 0 being black and 255 being white. All other grey levels lie between 0 and 255 which forms a fuzzy m_X structure. But in every image it is not mandatory that both the full black or white gray level shall remain present. But at least one must remain in every image. So we are considering m^*_X-structure and applying it on image analysis.

Let us consider an image I_k with some gray levels of different pixel values y_{ij} ,i,j=1,2,$\cdots$,n,n is any finite integer value. The same image is considered under different lighting facilities and considering the same pixel values their gray levels are studied. These collections of different image forms a space where due to brightness or darkness the pixel with gray level membership value 0 or 1 must lie in the collection. This collection is an m_X structure and the members are m^*_X open set. Suppose we have a collection of images $I = I_1, I_2, I_3, \cdots$. We are to determine $O^o_{I_k}(y_{ij})$ for all $I_k \in I$ with grey level at pixel y_{ij}. As from theorem 3.1.3, $O^o_{I_k}(y_{ij}) = O^{cl}_{I_k}(y^c_{ij})$ i.e. the open oscillation of an image at any pixel y_{ij} is same as the closed oscillation of the image at complementary pixel value i.e oscillation of negative image . To compare an unknown image with the known image with the help of fuzzy m^*_X-oscillation the following steps may be taken. At first we have to check the open oscillatory operator $O^o_{I_k}(y_{ij})$and then $O^{cl}_{I_k}(y_{ij})$.

For $O^o_{I_k}(y_{ij}) = \Lambda_{I_k}(y_{ij})$- $Int_{I_k}(y_{ij})$, the following cases may appear:

(1) $O^o_{I_k}(y_{ij}) = 0$ or 1

(2) $0 < O^o_{I_k}(y_{ij}) < 1$

(3) $O^o_{I_k}(y_{ij}) = \Lambda_{I_k}(y_{ij})$-$\phi$

(4)$O^o_{I_k}(y_{ij})$=Î- $\text{Int}_{I_k}(y_{ij})$

Case(1): Let if possible $O^o_{I_k}(y_{ij})= \Lambda_{I_k}(y_{ij})$-$\text{Int}_{I_k}(y_{ij})$ =0

$\iff \Lambda_{I_k}(y_{ij})=\text{Int}_{I_k}(y_{ij})$

$\iff$ infimum of all the intensities at the pixel (i,j) which are greater than or equal to the intensity of the unknown image at the pixel(i,j)= supremum of all the intensities at the pixel (i,j) which are less than or equal to the intensity of the unknown image at the pixel(i,j).

$\iff$ Intensity of the pixel(i,j) of the image I_k=Intensity at the pixel(i,j) of the unknown image.

$\iff$ image I_k and the unknown image are same at the pixel(i,j).

Now if possible let $O^o_{I_k}(y_{ij})= \Lambda_{I_k}(y_{ij})$-$\text{Int}_{I_k}(y_{ij})$=1.

$\iff \Lambda_{I_k}(y_{ij})$=1 and $\text{Int}_{I_k}(y_{ij})$=0.

i.e. the intensity of the unknown image at pixel(i,j) is not within a known position.

So this pixel is not compare with the pixel (i,j) of known image. So this pixel is undefined pixel.

Case(2): Let if possible 0< $O^o_{I_k}(y_{ij})$<1.

i.e. 0< $\Lambda_{I_k}(y_{ij})$-$\text{Int}_{I_k}(y_{ij})$<1

(i) If $O^o_{I_k}(y_{ij})\leq 0.1$, there may be some similarity between the two images at pixel (i,j).

(ii) If $O^o_{I_k}(y_{ij})\geq 0.1$,we have to check the difference between the intensity of the unknown image at pixel(i,j) and $\Lambda_{I_k}(y_{ij})$ or $\text{Int}_{I_k}(y_{ij})$.If this difference is$\leq$ 0.1 then also the images may be similar at the pixel(i,j).

Case(3): $O^o_{I_k}(y_{ij})= \Lambda_{I_k}(y_{ij})-\phi$.

In this case we may check the difference between the intensity of the unknown image at pixel(i,j) and $\Lambda_{I_k}(y_{ij})$.If this difference is $\leq$0.1 then, the images may be considered as similar at the pixel(i,j). Otherwise different.

Case(4): $O^o_{I_k}(y_{ij})$=Î- $\text{Int}_{I_k}(y_{ij})$,

In this case we may check the difference between the intensity of the unknown image at pixel(i,j) and $\text{Int}_{I_k}(y_{ij})$.If this difference is $\leq$0.1 then, the images may be considered as similar at the pixel(i,j).Otherwise different.

The case $O^{cl}_{I_k}(y_{ij})= \text{Cl}_{I_k}(y_{ij})$- $V_{I_k}(y_{ij})$ follows similarly as above cases.

Example 4.1. In this section the application of fuzzy m^*_X-structure is shown in the field of face recognition. Using syntex in MATLAB 7.0 we first display an image as given below:

Figure1: Image of a human face (I)

Let I (figure1) be a given image. Using Matlab software we can get all pixel values which are lying between [0,1]. Our aim is to compare an image of unknown object with the different intensity images of I due to effect of light. We take a part of this face (left eye side) and so we cut the other portion (figure2) and identified it as I_1(original image). Using other effect sharpen, edge, postrize we get I_2, $_3$, I_4 as given below(figure2, figure3, figure4, figure5):

I_1:(x(6,22),0.314),(x(26,22),0.765),(x(14,18),0.235),(x(14,26),0.388)

I_2:(x(6,22),0.337),(x(26,22),0.808),(x(14,18),0.29),(x(14,26),0.275)

I_3:(x(6,22),1.0),(x(26,22),0.4),(x(14,18),1.0),(x(14,26),1.0)

I_4:(x(6,22),0.227),(x(26,22),0.675),(x(14,18),0.227),(x(14,26),0.341)

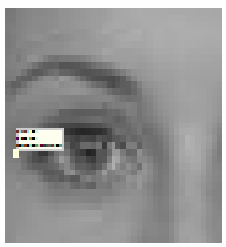

figure 2: I_1(soften)

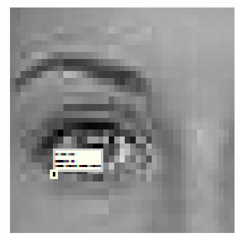

figure 3: I_2(Effect:sharpen)

figure4: I_3(Effect:edge):

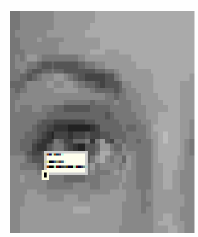

figure 5: I_4(Effect:postrize):

These I_1, I_2, I_3, I_4 are considered as open sets. Now let J be an image of an unknown object whose pixel values are: J: (y(6,22),0.5),(y(26,22),0.2),(y(14,18),0.7),(y(14,26),0.6)

The open oscillation operators of image J are: O^o_J[y(6,22)]= Λ_J(y(6,22))- Int_J(y(6,22))=1-0.337=0.663>0.1

O^o_J[y(26,22)]= Λ_J(y(26,22))- Int_J(y(26,22))=0.4-ϕ ,

Let d =0.4-0.2=0.2>0.1,so by case (3) pixels are different.

O^o_J[y(14,18)]= Λ_J(y(14,18))- Int_J(y(14,18))=1-0.29=0.71>0.1

O^o_J[y(14,26)]= Λ_J(y(14,26))- Int_J(y(14,26))=1-0.388=0.612>0.1 Since the open oscillation of each pixel is very high, we can say that the image of unknown object is different from image I.

Similarly closed oscillations may be calculated. The pixel values of negative image of image I are(figure7): (n(6,22),0.702),(n(26,22),0.0.251),(n(14,18),0.78),(n(14,26),0.627)

Remark 4.2. The negative image is the complementary image of original image and negative images may be considered as closed sets

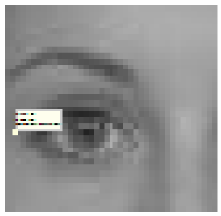

Figure6: Grey scale image of a part of human face(I_1)

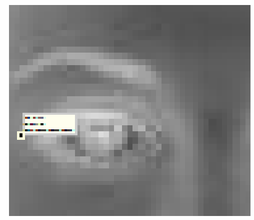

Figure7: Negative image of figure1

Now we check the closed oscillation of negative image as follows: The closed sets(i.e negative image) are:

I_1:(x^c(6,22),0.686),(x^c (26,22),0.235),(x^c (14,18),0.765),(x^c (14,26),0.612)

I_2:(x^c (6,22),0.663),(x^c (26,22),0.192),(x^c (14,18),0.71),(x^c (14,26),0.725)

I_3:(x^c (6,22),0.0),(x^c (26,22),0.6),(x^c (14,18),0.0),(x^c (14,26),0.0)

I_4:(x^c (6,22),0.773),(x^c (26,22),0.325),(x^c (14,18),0.773),(x^c (14,26),0.659)

The closed oscillation operators of image J are :

J^c=(y^c(6,22),0.5),(y^c (26,22),0.8),(y^c (14,18),0.3),(y^c (14,26),0.4)

O^{cl}_J[y^c (6,22)]= Cl(y^c (6,22))- V(y^c (6,22))=0.663-0=0.663

O^{cl}_J[y^c (26,22)]= Cl(y(26,22))- V(y^c (26,22))= I-0.6, Here d=0.8-0.6=0.2.

O^{cl}_J[y^c (14,18)]= Cl(y^c (14,18))- V(y^c (14,18))=0.71-0.0=0.71

O^{cl}_J[y^c (14,26)]= Cl(y^c (14,26))- V(y^c (14,26))=0.612-0.0=0.612 i.e. open oscillation of the pixels of image is same as the closed oscillation of negative image.

5. Conclusion

In the real world data it is not always possible to handle by rough set approach. So we introduce a new concept fuzzy m^*_X-structure and fuzzy m^*_X-oscillation. This concept may be very useful in the field of image analysis. The advantage of using fuzzy m^*_X-structure is that here we need not form topology i.e union of arbitrary elements and intersection of finite number of elements of fuzzy m^*_X-structure need not belong to fuzzy m^*_X-structure. It is more generalize form. We can work with any type of data set by m^*_X-structure. This concept will be very useful in the field of face recognition or criminal identification.

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