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L-fuzzy 0-(1- or 2- or 3-) 2-absorbing ideals in semirings

Ahmad Yousefian Darani, Ashraf Hashempoor

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Abstract. Let R be a commutative semiring. We define the concept of L-fuzzy $0-(1-$ or $2-$ or $3-)2-$absorbing ideals of R. Then we study some basic results concerning these classes of L-fuzzy ideals and discuss on the relationship among these classes of L-fuzzy ideals of R.

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Corresponding Author: Ahmad Yousefian Darani (yousefian@uma.ac.ir)

1. Introduction

There are many concepts of universal algebras generalizing an associative ring $(R, +, .)$. Some of them – in particular, semirings – have been proven very useful. Semirings (called also halfrings) are algebras $(R, +, .)$ sharing the same properties as a ring except that $(R, +)$ is assumed to be a semigroup rather than a commutative group. Semirings, ordered semirings and hemirings appear in a natural manner in some applications to the theory of automata (see for example [8] and [14]) and formal languages (see [2], [10] and [9]). The concept of semirings was introduced by H. S. Vandiver in 1935 and has studied extensively by many authors (e.g. [3, 4]). A nonempty set R together with two associative binary operations called addition and multiplication (denoted by + and ., respectively) will be called a semiring provided that:

(1) $(R, +)$ is a commutative monoid with identity element 0.
(2) $(R, .)$ is a monoid with identity element $1 \neq 0$.
(3) The multiplication is distributive with respect to the addition both from the left and from the right.
(4) $a.0 = 0.a = 0$ for all $a \in R$.

A semiring R is commutative if $(R, .)$ is a commutative semigroup.

A nonempty subset I of a semiring R will be called an ideal if $a, b \in I$ and $r \in R$ imply that $a + b \in I$ and $ra \in I$. A subtractive ideal (=k-ideal) I is an ideal of R such that if $a, a + b \in I$ then $b \in I$. Let A be a subset of R. The semiring ideal generated by A, denoted by A_d, is $\{r_1a_1 + r_2a_2 + ...r_sa_s | r_i \in R, a_i \in A, s \in \mathbb{N}_0\}$. The subtractive ideal (=k-ideal) generated by A, denoted by A_{sub}, is $\bigcap\{I | I \subseteq A$ and I is a subtractive ideal of $R\}$. It is proved in [11, Theorem 1.2] that $A_{sub} = \{x \in R | \exists a \in A_d$ such that $a + x \in A_d\}$. In this case, if A is an ideal of R then $A = A_d$, and A is a k-ideal of R if and only if $A = A_{sub}$. A prime ideal of R is a proper ideal P of R for which whenever $a, b \in R$ with $ab \in P$ then either $a \in P$ or $b \in P$. So P is a prime ideal of R if and only if for ideals A, B of R with $AB \subseteq P$, then $A \subseteq P$ or $B \subseteq P$, where $AB = \{ab | a \in A$ and $b \in B\}$ (see [4, Theorem 5]). R is called a prime semiring if 0 is a prime ideal of R.

In his classic paper [18], Zadeh introduced the notion of a fuzzy subset of a set X as a mapping from X to the interval $[0, 1]$. The fuzzy set theory developed by Zadeh himself and others can be found in mathematics and many applied areas. W. Liu [12] introduced and studied fuzzy subrings as well as fuzzy ideals in rings. Later, T. K. Mukherjee and M.K. Sen [13], K.L.N. Swamy and U.M. Swamy [15], and Zhang Yue [16] fuzzified certain standard concepts on rings and ideals. T. K. Dutta and B. K. Biswas [6] studied fuzzy ideals and fuzzy prime ideals of semirings, and they defined fuzzy k-ideals of semirings and characterized fuzzy prime k-ideals of semirings of non-negative integers. P. Dheena and S. Coumaressane in [5] initiated the notion of fuzzy $2-(0-$ or $1-)$prime ideal, fuzzy k-closure and fuzzy $m_2(m_0$ or $m_1)$-system and studied the relation ship among these classes of fuzzy ideals and fuzzy m-systems.

The first author, in [17], defined the concept of $0-(1-, 2-, 3-)$2-absorbing ideals of the semiring R. In this paper we generalize the concept of fuzzy $0-(1-, 2-)$ prime ideals of R.

2. Preliminaries

For the sake of completeness, we would like to recall some definitions and results proposed in this field earlier. Throughout this paper R stands for a semiring, and $L = (L, \leq, \wedge, \vee)$ will be a completely distributive lattice, which has the least and the greatest elements, say 0 and 1, respectively.

Definition 2.1. Let R be a semiring.

(1) A mapping $\mu : R \to L$ is called an L-fuzzy subset of R.

(2) For any two L-fuzzy subsets ν and μ of R, we say that μ is contained in ν and we write $\mu \subseteq \nu$ if $\mu(a) \leq \nu(a)$, for all $a \in R$.

(3) If μ is an L-fuzzy subset of R, then the image of μ denoted by $Im(\mu) = \{\mu(a) | a \in R\}$ and $|Im\mu|$ denotes the cardinality of $Im(\mu)$.

(4) For any L-fuzzy subset μ of R, μ_* denotes the subset $\{x \in R | \mu(x) = \mu(0)\}$ of R.

The set of all L-fuzzy subsets of R is called the L-power set of R and is denoted by L^R.

Definition 2.2. Let μ be any L-fuzzy subset of R. For $t \in L$ the set $\mu_t = \{x \in R | \mu(x) \geq t\}$ is called a t-level subset of μ.

Definition 2.3. Let μ, ν be any two L-fuzzy subsets of R. Then $\mu \cap \nu$, $\mu \cup \nu$, $\mu \cdot \nu$ and $\mu + \nu$ are L-fuzzy subsets of R defined by

$$(\mu \cap \nu)(x) = min\{\mu(x), \nu(x)\}$$
$$(\mu \cup \nu)(x) = max\{\mu(x), \nu(x)\}$$
$$(\mu.\nu)(x) = \begin{cases} sup_{x=yz}\{min\{\mu(y), \nu(z)\}\}, & \text{if x is expressed as x=yz;} \\ 0, & \text{otherwise.} \end{cases}$$
$$(\mu + \nu)(x) = \begin{cases} sup_{x=y+z}\{min\{\mu(y), \nu(z)\}\}, & \text{if x is expressed as x=y+z;} \\ 0, & \text{otherwise.} \end{cases}$$

By an L-fuzzy point x_r of R, $x \in R$; $r \in L \backslash \{0\}$, we mean $x_r \in L^R$ defined by

$$x_r(y) = \begin{cases} r, & \text{if y=x;} \\ 0, & \text{otherwise.} \end{cases}$$

If x_r is an L-fuzzy point of R and $x_r \subseteq \mu \in L^R$, we write $x_r \in \mu$. Note that $x_t \in \mu$ if and only if $x \in \mu_t$. Moreover for every L-fuzzy subset μ of R, it is easy to check that $\mu = \bigcup_{x_r \in \mu} x_r$. For $A \subseteq R$ the characteristic function of A, $\chi_A \in L^R$, is defined by

$$\chi_A(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{otherwise.} \end{cases}$$

Definition 2.4. An L-fuzzy subset μ of R is said to be an L-fuzzy left (resp. right) ideal of R if for all $x, y \in R$,
(1) $\mu(x + y) \geq min\{\mu(x), \mu(y)\}$; and
(2) $\mu(xy) \geq \mu(y)$ (resp. $\mu(xy) \geq \mu(y)$).
μ is an L-fuzzy ideal of R if it is both an L-fuzzy left and an L-fuzzy right ideal of R.

Lemma 2.5. *Let I be an ideal of R and $\alpha < \beta \in L$. Then the L-fuzzy subset of R defined by*

$$A(x) = \begin{cases} \alpha, & \text{if } x \in I; \\ \beta, & \text{otherwise.} \end{cases}$$

is an L-fuzzy ideal of R.

Proof. [5, Lemma 2.6]. □

Definition 2.6. An L-fuzzy ideal μ of R is said to be an L-fuzzy subtractive ideal (=k-ideal) of R if $\mu(x) \geq min\{\mu(x + y), \mu(y)\}$ for all $x, y \in R$.

Definition 2.7. If μ is any L-fuzzy subset of R, then μ_{sub} is defined as, for any $a \in R$, $\mu_{sub}(a) = sup_{x \in R}\{min\mu(a + x), \mu(x)\}\}$. μ_{sub} is called the L-fuzzy k-closure of μ (or L-fuzzy subtractive ideal of R generated by μ).

Clearly $\mu \leq \mu_{sub}$. Moreover, it is proved in [5, Lemma 2.18 and Lemma 2.19] that if μ is any L-fuzzy ideal of R, then μ_{sub} is an L-fuzzy k-ideal of R, and if μ is an L-fuzzy k-ideal of R, then $\mu = \mu_{sub}$. It has also proved in [5, Theorem 2.12] that the intersection of each two L-fuzzy k-ideals of R is again an L-fuzzy k-ideal of R.

Definition 2.8. Let R be a semiring, and let P be an ideal of R.

(i) P is called a $\mathfrak{0}-$prime (resp. $\mathfrak{2}-$prime) ideal of R if whenever I and J are ideals (resp. k-ideals) of R such that $IJ \subseteq P$, then either $I \subseteq P$ or $J \subseteq P$.

(ii) P is called a $\mathfrak{1}-$prime ideal of R if whenever I and J are ideals of R such that one of them is a k-ideal with $IJ \subseteq P$, then either $I \subseteq P$ or $J \subseteq P$.

Definition 2.9. Let R be a semiring, and let P be an L-fuzzy ideal of R.

(i) P is called an L-fuzzy $\mathfrak{0}-$prime (resp. L-fuzzy $\mathfrak{2}-$prime) ideal of R if whenever I and J are L-fuzzy ideals (resp. L-fuzzy k-ideals) of R such that $IJ \subseteq P$, then either $I \subseteq P$ or $J \subseteq P$.

(ii) P is called an L-fuzzy $\mathfrak{1}-$prime ideal of R if whenever I and J are L-fuzzy ideals of R such that one of them is an L-fuzzy k-ideal with $IJ \subseteq P$, then either $I \subseteq P$ or $J \subseteq P$.

Definition 2.10. Let R be a semiring, and let A be an ideal of R.

(i) A is called a $\mathfrak{0}-$2-absorbing (resp. $\mathfrak{3}-$2-absorbing) ideal of R if whenever I, J and K are ideals (resp. k-ideals) of R such that $IJK \subseteq A$, then $IJ \subseteq A$ or $IK \subseteq A$ or $JK \subseteq A$.

(ii) A is called a $\mathfrak{1}-$2-absorbing ideal of R if whenever I, J and K are ideals of R such that one of them is a k-ideal with $IJK \subseteq A$, then $IJ \subseteq A$ or $IK \subseteq A$ or $JK \subseteq A$.

(iii) A is called a $\mathfrak{2}-$2-absorbing ideal of R if whenever I, J and K are ideals of R such that two of them are k-ideals with $IJK \subseteq A$, then $IJ \subseteq A$ or $IK \subseteq A$ or $JK \subseteq A$.

3. L-fuzzy 2-absorbing ideals

Definition 3.1. Let R be a commutative semiring and let η be an L-fuzzy ideal of R. We say that η is a fuzzy 2-absorbing ideal of R if η is non-constant and for any fuzzy points $a_r, b_s, c_t \in L^R$, ($a, b, c \in R$ and $r, s, t \in L$), $a_r b_s c_t \in \eta$ implies that either $a_r b_s \in \eta$ or $a_r c_t \in \eta$ or $b_s c_t \in \eta$.

It is evident that every L-fuzzy prime ideal of R is L-fuzzy 2-absorbing. Now consider the following result:

Theorem 3.2. *Let η and μ be two distinct L-fuzzy prime ideals of R. Then $\eta \cap \mu$ is an L-fuzzy 2-absorbing ideal of R.*

Proof. Suppose that $a_r b_s c_t \in \eta \cap \mu$ for some L-fuzzy points $a_r, b_s, c_t \in L^R$, but $a_r b_s \notin \eta \cap \mu$ and $a_r c_t \notin \eta \cap \mu$. If $a_r b_s \notin \eta$ and $a_r c_t \notin \eta$, then, since η is an L-fuzzy prime ideal of R we have $c_t \in \eta$. In this case $a_r c_t \in \eta$ which is a contradiction. A similar argument leads us to a contradiction if $a_r b_s \notin \mu$ and $a_r c_t \notin \mu$. Therefore either $a_r b_s \notin \eta$ and $a_r c_t \notin \mu$ or $a_r b_s \notin \mu$ and $a_r c_t \notin \eta$. If the former case holds, then from $a_r b_s c_t \in \eta \cap \mu$ we get $c_t \in \eta$ and $b_s \in \mu$. Hence $b_s c_t \in \eta \cap \mu$. By a similar argument, we may show that $b_s c_t \in \eta \cap \mu$ if the latter case holds. Consequently, $\eta \cap \mu$ is an L-fuzzy 2-absorbing ideal of R. □

Theorem 3.3. *If η is an L-fuzzy 2-absorbing ideal of R, then η_t is a 2-absorbing ideal of R for every $t \in L$ with $\eta_t \neq R$.*

Proof. Let η be an L-fuzzy 2-absorbing ideal of R. Assume that $abc \in \eta_t$ for some $a, b, c \in R$. In this case from $\eta(abc) \geq t$ we have $a_t b_t c_t = (abc)_t \in \eta$. Since η is assumed to be an L-fuzzy 2-absorbing ideal of R, we have $(ab)_t = a_t b_t \in \eta$ or $(ac)_t = a_t c_t \in \eta$ or $(bc)_t = b_t c_t \in \eta$. Hence $ab \in \eta_t$ or $ac \in \eta_t$ or $bc \in \eta_t$. This shows that η_t is a 2-absorbing ideal of R. □

Corollary 3.4. *If η is an L-fuzzy 2-absorbing ideal of R, then η_* is a 2-absorbing ideal of R.*

Definition 3.5. Let $1 \neq \alpha \in L$. Then α is called a 2-absorbing element of L if $r \wedge s \wedge t \leq \alpha$ implies that $r \wedge s \leq \alpha$ or $r \wedge t \leq \alpha$ or $s \wedge t \leq \alpha$ for all $r, s, t \in L$.

Proposition 3.6. *If η is an L-fuzzy 2-absorbing ideal of R, then $\alpha = \eta(1)$ is a 2-absorbing element of L.*

Proof. Assume that $r \wedge s \wedge t \leq \alpha$ for some $r, s, t \in L$. In this case, $1_r, 1_s, 1_t$ are three fuzzy points of R with $1_r 1_s 1_t = 1_{r \wedge s \wedge t} \in \eta$. Now since η is assumed to be an L-fuzzy 2-absorbing ideal, we have $1_{r \wedge s} = 1_r 1_s \in \eta$ or $1_{r \wedge t} = 1_r 1_t \in \eta$ or $1_{s \wedge t} = 1_r 1_t \in \eta$. So $r \wedge s \leq \eta(1) = \alpha$ or $r \wedge t \leq \eta(1) = \alpha$ or $s \wedge t \leq \eta(1) = \alpha$, and the result follows. □

Theorem 3.7. *Let A be a 2-absorbing ideal of R and α a 2-absorbing element of L. Then the L-fuzzy subset of R defined by*

$$\mu(x) = \begin{cases} 1, & \textit{if } x \in A; \\ \alpha, & \textit{otherwise.} \end{cases}$$

is an L-fuzzy 2-absorbing ideal of R.

Proof. Since A is a 2-absorbing ideal of R we have $A \neq R$. Hence η is non-constant. Assume that $a_r b_s c_t \in \eta$ but $a_r b_s \notin \eta$ and $a_r c_t \notin \eta$ and $b_s c_t \notin \eta$, where $a_r, b_s, c_t \in F(R)$ are L-fuzzy points of R. Then $\eta(ab) = \alpha$ and so $ab \notin A$. Similarly, $ac \notin A$ and $bc \notin A$. But A is assumed to be a 2-absorbing ideal of R. So $abc \notin A$. So $\eta(abc) = \alpha$. Also from $(abc)_{r \wedge s \wedge t} = a_r b_s c_t \in \eta$ we have $r \wedge s \wedge t \leq \eta(abc) = \alpha$. Hence $r \wedge s \leq \alpha$ or $r \wedge t \leq \alpha$ or $s \wedge t \leq \alpha$, since α is a 2-absorbing element, which is a contradiction. Thus $a_r b_s \in \eta$ or $a_r c_t \in \eta$ or $b_s c_t \in \eta$. □

Example 3.8. As we mentioned previously, every L-fuzzy prime ideal of R is L-fuzzy 2-absorbing. In this example we show that the converse is not necessarily true. Let $\mathbb{N} = \{0, 1, 2, ...\}$ be the semiring with the usual addition and multiplication. The prime ideals of $\mathbb{N}$ are $0, p\mathbb{N}$ (p is a prime number), and $M = N \setminus \{1\}$. Every ideal of $\mathbb{N}$ has the form dA where A is an ideal of $\mathbb{N}$ containing all natural numbers larger than some natural number (See [1] for details). Now set $A = 6\mathbb{N}$. Than A is a 2-absorbing ideal of R which is not a prime ideal. Define $\eta : \mathbb{N} \to [0, 1]$ by

$$\eta(x) = \begin{cases} 1, & \text{if } x \in 6\mathbb{N}; \\ 0, & \text{otherwise.} \end{cases}$$

Then, by Theorem 3.7, η is a fuzzy 2-absorbing ideal of R. Moreover $\eta_0 = A$ is a 2-absorbing ideal of R that is not a prime ideal. Hence η is not a fuzzy prime ideal of R.

4. L-Fuzzy $0-(1-,2-,3-)2$-absorbing ideals

In this section we introduce the concepts of $0-(1-,2-,3-)2$-absorbing ideals of the semiring R and give some basic properties of these classes of ideals.

Definition 4.1. Let R be a semiring and assume that η is a fuzzy ideal of R. Then:

- η is called an L-fuzzy $0-2$-absorbing (resp. $3-2$-absorbing) ideal of R if η is non-constant and for any L-fuzzy ideals (resp. L-fuzzy k-ideals) λ, μ and ν of R, $\lambda\mu\nu \subseteq \eta$ implies that $\lambda\mu \subseteq \eta$ or $\mu\nu \subseteq \eta$ or $\lambda\nu \subseteq \eta$.
- η is called an L-fuzzy $1-2$-absorbing ideal of R if η is non-constant and whenever λ is a L-fuzzy k-ideal of R, andμ, ν are L-fuzzy ideals of R, then, $\lambda\mu\nu \subseteq \eta$ implies that $\lambda\mu \subseteq \eta$ or $\mu\nu \subseteq \eta$ or $\lambda\nu \subseteq \eta$.
- η is called an L-fuzzy $2-2$-absorbing ideal of R if η is non-constant and whenever λ, μ are L-fuzzy k-ideals of R and ν is a L-fuzzy ideal of R, then, $\lambda\mu\nu \subseteq \eta$ implies that $\lambda\mu \subseteq \eta$ or $\mu\nu \subseteq \eta$ or $\lambda\nu \subseteq \eta$.

Remark 4.2. The following diagram shows all implications among these classes of fuzzy ideals of a semiring (including the relations by transitivity):

$$
\begin{array}{ccccc}
 & & \text{L-Fuzzy prime} & & \\
 & & = \text{L-Fuzzy } 0-\text{ prime} & & \\
 & \swarrow & & \searrow & \\
\text{L-Fuzzy } 0-\text{2-absorbing} & & & & \text{L-Fuzzy } 1-\text{prime} \\
\downarrow & \searrow & & \swarrow & \downarrow \\
\text{L-Fuzzy 2-absorbing} & & \text{L-Fuzzy } 1-\text{2-absorbing} & & \text{L-Fuzzy } 2-\text{prime} \\
 & & \downarrow & & \\
 & & \text{L-Fuzzy } 2-\text{2-absorbing} & & \\
 & & \downarrow & & \\
 & & \text{L-Fuzzy } 3-\text{2-absorbing} & &
\end{array}
$$

Theorem 4.3. *Let η and μ be L-fuzzy ideals of R.*

(1) *If η and μ are L-fuzzy $0-$ prime ideals of R, then $\eta \cap \mu$ is an L-fuzzy $0-2-$absorbing ideal of R.*

(2) *If R is a commutative semiring, and if η and μ are L-fuzzy $1-$ prime ideals of R, then $\eta \cap \mu$ is an L-fuzzy $2-2-$absorbing ideal of R.*

(3) *If R is a commutative semiring, and if η and μ are L-fuzzy $2-$prime k-ideals of R, then $\eta \cap \mu$ is an L-fuzzy $3-2-$absorbing ideal of R.*

Proof. (1) Assume that λ, ν and θ are L-fuzzy ideals of R such that $\lambda\nu\theta \subseteq \eta \cap \mu$ but $\lambda\nu \nsubseteq \eta \cap \mu$ and $\lambda\theta \nsubseteq \eta \cap \mu$. Then we have the following cases:

Case 1. $\lambda\nu \nsubseteq \eta$ and $\lambda\theta \nsubseteq \eta$. Then it follows from $\lambda\nu\theta \subseteq \mu \cap \eta$ that $\theta \subseteq \eta$ since η is an L-fuzzy prime ideal of R. Therefore $\lambda\theta \subseteq \eta$ which is a contradiction.

Case 2. A similar argument as in the Case 1 leads us to a contradiction if $\lambda\nu \nsubseteq \mu$ and $\lambda\theta \nsubseteq \mu$.

Case 3. $\lambda\nu \nsubseteq \eta$ and $\lambda\theta \nsubseteq \mu$. In this case from $\lambda\nu\theta \subseteq \eta \cap \mu$ we have $\theta \subseteq \eta$ and $\nu \subseteq \mu$. Therefore $\nu\theta \subseteq \eta \cap \mu$.

Case 4. $\lambda\nu \nsubseteq \mu$ and $\lambda\theta \nsubseteq \eta$. By a similar argument as in the case 3 we may show that $\nu\theta \subseteq \eta \cap \mu$.

Consequently, $\eta \cap \mu$ is a fuzzy $0-2$-absorbing ideal of R.

(2) Let λ be an L-fuzzy ideal of R, and let ν and θ be L-fuzzy k-ideals of R such that $\lambda\nu\theta \subseteq \eta \cap \mu$ but $\lambda\nu \nsubseteq \eta \cap \mu$ and $\lambda\theta \nsubseteq \eta \cap \mu$. Then we have the following cases:

Case 1. $\lambda\nu \nsubseteq \eta$ and $\lambda\theta \nsubseteq \eta$. Then it follows from $\lambda\nu\theta \subseteq \mu$, $\lambda\nu \nsubseteq \eta$ and the fact that θ is an L-fuzzy k-ideal of R, that $\theta \subseteq \eta$ since η is an L-fuzzy $\mathfrak{o}-$prime ideal of R. Therefore $\lambda\theta \subseteq \eta$ which is a contradiction.

Case 2. A similar argument as in the Case 1 leads us to a contradiction if $\lambda\nu \nsubseteq \mu$ and $\lambda\theta \nsubseteq \mu$.

Case 3. $\lambda\nu \nsubseteq \eta$ and $\lambda\theta \nsubseteq \mu$. In this case from $\lambda\nu\theta \subseteq \eta$, $\lambda\nu \nsubseteq \eta$ and the fact that θ is an L-fuzzy k-ideal of R, we get $\theta \subseteq \eta$. In a similar way, from $\lambda\nu\theta \subseteq \mu$ we get $\nu \subseteq \mu$. Therefore $\nu\theta \subseteq \eta \cap \mu$.

Case 4. $\lambda\nu \nsubseteq \mu$ and $\lambda\theta \nsubseteq \eta$. By a similar argument as in the case 3 we may show that $\nu\theta \subseteq \eta \cap \mu$.

Therefore, $\eta \cap \mu$ is a fuzzy $\mathfrak{2}-2$-absorbing ideal of R.

(3) Let λ, ν and θ be L-fuzzy k-ideals of R such that $\lambda\nu\theta \subseteq \eta \cap \mu$ but $\lambda\nu \nsubseteq \eta \cap \mu$ and $\lambda\theta \nsubseteq \eta \cap \mu$. Then we have the following cases:

Case 1. $\lambda\nu \nsubseteq \eta$ and $\lambda\theta \nsubseteq \eta$. Since η is an L-fuzzy k-ideal of R, we have $(\lambda\nu)_{sub}\theta \subseteq \eta$ by [5, Lemma 2.20]. Moreover, by [5, Lemma 2.18], $(\lambda\nu)_{sub}$ is an L-fuzzy k-ideal of R. On the other hand $\lambda\nu \subseteq (\lambda\nu)_{sub}$; so $(\lambda\nu)_{sub} \nsubseteq \eta$. These imply that $\theta \subseteq \eta$ since η is an L-fuzzy $\mathfrak{2}-$prime ideal of R. Therefore $\lambda\theta \subseteq \eta$ which is a contradiction.

Case 2. A similar argument as in the Case 1 leads us to a contradiction if $\lambda\nu \nsubseteq \mu$ and $\lambda\theta \nsubseteq \mu$.

Case 3. $\lambda\nu \nsubseteq \eta$ and $\lambda\theta \nsubseteq \mu$. Since η is an L-fuzzy k-ideal of R, we have $(\lambda\nu)_{sub}\theta \subseteq \eta$ by [5, Lemma 2.20]. Moreover, by [5, Lemma 2.18], $(\lambda\nu)_{sub}$ is an L-fuzzy k-ideal of R. On the other hand $\lambda\nu \subseteq (\lambda\nu)_{sub}$; so $(\lambda\nu)_{sub} \nsubseteq \eta$. These imply that $\theta \subseteq \eta$ since η is an L-fuzzy $\mathfrak{2}-$prime ideal of R. By a similar argument, it follows from $\lambda\nu\theta \subseteq \mu$ that $\nu \subseteq \mu$. Therefore $\nu\theta \subseteq \eta \cap \mu$.

Case 4. $\lambda\nu \nsubseteq \mu$ and $\lambda\theta \nsubseteq \eta$. By a similar argument as in the case 3 we may show that $\nu\theta \subseteq \eta \cap \mu$.

Hence, $\eta \cap \mu$ is a fuzzy $\mathfrak{3}-2$-absorbing ideal of R. □

As it is mentioned previously, every L-fuzzy $\mathfrak{o}-2-$absorbing ideal of R is L-fuzzy $\mathfrak{3}-2-$absorbing, but the converse is not necessarily true. Now consider the following result.

Theorem 4.4. *Let μ be an L-fuzzy k-ideal of R. Then μ is an L-fuzzy $\mathfrak{o}-2$-absorbing ideal of R if and only if it is an L-fuzzy $\mathfrak{3}-2$-absorbing ideal of R.*

Proof. If μ is an L-fuzzy $\mathfrak{o}-2-$absorbing ideal of R, then obviously, it is an L-fuzzy $\mathfrak{3}-2-$absorbing ideal of R. Conversely, assume that μ is an L-fuzzy $\mathfrak{3}-2-$absorbing ideal of R and let λ, θ and ν be L-fuzzy ideas of R such that $\lambda\theta\nu \subseteq \mu$. Then from [5, Lemma 2.20] we have $\lambda_{sub}\theta_{sub}\nu_{sub} \subseteq \mu$ where $\lambda_{sub}, \theta_{sub}$ and ν_{sub} are fuzzy k-ideals of R by [5, Lemma 2.18]. Since μ is $\mathfrak{3}-2-$absorbing ideal of R, we have $\lambda_{sub}\theta_{sub} \subseteq \mu$ or $\theta_{sub}\nu_{sub} \subseteq \mu$ or $\lambda_{sub}\nu_{sub} \subseteq \mu$. Furthermore, $\lambda \subseteq \lambda_{sub}, \theta \subseteq \theta_{sub}$ and $\nu \subseteq \nu_{sub}$. So we have $\lambda\theta \subseteq \mu$ or $\theta\nu \subseteq \mu$ or $\lambda\nu \subseteq \mu$. That is μ is a fuzzy $\mathfrak{o}-2-$absorbing ideal of R. □

Theorem 4.5. *Let η be an L-fuzzy ideal of R. The following statements are equivalent:*

(i) *η is an L-fuzzy $3-2-$absorbing ideal of R.*

(ii) *For every $t \in L$, the t-level subset η_t of η is a $3-2-$absorbing ideal of R.*

Proof. (i)$\Rightarrow$(ii) Let η be an L-fuzzy $3-2-$absorbing ideal of R and let I, J and K be k-ideals of R such that $IJK \subseteq \eta_t$. Then from $IJK \subseteq \eta_t$ we have $1_I 1_J 1_L \subseteq \eta$. But $1_I, 1_J, 1_L$ are fuzzy k-ideals of R by [7, Theorem 2.2]. Since η is assumed to be an L-fuzzy $\ni-2-$absorbing ideal of R, we have $1_I 1_J \subseteq \eta$ or $1_I 1_K \subseteq \eta$ or $1_J 1_K \subseteq \eta$. Therefore, $IJ \subseteq \eta_t$ or $IK \subseteq \eta_t$ or $JK \subseteq \eta_t$. Hence η_t is a $3-2-$absorbing ideal of R.

(ii)$\Rightarrow$(i) Now assume that η_t is a $3-2-$absorbing ideal of R for every $t \in L$. Suppose that λ, μ, ν are L-fuzzy k-ideals of R such that $\lambda\mu\nu \subseteq \eta$. Then for every $t \in L$, we have $\lambda_t\mu_t\nu_t \subseteq \eta_t$. But by assumption η_t is a $3-2-$absorbing ideal of R. Consequently, $\lambda_t\mu_t \subseteq \eta_t$ or $\mu_t\nu_t \subseteq \eta_t$ or $\lambda_t\nu_t \subseteq \eta_t$, and so we have $\lambda\mu \subseteq \eta$ or $\mu\nu \subseteq \eta$ or $\lambda\nu \subseteq \eta$. So η is an L-fuzzy $3-2-$absorbing ideal of R. □

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Ahmad Yousefian Darani (yousefian@uma.ac.ir, youseffian@gmail.com)
Department of Mathematics and Applications, Faculty of Mathematical Sciences, University of Mohaghegh Ardabili, 56199-11367, Ardabil, Iran

Ashraf Hashempoor (hashempoor.ashraf@yahoo.com)
Department of Mathematics and Applications, Faculty of Mathematical Sciences, University of Mohaghegh Ardabili, 56199-11367, Ardabil, Iran