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Fuzzy detour g-centre in fuzzy graphs

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Abstract. In this paper, the fuzzy detour g-distance between two nodes, u and v of a fuzzy graph, is defined. The concepts of fuzzy detour g-eccentricity, fuzzy detour g-radius, fuzzy detour g-center, and fuzzy detour g-diameter are introduced. We demonstrate that for each pair a, b of positive real numbers with $a \leq b \leq 2a$, there exists a connected fuzzy graph G with $rad_{D_g}(G) = a$ and $diam_{D_g}(G) = b$. Construction of a fuzzy graph whose fuzzy detour g-center is the given fuzzy graph is discussed. Two characterisations of fuzzy detour $g-$self-centered fuzzy graphs are obtained. We establish that fuzzy trees are the only fuzzy g-detour graphs.

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Keywords: Fuzzy detour g-distance, Fuzzy detour g-eccentricity, Fuzzy detour g-radius, Fuzzy detour g-center, Fuzzy detour g-diameter, Fuzzy detour $g-$self centered fuzzy graph.

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1. Introduction

One of the remarkable mathematical inventions of the 20^{th} century is that of fuzzy sets by Zadeh in 1965[22]. The distinction between sets and fuzzy sets is that the sets divide the universal set into two subsets, namely members and non-members (dichotomy law), while the fuzzy set assigns a sequence of membership values to elements of the universal set ranging from zero to one. In 1975, Zadeh [23] introduced the notion of an interval-valued fuzzy subset as an extension of the fuzzy set, in which the values of the membership degrees are interval of numbers instead of numbers. Hongmei and Lianhua defined an interval-valued fuzzy graph in [9]. Several important works on interval-valued fuzzy graphs can be found in [20]. The idea of fuzzy set theory was introduced into fuzzy graph theory by Rosenfeld in 1975 [14]. During this same period, Yeh and Bang also introduced various concepts of fuzzy graphs [21]. Fuzzy graph theory has numerous applications in various fields, especially in the field of clustering analysis, neural networks, pattern recognition, decision making, and expert systems. The fuzzy analogs of several graph theoretical concepts, including sub-graphs, paths and connectedness, cliques, bridges and cut nodes, and forest and trees are defined by Rosenfeld. Bhutani and Rosenfeld [2] defined fuzzy end nodes in fuzzy graphs, and studied several properties of fuzzy end nodes in fuzzy trees. They introduced the concept of strong arcs and strong paths [1] and $g-$distance in fuzzy graphs [3]. The standard $g-$distance between two vertices, u and v in a connected fuzzy graph, is the length of the shortest strong $u-v$ path in G [3]. The standard $g-$distance has also been studied in [15], [16], and [17]. The detour distance between two vertices u and v in a connected graph G is the length of the longest $u-v$ path in G [7]. The detour distance, similar to standard distance, is a metric on the vertex set of G [8], [6], and [5]. The concept of $\mu-$distance was introduced by Rosenfeld [14], and further studied by Sunitha and Vijayakumar [18]. In 2010, Nagoorgani and Umamaheswari [13] introduced the concept of fuzzy detour $\mu-$distance. Based on this $\mu-$distance, they defined the fuzzy detour $\mu-$center and its properties. We extend these ideas using the fuzzy detour $g-$ distance in fuzzy graphs.

2. Preliminaries

This section includes a quick review of the basic definitions in fuzzy graph theory which is referred in this paper. Rosenfeld defined a fuzzy graph as follows.

Definition 2.1 ([11])**.** A fuzzy graph is denoted by $G : (V, \sigma, \mu)$ where V is a vertex set, σ is a fuzzy subset of V and μ is a fuzzy relation on σ. i.e., $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)\ \forall\ x, y \in V$. It is assume that V is finite and nonempty, μ is reflexive (i.e.,$\mu(x, x) = \sigma(x), \forall x$) and symmetric(i.e., $\mu(x, y) = \mu(y, x), \forall(x, y)$). In all the examples σ is chosen suitably.

Also, we denote the underlying crisp graph by $G^* : (\sigma^*, \mu^*)$ where

$$\sigma^* = \{u \in V : \sigma(u) > 0\} \text{ and } \mu^* = \{(u, v) \in V \times V : \mu(u, v) > 0\}.$$

We assume $\sigma^* = V$.

Definition 2.2 ([11])**.** The fuzzy graph $H : (\tau, \nu)$ is said to be a partial fuzzy subgraph of $G : (\sigma, \mu)$ if $\tau \subseteq \sigma$ and $\nu \subseteq \mu$. In particular H is called a fuzzy subgraph of G if $\tau(x) = \sigma(x)\ \forall\ x \in \tau^*$, $\nu(x, y) = \mu(x, y)\ \forall(x, y)\epsilon\nu^*$. Let $P \subseteq V$, the fuzzy graph $H : (P, \tau, \nu)$ is called a fuzzy subgraph of $G : (V, \sigma, \mu)$ induced by P if $\tau(x) = \sigma(x)\ \forall\ x \in$ P and $\nu(x, y) = \mu(x, y)\ \forall\ x, y \in$ P. The fuzzy graph $H : (\tau, \nu)$ is said to be a spanning fuzzy subgraph of $G : (V, \sigma, \mu)$ if $\tau(x) = \sigma(x)\ \forall\ x$ and $\nu(x, y) = \mu(x, y)$ for every $(x, y) \in \nu^*$. $G : (V, \sigma, \mu)$ is called trivial if $|\sigma^*| = 1$.

Definition 2.3 ([11])**.** In a fuzzy graph $G : (V, \sigma, \mu)$, a path P of length n is a sequence of distinct nodes $u_0, u_1, ..., u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, ..., n$ and the degree of membership of a weakest arc is defined as its strength. If $u_0 = u_n$ and $n \geq 3$ then P is called a cycle and P is called a fuzzy cycle, if it contains more than one weakest arc. The strength of a cycle is the strength of the weakest arc in it.

Definition 2.4 ([11])**.** Let $G : (V, \sigma, \mu)$ be a fuzzy graph. The strength of connectedness between two nodes x and y is defined as the maximum of the strength of all paths between x and y and is denoted by $CONN_G(x, y)$. A fuzzy graph $G : (V, \sigma, \mu)$ is connected if for every x, y in $\sigma^*, CONN_G(x, y) > 0$.

Definition 2.5 ([11])**.** A fuzzy graph $G : (V, \sigma, \mu)$ is said to be complete if $\mu(x, y) = \sigma(x) \wedge \sigma(y), \forall x, y \in \sigma^*$.

Definition 2.6 ([1])**.** An arc of a fuzzy graph $G : (V, \sigma, \mu)$ is called strong if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted.

Depending on $CONN_G(x, y)$ of an arc (x, y) in a fuzzy graph $G : (V, \sigma, \mu)$, Sunil Mathew and M.S.Sunitha [10] defined three different types of arcs. Note that $CONN_{G-(x,y)}(x, y)$ is the the strength of connectedness between x and y in the fuzzy subgraph obtained from G by deleting the arc (x, y).

Definition 2.7 ([10])**.** An arc (x, y) in G is α-strong if $\mu(x, y) > CONN_{G-(x,y)}(x, y)$. An arc (x, y) in G is $\beta-$ strong if $\mu(x, y) = CONN_{G-(x,y)}(x, y)$. An arc (x, y) in G is $\delta-$ arc if $\mu(x, y) < CONN_{G-(x,y)}(x, y)$. Thus an arc (x,y) is strong arc if it is either $\alpha-$ strong or $\beta-$ strong. An $x - y$ path P is called strong path if P contains only strong arcs.

Definition 2.8 ([11])**.** A fuzzy cutnode w is a node in G whose removal from G reduces the strength of connectedness between some pair of nodes other than w.

Definition 2.9. A fuzzy graph G is said to be a block if it is connected and has no cutnodes.

Throughout we assume that G is connected.

Definition 2.10 ([3])**.** Let $G : (V, \sigma, \mu)$ be a fuzzy graph. A strong path P from x to y in G is an $x - y$ geodesic if there is no shorter strong path from x to y and the length of an $x - y$ geodesic is the geodesic distance from x to y denoted by $d_g(x, y)$. We refer $d_g(x, y)$ as the standard $g-$distance from x to y.

3. Fuzzy detour $g-$distance

In this section the fuzzy detour g-distance is defined and depending on fuzzy detour g-distance we define fuzzy detour g-eccentricity, fuzzy detour g-radius and fuzzy detour g-diameter.

Definition 3.1. The length of a longest strong $u - v$ path between two nodes u and v in a connected fuzzy graph G is called fuzzy detour g-distance from u to v, denoted by $D_g(u, v)$.

Example 3.2. Consider the fuzzy graph given in Fig. 1.

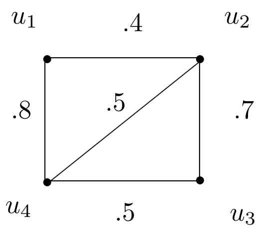

Fig. 1

For the fuzzy graph of Fig. 1, note that all arcs except (u_1, u_2) are strong and the standard $g-$distance and fuzzy detour g-distance between two nodes are as follows.

$d_g(u_1, u_2) = 2$, $d_g(u_1, u_3) = 2$, $d_g(u_1, u_4) = 1$, $d_g(u_2, u_3) = 1$, $d_g(u_2, u_4) = 1$, $d_g(u_3, u_4) = 1$ while $D_g(u_1, u_2) = 3$, $D_g(u_1, u_3) = 3$, $D_g(u_1, u_4) = 1$, $D_g(u_2, u_3) = 2$, $D_g(u_2, u_4) = 2$, $D_g(u_3, u_4) = 2$.

Definition 3.3. Any $u - v$ strong path of length $D_g(u, v)$ is called a $u - v$ fuzzy g-detour. A fuzzy graph G is called fuzzy g-detour graph if $D_g(u, v) = d_g(u, v)$ for every pair u and v of nodes of G.

Example 3.4. Consider the fuzzy graph given in Fig. 2.

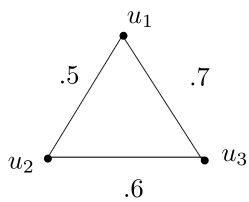

Fig. 2

For the fuzzy graph of Fig. 2, note that all arcs except (u_1, u_2) are strong and the standard $g-$distance and fuzzy detour g-distance between two nodes are as follows.

$d_g(u_1, u_2) = 2 = D_g(u_1, u_2)$, $d_g(u_1, u_3) = 1 = D_g(u_1, u_3)$, $d_g(u_2, u_3) = 1 = D_g(u_2, u_3)$. Here $D_g(u, v) = d_g(u, v)$ for every pair u and v of nodes of G. Hence G is a fuzzy g-detour graph.

In section 6 we prove that fuzzy trees are the only fuzzy g-detour graphs.

Proposition 3.5. *If u and v are any two nodes in a connected fuzzy graph $G : (V, \sigma, \mu)$, then $0 \leq d_g(u, v) \leq D_g(u, v) < \infty$.*

Proposition 3.6. *If u and v are any two nodes in a connected fuzzy graph $G : (V, \sigma, \mu)$, then $D_g(u, v) = 0$ if and only if $d_g(u, v) = 0$ if and only if $u = v$.*

Theorem 3.7. *Fuzzy detour g-distance is a metric on the node set of every connected fuzzy graph.*

Proof. Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph. Note that (1) $D_g(u, v) \geq 0$, (2) $D_g(u, v) = 0$ if and only if $u = v$ and (3) $D_g(u, v) = D_g(v, u)$ for every pair u, v of nodes of G. It remains only to show that fuzzy detour g-distance satisfies the triangle inequality. Let u, v and w be any three nodes of G. Since the inequality

$D_g(u,w) \leq D_g(u,v) + D_g(v,w)$ holds if any two of these three nodes are the same node, we assume that u, v and w are distinct. Let P be a $u-w$ fuzzy $g-detour$ in G of length $D_g(u,w) = k$. Then there exists two cases.

Case 1. v lies on P.
Let P_1 be the $u-v$ sub path of P and P_2 be the $v-w$ sub path of P. Suppose that the length of P_1 is s and the length of P_2 is t. Then $s+t=k$.
Therefore $D_g(u,w) = k = s+t \leq D_g(u,v) + D_g(v,w)$.

Case 2. v does not lie on P.
Since there exists a strong path between every pair of nodes, there is a shortest strong path Q from v to a node of P. Let x be any node on P and Q be the $v-x$ geodesic such that no other node of Q lies on P. Let r be the length of Q. Thus $r > 0$. Let the $u-x$ sub path $P^{'}$ of P has length a and the $x-w$ sub path $P^{''}$ of P has length b. Then $a \geq 0$ and $b \geq 0$. Therefore $D_g(u,v) \geq a+r$ and $D_g(v,w) \geq b+r$. So $D_g(u,w) = k = a+b < (a+r)+(b+r) \leq D_g(u,v) + D_g(v,w)$. So the triangle inequality holds. □

The fuzzy detour g-eccentricity, $e_{D_g}(u)$ of a node u is the fuzzy detour g-distance from u to a node farthest from u. Let $u^*_{D_g}$ denote set of all fuzzy detour g-eccentric nodes of u. The fuzzy detour g-radius of G, $rad_{D_g}(G)$ is the minimum fuzzy detour g-eccentricity among the nodes of G. A node u in G is a fuzzy detour g-central node if, $e_{D_g}(u) = rad_{D_g}(G)$. The fuzzy detour g-diameter of G, $diam_{D_g}(G)$ is the maximum fuzzy detour g-eccentricity among the nodes of G. A node u in a connected fuzzy graph G is called fuzzy detour g-peripheral node if $e_{D_g}(u) = diam_{D_g}(G)$.

Example 3.8. Consider the fuzzy graph given in Fig. 1. Here $e_{D_g}(u_1) = 3$, $e_{D_g}(u_2) = 3$, $e_{D_g}(u_3) = 3$, $e_{D_g}(u_4) = 2$ and $rad_{D_g}(G) = 2$, $diam_{D_g}(G) = 3$.

Definition 3.9. The fuzzy subgraph of G induced by the fuzzy detour $g-$central nodes is called fuzzy detour $g-$centre of G, denoted by $C_{D_g}(G)$. If every node of G is fuzzy detour $g-$central node, then $C_{D_g}(G) = G$, and G is called fuzzy detour $g-$self centered. Note that for fuzzy detour $g-$self centered fuzzy graph $rad_{D_g}(G) = diam_{D_g}(G)$.

Example 3.10. Consider the fuzzy graph given in Fig. 3.

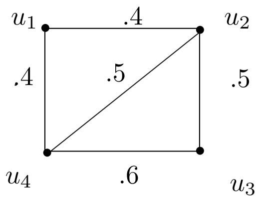

Fig. 3

For the fuzzy graph of Fig. 3, note that all arcs are strong and the fuzzy detour g-distance between two nodes are as follows.

$D_g(u_1,u_2) = 3$, $D_g(u_1,u_3) = 3$, $D_g(u_1,u_4) = 3$, $D_g(u_2,u_3) = 3$, $D_g(u_2,u_4) = 2$, $D_g(u_3,u_4) = 3$.

$e_{D_g}(u_1) = 3$, $e_{D_g}(u_2) = 3$, $e_{D_g}(u_3) = 3$, $e_{D_g}(u_4) = 3$

and $rad_{D_g}(G) = diam_{D_g}(G) = 3$. Therefore $C_{D_g}(G) = G$, and G is a fuzzy detour $g-$self centered fuzzy graph.

Theorem 3.11. *For every non-trivial connected fuzzy graph $G : (V, \sigma, \mu)$, $rad_{D_g}(G) \le diam_{D_g}(G) \le 2\, rad_{D_g}(G)$.*

Proof. The inequality $rad_{D_g}(G) \le diam_{D_g}(G)$ follows from definition. Let u, v be two nodes such that $D_g(u, v) = diam_{D_g}(G)$. Let w be a fuzzy detour $g-$central node of G. Hence the fuzzy detour $g-$distance between w and any other node of G is atmost fuzzy detour $g-$radius of G.
By triangle inequality,
$diam_{D_g}(G) = D_g(u, v) \le D_g(u, w) + D_g(w, v)$
$\le rad_{D_g}(G) + rad_{D_g}(G) = 2\, rad_{D_g}(G)$. □

4. Fuzzy detour $g-$centre

In crisp graph center lies in a block of G (see [4]). In this section we extend this idea to fuzzy detour $g-$centre.

Theorem 4.1. *The fuzzy detour $g-$centre, $C_{D_g}(G)$ of every connected fuzzy graph $G : (V, \sigma, \mu)$ lie in a single block of G.*

Proof. Assume, to the contrary, that $G : (V, \sigma, \mu)$ is a connected fuzzy graph whose fuzzy detour $g-$centre $C_{D_g}(G)$, is not a subgraph of a single block of G. Then there is a cut node v of G such that $G - v$ contains two components G_1 and G_2 each of which contain nodes of $C_{D_g}(G)$. Let u be a node of G such that $D_g(u, v) = e_{D_g}(v)$, and let P_1 be a $u - v$ fuzzy $g-$detour in G. At least one of G_1 or G_2 contains no node of P_1. Let w be a fuzzy detour $g-$central node of G that belong to G_2 and let P_2 be a $u - w$ fuzzy $g-$detour . Then P_1 followed by P_2 produces a $u - w$ fuzzy $g-$detour whose length is greater than that of P_1. Hence $e_{D_g}(w) > e_{D_g}(v)$. Which contradicts the fact that w is a fuzzy detour $g-$central node of G. □

Corollary 4.2. *Let $G : (V, \sigma, \mu)$ be a fuzzy graph with atleast one fuzzy cut node which is not a cut node of G^* and let v be a node of G. If $e_{D_g}(v) = rad_{D_g}(G)$, then v is not a fuzzy cut node of G.*

Proof. $e_{D_g}(v) = rad_{D_g}(G)$, implies that v is a fuzzy detour $g-$central node of G, that is $v \epsilon C_{D_g}(G)$. Since the fuzzy detour $g-$centre node of G, $C_{D_g}(G)$, of every connected fuzzy graph G is a fuzzy subgraph of some block of G and this block contain no fuzzy cut node of G, v cannot be a fuzzy cut node of G. □

Embedding problem

Theorem 4.3. *Every fuzzy graph is the fuzzy detour $g-$centre of some fuzzy graph.*

Proof. Let $G : (V, \sigma, \mu)$ be a fuzzy graph with n nodes where V = $\{ u_1, u_2, ..., u_n \}$, and let $H : (V^|, \sigma^|, \mu^|)$ be the fuzzy graph obtained by adding $n + 1$ nodes $\{ w_1, ..., w_n, w_{n+1} \}$ to G as follows.
$V^| = V \cup \{ w_1, ..., w_n, w_{n+1} \}$.
$\sigma^| = \sigma$, for all $u_i \epsilon G$, i = 1,..., n.
$\mu^| = \mu$, for all $(u_i, u_j) \in G$.

Let $c = \bigwedge \sigma(u_i)$, i = 1,..., n.
$\sigma^{|}(w_j) = t, 0 < t \leq c$, j = 1,..., n+1.
$\mu^{|}(w_j, u_i) = t$, for all u_i and w_j, i = 1,..., n, j = 1,..., n+1. Thus all arcs (w_j, u_i) are strong.
Here $\forall u_i \in G$, $e_{D_g}(u_i) = 2n - 1$ and $\forall w_j$, $e_{D_g}(w_j) = 2n$. Hence $rad_{D_g}(G) = 2n - 1$. Thus $H : (V^{|}, \sigma^{|}, \mu^{|})$ is a fuzzy graph with $G : (V, \sigma, \mu)$ as its fuzzy detour $g-$centre. □

Theorem 4.4. *For each pair a, b of positive real numbers with $a \leq b \leq 2a$ there exists a connected fuzzy graph G with $rad_{D_g}(G) = a$ and $diam_{D_g}(G) = b$.*

Proof. For $a = b = k \geq 1$ the complete fuzzy graph on k+1 nodes has the desired property. For $a < b \leq 2a$, let H_1 and H_2 be any two fuzzy graphs such that H_1 is of order $a + 1$ and H_2 is of order $b - a + 1$ and also such that H_1^* and H_2^* are complete and all arcs in H_1 and H_2 are strong. Now G be a fuzzy graph of order $b + 1$ obtained by identifying a node v of H_1 and a node of H_2. Since $b \leq 2a$ it follows that $b - a + 1 \leq a + 1$. Then $e_{D_g}(v) = a$. Since there is a strong path in G which passes through every other nodes of G with initial node x, where $x \epsilon G - v$, it follows that $e_{D_g}(x) = b$. Hence $rad_{D_g}(G) = a$ and $diam_{D_g}(G) = b$. □

Example 4.5. Consider the fuzzy graph given in Fig. 4.

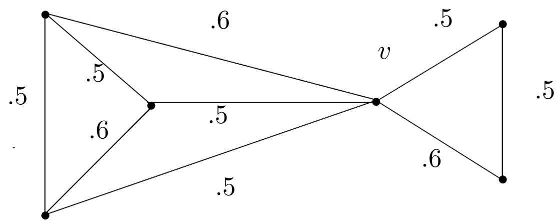

Fig. 4

In Fig. 4, fuzzy graph G is constructed from H_1 and H_2 by identifying a node v of H_1 and a node of H_2 for a =3 and b =5. Hence $rad_{D_g}(G) = 3$ and $diam_{D_g}(G) = 5$.

5. Fuzzy detour $g-$self centered fuzzy graph

With respect to standard $g-$distance note that a necessary condition for a $g-$self centered fuzzy graph is that each node is $g-$eccentric, which is not sufficient. But we observed that in fuzzy detour $g-$distance it is sufficient also, which is discussed as follows.

Theorem 5.1. *A fuzzy graph $G : (V, \sigma, \mu)$ is fuzzy detour $g-$self centered fuzzy graph if and only if each node of G is fuzzy detour $g-$ eccentric.*

Proof. Assume $G : (V, \sigma, \mu)$ is fuzzy detour $g-$self centered fuzzy graph and let v be any node of G. Let $u \epsilon v^*_{D_g}$. Then $e_{D_g}(v) = D_g(u, v)$ and G being fuzzy detour $g-$self centered fuzzy graph $e_{D_g}(u) = e_{D_g}(v) = D_g(u, v)$, which shows that $v \epsilon u^*_{D_g}$, and v is fuzzy detour $g-$ eccentric.

Conversely assume that each node of G is fuzzy detour $g-$ eccentric. To prove that G is fuzzy detour $g-$self centered fuzzy graph. Assume to the contrary, that G is not fuzzy detour $g-$self centered, ie, $rad_{D_g}(G) \neq diam_{D_g}(G)$. Let y be a node in G such that $e_{D_g}(y) = diam_{D_g}(G)$, and let $z \epsilon y^*_{D_g}$. Let P be a $y-z$ fuzzy $g-$detour in G. Then there must exists a node w on P such that w is not fuzzy detour $g-$ eccentric node of any node of P. Also w is not a fuzzy detour $g-$ eccentric node of any other node. Otherwise if w is a fuzzy detour $g-$ eccentric node of any other node u (say), i.e., $w \epsilon u^*_{D_g}$, then we can extend $u - w$ fuzzy $g-$detour to longer path(to y or to z or to both), which is a contradiction to $w \epsilon u^*_{D_g}$. Therefore $rad_{D_g}(G) = diam_{D_g}(G)$ and G is fuzzy detour $g-$self centered fuzzy graph. □

Theorem 5.2. *For a fuzzy detour $g-$self centered fuzzy graph $G : (V, \sigma, \mu)$ $rad_D(G) = diam_D(G) = n - 1$, where $n = |V|$.*

Proof. Assume $G : (V, \sigma, \mu)$ is fuzzy detour $g-$self centered. To prove $diam_D(G) = n - 1$. Assume to the contrary that $diam_D(G) = k < n - 1$.

Claim: There exists a node x in G which is common to all fuzzy detour peripheral paths.

If not let P_1 and P_2 be two fuzzy detour peripheral paths such that P_1 and P_2 share no common node. Let $y \epsilon P_1$ and $z \epsilon P_2$. Since G is connected there exists a strong path from z to y. Then, there exist nodes on P_1 and P_2 with eccentricity greater than k, which is not possible. Hence the claim.

Since x is on every fuzzy detour peripheral path, $e_{D_g}(x) < k$, which is a contradiction to our assumption that G is fuzzy detour $g-$self centered. □

Theorem 5.3. *A connected fuzzy graph $G : (V, \sigma, \mu)$ on n nodes such that G^* is complete is fuzzy detour $g-$self centered if each arc is strong, further $rad_{D_g}(G) = n - 1$.*

Proof. Let $\sigma^* = \{v_1, v_2, v_3, ..., v_n\}$. Since G^* is complete each node v_i is incident with exactly $n-1$ arcs, and all arcs are strong. Hence $e_{D_g}(v_i) = n-1$, $\forall i = 1, 2, ..., n$ and G is a fuzzy detour $g-$self centered fuzzy graph with $rad_{D_g}(G) = n - 1$. □

Remark 5.4. The condition in Theorem 5.3 is not necessary.

Example 5.5. Consider the fuzzy graph given in Fig. 5.

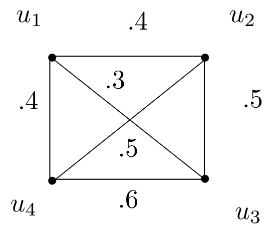

Fig. 5

Here $e_{D_g}(u_1) = 3$, $e_{D_g}(u_2) = 3$, $e_{D_g}(u_3) = 3$, $e_{D_g}(u_4) = 3$ and G is a fuzzy detour $g-$self centered fuzzy graph with G^* complete, but the arc (u_1, u_3) is not strong.

Corollary 5.6. *A complete fuzzy graph on n nodes is fuzzy detour $g-$self centered and $rad_{D_g}(G) = n - 1$.*

A characterization of fuzzy detour $g-$self centered fuzzy graph.

Theorem 5.7. *Let $G:(V,\sigma,\mu)$ be a fuzzy graph and let $u \epsilon v^*_{D_g}$. Then G is fuzzy detour $g-$self centered if and only if $v \epsilon u^*_{D_g}$.*

Proof. Let $G:(V,\sigma,\mu)$ be a fuzzy graph and u and v be two nodes of G. Let $u \epsilon v^*_{D_g}$. Assume G is fuzzy detour $g-$self centered. Required to prove that $v \epsilon u^*_{D_g}$. Now $e_{D_g}(u) = e_{D_g}(v)$, $u \neq v$. ....(1) and $D_g(v,u) = e_{D_g}(v)$. ....(2)
From (1) and (2) $e_{D_g}(u) = D_g(v,u)$ and thus $v \epsilon u^*_{D_g}$.

Conversely let G be a fuzzy graph and u, v be any two nodes of G such that $u \epsilon v^*_{D_g}$ and by assumption $v \epsilon u^*_{D_g}$. Then $e_{D_g}(u) = e_{D_g}(v)$, $u \neq v$. Therefore G is fuzzy detour $g-$self centered. □

6. Fuzzy detour $g-$distance in fuzzy trees

A connected fuzzy graph $G:(V,\sigma,\mu)$ is a fuzzy tree if it has a fuzzy spanning subgraph $F:(V,\sigma,\nu)$, which is a tree where for all arcs (x,y) not in F there exists a path from x to y in F whose strength is more than $\mu(x,y)$ [11]. Note that here F is a tree which contain all nodes of G and hence is a spanning tree of G. Also note that F is the unique maximum spanning tree of G [19].

Based on standard $g-$distance, Sameena.K and M.S.Sunitha [16] have studied some properties of fuzzy tree G and its associated maximum spanning tree F. Since there exists unique strong path between every pair of nodes in G [1], we establish that $g-$distance and fuzzy detour $g-$distance coincide in case of fuzzy trees in the following theorem. Thus the result in [16] hold good for fuzzy detour $g-$distance in fuzzy trees.

Theorem 6.1. *A connected fuzzy graph $G:(V,\sigma,\mu)$ is a fuzzy g-detour graph if and only if G is a fuzzy tree.*

Proof. Assume G is a fuzzy tree. Then there exists unique strong path between every pair of nodes in G [1]. Hence $D_g(u,v) = d_g(u,v)$ for every pair u and v of nodes of G. Therefore G is a fuzzy g-detour graph.

Conversely, assume G is a fuzzy g-detour graph on n nodes. That is $D_g(u,v) = d_g(u,v)$ for every pair u and v of nodes of G. When $n = 2$, the result is trivial and G is a fuzzy tree. So let $n \geq 3$. Assume on the contrary that G is not a fuzzy tree. Then there exists atleast one pair of nodes u_1, v_1 and more than one strong path from u_1 to v_1. Let P_1 and P_2 be two u_1-v_1 strong paths. Then union of P_1 and P_2 contains atleast one cycle (say) C in G. Let u and v be two adjacent nodes in C. Then $d_g(u,v) = 1$ and $D_g(u,v) > 1$, which is a contradiction to the assumption that $D_g(u,v) = d_g(u,v)$, and hence G is a fuzzy tree. □

7. Conclusions

In this paper, we introduced the fuzzy detour g-distance, fuzzy detour g-eccentricity, and fuzzy detour g-center of fuzzy graphs. We established that every fuzzy graph is the fuzzy detour $g-$center of some fuzzy graph. Also, for each pair a, b of positive real numbers with $a \leq b \leq 2a$, there exists a connected fuzzy graph H with $rad_{D_g}(H) = a$ and $diam_{D_g}(H) = b$. We observed that fuzzy trees are the only fuzzy g-detour graphs.

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