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Characterizations of falling fuzzy positive implicative ideals in BCK-algebras

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Abstract. In the paper [Y. B. Jun and M. S. Kang, Fuzzy positive implicative ideals of BCK-algebras based on the theory of falling shadows, Comput. Math. Appl. 61 (2011), 62–67], they showed that a falling fuzzy ideal is not a falling fuzzy positive implicative ideal. In this article, conditions for a falling fuzzy ideal to be a falling fuzzy positive implicative ideal are provided. Characterizations of a falling fuzzy positive implicative ideal are established.

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1. Introduction

In the study of a unified treatment of uncertainty modelled by means of combining probability and fuzzy set theory, Goodman [1] pointed out the equivalence of a fuzzy set and a class of random sets. Wang and Sanchez [12] introduced the theory of falling shadows which directly relates probability concepts with the membership function of fuzzy sets. Falling shadow representation theory shows us the way of selection relaid on the joint degrees distributions. It is reasonable and convenient approach for the theoretical development and the practical applications of fuzzy sets and fuzzy logics. The mathematical structure of the theory of falling shadows is formulated in (Wang, [11]). Tan et al. [9, 10] established a theoretical approach to define a fuzzy inference relation and fuzzy set operations based on the theory of falling shadows. Jun and Park discussed the notion of a falling fuzzy subalgebra/ideal of a BCK/BCI-algebra. Jun et al. [4] also considered falling fuzzy positive implicative ideals. They introduced the notion of a falling fuzzy positive implicative ideal of a BCK-algebra based on the theory of a falling shadow, and provided relations between falling fuzzy positive implicative ideals and falling fuzzy ideals. Generally, we know that every falling fuzzy positive implicative ideal is a falling fuzzy ideal, but the converse may not be true (see [4]). In [13], Zhan et al. introduced the notions of falling fuzzy (implicative) filters of R_0-algebras based on the theory of falling shadows and fuzzy sets. They provided relations between fuzzy (implicative) filters and falling fuzzy (implicative) filters, and applied the concept of falling fuzzy inference relations to R_0-algebras and obtained some related results. Jun and Song [6] used the theory of falling shadows to establish a falling fuzzy quasi-associative ideal in a BCI-algebra as a generalization of a fuzzy quasi-associative ideal in BCI-algebras. They provided relations between falling fuzzy quasi-associative ideals and falling fuzzy ideals.

In this paper, we discuss conditions for a falling fuzzy ideal to be a falling fuzzy positive implicative ideal. We establish characterizations of a falling fuzzy positive implicative ideal.

2. Preliminaries

A BCK/ BCI-algebra is an important class of logical algebras introduced by K. Iséki and was extensively investigated by several researchers.

An algebra $(X;*,0)$ of type $(2,0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(I) $(\forall x,y,z\in X)\ (((x*y)*(x*z))*(z*y)=0)$,
(II) $(\forall x,y\in X)\ ((x*(x*y))*y=0)$,
(III) $(\forall x\in X)\ (x*x=0)$,
(IV) $(\forall x,y\in X)\ (x*y=0, y*x=0 \Rightarrow x=y)$.

If a BCI-algebra X satisfies the following identity:

(V) $(\forall x\in X)\ (0*x=0)$,

then X is called a *BCK-algebra*. Any BCK/BCI-algebra X satisfies the following axioms:

(a1) $(\forall x\in X)\ (x*0=x)$,
(a2) $(\forall x,y,z\in X)\ (x\le y \Rightarrow x*z\le y*z,\ z*y\le z*x)$,
(a3) $(\forall x,y,z\in X)\ ((x*y)*z=(x*z)*y)$,

where $x\le y$ if and only if $x*y=0$.

A subset I of a BCK/BCI-algebra X is called an *ideal* of X if it satisfies:

(b1) $0\in I$.
(b2) $(\forall x\in X)\ (\forall y\in I)\ (x*y\in I \implies x\in I)$.

Every ideal I of a BCK/BCI-algebra X has the following assertion:

$$(\forall x\in X)\,(\forall y\in I)\,(x\le y \implies x\in I). \tag{2.1}$$

A subset I of a BCK-algebra X is called a *positive implicative ideal* of X if it satisfies (b1) and

(b3) $(\forall x,y,z\in X)\ ((x*y)*z\in I,\ y*z\in I \Rightarrow x*z\in I)$.

We refer the reader to the paper [2] and book [7] for further information regarding BCK-algebras.

A fuzzy set μ in a BCK/BCI-algebra X is called a *fuzzy ideal* of X (see [8]) if it satisfies:

(c1) $(\forall x \in X)$ $(\mu(0) \geq \mu(x))$.
(c2) $(\forall x, y \in X)$ $(\mu(x) \geq \min\{\mu(x * y), \mu(y)\})$.

A fuzzy set μ in a BCK-algebra X is called a *fuzzy positive implicative ideal* of X (see [3]) if it satisfies (c1) and

(c3) $(\forall x, y, z \in X)$ $(\mu(x * z) \geq \min\{\mu((x * y) * z), \mu(y * z)\})$.

We now display the basic theory of falling shadows. We refer the reader to the papers [1, 9, 10, 11, 12] for further information regarding falling shadows.

Given a universe of discourse U, let $\mathscr{P}(U)$ denote the power set of U. For each $u \in U$, let

$$\dot{u} := \{E \mid u \in E \text{ and } E \subseteq U\}, \tag{2.2}$$

and for each $E \in \mathscr{P}(U)$, let

$$\dot{E} := \{\dot{u} \mid u \in E\}. \tag{2.3}$$

An ordered pair $(\mathscr{P}(U), \mathscr{B})$ is said to be a *hyper-measurable structure* on U if $\mathscr{B}$ is a σ-field in $\mathscr{P}(U)$ and $\dot{U} \subseteq \mathscr{B}$. Given a probability space $(\Omega, \mathscr{A}, P)$ and a hyper-measurable structure $(\mathscr{P}(U), \mathscr{B})$ on U, a *random set* on U is defined to be a mapping $\xi : \Omega \to \mathscr{P}(U)$ which is $\mathscr{A}$-$\mathscr{B}$ measurable, that is,

$$(\forall C \in \mathscr{B})\,(\xi^{-1}(C) = \{\omega \mid \omega \in \Omega \text{ and } \xi(\omega) \in C\} \in \mathscr{A}). \tag{2.4}$$

Suppose that ξ is a random set on U. Let

$$\tilde{H}(u) := P(\omega \mid u \in \xi(\omega))$$

for each $u \in U$. Then $\tilde{H}$ is a kind of fuzzy set in U. We call $\tilde{H}$ a *falling shadow* of the random set ξ, and ξ is called a *cloud* of $\tilde{H}$.

For example, $(\Omega, \mathscr{A}, P) = ([0, 1], \mathscr{A}, m)$, where $\mathscr{A}$ is a Borel field on $[0, 1]$ and m is the usual Lebesgue measure. Let $\tilde{H}$ be a fuzzy set in U and $\tilde{H}_t := \{u \in U \mid \tilde{H}(u) \geq t\}$ be a t-cut of $\tilde{H}$. Then

$$\xi : [0, 1] \to \mathscr{P}(U),\; t \mapsto \tilde{H}_t$$

is a random set and ξ is a cloud of $\tilde{H}$. We shall call ξ defined above as the *cut-cloud* of $\tilde{H}$ (see [1]).

3. Characterizations of falling fuzzy positive implicative ideals

In what follows let X denote a BCK-algebra unless otherwise specified.

Definition 3.1 ([4, 5])**.** Let $(\Omega, \mathscr{A}, P)$ be a probability space, and let

$$\xi : \Omega \to \mathscr{P}(X)$$

be a random set. If $\xi(\omega)$ is a (positive implicative) ideal of X for any $\omega \in \Omega$, then the falling shadow $\tilde{H}$ of the random set ξ, that is,

$$\tilde{H}(x) = P(\omega \mid x \in \xi(\omega)) \tag{3.1}$$

is called a *falling fuzzy (positive implicative) ideal* of X.

Let $(\Omega, \mathscr{A}, P)$ be a probability space and let

$$F(X) := \{f \mid f : \Omega \to X \text{ is a mapping}\}.$$

Define an operation $\circledast$ on $F(X)$ by

$$(\forall \omega \in \Omega)\,((f \circledast g)(\omega) = f(\omega) * g(\omega))$$

for all $f, g \in F(X)$. Let $\theta \in F(X)$ be defined by $\theta(\omega) = 0$ for all $\omega \in \Omega$. Then $(F(X); \circledast, \theta)$ is a BCK-algebra.

Let $(\Omega, \mathscr{A}, P)$ be a probability space and $\tilde{H}$ a falling shadow of a random set $\xi : \Omega \to \mathscr{P}(X)$. For any $x \in X$, let

$$\Omega(x; \xi) := \{\omega \in \Omega \mid x \in \xi(\omega)\}. \tag{3.2}$$

Then $\Omega(x; \xi) \in \mathscr{A}$.

The following theorem is a characterization of a falling fuzzy ideal in a BCK-algebra.

Theorem 3.2. *Let $\tilde{H}$ be a falling shadow of a random set $\xi : \Omega \to \mathscr{P}(X)$. Then $\tilde{H}$ is a falling fuzzy ideal of X if and only if the following condition is valid:*

$$(\forall x, y, z \in X)\,(x * y \leq z \Rightarrow \Omega(y; \xi) \cap \Omega(z; \xi) \subseteq \Omega(x; \xi))\,. \tag{3.3}$$

Proof. Let $\tilde{H}$ be a falling fuzzy ideal of X. Assume that $x * y \leq z$ for all $x, y, z \in X$. Let $\omega \in \Omega$ be such that $\omega \in \Omega(y; \xi) \cap \Omega(z; \xi)$. Then $y \in \xi(\omega)$ and $z \in \xi(\omega)$. Since $\xi(\omega)$ is an ideal of X, it follows from (2.1) and (b2) that $x \in \xi(\omega)$ so that $\omega \in \Omega(x; \xi)$. Hence $\Omega(y; \xi) \cap \Omega(z; \xi) \subseteq \Omega(x; \xi)$ for all $x, y, z \in X$ with $x * y \leq z$.

Conversely, suppose the condition (3.3) holds. Let $\omega \in \Omega$ be such that $\xi(\omega) \neq \varnothing$. Then there exists $x \in \xi(\omega)$, and so $\omega \in \Omega(x; \xi)$. Since $0 * x = 0 \leq x$, it follows from (3.3) that

$$\Omega(x; \xi) = \Omega(x; \xi) \cap \Omega(x; \xi) \subseteq \Omega(0; \xi)$$

so that $\omega \in \Omega(0; \xi)$, that is, $0 \in \xi(\omega)$. Let $x, y \in X$ be such that $x * y \in \xi(\omega)$ and $y \in \xi(\omega)$. Then $\omega \in \Omega(x * y; \xi) \cap \Omega(y; \xi)$. Note that $x * (x * y) \leq y$. Hence $\Omega(x * y; \xi) \cap \Omega(y; \xi) \subseteq \Omega(x; \xi)$ by (3.3), and so $\omega \in \Omega(x; \xi)$, that is, $x \in \xi(\omega)$. Therefore $\tilde{H}$ is a falling fuzzy ideal of X. □

Proposition 3.3. *Every falling fuzzy ideal $\tilde{H}$ of X satisfies the following condition:*

$$\Omega((x * y) * z; \xi) \cap \Omega(y * z; \xi) \subseteq \Omega((x * z) * z; \xi) \tag{3.4}$$

for all $x, y, z \in X$.

Proof. Let $\omega \in \Omega((x * y) * z; \xi) \cap \Omega(y * z; \xi)$. Then $(x * y) * z \in \xi(\omega)$ and $y * z \in \xi(\omega)$. Using (I) and (a3), we have

$$((x * z) * z) * ((x * y) * z) \leq (x * z) * (x * y) \leq y * z.$$

Since $\xi(\omega)$ is an ideal of X, it follows from (1) that $((x * z) * z) * ((x * y) * z) \in \xi(\omega)$ so from (b2) that $(x * z) * z \in \xi(\omega)$, i.e., $\omega \in \Omega((x * z) * z; \xi)$. Hence

$$\Omega((x * y) * z; \xi) \cap \Omega(y * z; \xi) \subseteq \Omega((x * z) * z; \xi)$$

for all $x, y, z \in X$. □

We provide conditions for a falling shadow to be a falling fuzzy ideal.

Theorem 3.4. *Let $\tilde{H}$ be a falling shadow of a random set $\xi : \Omega \to \mathscr{P}(X)$ such that $0 \in \xi(\omega)$ for all $\omega \in \Omega$. If the condition (3.4) holds, then $\tilde{H}$ is a falling fuzzy ideal of X.*

Proof. Let $\omega \in \Omega$ and $x, y \in X$ be such that $x * y \in \xi(\omega)$ and $y \in \xi(\omega)$. Then

$$\begin{aligned}\omega \in \Omega(x * y; \xi) \cap \Omega(y; \xi) &= \Omega((x * y) * 0; \xi) \cap \Omega(y * 0; \xi)\\ &\subseteq \Omega((x * 0) * 0; \xi) = \Omega(x; \xi)\end{aligned}$$

by using (a1) and (3.4), and so $x \in \xi(\omega)$. Therefore $\tilde{H}$ is a falling fuzzy ideal of X. □

Theorem 3.5. *Let $\tilde{H}$ be a falling shadow of a random set $\xi : \Omega \to \mathscr{P}(X)$ such that $0 \in \xi(\omega)$ for all $\omega \in \Omega$. If the condition*

$$(\forall x, y, z \in X)\,(\Omega(((x * y) * y) * z; \xi) \cap \Omega(z; \xi) \subseteq \Omega(x * y; \xi)) \tag{3.5}$$

is valid, then $\tilde{H}$ is a falling fuzzy ideal of X.

Proof. Let $\omega \in \Omega$ and $x, y \in X$ be such that $x * y \in \xi(\omega)$ and $y \in \xi(\omega)$. Using (a1) and (3.5), we have

$$\begin{aligned}\omega \in \Omega(x * y; \xi) \cap \Omega(y; \xi) &= \Omega(((x * 0) * 0) * y; \xi) \cap \Omega(y; \xi)\\ &\subseteq \Omega(x * 0; \xi) = \Omega(x; \xi),\end{aligned}$$

and so $x \in \xi(\omega)$. Hence $\tilde{H}$ is a falling fuzzy ideal of X. □

Proposition 3.6. *If $\tilde{H}$ is a falling fuzzy positive implicative ideal of X, then*

$$(\forall x, y \in X)\,(\Omega((x * y) * y; \xi) \subseteq \Omega(x * y; \xi))\,, \tag{3.6}$$

$$(\forall x, y, z \in X)\,(\Omega((x * y) * z; \xi) \subseteq \Omega((x * z) * (y * z); \xi))\,. \tag{3.7}$$

Proof. Let $\omega \in \Omega((x * y) * y; \xi)$. Then $(x * y) * y \in \xi(\omega)$. Since $\xi(\omega)$ is a positive implicative ideal of X, we have $y * y = 0 \in \xi(\omega)$ and $x * y \in \xi(\omega)$ by (b3). Hence $\omega \in \Omega(x*y; \xi)$, which proves (3.6). Now let $\omega \in \Omega((x*y)*z; \xi)$. Then $(x*y)*z \in \xi(\omega)$. Note that

$$((x * (y * z)) * z) * z = ((x * z) * (y * z)) * z \leq (x * y) * z \in \xi(\omega).$$

Since $\xi(\omega)$ is a positive implicative ideal and hence an ideal of X, it follows from (1) that $((x * (y * z)) * z) * z \in \xi(\omega)$. Using (3.6) and (a3), we have

$$\begin{aligned}\omega &\in \Omega(((x * (y * z)) * z) * z; \xi)\\ &\subseteq \Omega((x * (y * z)) * z; \xi)\\ &= \Omega((x * z) * (y * z); \xi).\end{aligned}$$

Therefore $\Omega((x * y) * z; \xi) \subseteq \Omega((x * z) * (y * z); \xi)$ for all $x, y, z \in X$. □

Proposition 3.7. *If $\tilde{H}$ is a falling fuzzy ideal of X, then conditions (3.5), (3.6) and (3.7) are equivalent.*

Proof. Assume that the condition (3.6) holds. Let $\omega \in \Omega((x*y)*z;\xi)$. Then

$$((x*(y*z))*z)*z = ((x*z)*(y*z))*z \leq (x*y)*z \in \xi(\omega),$$

and so $((x*(y*z))*z)*z \in \xi(\omega)$ since $\xi(\omega)$ is an ideal of X. It follows from (3.6) that

$$\begin{aligned} \omega &\in \Omega(((x*(y*z))*z)*z;\xi) \\ &\subseteq \Omega((x*(y*z))*z;\xi) \\ &= \Omega((x*z)*(y*z);\xi). \end{aligned}$$

Hence $\Omega((x*y)*z;\xi) \subseteq \Omega((x*z)*(y*z);\xi)$ for all $x,y,z \in X$.

Now suppose that the condition (3.7) is valid. Let

$$\omega \in \Omega(((x*y)*y)*z;\xi) \cap \Omega(z;\xi).$$

Then $\omega \in \Omega(z;\xi)$ and

$$\begin{aligned} \omega &\in \Omega(((x*y)*y)*z;\xi) = \Omega(((x*z)*y)*y;\xi) \\ &= \Omega(((x*z)*y)*(y*y);\xi) = \Omega((x*z)*y;\xi) \\ &= \Omega((x*y)*z;\xi). \end{aligned}$$

Thus $z \in \xi(\omega)$ and $(x*y)*z \in \xi(\omega)$. Since $\xi(\omega)$ is an ideal of X, it follows that $x*y \in \xi(\omega)$, that is, $\omega \in \Omega(x*y;\xi)$. Hence (3.5) is valid.

Finally, suppose that the condition (3.5) holds. Let $(x*y)*z \in \xi(\omega)$ and $y*z \in \xi(\omega)$ for all $x,y,z \in X$ and $\omega \in \Omega$. Then

$$((x*z)*z)*(y*z) \leq (x*z)*y = (x*y)*z \in \xi(\omega).$$

Since $\xi(\omega)$ is an ideal of X, we have $((x*z)*z)*(y*z) \in \xi(\omega)$ by (1), and so $(x*z)*z \in \xi(\omega)$ by (b2). it follows that $((x*z)*z)*0 = (x*z)*z \in \xi(\omega)$ and $0 \in \xi(\omega)$. Hence

$$\omega \in \Omega(((x*z)*z)*0;\xi) \cap \Omega(0;\xi) \subseteq \Omega(x*z;\xi)$$

by (3.5). Thus $x*z \in \xi(\omega)$, and therefore $\tilde{H}$ is a falling fuzzy positive implicative ideal of X. By Proposition 3.6, the condition (3.6) is valid. □

We provide conditions for a falling shadow to be a falling fuzzy positive implicative ideal.

Theorem 3.8. *Every falling fuzzy ideal $\tilde{H}$ of X satisfying the condition (3.5) is a falling fuzzy positive implicative ideal of X.*

Proof. Let $x,y,z \in X$ and $\omega \in \Omega$ be such that $(x*y)*z \in \xi(\omega)$ and $y*z \in \xi(\omega)$. Then

$$((x*z)*z)*(y*z) \leq (x*z)*y = (x*y)*z \in \xi(\omega),$$

and so $((x*z)*z)*(y*z) \in \xi(\omega)$ since $\xi(\omega)$ is an ideal of X. Using (3.5), we have

$$\omega \in \Omega(((x*z)*z)*(y*z);\xi) \cap \Omega(y*z;\xi) \subseteq \Omega(x*z;\xi).$$

Thus $x*z \in \xi(\omega)$, which shows that $\tilde{H}$ is a falling fuzzy positive implicative ideal of X. □

Proposition 3.7 and Theorem 3.8 are combined to form the following corollary.

Corollary 3.9. *Every falling fuzzy ideal $\tilde{H}$ of X satisfying (3.6) or (3.7) is a falling fuzzy positive implicative ideal of X.*

Lemma 3.10 ([4]). *Every falling fuzzy positive implicative ideal is a falling fuzzy ideal.*

Lemma 3.11 ([4]). *If a falling shadow $\tilde{H}$ of a random set $\xi : \Omega \to \mathscr{P}(X)$ is a falling fuzzy positive implicative ideal of X, then*

$$(\forall x, y, z \in X) \ \ (\Omega((x * y) * z; \xi) \cap \Omega(y * z; \xi) \subseteq \Omega(x * z; \xi)) . \tag{3.8}$$

Theorem 3.12. *Let $\tilde{H}$ be a falling shadow of a random set $\xi : \Omega \to \mathscr{P}(X)$. Then $\tilde{H}$ is a falling fuzzy positive implicative ideal of X if and only if $\tilde{H}$ is a falling fuzzy ideal of X that satisfies the condition (3.8).*

Proof. The necessity follows from Lemmas 3.10 and 3.11.

Conversely, assume that $\tilde{H}$ is a falling fuzzy ideal of X that satisfies the condition (3.8). Let $\omega \in \Omega$ and $x, y, z \in X$ be such that $(x * y) * z \in \xi(\omega)$ and $y * z \in \xi(\omega)$. Then

$$\omega \in \Omega((x * y) * z; \xi) \cap \Omega(y * z; \xi) \subseteq \Omega(x * z; \xi)$$

by (3.8), and so $x * z \in \xi(\omega)$. Consequently, $\tilde{H}$ is a falling fuzzy positive implicative ideal of X. □

4. Conclusion

We have investigated relations between a falling fuzzy ideal and a falling fuzzy positive implicative ideal. In general, a falling fuzzy positive implicative ideal is a falling fuzzy ideal, but not converse. We have provided conditions for a a falling fuzzy ideal to be a a falling fuzzy positive implicative ideal. We have discussed characterizations of a falling fuzzy positive implicative ideal.

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