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New structures on partially ordered soft sets and soft Scott topology

Bekir Tanay, Gözde Yaylali

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Abstract. In this paper, we give some new structures for the partially ordered soft sets such as directed soft set, directed complete soft set. Also, we introduce the soft Scott topology by using soft set relation.

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Corresponding Author: Bekir Tanay (btanay@mu.edu.tr)

1. Introduction

In the real world, most of times exact solution can not be found for the problems, so the real world problems can be solved approximately. On the other hand, D.Molodtsov [9] introduced new approach for the real world problems in the economics, engineering, and environmental areas, which approximate the initial universe of the problems. Molodtsov's soft set theory was proposed for dealing with uncertainty. Many operators for soft set theory are introduced by Maji et. al. [7] in detail. Furthermore, Babitha [1] gave definitions for the soft set relation. After that, Babitha and Sunil [2] introduced ordering on soft sets, and they defined the partially ordered soft set. Some further definitions about soft set relation, such as kernels and closures of soft sets were introduced by Yang and Guo [15]. Moreover, Park et. al. [11] studied equivalence relation and they proved that poset of the equivalence soft set relations on a soft set (F, A) is a complete lattice.

Shabir and Naz [13] introduced soft topological spaces, defined over an initial universe with a fixed parameter set but Min [8] made some corrections on [13]. Zorlutuna et. al.[16] introduced the concepts of soft interior point, soft interior, soft neighborhood, soft continuity and soft compactness and also Nazmul et. al. [10] presented some neighborhood properties in soft topological space. Husain and Ahmad [6] strengthened theory of soft topological spaces which is defined in [13] by defining

it on a fixed initial universe. Varol et. al. [14] gave some properties of soft Hausdorff spaces and introduced some new concepts such as convergence of sequences. Furthermore Cagman et. al. [3] defined soft topology by modifying the definition of a soft set. Roy and Samanta [12] strengthen the definition of a soft topological space which is defined by Cagman et. al. [3] and they gave same results relative to base and subbase.
In order to refresh the fundamental concepts of set theory we refer to [5], [4]. Scott topology is well known in theoretical computer science and topological lattice theory. In this study we introduce soft Scott topology by using soft set relation. To define soft Scott topology we introduce directed and directed complete soft set. Also, we give some new structures for the partially ordered soft set (F, A, R) such as, infimum and supremum of a soft set.

2. Preliminaries and basic definitions

Definition 2.1 ([9])**.** Let U be an initial universe, E be the set of parameters. Let $\mathcal{P}(U)$ be the set of all subsets of U and A be a subset of E. A pair (F, A) is called a soft set over U where $F : A \longrightarrow \mathcal{P}(U)$ is a set-valued function.

Definition 2.2 ([7])**.** A soft set (F, A) over U is said to be a Null soft set denoted by Φ, if for every $\epsilon \in A$, $F(\epsilon) = \varnothing$

Definition 2.3 ([7])**.** For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) and is denoted by $(F, A)\widetilde{\subset}(G, B)$ if
(i) $A \subset B$ and,
(ii) $\forall \epsilon \in A$, $F(\epsilon)$ and $G(\epsilon)$ are identical approximations, which means $F(\epsilon) = G(\epsilon)$

Definition 2.4 ([3])**.** The soft complement $(F, A)^c$ of (F, A) is defined by $(F, A)^c = (F^c, A)$, where $F^c(x) = U - F(x)$, $\forall x \in A$.

Definition 2.5 ([7])**.** Union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$, and for each $e \in C$,

$$H(e) = \begin{cases} F(e), & if\, e \in A - B, \\ G(e), & if\, e \in B - A, \\ F(e) \cup G(e), & if\, e \in A \cap B \end{cases}$$

We write $(F, A)\widetilde{\cup}(G, B) = (H, C)$.

Definition 2.6 ([7])**.** Intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cap B$, and for each $e \in C$, $H(e) = F(e) \cap G(e)$. We write $(F, A)\widetilde{\cap}(G, B) = (H, C)$.

Definition 2.7 ([12])**.** A *soft topology* $\widetilde{\tau}$ on a soft set (F, A) is a family of soft subsets of (F, A) satisfying the following properties
i) Φ, $(F, A) \in \widetilde{\tau}$;
ii) If (G, B), $(H, C) \in \widetilde{\tau}$, then $(G, B)\widetilde{\cap}(H, C) \in \widetilde{\tau}$;
iii) If $(F_\alpha, A_\alpha) \in \widetilde{\tau}$ for all $\alpha \in \Lambda$, an index set, then $\widetilde{\bigcup}_{\alpha\in\Lambda}(F_\alpha, A_\alpha) \in \widetilde{\tau}$
If $\widetilde{\tau}$ is a soft topology on a soft set (F, A), $(F, A, \widetilde{\tau})$ is called the *soft topological space*.

Definition 2.8 ([12])**.** If $\widetilde{\tau}$ is a soft topology on (F, A), then the member of $\widetilde{\tau}$ is called an *open soft set* in $(F, A, \widetilde{\tau})$

Definition 2.9 ([3])**.** Let $(F, A, \widetilde{\tau})$ be a soft topological space and $(G, B)\widetilde{\subset}(F, A)$. Then, (G, B) is said to be *closed soft set* if the soft set $(G, B)^c$ is open soft set.

Definition 2.10 ([1])**.** Let (F, A) and (G, B) be two soft sets over U, then the cartesian product of (F, A) and (G, B) is defined as, $(F, A) \times (G, B) = (H, A \times B)$ where $H : A \times B \to \mathcal{P}(U \times U)$ and $H(a, b) = F(a) \times G(b)$, where $(a, b) \in A \times B$ i.e. $H(a, b) = \{(h_i, h_j)|h_i \in F(a), h_j \in G(b)\}$

Definition 2.11 ([1])**.** Let (F, A) and (G, B) be two soft sets over U, then a relation R from (F, A) to (G, B) is a soft subset of $(F, A) \times (G, B)$.
In other words, a relation R from (F, A) to (G, B) is of the form $R = (H_1, S)$ where $S \subset A \times B$ and $H_1(a, b) = H(a, b)$ for all $(a, b) \in S$ where $(H, A \times B) = (F, A) \times (G, B)$.

Definition 2.12 ([1])**.** Let R be a soft set relation from (F, A) to (G, B). Then the domain of R is defined as the soft set (D, A_1) where
$A_1 = \{a \in A : H(a, b) \in R \, for\, some\, b \in B\}$ and $D(a_1) = F(a_1)$, for all $a_1 \in A_1$.
The range of R is defined as the soft set (RG, B_1), where $B_1 \subset B$ and $B_1 = \{b \in B : H(a, b) \in R \, for\, some\, a \in A\}$ and $RG(b_1) = G(b_1)$ $\forall b_1 \in B_1$

Definition 2.13 ([1])**.** Let R be a relation on (F, A), then

(1) R is reflexive if $H_1(a, a) \in R$, $\forall a \in A$.
(2) R is symmetric if $H_1(a, b) \in R \Rightarrow H_1(b, a) \in R$.
(3) R is transitive if $H_1(a, b) \in R$, $H_1(b, c) \in R \Rightarrow H_1(a, c) \in R$ for every $a, b, c \in A$.

Definition 2.14 ([2])**.** A binary soft set relation R on (F, A) is an antisymmetric if $F(a) \times F(b) \in R$ and $F(b) \times F(a) \in R$ for every $F(a), F(b) \in (F, A)$ imply $F(a) = F(b)$.

Definition 2.15 ([2])**.** A binary soft set relation $\leq$ on (F, A) which is reflexive, antisymmetric and transitive is called a partial ordering of (F, A).
The triple $(F, A, \leq)$ is called a partially ordered soft set.

Definition 2.16 ([2])**.** Let $\leq$ be an ordering of (F, A) and $F(a)$ and $F(b)$ be any two elements in (F, A). We say that $F(a)$ and $F(b)$ are comparable in the ordering if $F(a) \leq F(b)$ or $F(b) \leq F(a)$. We say that $F(a)$ and $F(b)$ are incomparable if they are not comparable.

Definition 2.17 ([2])**.** Let $\leq$ be a partially ordering of the soft set (F, A). Then $\leq$ is called a total or linear ordering on (F, A) if every element in (F, A) is comparable in the ordering $\leq$.

3. Some results on orderings on soft sets

Definition 3.1. Let $B \subset A$ and $(G, B)\widetilde{\subset}(F, A)$ where (F, A) is ordered by R. (G, B) is a *chain* in (F, A) if any two element in (G, B) are comparable.

Definition 3.2 ([2])**.** Let $(G, B, \leq)$ be a partially ordered soft set. Then,

a) For $b \in B$, $G(b)$ is the least element of (G, B)in the ordering '$\leq$' if $G(b) \leq G(x)$ for all $x \in B$

b) For $b \in B$, $G(b)$ is a minimal element of (G, B) in the ordering '$\leq$' if there exists no $x \in B$ such that $G(x) \leq G(b)$ and $G(x) \neq G(b)$.

a') For $b \in B$, $G(b)$ is the greatest element of (G, B) in the ordering '$\leq$' if for every $x \in B$ $G(x) \leq G(b)$.

b') For $b \in B$, $G(b)$ is a maximal element of (G, B) in the ordering '$\leq$' if there exists no $x \in B$ such that $G(b) \leq G(x)$ and $G(x) \neq G(b)$.

Example 3.3. Consider soft sets (F, A) and (G, B) as follows: The universe set $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and the parameter set $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$; $A = \{e_1, e_2, e_3, e_5, e_6\}$ and $B = \{e_1, e_3, e_5\}$ $F(e_1) = \{h_1, h_4\}$, $F(e_2) = \{h_1, h_3\}$, $F(e_3) = \{h_3, h_4, h_5\}$, $F(e_5) = \{h_1\}$, $F(e_6) = \{h_1, h_6\}$ and $G(e_1) = \{h_1, h_4\}$, $G(e_3) = \{h_3, h_4, h_5\}$, $G(e_5) = \{h_1\}$. Hence $(G, B)\widetilde{\subset}(F, A)$. Now define a relation $\leq$ on (F, A) as

$$\begin{aligned}\leq = \{&F(e_1) \times F(e_1),\ F(e_2) \times F(e_2),\ F(e_3) \times F(e_3),\ F(e_5) \times F(e_5),\\ &F(e_6) \times F(e_6),\ F(e_1) \times F(e_3),\ F(e_1) \times F(e_5),\ F(e_1) \times F(e_6),\\ &F(e_2) \times F(e_3),\ F(e_2) \times F(e_5),\ F(e_2) \times F(e_6),\ F(e_3) \times F(e_5),\\ &F(e_3) \times F(e_6),\ F(e_5) \times F(e_6)\}\end{aligned}$$

Then $F(e_1)$ and $F(e_2)$ are minimal elements of (F, A) and $F(e_6)$ is the maximal element and the greatest element of (F, A). Similarly since $G(e_1) \times G(e_1)$, $G(e_3) \times G(e_3)$, $G(e_5) \times G(e_5)$, $G(e_1) \times G(e_3)$, $G(e_3) \times G(e_5)$, $G(e_1) \times G(e_5) \in \leq$, we can say $G(e_1)$ is a minimal element and the least element; and also $G(e_5)$ is a maximal element and the greatest element of (G, B). Besides (G, B) is a chain in (F, A) because any two element of (G, B) are comparable.

Definition 3.4. Let $\leq$ be an ordering of (F, A), let $(G, B)\widetilde{\subset}(F, A)$.

- For $a \in A$, $F(a)$ is a lower bound of (G, B) in the ordered soft set $(F, A, \leq)$ if $F(a) \leq G(x)$ for all $x \in B$.
- For $a \in A$, $F(a)$ is called infimum of (G, B) in $(F, A, \leq)$ (or the greatest lower bound) if it is the greatest element of the set of all lower bounds of (G, B) in $(F, A, \leq)$.

Similarly,

- For $a \in A$, $F(a)$ is an upper bound of (G, B) in the ordered soft set $(F, A, \leq)$ if $G(x) \leq F(a)$ for all $x \in B$.
- For $a \in A$, $F(a)$ is called supremum of (G, B) in $(F, A, \leq)$ (or the least upper bound) if it is the least element of the set of all upper bounds of (G, B) in $(F, A, \leq)$.

Example 3.5. Consider the soft sets (F, A) and (G, B) as presented in Example 3.3. $F(e_5)$ and $F(e_6)$ are upper bounds of (G, B) in the ordered soft set (F,A,$\leq$). $F(e_5)$ is the supremum of (G, B). $F(e_1)$ is the lower bound of (G, B) and so it is the infimum of (G, B) in $(F, A, \leq)$.

Definition 3.6. Consider a soft set (F, A) equipped with reflexsive, transitive relation $\leq$. This relation is called preorder and (F, A) is a preordered soft set.

Definition 3.7. Let (F, A) be a soft set. (F, A) is called a *finite soft set*, if it is a soft set with a finite parameter set.

Definition 3.8. Let (F, A) be a preordered soft set. A soft subset (G, B) of (F, A) is directed provided it is nonnull and every finite soft subset of (G, B) has an upperbound in (G, B).

Example 3.9. Consider the soft set (F, A) over U, where $U = \{c_1, c_2, c_3, c_4, c_5, c_6\}$, $A = \{a_1, a_2, a_3\}$ and $F(a_1) = \{c_1, c_2\}$, $F(a_2) = \{c_2\}$, $F(a_3) = \{c_4, c_5, c_6\}$ and

$$\leq = \{F(a_1) \times F(a_1), F(a_2) \times F(a_2), F(a_3) \times F(a_3),$$
$$F(a_1) \times F(a_2), F(a_2) \times F(a_3), F(a_1) \times F(a_3)\}.$$

(F, A) is directed soft set with the relation $\leq$.

Definition 3.10. Let (F, A) be a soft set with a preorder $\leq$. For $(G, B)\widetilde{\subset}(F, A)$

i) $\downarrow (G, B) = (H, C)$ where $C = \{a \in A : F(a) \leq G(b)\, for\, some\, b \in B\}$ and $H = F|_C$.

ii) $\uparrow (G, B) = (K, D)$ where $D = \{a \in A : G(b) \leq F(a)\, for\, some\, b \in B\}$ and $K = F|_D$.

iii) (G, B) is a lower soft set iff $(G, B) = \downarrow (G, B)$.

iv) (G, B) is an upper soft set iff $(G, B) = \uparrow (G, B)$.

v) (G, B) is an ideal iff it is a directed lower soft set.

Example 3.11. Consider the soft set (F, A) and the soft set relation $\leq$ as presented in Example 3.9.
Let $B = \{a_1, a_2\}$ and $G : B \to P(U)$ such that $G(a_1) = \{c_1, c_2\}$, $G(a_2) = \{c_2\}$. Thus $(G, B)\widetilde{\subset}(F, A)$.
$\uparrow (G, B) = (H, C)$ where $C = \{a \in A : G(b) \leq F(a)\, for\, some\, b \in B\} = \{a_1, a_2, a_3\}$ and $H = F|_C$.
$\downarrow (G, B) = (K, D)$ where $D = \{a \in A : F(a) \leq G(b)\, for\, some\, b \in B\} = \{a_1, a_2\}$ and $K = F|_D$.
Since $D = B$, we have $\downarrow (G, B) = (G, B)$. Hence (G, B) is a lower soft set.

Definition 3.12. A partially ordered soft set is said to be a directed complete soft set if every directed soft subset has a supremum.

Example 3.13. Take $A = \mathbb{Z}^+$ and let $U = \mathbb{R}^*$ extended real numbers and $F : \mathbb{Z} \to \mathcal{P}(\mathbb{R}^*)$ such that $F(a) = (1, a + 1]$ for $a \in A$. So (F, A) is a soft set. It is a partially ordered soft set by inclusion.

Lemma 3.14. *Let (F, A) be a directed complete partially ordered soft set and let $(G_1, B_1)\widetilde{\subseteq}(F, A)$, $(G_2, B_2)\widetilde{\subseteq}(F, A)$, such that $\uparrow (G_1, B_2) = (G_1, B_1)$, $\uparrow (G_2, B_2) = (G_2, B_2)$ and $(G_i, B_i)\widetilde{\subseteq}(F, A)$ for all i in an index set I. Then,*

i) $\uparrow ((G_1, B_1)\widetilde{\cap}(G_2, B_2)) = \uparrow (G_1, B_1)\widetilde{\cap} \uparrow (G_2, B_2)$.

ii) $\uparrow \big(\widetilde{\bigcup_{i\in I}}(G_i, B_i)\big) = \widetilde{\bigcup_{i\in I}} \uparrow (G_i, B_i)$.

Proof. (i) From Definition 2.6 and Definition 3.10 we have

$$\uparrow \big((G_1, B_1)\widetilde{\cap}(G_2, B_2)\big) = \uparrow (G, B) = (K, D),$$

where $B = B_1 \cap B_2$. Let $K(d) \in (K, D)$. Then there exists $b \in B$ such that $G(b) \leq K(d)$, where $G(b) = G_1(b_1) \cap G_2(b_2) = F(b)$. From Definition 2.3 and $b \in B_1$ and $b \in B_2$, it follows $G_1(b) \leq K(d)$ and $G_2(b) \leq K(d)$. Therefore $K(d) \in \uparrow (G_1, B_1) = (K_1, D_1)$ and $K(d) \in \uparrow (G_2, B_2) = (K_2, D_2)$. Thus $K(d) \in \uparrow (G_1, B_1)\widetilde{\cap} \uparrow (G_2, B_2)$

since $d \in D_1 \cap D_2$ and $K_1(d) \cap K_2(d) = F(d) \cap F(d) = F(d) = K(d)$. Conversely, let $K(d) \in \uparrow (G_1, B_1) \widetilde{\cap} \uparrow (G_2, B_2)$. Then we have $K(d) \in (G_1, B_1)\widetilde{\cap}(G_2, B_2)$, since $\uparrow (G_1, B_1) = (G_1, B_1)$ and $\uparrow (G_2, B_2) = (G_2, B_2)$. Thus $K(d) \in \uparrow (G_1, B_1)\widetilde{\cap}(G_2, B_2)$.

(ii) Similar to (i). □

4. Soft Scott topology

Definition 4.1. Let (F, A) be a directed complete partially ordered soft set and $(G, B)\widetilde{\subseteq}(F, A)$. Then (G, B) is called a *Scott open soft set* iff the following two conditions are satisfied:

i) $(G, B) = \uparrow (G, B)$;
ii) $\sup (D, C) \in (G, B)$ implies $(D, C)\widetilde{\cap}(G, B) \neq \Phi$ for all directed complete soft sets $(D, C)\widetilde{\subset}(F, A)$.

Theorem 4.2. *The collection of all Scott open soft sets of* (F, A) *is a soft topology.*

Proof. Let $\widetilde{\tau}$ be the set of all Scott open soft sets of (F, A).

(i) $\Phi, (F, A) \in \widetilde{\tau}$ since they satisfy two conditions of the Definition 4.1.

(ii) Let $(G_1, B_1), (G_2, B_2) \in \widetilde{\tau}$ and let $(G, B) = (G_1, B_1)\widetilde{\cap}(G_2, B_2)$. Since

$$\uparrow (G_1, B_1) = (G_1, B_1) \text{ and } \uparrow (G_2, B_2) = (G_2, B_2),$$

then $\uparrow (G, B) = (G, B)$. Suppose $\sup(D, C) \in (G, B)$ for a directed soft set (D, C), then $\sup(D, C) \in (G_1, B_1)$ and $\sup(D, C) \in (G_2, B_2)$, then there exists $D(c_1)$ in (G_1, B_1) and there exists $D(c_2)$ in (G_2, B_2) where $D(c_1), D(c_2)$ are elements of (D, C). Since (D, C) is a directed soft set then $\sup\{D(c_1), D(c_2)\} = D(c)$ is in (D, C). By the condition (ii) in the Definition 4.1, $D(c)$ is in (G_1, B_1) and (G_2, B_2). So $(D, C)\widetilde{\cap}(G, B) \neq \Phi$.

(iii) Let λ be index set and let for $i \in \lambda$, $(F_i, A_i) \in \widetilde{\tau}$. Then

$$\widetilde{\bigcup}_{i\in\lambda}(F_i, A_i) = \uparrow \big(\widetilde{\bigcup}_{i\in\lambda}(F_i, A_i)\big),$$

since for all $e \in A_j \setminus \bigcup_{i\neq j} A_i$, $F_j(e) = F(e)$ and for all $e \in \bigcap_{i\in\lambda} A_i$, $F_i(e) = F(e)$ and for all $i \in \lambda$, $(F_i, A_i) \in \widetilde{\tau}$. Suppose $\sup(D, C) \in \widetilde{\bigcup}_{i\in\lambda}(F_i, A_i)$ for a directed soft set (D, C). Then for some $i \in \lambda$, $\sup(D, C) \in (F_i, A_i)$. Since for all $i \in \lambda$, (F_i, A_i) satisfies the condition (ii) in the Definition 4.1, $(D, C)\widetilde{\cap}(F_i, A_i) \neq \Phi$, then $(D, C)\widetilde{\cap}\big(\widetilde{\bigcup}_{i\in\lambda}(F_i, A_i)\big) \neq \Phi$. Therefore $\widetilde{\bigcup}_{i\in\lambda}(F_i, A_i) \in \widetilde{\tau}$. Thus $\widetilde{\tau}$ is a soft topology. □

Definition 4.3. The collection of all Scott open soft sets of (F, A) is called *Soft Scott Topology* on (F, A).

Definition 4.4. Let $(F, A, \widetilde{\tau})$ be a soft Scott topology, and $(G, B)\widetilde{\subseteq}(F, A)$. Then (G, B) is called *Scott closed soft set* if $(G, B)^c$ is Scott open soft set.

We say that a soft subset (G, B) of a directed complete partially ordered soft set (F, A) has the *property* (S) provided that the following condition is satisfied:

(S): If $\sup(D, C) \in (G, B)$ for any directed soft set (D, C), then there is a $D(c) \in (D, C)$ such that $G(b) \in (G, B)$ for all $G(b) \in (D, C)$ with $G(b) \geq D(c)$.

Example 4.5. Let $U = \{c_1, c_2, c_3, c_4, c_5, c_6\}$ be the initial universe and let $A = \{a_1, a_2, a_3\}$. Consider soft set (F, A) defined by

$$F(a_1) = \{c_1, c_2\}, F(a_2) = \{c_2\}, F(a_3) = \{c_4, c_5, c_6\}.$$

Now define a soft set relation $\leq$ on (F, A) as

$$\leq = \{F(a_1) \times F(a_1), F(a_2) \times F(a_2), F(a_3) \times F(a_3), F(a_1) \times F(a_2), F(a_2) \times F(a_3), F(a_1) \times F(a_3)\}.$$

We can also express this relation as $F(a_1) \leq F(a_1)$, $F(a_2) \leq F(a_2)$, $F(a_3) \leq F(a_3)$, $F(a_1) \leq F(a_2)$, $F(a_2) \leq F(a_3)$, $F(a_1) \leq F(a_3)$. (F, A) is a directed complete partially ordered soft set with the relation $\leq$. When we examine the all soft subsets of (F, A), we obtain for a soft set (F_1, B_1) where $B_1 = \{a_3\}$ and $F_1 = F|_{B_1}$, that $\uparrow (F_1, B_1) = (F_1, B_1)$. Since there is no $F(a_i)$, $a_i \in A$ except $i = 3$ satisfying $F(a_3) \leq F(a_i)$. Similarly we have $\uparrow (F_2, B_2) = (F_2, B_2)$ where $B_2 = \{a_2, a_3\}$, $F_2 = F|_{B_2}$ and $\uparrow (F, A) = (F, A)$, $\uparrow \Phi = \Phi$. Consequently, the family of Scott open soft sets is obtained as follows $\{(F_1, B_1), (F_2, B_2), (F, A), \Phi\}$.

Remark 4.6. In any directed complete partially ordered soft set, a set is Scott open soft set iff it is an upper soft set satisfying (S).

Example 4.7. Let $U = (-\infty, 0]$ be the initial universe and let $A = \mathbb{Z}^-$ be the parameter set and let (F, A) be a soft set, defined by $(F, A) = \{F(a) = (a, 0] \mid a \in A\}$. Consider the soft set relation $\leq$ on (F, A), which is defined by $F(a) \leq F(b) :\Leftrightarrow a \leq b$. (F, A) is a directed complete partially ordered soft set with the relation $\leq$. By examining all soft subsets of (F, A). We obtained that the only soft subset (G, B) of (F, A) satisfying $\uparrow (G, B) = (G, B)$ are (F, A), Φ and (G, B_x); which is defined by $B_x = \{b \in \mathbb{Z}^- \mid x \leq b\}$ and $G = F|_{B_x}$ for $x \in \mathbb{Z}^-$.

Let (D, C) be a directed partially ordered soft set such that $\sup (D, C) \in (G, B)$ and $\sup (D, C) = G(b)$. $G(b) \in (G, B)$ implies $b \in B$. Then for all $c \in C$, $D(c) \leq G(b)$. This implies for all $c \in C$, $c \leq b$, then $b \in C$. Thus, there exists at least $G(b)$ in $(D, C)\widetilde{\cap}(G, B)$. Then $(D, C)\widetilde{\cap}(G, B) \neq \Phi$ is satisfied. Eventually, these soft subsets (G, B) are Scott open soft set for a directed complete partially ordered soft set (F, A).

Example 4.8. Let $U = [-\infty, 0]$ be the initial universe and $A = \mathbb{Z}^-$ be the parameter set. A soft set (F, A) is defined by
$(F, A) = \{F(a) \mid a \in A\} = \{F(a) = (a, 0] \mid a \in A\}$. Consider a soft set relation $\leq$ defined by $F(a) \leq F(b) :\Leftrightarrow F(a) \supseteq F(b)$ on (F, A).

(1) $(a, 0] \subseteq (a, 0]$ implies $F(a) \leq F(a)$.
(2) $F(a) \leq F(b)$ and $F(b) \leq F(a)$ implies $(b, 0] \subseteq (a, 0]$ and $(a, 0] \subseteq (b, 0]$, then we get $(a, 0] = (b, 0]$. Then $a = b$. Thus $F(a) = F(b)$.
(3) Let $F(a) \leq F(b)$ and $F(b) \leq F(c)$. This implies $(b, 0] \subseteq (a, 0]$ and $(c, 0] \subseteq (b, 0]$. Then we obtain $(c, 0] \subseteq (a, 0]$ and $F(a) \leq F(c)$.

Therefore (F, A) is a partially ordered soft set with the soft set relation $\leq$.
Let $D, C)$ be a directed complete partially ordered soft set. Since $\mathbb{Z}^-$ is a directed complete partially ordered set, then $\sup (D, C)$ exists and $\sup (D, C) \in (F, A)$. Thus (F, A) is a directed complete partially ordered soft set. When we examine soft subsets of (F, A), for $F(x) \in (F, A)$,

$$(G, B) = \{G(b) \mid F(x) \leq G(b)\} = \{G(b) \mid (b, 0] \subseteq (x, 0]\}$$

that is; $B = \{b \in \mathbb{Z}^- \mid x \leq b\}$ and $G = F|_B$. Then $\uparrow (G, B) = (G, B)$. Also, for all directed soft sets with $\sup (D, C) \in (G, B)$, $(D, C)\widetilde{\cap}(G, B) \neq \Phi$ should be shown. Let $\sup (D, C) = G(b)$. Since $G(b) \in (G, B)$ then $b \in B$. From this for all $c \in C$, $D(c) \leq G(b)$. Then for all $c \in C$, $c \leq b$, then $b \in C$. Thus $(D, C)\widetilde{\cap}(G, B)$ contains at least $G(b)$. So $(D, C)\widetilde{\cap}(G, B) \neq \Phi$. Therefore these soft subsets (G, B) of (F, A) are the Scott open soft set sets.

5. Conclusion

In this paper, we improved partially ordered soft sets, and we gave some definitions for the ordered soft set relation. Also, soft Scott topology was introduced and some illustrative applications on soft Scott topology were given in this study. Moreover, one can construct the soft version of Isbell Topology, which is well known topology on function spaces by using the concepts proposed in this paper.

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Bekir Tanay (btanay@mu.edu.tr)
Department of Mathematics, Faculty of Arts and Sciences, Mugla Sıtkı Koçman University, postal code 48000 Muğla, Turkey

Gözde Yaylali (gozdeyaylali@mu.edu.tr)
Department of Mathematics, Faculty of Arts and Sciences, Mugla Sıtkı Koçman University, postal code 48000 Muğla, Turkey