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Intuitionistic fuzzy totally weakly generalized continuous mappings

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Abstract. The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy totally weakly generalized continuous mappings and intuitionistic fuzzy totally weakly generalized open mappings in intuitionistic fuzzy topological space. Some of their properties are explored.

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1. Introduction

Fuzzy set (FS) as proposed by Zadeh ([16]) in 1965, is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang ([2]) in 1968, there have been several generalizations of notions of fuzzy sets and fuzzy topology. By adding the degree of non-membership to FS, Atanassov ([1]) proposed intuitionistic fuzzy set (IFS) in 1986 which appeals more accurate to uncertainty quantification and provides the opportunity to precisely model the problem, based on the existing knowledge and observations. In 1997, Coker ([3]) introduced the concept of intuitionistic fuzzy topological space. In this paper, we introduce the notion of intuitionistic fuzzy totally weakly generalized continuous mappings and intuitionistic fuzzy totally weakly generalized open mappings in intuitionistic fuzzy topological space and study some of their properties. We provide some characterizations of intuitionistic fuzzy totally weakly generalized continuous mappings and establish the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

2. Preliminaries

Definition 2.1 ([1]). Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element x ∈ X to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

Definition 2.2 ([1]). Let A and B be IFSs of the forms $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$.
Then
(a)$A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,
(b)$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c)$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$,
(d)$A \cap$ B $= \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$,
(e)$A \cup$ B $= \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, the notation $A = \langle x, \mu_A, \nu_A\rangle$ shall be used instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are the *empty set* and the *whole set* of X, respectively.

Definition 2.3 ([3]). An *intuitionistic fuzzy topology* (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:
(a)$0_\sim, 1_\sim \in \tau$,
(b)$G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(c)$\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4 ([3]). Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

int(A)$= \cup\{G / G\ is\ an\ IFOS\ in\ X\ and\ G \subseteq A\}$,

cl(A)$= \cap\{K / K\ is\ an\ IFCS\ in\ X\ and\ A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5. An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ in an IFTS (X, τ) is said to be
(a)([6]) *intuitionistic fuzzy semi closed set* (IFSCS in short) if $int(cl(A)) \subseteq A$,
(b)([6]) *intuitionistic fuzzy α-closed set* (IFαCS in short) if $cl(int(cl(A))) \subseteq A$,

(c)([6]) *intuitionistic fuzzy pre-closed set* (IFPCS in short) if $cl(int(A)) \subseteq A$,
(d)([6]) *intuitionistic fuzzy regular closed set* (IFRCS in short) if $cl(int(A)) = A$,
(e)([15]) *intuitionistic fuzzy generalized closed set* (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,
(f)([13]) *intuitionistic fuzzy generalized semi closed set* (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,
(g)([12]) *intuitionistic fuzzy α generalized closed set* (IFαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,
(h)([5]) *intuitionistic fuzzy γ closed set* (IFγCS in short) if $int(cl(A)) \cap cl(int(A)) \subseteq A$.

An IFS A is called *intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set, intuitionistic fuzzy α generalized open set and intuitionistic fuzzy γ open set* (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS, IFαGOS and IFγOS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS, IFαGCS and IFγCS respectively.

Definition 2.6 ([8])**.** An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy weakly generalized closed set* (IFWGCS in short) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

The family of all IFWGCSs of an IFTS (X, τ) is denoted by IFWGC(X).

Definition 2.7 ([8])**.** An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ is said to be an *intuitionistic fuzzy weakly generalized open set* (IFWGOS in short) in an IFTS (X, τ) if the complement A^c is an IFWGCS in X.

The family of all IFWGOSs of an IFTS (X, τ) is denoted by IFWGO(X).

Remark 2.8 ([8])**.** Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFWGCS but the converses need not be true in general.

Definition 2.9 ([9])**.** Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy weakly generalized interior* and an *intuitionistic fuzzy weakly generalized closure* are defined by

wgint(A)= $\cup\{G/G\, is\, an\, IFWGOS\, in\, X\, and\, G \subseteq A\}$,
wgcl(A)= $\cap\{K/K\, is\, an\, IFWGCS\, in\, X\, and\, A \subseteq K\}$.

Definition 2.10 ([3])**.** Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle / y \in Y\}$ is an IFS in Y, then the *pre-image* of B under f is the IFS in X defined by $f^{-1}(B) = \left\{\left\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\right\rangle / x \in X\right\}$, where $f^{-1}(\mu_B(x)) = \mu_B(f(x))$.

If $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ is an IFS in X, then the *image* of A under f denoted by f(A), is the IFS in Y defined by $f(A) = \{\langle y, f(\mu_A(y)), f_-(\nu_A(y))\rangle / y \in Y\}$ where $f_-(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.11. Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be
(a)([4]) *intuitionistic fuzzy continuous* if $f^{-1}(B)$ is an IFCS in X for every IFCS B in Y,
(b)([10]) *intuitionistic fuzzy weakly generalized continuous* if $f^{-1}(B)$ is an IFWGCS in X for every IFCS B in Y,
(c)([11]) *intuitionistic fuzzy perfectly weakly generalized continuous mapping* if the inverse image of every IFWGCS in Y is an intuitionistic fuzzy clopen set in X,
(d)([7]) *intuitionistic fuzzy totally continuous mapping* if the inverse image of every IFCS in Y is an intuitionistic fuzzy clopen set in X,
(e)([9]) *intuitionistic fuzzy weakly generalized irresolute mapping* if $f^{-1}(B)$ is an IFWGCS in X for every IFWGCS B in Y,
(f)([14]) *intuitionistic fuzzy open mapping* if $f(A)$ is an IFOS in Y for every IFOS A in X.

Definition 2.12 ([8])**.** An IFTS (X, τ) is said to be an *intuitionistic fuzzy* $_wT_{1/2}$ *space* (IF$_wT_{1/2}$ space) if every IFWGCS in X is an IFCS in X.

Definition 2.13 ([8])**.** An IFTS (X, τ) is said to be an *intuitionistic fuzzy* $_{wg}T_q$ *space* (IF$_{wg}T_q$ space) if every IFWGCS in X is an IFPCS in X.

3. Intuitionistic fuzzy totally weakly generalized continuous mappings

In this section, we introduce intuitionistic fuzzy totally weakly generalized continuous mapping in an intuitionistic fuzzy topological space and study some of their properties.

Definition 3.1. An IFS A is called *intuitionistic fuzzy weakly generalized clopen set* in (X, τ) if it is both intuitionistic fuzzy weakly generalized open and intuitionistic fuzzy weakly generalized closed in (X, τ).

Definition 3.2. A mapping $f : (X, \tau) \to (Y, \sigma)$ from an IFTS (X, τ) into an IFTS (Y, σ) is said to be an *intuitionistic fuzzy totally weakly generalized continuous mapping* if the inverse image of every IFOS in Y is an intuitionistic fuzzy weakly generalized clopen set in X.

Theorem 3.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.*
(a) f is an intuitionistic fuzzy totally weakly generalized continuous mapping,
(b) $f^{-1}(B)$ is an intuitionistic fuzzy weakly generalized clopen set in X for each IFCS B in Y.

Proof. (a) $\Rightarrow$ (b): Let B be an IFCS in Y. Then B^c is an IFOS in Y. Since f is an intuitionistic fuzzy totally weakly generalized continuous mapping, $f^{-1}(B^c) = (f^{-1}(B))^c$ is an intuitionistic fuzzy weakly generalized clopen set in X. Hence $f^{-1}(B)$ is an intuitionistic fuzzy weakly generalized clopen set in X.
(b) $\Rightarrow$ (a): Let B be an IFOS in Y. Then B^c is an IFCS in Y. By hypothesis, $f^{-1}(B^c) = (f^{-1}(B))^c$ is an intuitionistic fuzzy weakly generalized clopen set in X, which implies $f^{-1}(B)$ is an intuitionistic fuzzy weakly generalized clopen set in X. Hence f is an intuitionistic fuzzy totally weakly generalized continuous mapping. $\square$

Theorem 3.4. *Every intuitionistic fuzzy totally weakly generalized continuous mapping is an intuitionistic fuzzy weakly generalized continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy totally weakly generalized continuous mapping and B be an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy weakly generalized clopen set in X. That is, $f^{-1}(B)$ is an IFWGOS and IFWGCS in X. Hence f is an intuitionistic fuzzy weakly generalized continuous mapping. □

Example 3.5. Let X = $\{a, b\}$, Y = $\{u, v\}$ and $T_1 = \langle x, (0.4, 0.3), (0.6, 0.7)\rangle$, $T_2 = \langle y, (0.4, 0.3), (0.6, 0.7)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy weakly generalized continuous mapping but not an intuitionistic fuzzy totally weakly generalized continuous mapping, since the IFS $T_2^c = \langle y, (0.6, 0.7), (0.4, 0.3)\rangle$ is an IFCS in Y but $f^{-1}(T_2^c) = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle$ is not an intuitionistic fuzzy weakly generalized clopen set in X.

Theorem 3.6. *Every intuitionistic fuzzy totally continuous mapping is an intuitionistic fuzzy totally weakly generalized continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy totally continuous mapping and B be an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Since every IFOS and IFCS is an IFWGOS and IFWGCS, $f^{-1}(B)$ is an IFWGOS and IFWGCS in X. Thus $f^{-1}(B)$ is an intuitionistic fuzzy weakly generalized clopen set in X. Hence f is an intuitionistic fuzzy totally weakly generalized continuous mapping. □

Example 3.7. Let X = $\{a, b\}$, Y = $\{u, v\}$ and $T_1 = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle$, $T_2 = \langle y, (0.5, 0.5), (0.5, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy totally weakly generalized continuous mapping but not an intuitionistic fuzzy totally continuous mapping, since the IFS $T_2^c = \langle y, (0.5, 0.5), (0.5, 0.5)\rangle$ is an IFCS in Y but $f^{-1}(T_2^c) = \langle x, (0.5, 0.5), (0.5, 0.5)\rangle$ is not an intuitionistic fuzzy clopen set in X.

Theorem 3.8. *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy totally weakly generalized continuous mapping from an from an IFTS (X, τ) into an IFTS (Y, σ) and (X, τ) an $IF_wT_{1/2}$ space. Then f is an intuitionistic fuzzy totally continuous mapping.*

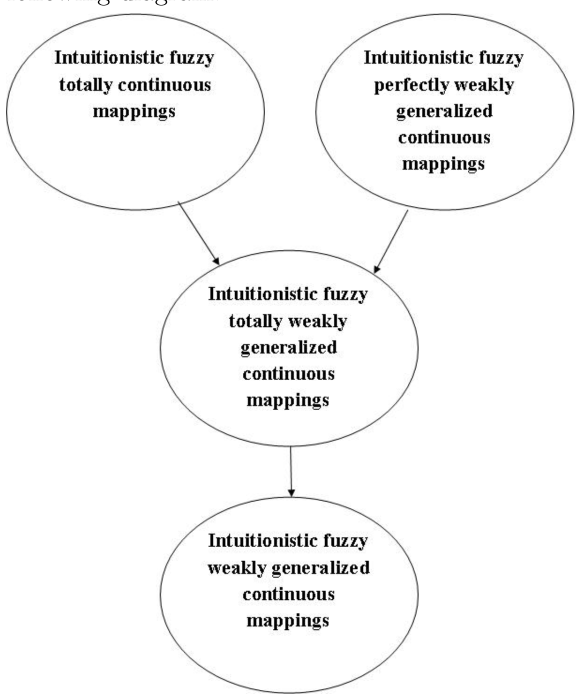
Proof. Let B be an IFOS in Y. Since f is an intuitionistic fuzzy totally weakly generalized continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy weakly generalized clopen set in X. Since (X, τ) is an $IF_wT_{1/2}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Hence f is an intuitionistic fuzzy totally continuous mapping. □

Theorem 3.9. *Every intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy totally weakly generalized continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping and B be an IFOS in Y. Since every IFOS is an IFWGOS in Y, B is an IFWGOS in Y. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Since every IFCS and IFOS are IFWGCS and IFWGOS,$f^{-1}(B)$ is an intuitionistic fuzzy weakly generalized clopen set in X. Hence f is an intuitionistic fuzzy totally weakly generalized continuous mapping. □

Example 3.10. Let X = $\{a, b\}$, Y = $\{u, v\}$ and $T_1 = \langle x, (0.6, 0.6), (0.3, 0.3)\rangle$, $T_2 = \langle y, (0.5, 0.5), (0.5, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy totally weakly generalized continuous mapping but not an intuitionistic fuzzy perfectly weakly genralized continuous mapping, since the IFS $T_2^c = \langle y, (0.5, 0.5), (0.5, 0.5)\rangle$ is an IFWGCS in Y but $f^{-1}(T_2^c) = \langle x, (0.5, 0.5), (0.5, 0.5)\rangle$ is not an intuitionistic fuzzy clopen set in X.

The relation among various types of intuitionistic fuzzy continuities are given in the following diagram.



The reverse implications are not true in general in the above diagram. In this diagram by "$A \to B$"we mean A implies B but not conversely.

Theorem 3.11. *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy totally weakly generalized continuous mapping from an from an IFTS (X, τ) into an IFTS (Y, σ) and (X, τ), (Y, σ) be $IF_wT_{1/2}$ spaces. Then f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping.*

Proof. Let B be an IFWGOS in Y. Since (Y, σ) is an $IF_wT_{1/2}$ space, B is an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy weakly generalized clopen set in X. Since (X, τ) is an $IF_wT_{1/2}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Hence f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping. □

Theorem 3.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy totally weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then $f(wgcl(A)) \subseteq cl(f(A))$ for every IFS A in X.*

Proof. Let A be an IFS in X. Then $cl(f(A))$ is an IFCS in Y. Since f is an intuitionistic fuzzy totally weakly generalized continuous mapping, $f^{-1}(cl(f(A)))$ is an intuitionistic fuzzy weakly generalized clopen set in X. Thus $f^{-1}(cl(f(A)))$ is an IFWGCS in X. Clearly $A \subseteq f^{-1}(cl(f(A)))$. Therefore $wgcl(A) \subseteq wgcl(f^{-1}(cl(f(A))))$ $= f^{-1}(cl(f(A)))$. Hence $f(wgcl(A)) \subseteq cl(f(A))$ for every IFS A in X. □

Theorem 3.13. *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy totally weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then $wgcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for every IFS B in Y.*

Proof. Let B be an IFS in Y. Then $cl(B)$ is an IFCS in Y. By hypothesis, $f^{-1}(cl(B))$ is an intuitionistic fuzzy weakly generalized clopen set in X. Thus $f^{-1}(cl(B))$ is an IFWGCS in X. Clearly $B \subseteq cl(B)$ implies $f^{-1}(B) \subseteq f^{-1}(cl(B))$. Therefore $wgcl(f^{-1}(B)) \subseteq wgcl(f^{-1}(cl(B))) = f^{-1}(cl(B))$. Hence $wgcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for every IFS B in Y. □

Theorem 3.14. *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy totally weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then $f^{-1}(int(B)) \subseteq wgint(f^{-1}(B))$ for every IFS B in Y.*

Proof. Let B be an IFS in Y. Then $int(B)$ is an IFOS in Y. By hypothesis, $f^{-1}(int(B))$ is an intuitionistic fuzzy weakly generalized clopen set in X. Thus $f^{-1}(int(B))$ is an IFWGOS in X. Clearly $int(B) \subseteq B$ implies $f^{-1}(int(B)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(int(B)) = wgint(f^{-1}(int(B))) \subseteq wgint(f^{-1}(B))$. Hence $f^{-1}(int(B)) \subseteq wgint(f^{-1}(B))$ for every IFS B in Y. □

Theorem 3.15. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be two intuitionistic fuzzy totally weakly generalized continuous mappings. Then their composition $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized continuous mapping if (Y, σ) an $IF_wT_{1/2}$ space.*

Proof. Let A be an IFCS in Z. Then $g^{-1}(A)$ is an intuitionistic fuzzy weakly generalized clopen set in Y, by hypothesis. Since (Y, σ) is an $IF_wT_{1/2}$ space, $g^{-1}(A)$ is an intuitionistic fuzzy clopen set in Y and hence an IFCS in Y. Further, since f is an intuitionistic fuzzy totally weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an intuitionistic fuzzy weakly generalized clopen set in X. Hence $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized continuous mapping. □

Theorem 3.16. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two mappings, then the following statements hold.*

(i) Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy totally weakly generalized continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ an intuitionistic fuzzy continuous mapping. Then their composition $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized continuous mapping.

(ii) Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy totally weakly generalized continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$an intuitionistic fuzzy weakly generalized continuous mapping. Then their composition $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized continuous mapping if (Y, σ) is an $IF_wT_{1/2}$ space.

(iii) Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy weakly generalized irresolute mapping and $g : (Y, \sigma) \to (Z, \delta)$ an intuitionistic fuzzy totally weakly generalized continuous mapping. Then their composition $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized continuous mapping.

Proof. (i) Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFCS in Y. Since f is an intuitionistic fuzzy totally weakly generalized continuous mapping, $f^{-1}(g^{-1}(A))$ $= (gof)^{-1}(A)$ is an intuitionistic fuzzy weakly generalized clopen set in X. Hence $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized continuous mapping.

(ii) Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFWGCS in Y. Thus $g^{-1}(A)$ is an IFCS in Y, as (Y, σ) is an $IF_wT_{1/2}$ space. Since f is an intuitionistic fuzzy totally weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an intuitionistic fuzzy weakly generalized clopen set in X. Hence $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized continuous mapping.

(iii) Let A be an IFCS in Z. Since g is an intuitionistic fuzzy totally weakly generalized continuous mapping, $g^{-1}(A)$ is an intuitionistic fuzzy weakly generalized clopen set in Y. Further, since f is an intuitionistic fuzzy weakly generalized irresolute mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an intuitionistic fuzzy weakly generalized clopen set in X. Hence $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized continuous mapping. □

4. Intuitionistic fuzzy totally weakly generalized open mappings

In this section, we introduce intuitionistic fuzzy totally weakly generalized open mappings in intuitionistic fuzzy topological space and study some of their properties.

Definition 4.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an *intuitionistic fuzzy totally weakly generalized open mapping* if the image of every IFOS in X is an intuitionistic fuzzy weakly generalized clopen set in Y.

Theorem 4.2. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.*

(a) f is an intuitionistic fuzzy totally weakly generalized open mapping,

(b) $f(B)$ is an intuitionistic fuzzy weakly generalized clopen set in Y for each IFCS B in X.

Proof. (a) ⇒ (b): Let B be an IFCS in X. Then B^c is an IFOS in X. Since f is an intuitionistic fuzzy totally weakly generalized open mapping, $f(B^c) = (f(B))^c$ is an intuitionistic fuzzy weakly generalized clopen set in Y. Hence $f(B)$ is an intuitionistic fuzzy weakly generalized clopen set in Y.

(b) ⇒ (a): Let B be an IFOS in X. Then B^c is an IFCS in X. By assumption, $f(B^c) = (f(B))^c$ is an intuitionistic fuzzy weakly generalized clopen set in Y, which implies $f(B)$ is an intuitionistic fuzzy weakly generalized clopen set in Y. Hence f is an intuitionistic fuzzy totally weakly generalized open mapping. □

Theorem 4.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.*

(a) Inverse of f is an intuitionistic fuzzy totally weakly generalized continuous mapping.

(b) f is an intuitionistic fuzzy totally weakly generalized open mapping.

Proof. (a) ⇒ (b): Let A be an IFOS in X. By assumption, $(f^{-1})^{-1}(A) = f(A)$ is an intuitionistic fuzzy weakly generalized clopen set in Y. Hence f is an intuitionistic fuzzy totally weakly generalized open mapping.

(b) ⇒ (a): Let B be an IFOS in X. Then f(B) is an intuitionistic fuzzy weakly generalized clopen set in Y. That is, $(f^{-1})^{-1}(B) = f(B)$ is an intuitionistic fuzzy weakly generalized clopen set in Y. Hence f^{-1} is an intuitionistic fuzzy totally weakly generalized continuous mapping. □

Theorem 4.4. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be two intuitionistic fuzzy totally weakly generalized open mappings. Then their composition $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized open mapping if (Y, σ) an $IF_wT_{1/2}$ space.*

Proof. Let A be an IFOS in X. Then $f(A)$ is an intuitionistic fuzzy weakly generalized clopen set in Y, by hypothesis. Since (Y, σ) is an $IF_wT_{1/2}$ space, $f(A)$ is an intuitionistic fuzzy clopen set in Y and hence an IFOS in Y. Further, since g is an intuitionistic fuzzy totally weakly generalized open mapping, $g(f(A)) = (gof)(A)$ is an intuitionistic fuzzy weakly generalized clopen set in Z. Hence $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized open mapping. □

Theorem 4.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy open mapping and $g : (Y, \sigma) \to (Z, \delta)$ an intuitionistic fuzzy totally weakly generalized open mapping. Then their composition $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized open mapping.*

Proof. Let A be an IFOS in X. By hypothesis, $f(A)$ is an IFOS in Y. Since g is an intuitionistic fuzzy totally weakly generalized open mapping, $g(f(A)) = (gof)(A)$ is an intuitionistic fuzzy weakly generalized clopen set in Z. Hence $gof : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy totally weakly generalized open mapping. □

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