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On fuzzy m-systems and n-systems of ordered semigroup

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Abstract. In this paper, we introduce the notions of fuzzy m-systems and fuzzy n-systems of ordered semigroup. We characterize weakly fuzzy prime ideal of ordered semigroup through fuzzy points, fuzzy m-system, m-system, and prime ideal in ordered semigroup. We introduce the concept of weakly fuzzy semiprime ideals of ordered semigroup. We characterize weakly fuzzy semiprime ideal of ordered semigroup through fuzzy points, fuzzy n-system, n-system, and semiprime ideal in ordered semigroup.

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1. Introduction

The important concept of a fuzzy set put forth by Zadeh in 1965[11] has opened up keen insights and applications in a wide range of scientific fields. Since then, many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, groupoids, topology, etc. The theory of fuzzy sets on ordered semigroups has been recently developed. Fuzzy sets in ordered semigroup were first studied by Kehayopulu and Tsingelis in[5], then they defined fuzzy analogies for several notations, which have proven useful in the theory of ordered semigroups. Dheena and Mohanraj[1] generalized fuzzy prime ideals in a semiring R to $(\lambda, \mu)-$fuzzy prime ideals. Kuroki[8] introduced fuzzy semiprime ideals of ordered semigroup. Xie[10] introduced the notion of ordered fuzzy points of an ordered semigroup and gave the characterization of prime fuzzy ideals of ordered semigroup. Kehayopulu[6] introduced the concepts of m-systems and n-systems in ordered semigroup and found the relations between m-systems and weakly prime ideals and between n-systems and weakly semiprime ideals.

Motivated by Xie, Dheena and Kehayopulu's work, we introduce the notions of fuzzy m-systems and fuzzy n-systems of ordered semigroup. We characterize weakly fuzzy prime ideal of ordered semigroup through fuzzy points, fuzzy m-system, m-system, and prime ideal in ordered semigroup. We introduce the notion of weakly fuzzy semiprime ideals of ordered semigroup. We characterize weakly fuzzy semiprime ideal of ordered semigroup through fuzzy points, fuzzy n-system, n-system, and semiprime ideal in ordered semigroup.

2. Preliminaries

Definition 2.1. By an ordered semigroup(po-semigroup), we mean a structure $(S, ., \leq)$ in which the following conditions are satisfied:

(OS1) $(S, \cdot)$is an semigroup.

(OS2) $(S, \leq)$is a poset.

(OS3) $a \leq b \Rightarrow xa \leq xb$ and $ax \leq bx$ for all $a, b, x \in S$.

For $A \subseteq S$, we denote $(A] := \{t \in S | t \leq h \ for \ some \ h \in A\}$. If $A = \{a\}$, then we write $(a]$ instead of $\{(a]\}$. For non-empty subsets A, B of S, we denote

$$AB := \{ab | a \in A, b \in B\}$$

Definition 2.2. Let $(S, \cdot, \leq)$ be an ordered semigroup. A non-empty subset A of S is called an ideal of S if

1. For all $a \in A$, for all $s \in S$, $s \leq a$ implies $s \in A$.
2. $AS \subseteq A$ and $SA \subseteq A$.

Definition 2.3. Let $(S, ., \leq)$ be an ordered semigroup. An ideal P in S is said to be prime ideal if $A \cdot B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$, for the ideals $A, B$of S.

By a fuzzy set μ of S, we mean a mapping $\mu : S \longrightarrow [0, 1]$.

Definition 2.4. A fuzzy set μ of S of the form

$$\mu(y) = \begin{cases} t \in (0, 1] & \text{if } \ y = x, \\ 0 & \text{if } \ y \neq x. \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t.

For all $t \in [0, 1]$, a fuzzy point x_t is said to belong to a fuzzy set μ if $\mu(x) \geq t$ and it is denoted as $x_t \in \mu$. We denote $x_t \bar{\in} \mu$ if $\mu(x) < t$.

Definition 2.5. Let μ be a fuzzy set of an ordered semigroup S. Then the level set μ_t and strong level set $\overline{\mu_t}$ are defined as follows:

$$\mu_t = \{x | \mu(x) \geq t\}$$

$$\overline{\mu_t} = \{x | \mu(x) > t\}$$

Definition 2.6. Let μ be a fuzzy set of an ordered semigroup S. Then μ is said to be a fuzzy ideal of S if

1. $x \leq y \Rightarrow \mu(x) \geq \mu(y)$.
2. $\mu(xy) \geq \mu(x) \vee \mu(y)$, for all $x, y \in S$.

Definition 2.7. Let A be a non-empty subset of an ordered semigroup S. Then the characteristic function $\chi_A(x)$ is a fuzzy set of S defined as follows:

$$\chi_A(x) = \begin{cases} 1 & \text{if } \; x \in A \\ 0 & \text{otherwise} \end{cases}$$

The characteristic function of S is denoted by 1. Clearly $t\chi_A$ is a generalization of the characteristic mapping χ_A of A.

Definition 2.8. In ordered semigroups, a fuzzy point a_t is defined as $t\chi_{(a]}$. We denote $(a_t) = t\chi_{(a)}$, where (a) is the ideal generated by a.

Definition 2.9. [9] For $a \in S$, we define

$$A_a = \{(y, z) \in S \times S | a \leq yz\}$$

Definition 2.10. Let λ and μ be any fuzzy sets of an ordered semigroup S. The fuzzy product of λ and μ denoted by $\lambda \circ \mu$ is defined as follows:

$$(\lambda \circ \mu)(a) = \begin{cases} \bigvee\limits_{(y,z) \in A_a} \{\lambda(y) \wedge \mu(z)\} & if \; A_a \neq \varnothing \\ 0 & if \; A_a = \varnothing. \end{cases}$$

Lemma 2.11 ([9])**.** *Let S be an ordered semigroup and μ a fuzzy set of S. Then μ is a fuzzy ideal of S if and only if*

(1) $x \leq y$ implies $\mu(x) \geq \mu(y)$, for all $x, y \in S$.

(2) $\mu \circ 1 \subseteq \mu$ and $1 \circ \mu \subseteq \mu$.

Lemma 2.12 ([10])**.** *Let A, B be any non-empty subset of S. Then for any $t \in (0, 1]$, the following statements are true.*

1. $t\chi_A \circ t\chi_B = t\chi_{(AB]}$

2. $t\chi_A \cup t\chi_B = t\chi_{A \cup B}$

3. $1 \circ t\chi_A = t\chi_{(SA]}, t\chi_A \circ 1 = t\chi_{(AS]}$ and $1 \circ t\chi_A \circ 1 = t\chi_{(SAS]}$.

4. If A is an ideal(right,left) of S, then $t\chi_A$ is a fuzzy ideal(fuzzy left, fuzzy right) of S.

3. Fuzzy m-system and weakly fuzzy prime ideal

Hereafter S denotes an ordered semigroup unless otherwise specified.

Definition 3.1. Let μ be a fuzzy ideal of S. Then μ is a fuzzy prime ideal of S if $\lambda \circ \sigma \subseteq \mu$ implies $\lambda \subseteq \mu$ or $\sigma \subseteq \mu$, for the fuzzy ideals λ and σ of S.

Definition 3.2. A fuzzy ideal μ of S is said to be a weakly fuzzy prime ideal if $t\chi_A \circ t\chi_B \subseteq \mu$ implies $t\chi_A \subseteq \mu$ or $t\chi_B \subseteq \mu$, for the ideals A and B in S and for all $t \in [0, 1]$.

Theorem 3.3 ([10])**.** *Let μ be a fuzzy ideal of S. Then μ is a weakly fuzzy prime ideal of S if and only if, for any $x, y \in S$ and $t > 0$, $x_t \circ 1 \circ y_t \subseteq \mu$ implies $x_t \in \mu$ or $y_t \in \mu$.*

Definition 3.4. A non-empty subset A of S is called an m-system in S if for each $a, b \in A$ there exist $c \in A$ and $x \in S$ such that $c \leq axb$.

Definition 3.5. Let μ be a fuzzy set of S. Then μ is said to be a fuzzy m-system of S if $\mu(x) > t$ and $\mu(y) > t$ imply there exists $c \in S$ such that $\mu(c) > t$ and $c \leq xsy$.

Theorem 3.6 ([10]). *Let μ be a fuzzy set of S. Then μ is a fuzzy ideal of S if and only if μ_t is an ideal in S for $t \in [0,1]$ whenever non-empty.*

Lemma 3.7. *Let μ be a fuzzy set of S. Then $\overline{(1-\mu)_t} = \mu^c_{1-t}$.*

Proof. Let $x \in S$. Then,

$$\begin{aligned} x \in \overline{(1-\mu)_t} &\Leftrightarrow 1-\mu(x) > t \\ &\Leftrightarrow -1+\mu(x) < -t \\ &\Leftrightarrow \mu(x) < 1-t \\ &\Leftrightarrow x \in \mu^c_{1-t}. \end{aligned}$$

□

Lemma 3.8. *Let A be a subset of S and μ be a fuzzy ideal of S. Then the following statements are equivalent.*

1. $t\chi_A \subseteq \mu,\ t \in [0,1]$.
2. $A \subseteq \mu_t,\ t \in [0,1]$.

Proof. (1) $\Rightarrow$ (2) Let $t\chi_A \subseteq \mu,\ t \in [0,1]$. Then $\mu(x) \geq t\chi_A(x)$, for all $x \in S$. Let $x \in A$. Then $t\chi_A(x) = t$. Thus $\mu(x) \geq t$ implies $x \in \mu_t$. Therefore $A \subseteq \mu_t$.

(2) $\Rightarrow$ (1) Let $x \in A$. Then $t\chi_A(x) = t,\ t \in [0,1]$. Now, $A \subseteq \mu_t$ implies $\mu(x) \geq t = t\chi_A(x)$. If $x \notin A$, then $t\chi_A(x) = 0$. Hence $\mu(x) \geq 0 = t\chi_A(x)$, for all x. Therefore $t\chi_A \subseteq \mu$. □

Theorem 3.9. *Let μ be a fuzzy ideal of S. Then the following statements are equivalent.*

(1)μ *is a weakly fuzzy prime ideal of* S.
(2)$a_t \circ 1 \circ b_t \subseteq \mu$ *implies* $a_t \in \mu$ *or* $b_t \in \mu$ *for* $a, b \in S,\ t \in [0,1]$.
(3)$1-\mu$ *is a fuzzy m-system of* S.
(4)$\overline{(1-\mu)_t}$ *is a m-system for all* $t \in [0,1]$ *whenever non-empty.*
(5)μ^c_t *is a m-system for all* $t \in [0,1]$ *whenever non-empty.*
(6)μ_t *is a prime ideal in* S *for all* $t \in [0,1]$ *whenever non-empty.*

Proof. (1) $\Rightarrow$ (2) By Theorem 3.3, $a_t \circ 1 \circ b_t \subseteq \mu$ implies $a_t \in \mu$ or $b_t \in \mu$ for $a, b \in S,\ t \in [0,1]$.

(2) $\Rightarrow$ (3) Let $1-\mu(x) > t$ and $1-\mu(y) > t,\ t \in [0,1]$. Then by Theorem 3.3, $x_{1-t}\overline{\in}\mu$ and $y_{1-t}\overline{\in}\mu$ imply $x_{1-t} \circ 1 \circ y_{1-t} \nsubseteq \mu$. Then there exists $c \in S$ such that $(x_{1-t} \circ 1 \circ y_{1-t})(c) = 1-t > \mu(c)$ and $c \in (xSy]$. Now, $1-t > \mu(c)$ and $c \in (xSy]$ imply $1-\mu(c) > t$ and $c \leq xsy$. Thus $1-\mu$ is a fuzzy m-system.

(3) $\Rightarrow$ (4) Let $x, y \in \overline{(1-\mu)_t}$ for $t \in [0,1]$. Then $1-\mu(x) > t$ and $1-\mu(y) > t$ imply there exists $c \in S$ such that $1-\mu(c) > t$ and $c \leq xsy$. Thus $\overline{(1-\mu)_t}$ is a m-system for all $t \in [0,1]$ whenever non-empty.

(4) $\Rightarrow$ (5) By Lemma 3.7, $\overline{(1-\mu)_t} = \mu^c_{1-t} = \mu^c_s$, where $s = 1-t \in [0,1]$. Then μ^c_t is a m-system for all $t \in [0,1]$ whenever non-empty.

(5) $\Rightarrow$ (6) By Theorem 3.6, μ_t is an ideal whenever non-empty. If there exist ideals A and B in S such that $A \cdot B \subseteq \mu_t$ with $A \nsubseteq \mu_t$ and $B \nsubseteq \mu_t$, then there exist

$x \in A$ and $y \in B$ such that $x \notin \mu_t$ and $y \notin \mu_t$. Then $x, y \in \mu_t^c$. Now, $x, y \in \mu_t^c$ implies there exists $c \in \mu_t^c$ such that $c \leq xsy$. Then $c \in (xSy] \subseteq (A](B] = (AB] = AB \subseteq \mu_t$, which is a contradiction. Therefore μ_t is a prime ideal of S for all $t \in [0,1]$ whenever non-empty.

(6) $\Rightarrow$ (1) Let A and B be ideals in S such that $t\chi_A \circ t\chi_B \subseteq \mu$. Then by Lemma 2.12, $t\chi_{(AB]} \subseteq \mu$. Thus $A \cdot B \subseteq (AB] \subseteq \mu_t$ implies $A \subseteq \mu_t$ or $B \subseteq \mu_t$. Then by Lemma 3.8, $t\chi_A \subseteq \mu$ or $t\chi_B \subseteq \mu$. Thus μ is a weakly fuzzy prime ideal of S. □

4. Fuzzy n-system and weakly fuzzy semiprime ideal

Definition 4.1. An ideal I in S is said to be a semiprime ideal if $A \cdot A \subseteq I$ implies $A \subseteq I$, for an ideal A in S.

Definition 4.2. A fuzzy ideal μ is said to be a fuzzy semiprime ideal of S if $\lambda \circ \lambda \subseteq \mu$ implies $\lambda \subseteq \mu$ for fuzzy ideal λ of S.

Definition 4.3. A fuzzy ideal μ is said to be a weakly fuzzy semiprime ideal of S if $t\chi_A \circ t\chi_A \subseteq \mu$ implies $t\chi_A \subseteq \mu$ for an ideal A in S and for all $t \in [0,1]$.

Theorem 4.4. *Let A be a subset of S and μ be a fuzzy set of S. Then μ is a weakly fuzzy semiprime ideal of S if and only if $(t\chi_A)^n \subseteq \mu$ implies $t\chi_A \subseteq \mu$ for all $n \in \mathbb{N}$.*

Proof. Assume that μ is a weakly fuzzy semiprime ideal of S. Then $(t\chi_A)^2 = t\chi_A \circ t\chi_A \subseteq \mu$ implies $t\chi_A \subseteq \mu$. Thus the result is true for $n = 2$. Assume the result is true for all k, $k < n$. If n is even, then $n = 2m$ and $(t\chi_A)^{2m} \subseteq \mu$ implies $(t\chi_A)^m \circ (t\chi_A)^m \subseteq \mu$ implies $(t\chi_A)^m \subseteq \mu$. Then by our assumption, $t\chi_A \subseteq \mu$. If n is odd, then $n = 2m + 1$. Now $(t\chi_A)^{2m+1} \subseteq \mu$. Then $(t\chi_A) \circ (t\chi_A)^{2m+1} \subseteq 1 \circ (t\chi_A)^{2m+1} \subseteq (1 \circ t\chi_A) \circ (t\chi_A)^{2m} \subseteq t\chi_A \circ (t\chi_A)^{2m} = (t\chi_A)^{2m+1} \subseteq \mu$. Thus $(t\chi_A)^{2m+2} \subseteq \mu$, where $m < 2m = n-1$. Therefore $(t\chi_A)^{m+1} \circ (t\chi_A)^{m+1} \subseteq \mu$, implies $(t\chi_A)^{m+1} \subseteq \mu$. By assumption, $(t\chi_A)^{m+1} \subseteq \mu$ implies $t\chi_A \subseteq \mu$. Thus $(t\chi_A)^n \subseteq \mu$ implies $t\chi_A \subseteq \mu$. By induction, $(t\chi_A)^n \subseteq \mu$ implies $t\chi_A \subseteq \mu$ for all n.

Conversely, by taking $n = 2$, $(t\chi_A)^2 = t\chi_A \circ t\chi_A \subseteq \mu$ implies $t\chi_A \subseteq \mu$. Therefore μ is a weakly fuzzy semiprime ideal of S. □

Definition 4.5. A non-empty subset A of S is called an n-system if for each $a \in A$ there exist $c \in A$ and $s \in S$ such that $c \leq asa$.

Definition 4.6. Let μ be a fuzzy set of S. Then μ is said to be a fuzzy n-system of S if $\mu(x) > t$ implies there exists $c \in S$ such that $\mu(c) > t$ and $c \leq xsx$.

Theorem 4.7. *Let μ be a fuzzy ideal of S. Then the following statements are equivalent.*

(1) μ is a weakly fuzzy semiprime ideal of S.
(2) $a_t \circ 1 \circ a_t \subseteq \mu$ implies $a_t \in \mu$ for $a \in S,\ t \in [0,1]$.
(3) $1 - \mu$ is a fuzzy n-system of S.
(4) $\overline{(1-\mu)}_t$ is a n-system for all $t \in [0,1]$ whenever non-empty.
(5) μ_t^c is a n-system for all $t \in [0,1]$ whenever non-empty.
(6) μ_t is a semiprime ideal in S for all $t \in [0,1]$ whenever non-empty.

Proof. (1) $\Rightarrow$ (2) Let $a_t \circ 1 \circ a_t \subseteq \mu$. Then by Lemma 2.12,

$$\begin{aligned} t\chi_{(SaS]} \circ t\chi_{(SaS]} &= (1 \circ a_t \circ 1) \circ (1 \circ a_t \circ 1) \\ &= 1 \circ a_t \circ (1 \circ 1) \circ a_t \circ 1 \\ &\subseteq 1 \circ a_t \circ 1 \circ a_t \circ 1 \\ &\subseteq 1 \circ \mu \circ 1 \\ &\subseteq \mu. \end{aligned}$$

Thus $t\chi_{(SaS]} \subseteq \mu$. Therefore $1 \circ a_t \circ 1 \subseteq \mu$. Now,

$$\begin{aligned} (a_t) \circ (a_t) \circ (a_t) = {} & (a_t \cup 1 \circ a_t \cup a_t \circ 1 \cup 1 \circ a_t \circ 1) \circ (a_t \cup 1 \circ a_t \cup a_t \circ 1 \cup 1 \circ a_t \circ 1) \\ & \circ (a_t \cup 1 \circ a_t \cup a_t \circ 1 \cup 1 \circ a_t \circ 1) \\ & \subseteq 1 \circ (a_t \cup 1 \circ a_t \cup a_t \circ 1 \cup 1 \circ a_t \circ 1) \circ 1 \\ & = (1 \circ a_t \cup 1 \circ 1 \circ a_t \cup 1 \circ a_t \circ 1 \cup 1 \circ 1 \circ a_t \circ 1) \circ 1 \\ & = (1 \circ a_t \circ 1) \cup (1 \circ 1 \circ a_t \circ 1) \cup (1 \circ a_t \circ 1 \circ 1) \cup (1 \circ 1 \circ a_t \circ 1 \circ 1) \\ & \subseteq 1 \circ a_t \circ 1 \\ & \subseteq \mu. \end{aligned}$$

Hence by Theorem 4.4, $(a_t)^3 \subseteq \mu$ implies $a_t \in \mu$.

(2) $\Rightarrow$ (3) Let $1 - \mu(x) > t, t \in [0,1]$. Then by Theorem 3.3, $x_{1-t} \bar{\in} \mu$ implies $x_{1-t} \circ 1 \circ x_{1-t} \nsubseteq \mu$. Then there exists $c \in S$ such that $(x_{1-t} \circ 1 \circ x_{1-t})(c) = 1-t > \mu(c)$ and $c \in (xSx]$. Now, $1 - t > \mu(c)$ and $c \in (xSx]$ imply $1 - \mu(c) > t$ and $c \leq xsx$. Thus $1 - \mu$ is a fuzzy n-system.

(3) $\Rightarrow$ (4) Let $x \in \overline{(1 - \mu)}_t$ for $t \in [0,1]$. Then $1 - \mu(x) > t$ implies there exists $c \in S$ such that $1 - \mu(c) > t$ and $c \leq xsx$. Thus $c \in \overline{(1 - \mu)}_t$ and $c \leq xsx$. Therefore $\overline{(1 - \mu)}_t$ is a n-system for all $t \in [0,1]$ whenever non-empty.

(4) $\Rightarrow$ (5)By Lemma 3.7, $\overline{(1 - \mu)}_t = \mu^c_{1-t} = \mu^c_s$, where $s = 1 - t \in [0,1]$. Then μ^c_t is a m-system for for all $t \in [0,1]$ whenever non-empty.

(5) $\Rightarrow$ (6) Let A be an ideal in S such that $A \cdot A \subseteq \mu_t,\ t \in [0,1]$. If there exists $a \in A$ but $a \notin \mu_t$, then $a \in \mu^c_t$. Now, $a \in \mu^c_t$ implies there exists $c \in S$ such that $c \in \mu^c_t$ and $c \leq axa,\ x \in S$. Then $c \in (aSa] \subseteq A \cdot A \subseteq \mu_t$ which is a contradiction. Thus $A \subseteq \mu_t$. Therefore μ_t is a semiprime ideal in S for all $t \in [0,1]$ whenever non-empty.

(6) $\Rightarrow$ (1) Let A be an ideal of S such that $t\chi_A \circ t\chi_A \subseteq \mu$. Then by Lemma 2.12, $t\chi_A \circ t\chi_A = t\chi_{(A \cdot A]} = t\chi_{A \cdot A} \subseteq \mu$, implies $A \cdot A \subseteq \mu_t$. Then $A \subseteq \mu_t$. Thus by Lemma 3.8, $t\chi_A \subseteq \mu$. Therefore μ is a weakly fuzzy semiprime ideal of S. □

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