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Intuitionistic fuzzy soft matrix theory and its application in medical diagnosis

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Abstract. This paper is an attempt to introduce the basic concept of intuitionistic fuzzy soft matrix theory. Further the concept of intuitionistic fuzzy soft matrix product has been applied to solve a problem in medical diagnosis.

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1. Introduction

Fuzzy set introduced by Zadeh [14] consist of membership function of a certain set of data related to problem but intuitionistic fuzzy set introduced by Atanassov [2] consist of membership function as well as non membership function of a certain set of data related to problem. Thus one can consider fuzzy sets as generalization of classical or crisp sets and intuitionistic fuzzy sets are generalization of fuzzy sets. Hence intuitionistic fuzzy sets can be more relevant for application for solutions of decision making problems particularly in medical diagnosis, marketing and financial services where there is more chances of non null hesitation factor. In 1999, Molodtsov [10] introduced the theory of soft sets, which is a new approach to vagueness. In 2003, Maji et al. [6] studied the theory of soft sets initiated by Molodtsov [10] and developed several basic notions of Soft Set Theory. At present, researchers are contributing a lot on the extension of soft set theory. In 2005, Pei and Miao [13] and Chen et al. [6] studied and improved the findings of Maji et al. [8]. Maji et al. [7] initiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan's Law etc. These results were further revised and improved by Ahmad and Kharal [1]. Moreover Maji et al. [9] extended soft sets to intuitionistic fuzzy soft sets. Intuitionistic fuzzy soft set theory is a combination of soft sets and intuitionistic fuzzy sets initiated by Atanassov [2]. One of the important theory of mathematics which has a vast application in science and engineering is the theory of matrices. But the classical matrix theory has some restrictions in solving the problems involving uncertainties. In [11], Neog and Sut proposed a matrix representation of a fuzzy soft set using the notion of extended fuzzy set initiated by Baruah [4] and successfully applied the proposed notion of fuzzy soft matrix in certain decision making problems [11,12].In [5] Baruah expresses fuzziness in terms of randomness in the measure theoretic sense. In this paper, we extend the notion of intuitionistic fuzzy soft matrices which is supported by a decision making problem in medical diagnosis.

2. Preliminaries

In this section, we first recall some basic concepts which would be used in the sequel.

Definition 2.1. [2] Let $A = (x, \mu_A(x), \nu_B(x)); x \in U)$ and
$$B = (x, \mu_B(x), \nu_B(x)); x \in U)$$
be two intuitionistic fuzzy sets defined over the same universe U. Then the operations union and intersection are defined as
$A \cup B = \{x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x)); x \in U\}$
$A \cap B = \{x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x)); x \in U\}$

Definition 2.2. [2] An intuitionistic fuzzy set A over the universe U defined as $A = (x, 0, 1) : x \in U)$ is said to be intuitionistic fuzzy null set and is denoted by $\overline{0}$.

Definition 2.3. [2] An intuitionistic fuzzy set A over the universe U defined as $A = (x, 0, 1) : x \in U)$ is said to be intuitionistic fuzzy absolute set and is denoted by $\overline{1}$.

Definition 2.4. [10] A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U. In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\epsilon), \epsilon \in E$, from this family may be considered as the set of ϵ -element of the soft set (F, E) or as the set of ϵ -approximate elements of the soft set.

Definition 2.5. [9] Let U be an initial universe set and E be the set of parameters. Let IF^U denote the collection of all intuitionistic fuzzy subsets of U. Let $A \subseteq E$. A pair (F, A) is called an intuitionistic fuzzy soft set over U where F is a mapping given by $F : A \to IF^U$.

Definition 2.6. [9] Union of two intuitionistic fuzzy soft sets (F, A) and (G, B) over (U, E) is an Intuitionistic fuzzy soft set (H, C) where $C = A \cup B$ and $\forall \epsilon \in C$,

$$H(\epsilon) = \begin{cases} F(\epsilon) & if \epsilon \in A - B \\ G(\epsilon) & if \epsilon \in A - B \\ F(\epsilon) \cup G(\epsilon) & if \epsilon \in A \cap B \end{cases}$$

and is written as $(F, A)\widetilde{\cup}(G, B) = (H, C)$

Definition 2.7. [9] Let (F, A) and (G, B) be two intuitionistic fuzzy soft sets over (U, E). Then intersection (F, A) and (G, B) is an intuitionistic fuzzy soft set (H, C) where $C = A \cap B$ and $\forall \epsilon \in C, H(\epsilon) = F(\epsilon) \cap G(\epsilon)$. We write $(F, A) \widetilde{\cap} (G, B) = (H, C)$.

Definition 2.8. [3] A intuitionistic fuzzy soft set (F, A) over U is said to be null intuitionistic fuzzy soft set $\forall \epsilon \in A, F(\epsilon)$ is the null intuitionistic fuzzy set φ. In other words, for an absolute fuzzy soft set (F, A), $\forall \epsilon \in A, F(\epsilon) = \{x, 0, 1; x \in U\}$.

Definition 2.9. [3] A intuitionistic fuzzy soft set (F, A) over U is said to be absolute intuitionistic fuzzy soft set if $\forall \epsilon \in A, F(\epsilon)$ is the absolute intuitionistic fuzzy set. In other words, for an absolute intuitionistic fuzzy soft set $(F, A), \forall \epsilon \in A, F(\epsilon) = \{x, 1, 0; x \in U\}$.

Definition 2.10. [9] The complement of an intuitionistic fuzzy soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \rightarrow IF^U$ is a mapping given by $F^c(\sigma) = (F(-\sigma))^c$ for all $\sigma \in \rceil A$.

3. Intuitionistic fuzzy soft matrices

Definition 3.1. Let $U = \{c_1, c_2, c_3, \dots c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \dots e_n\}$. Then the intuitionistic fuzzy soft set can be expressed in matrix form $A = [a_{ij}]_{m \times n}$ or simply by $[a_{ij}], i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n$ and $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$; where $\mu_j(c_i)$ and $\nu_j(c_i)$ represent the intuitionistic fuzzy membership value and intuitionistic fuzzy non membership value respectively of c_i. We can represent an intuitionistic fuzzy soft set with its intuitionistic fuzzy soft matrix. The set of all $m \times n$ intuitionistic fuzzy soft matrices over U will be denoted by $IFSM_{m \times n}$.

Definition 3.2. Let $A = [a_{ij}] \in IFSM_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. If m = 1, then A is called an intuitionistic fuzzy soft row matrix. In other words, if the universe under consideration contains only one single element, we get an intuitionistic fuzzy soft row matrix.

Definition 3.3. Let $A = [a_{ij}] \in IFSM_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. If n = 1, then A is called an intuitionistic fuzzy soft column matrix. In other words, if the set of parameters under consideration contains only one single parameter, we get an intuitionistic fuzzy soft column matrix.

Definition 3.4. Let $A = [a_{ij}] \in IFSM_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. If $m \neq n$ then A is called an intuitionistic fuzzy soft rectangular matrix.

Definition 3.5. Let $A = [a_{ij}] \in IFSM_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. If m = n, then A is called an intuitionistic fuzzy soft square matrix.

Definition 3.6. Let $A = [a_{ij}] \in IFSM_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$, then the elements $a_{11}, a_{12}, \dots a_{mm}$ are called the diagonal elements and the line along which they lie is called the principal diagonal of the intuitionistic fuzzy soft matrix.

Definition 3.7. Let $A = [a_{ij}] \in IFSM_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. Then A is called intuitionistic fuzzy soft diagonal matrix if m = n and $\mu_j(c_i) = \nu_j(c_i) = 0 \forall i \neq j$ i.e m = n and $a_{ij} = (0, 0) \forall i \neq j$.

Definition 3.8. Let $A = [a_{ij}] \in IFSM_{m\times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. Then A is called intuitionistic fuzzy soft scalar matrix if $m = n$, $a_{ij} = (\mu_j(c_i), \nu_j(c_i)) \forall i = j$ and $\mu_j(c_i) = \nu_j(c_i) = 0 \forall i \neq j$.

Definition 3.9. Let $A = [a_{ij}] \in IFSM_{m\times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. Then A is called intuitionistic fuzzy soft zero (or null) matrix denoted by $[0]_{m\times n}$, or simply by [0] if $\mu_j(c_i) = \nu_j(c_i) = 0$ for all i and j i.e $a_{ij} = (0, 0) \forall i, j$.

Definition 3.10. Let $A = [a_{ij}] \in IFSM_{m\times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. Then A is called intuitionistic fuzzy soft unit or identity matrix if $m = n$, $a_{ij} = (\mu_j(c_i), \nu_j(c_i)) = (0, 0)$ for all $i \neq j$ and $a_{ij} = (1, 0) \forall i = j$.

Definition 3.11. Let $A = [a_{ij}] \in IFSM_{m\times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. Then A is called intuitionistic fuzzy soft upper triangular matrix if $m = n$, $a_{ij} = (0, 0) \forall i > j$.

Definition 3.12. Let $A = [a_{ij}] \in IFSM_{m\times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. Then A is called intuitionistic fuzzy soft lower triangular matrix if $m = n$,$a_{ij} = (0, 0) \forall i < j$. An intuitionistic fuzzy soft matrix is said to be triangular if it is either intuitionistic fuzzy soft lower or intuitionistic fuzzy soft upper triangular matrix.

Definition 3.13. Let the intuitionistic fuzzy soft matrices corresponding to the intuitionistic fuzzy soft sets (F, E) and (G, E) be $A = [a_{ij}], B = [b_{ij}] \in IFSM_{m\times n}; a_{ij} = (\mu_{j1}(c_i), \nu_{j1}(c_i))$ and $b_{ij} = (\mu_{j2}(c_i), \nu_{j2}(c_i)), i = 1, 2, 3 \ldots m; j = 1, 2, 3 \ldots n$. Then A and B are called intuitionistic fuzzy soft equal matrices denoted by A = B, if $\mu_{j1}(c_i) = \mu_{j2}(c_i)$ and $\nu_{j1}(c_i) = \nu_{j2}(c_i) \forall i, j$.

Definition 3.14. Let $U = \{c_1, c_2, c_3 \ldots c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3 \ldots e_n\}$. Let the set of all $m \times n$ intuitionistic fuzzy soft matrices over U be $IFSM_{m\times n}$. Let $A, B \in IFSM_{m\times n}$, where $A = \lfloor a_{ij} \rfloor_{m\times n}, a_{ij} = (\mu_{j1}(c_i), \nu_{j1}(c_i))$ and $B = \lfloor b_{ij} \rfloor_{m\times n}, b_{ij} = (\mu_{j2}(c_i), \nu_{j2}(c_i))$. We define the operation 'addiion (+)' between A and B as $A + B = C$, where $C = \lfloor c_{ij} \rfloor_{m\times n}, c_{ij} = (max(\mu_{j1}(c_i), \mu_{j2}(c_i)), min(\nu_{j1}(c_i), \nu_{j2}(c_i)))$.

Example 3.15. Let $U = \{c_1, c_2, c_3, c_4\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3\}$. We consider the intuitionistic fuzzy soft sets

$$
\begin{aligned}
(F,E) = \Big\{ F(e_1) &= \{(c_1, 0.3, 0.5), (c_2, 0.5, 0.4), (c_3, 0.6, 0.1), (c_4, 0.5, 0.3)\} \\
F(e_2) &= \{(c_1, 0.7, 0.2), (c_2, 0.9, 0.05), (c_3, 0.7, 0.09), (c_4, 0.8, 0.04)\} \\
F(e_3) &= \{(c_1, 0.6, 0.3), (c_2, 0.7, 0.08), (c_3, 0.7, 0.03), (c_4, 0.3, 0.4)\} \Big\}
\end{aligned}
$$

$$
\begin{aligned}
(G,E) = \Big\{ G(e_1) &= \{(c_1, 0.8, 0.2), (c_2, 0.7, 0.1), (c_3, 0.5, 0.3), (c_4, 0.4, 0.3)\} \\
G(e_2) &= \{(c_1, 0.9, 0.1), (c_2, 0.9, 0.05), (c_3, 0.8, 0.07), (c_4, 0.7, 0.06)\} \\
G(e_3) &= \{(c_1, 0.5, 0.35), (c_2, 0.9, 0.06), (c_3, 0.6, 0.2), (c_4, 0.8, 0.15)\} \Big\}
\end{aligned}
$$

The intuitionistic fuzzy soft matrices representing these two intuitionistic fuzzy soft sets are respectively

$$A = \begin{pmatrix} (0.3, 0.5) & (0.7, 0.2) & (0.6, 0.3) \\ (0.5, 0.4) & (0.9, 0.05) & (0.7, 0.08) \\ (0.6, 0.3) & (0.7, 0.09) & (0.7, 0.03) \\ (0.5, 0.3) & (0.8, 0.04) & (0.3, 0.4) \end{pmatrix}_{4\times 3}$$

and

$$B = \begin{pmatrix} (0.8, 0.2) & (0.9, 0.1) & (0.5, 0.35) \\ (0.7, 0.1) & (0.9, 0.05) & (0.9, 0.06) \\ (0.5, 0.3) & (0.8, 0.07) & (0.6, 0.2) \\ (0.4, 0.3) & (0.7, 0.06) & (0.8, 0.15) \end{pmatrix}_{4\times 3}$$

Here $A + B = \begin{pmatrix} (0.8, 0.2) & (0.9, 0.1) & (0.6, 0.3) \\ (0.7, 0.1) & (0.9, 0.05) & (0.9, 0.06) \\ (0.6, 0.3) & (0.8, 0.07) & (0.7, 0.03) \\ (0.5, 0.3) & (0.8, 0.04) & (0.8, 0.15) \end{pmatrix}_{4\times 3}$

Proposition 3.16. *Let $A, B, C \in IFSM_{m\times n}$. Then the following results hold.*
(i) $A + B = B + A$
(ii) $(A + B) + C = A + (B + C)$
(iii) $A + [0] = A = [0] + A$.

Proof. (i) Let $A = \lfloor(\mu_{j1}(c_i), \nu_{j1}(c_i))\rfloor, B = \lfloor(\mu_{j2}(c_i), \nu_{j2}(c_i))\rfloor$
Now

$$\begin{aligned} A + B &= \lfloor(max(\mu_{j1}(c_i), \mu_{j2}(c_i)), (min(\nu_{j1}(c_i), \nu_{j2}(c_i)))\rfloor \\ &= \lfloor(max(\mu_{j2}(c_i), \mu_{j1}(c_i)), (min(\nu_{j2}(c_i), \nu_{j1}(c_i)))\rfloor \\ &= B + A \end{aligned}$$

(ii) Let $A = \lfloor(\mu_{j1}(c_i), \nu_{j1}(c_i))\rfloor, B = \lfloor(\mu_{j2}(c_i), \nu_{j2}(c_i))\rfloor, C = \lfloor(\mu_{j3}(c_i), \nu_{j3}(c_i))\rfloor$
Now
$(A{+}B){+}C = \lfloor(max(\mu_{j1}(c_i), \mu_{j2}(c_i)), (min(\nu_{j1}(c_i), \nu_{j2}(c_i)))\rfloor + \lfloor(\mu_{j3}(c_i), \nu_{j3}(c_i))\rfloor$
$= \lfloor(max(\mu_{j1}(c_i), (\mu_{j2}(c_i), \mu_{j3}(c_i))), (min(\nu_{j1}(c_i), (\nu_{j2}(c_i), \nu_{j3}(c_i)))\rfloor$
$= A + (B + C)$.
(iii) Let $A = \lfloor(\mu_{j1}(c_i), \nu_{j1}(c_i))\rfloor$, Also $[0] = \lfloor(\mu_j(c_i), \nu_j(c_i))\rfloor$ so that $\delta_{(0)ij} = 0\forall i, j$
Now

$$\begin{aligned} A + [0] &= \lfloor(max(\mu_{j1}(c_i), \mu_j(c_i)), (min(\nu_{j1}(c_i), \nu_j(c_i)))\rfloor \\ &= \lfloor(\mu_{j1}(c_i), \mu_j(c_i))\rfloor \\ &= A \end{aligned}$$

□

Definition 3.17. Let $A = [a_{ij}]_{m\times n}, a_{ij} = (\mu_{j1}(c_i), \nu_{j1}(c_i))$; where $\mu_{j1}(c_i)$ and $\nu_{j1}(c_i)$ represent the intuitionistic fuzzy membership value and intuitionistic fuzzy non membership value respectively of c_i. Also let $k =\in [0]$ be a scalar. Then kA is defined as

$$\begin{aligned} kA &= k[a_{ij}]_{m\times n} \\ &= k[(\mu_{j1}(c_i), \nu_{j1}(c_i))]_{m\times n} \\ &= \lfloor(min(k, \mu_{j1}(c_i)), (max(k, \nu_{j1}(c_i)))\rfloor_{m\times n} \end{aligned}$$

Definition 3.18. Let $A = [a_{ij}]_{m\times n}, a_{ij} = (\mu_{j1}(c_i), \nu_{j1}(c_i))$; where $\mu_{j1}(c_i)$ and $\nu_{j1}(c_i)$ represent the intuitionistic fuzzy membership value and intuitionistic fuzzy non membership value respectively of c_i. Also let $B = [b_{jk}]_{n\times p}, b_{jk} = (\mu_{k2}(c_j), \nu_{k2}(c_j))$; where $\mu_{k2}(c_j)$ and$\nu_{k2}(c_j)$ represent the intuitionistic fuzzy membership value and intuitionistic fuzzy non membership value respectively of c_j. We now define A.B, the product of A and B as,

$$\begin{aligned} A.B &= [d_{ik}]_{m\times p} \\ &= \lfloor(maxmin(\mu_{j1}(c_i), \mu_{k2}(c_j)), minmax(\nu_{j1}(c_i), \nu_{k2}(c_j))\rfloor_{m\times p} \end{aligned}$$

where $1 \leq i \leq m, 1 \leq k \leq p$, for $j = 1, 2, 3 \ldots n$

Remark

3.1 The matrices A and B are defined over two different universes respectively.

3.2 Product of two intuitionistic fuzzy soft matrices A and B representing intuitionistic fuzzy soft sets over the same universe is defined only when the matrices are square matrices.

3.3 If the product $A.B$ is defined then $B.A$ may not be defined.

3.4 Using product of two intuitionistic fuzzy soft matrices, we can find out the positive integral powers of intuitionistic fuzzy soft square matrices.

Definition 3.19. Let $A = [a_{ij}]_{m\times n}, a_{ij} = (\mu_{j1}(c_i), \nu_{j1}(c_i))$; where $\mu_{j1}(c_i)$ and $\nu_{j1}(c_i)$ represent the intuitionistic fuzzy membership value and intuitionistic fuzzy non membership value respectively of c_i.
We define $A^T = [a^T_{ij}]_{n\times m} \in IFSM_{n\times m}$, where $a^T_{ij} = a_{ji}$.

Proposition 3.20. *Let $A, B \in IFSM_{m\times n}$.Then the following result hold.*
(i) $(A^T)^T = A$
(ii)$(A + B)^T = A^T + B^T$

Proof. (i) Let $A = \lfloor(\mu_{j1}(c_i), \nu_{j1}(c_i))\rfloor$
Now

$$\begin{aligned} A^T &= [(\mu_{j1}(c_i), \nu_{j1}(c_i))]^T \\ &= \lfloor(\mu_{i1}(c_j), \nu_{i1}(c_j))\rfloor \\ (A^T)^T &= [(\mu_{i1}(c_j), \nu_{i1}(c_j))]^T \\ &= \lfloor(\mu_{j1}(c_i), \nu_{j1}(c_i))\rfloor \\ &= A \end{aligned}$$

(ii) Let $A = \lfloor(\mu_{j1}(c_i), \nu_{j1}(c_i))\rfloor, B = \lfloor(\mu_{j2}(c_i), \nu_{j2}(c_i))\rfloor$
Now

$$\begin{aligned}
A+B &= \lfloor(max(\mu_{j1}(c_i), \mu_{j2}(c_i)), min(\nu_{j1}(c_i), \nu_{j2}(c_i)))\rfloor \\
(A+B)^T &= \lfloor(max(\mu_{j1}(c_i), \mu_{j2}(c_i)), min(\nu_{j1}(c_i), \nu_{j2}(c_i)))\rfloor^T \\
&= \lfloor(max(\mu_{i1}(c_j), \mu_{i2}(c_j)), min(\nu_{i1}(c_j), \nu_{i2}(c_j)))\rfloor \\
&= \lfloor(\mu_{i1}(c_j), \nu_{i1}(c_j))\rfloor + \lfloor(\mu_{i2}(c_j), \nu_{i2}(c_j))\rfloor \\
&= \lfloor(\mu_{j1}(c_i), \nu_{j1}(c_i))\rfloor^T + \lfloor(\mu_{j2}(c_i), \nu_{j2}(c_i))\rfloor^T \\
&= A^T + B^T
\end{aligned}$$

□

Definition 3.21. Let $A = [a_{ij}]_{m\times n}, a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. Then A is said to be a intuitionistic fuzzy soft symmetric matrix if $A^T = A$.

Example 3.22. Let $A = \begin{pmatrix} (0.1, 0.8) & (0.4, 0.5) & (0.2, 0.6) \\ (0.4, 0.5) & (0.5, 0.4) & (0.6, 0.2) \\ (0.2, 0.6) & (0.6, 0.2) & (0.3, 0.1) \end{pmatrix}$ be an intuitionistic fuzzy soft square matrix. We see that

$$A^T = \begin{pmatrix} (0.1, 0.8) & (0.4, 0.5) & (0.2, 0.6) \\ (0.4, 0.5) & (0.5, 0.4) & (0.6, 0.2) \\ (0.2, 0.6) & (0.6, 0.2) & (0.3, 0.1) \end{pmatrix} = A$$

By definition, A is an intuitionistic fuzzy soft symmetric matrix.

Definition 3.23. Let $A = [a_{ij}]_{m\times m}, a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. Then A is said to be an intuitionistic fuzzy soft idempotent matrix if $A^2 = A$.

Example 3.24. Let $A = \begin{pmatrix} (0.3, 0.6) & (0.4, 0.3) & (0.2, 0.5) \\ (0.2, 0.7) & (0.5, 0.2) & (0.2, 0.7) \\ (0.3, 0.6) & (0.5, 0.2) & (0.3, 0.4) \end{pmatrix}$ be an intuitionistic fuzzy soft square matrix. Then

$A \cdot A = A^2$

$$= \begin{pmatrix} (0.3, 0.6) & (0.4, 0.3) & (0.2, 0.5) \\ (0.2, 0.7) & (0.5, 0.2) & (0.2, 0.7) \\ (0.3, 0.6) & (0.5, 0.2) & (0.3, 0.4) \end{pmatrix} \cdot \begin{pmatrix} (0.3, 0.6) & (0.4, 0.3) & (0.2, 0.5) \\ (0.2, 0.7) & (0.5, 0.2) & (0.2, 0.7) \\ (0.3, 0.6) & (0.5, 0.2) & (0.3, 0.4) \end{pmatrix}$$

$$= \begin{pmatrix} (0.3, 0.6) & (0.4, 0.3) & (0.2, 0.5) \\ (0.2, 0.7) & (0.5, 0.2) & (0.2, 0.7) \\ (0.3, 0.6) & (0.5, 0.2) & (0.3, 0.4) \end{pmatrix} = A$$

Thus $A \cdot A = A^2 = A$ It follows that is an intuitionistic fuzzy soft idempotent matrix.

4. Intuitionistic fuzzy soft set in medical diagnosis

Analogous to the Sanchez's notion of medical knowledge we form two matrices M_1 and M_2 as medical knowledge of an intuitionistic fuzzy soft set (F_1, D) and its complement $(F_1, D)^c$ respectively over S the set of symptoms of diseases, where D represents the set of diseases. Similarly we form two matrices N_1 and N_2 as medical knowledge of an intuitionistic fuzzy soft set (F_2, S) and its complement $(F_2, S)^c$

respectively over P the set of symptoms of patients. Then we obtain two matrices T_1 and T_2 using our definition of product of two intuitionistic fuzzy soft matrices as $T_1 = N_1 M_1$ and $T_2 = N_2 M_2$. Now find membership value and non membership value matrices. Then comparing the membership value and non membership value individually, the disease of the patient is diagnosed.

ALGORITHM

(1) Input the intuitionistic fuzzy soft set (F_1, D) and find $(F_1, D)^c$.Also find the corresponding matrices M_1 and M_2.

(2) Input the intuitionistic fuzzy soft set (F_2, S) and find $(F_2, S)^c$.Also find the corresponding matrices N_1 and N_2.

(3) Find $T_1 = N_1 M_1$ and $T_2 = N_2 M_2$.

(4) Find the membership value and non membership value matrices T_1^{μ} and T_1^{ν} of T_1 and T_2^{μ} and T_2^{ν} of T_2.

(5) Compare the membership value and non membership value of T_1 and T_2 individually.

CASE STUDY

Consider three patients p_1, p_2 and p_3 admitted in a hospital with symptoms of headache,diarrhoea, abdominal pain and vomiting. Suppose possible diseases with these symptoms be jaundice and typhoid. Let e_1, e_2, e_3 and e_4 represents the symptoms headache,diarrhoea, abdominal pain and vomiting respectively. Let d_1 and d_2 represents the diseases jaundice and typhoid respectively. Let $S = e_1, e_2, e_3, e_4$ and $D = d_1, d_2$ be the parameter set representing the symptoms and diseases respectively. Also let $P = p_1, p_2, p_3$ be the set of patient.

Let (F_1, D) be an intuitionistic fuzzy soft sets over S, where F_1 is a mapping $F_1 : D \to \widetilde{F_1}(S)$, gives an approximate description of intuitionistic fuzzy soft medical knowledge of the two diseases and their symptoms.

$$(F_1, D) = \Big\{ F_1(d_1) = \{(e_1, 0.45, 0.50), (e_2, 0.20, 0.75), (e_3, 0.90, 0.03), (e_4, 0.85, 0.05)\}$$
$$F_1(d_2) = \{(e_1, 0.65, 0.25), (e_2, 0.95, 0.01), (e_3, 0.15, 0.85), (e_4, 0.03, 0.80)\} \Big\}$$

Now

$$(F_1, D)^c = \Big\{ F_1^c(d_1) = \{(e_1, 0.50, 0.45), (e_2, 0.75, 0.20), (e_3, 0.03, 0.90), (e_4, 0.05, 0.85)\}$$
$$F_1^c(d_2) = \{(e_1, 0.25, 0.65), (e_2, 0.01, 0.95), (e_3, 0.85, 0.15), (e_4, 0.80, 0.03)\} \Big\}$$

This set represents the complement of the intuitionistic fuzzy soft set (F_1, D). Now we will represent the intuitionistic fuzzy soft set (F_1, D) and $(F_1, D)^c$ by the matrices M_1 and M_2 as follows

$$M_1 = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & \quad\quad d_2 \end{array} \\ \begin{pmatrix} (0.45, 0.50) & (0.65, 0.25) \\ (0.20, 0.75) & (0.95, 0.01) \\ (0.90, 0.03) & (0.15, 0.85) \\ (0.85, 0.05) & (0.03, 0.80) \end{pmatrix} \end{array} \text{ and } M_2 = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & \quad\quad d_2 \end{array} \\ \begin{pmatrix} (0.50, 0.45) & (0.25, 0.65) \\ (0.75, 0.20) & (0.01, 0.95) \\ (0.03, 0.90) & (0.85, 0.15) \\ (0.05, 0.85) & (0.80, 0.03) \end{pmatrix} \end{array}$$

Again let us consider another intuitionistic fuzzy soft set (F_2, S) over P, where $F_2 : S \to \widetilde{F_2}(P)$, gives an approximate description of intuitionistic fuzzy soft medical knowledge of the symptoms in patients.

$$
\begin{aligned}
(F_2,S)=\Big\{ F_2(e_1) &= \{(p_1,0.80,0.15),(p_2,0.20,0.75),(p_3,0.90,0.09)\}\\
F_1(e_2) &= \{(p_1,0.86,0.10),(p_2,0.05,0.70),(p_3,0.73,0.14)\}\\
F_1(e_3) &= \{(p_1,0.07,0.67),(p_2,0.86,0.05),(p_3,0.10,0.78)\}\\
F_1(e_4) &= \{(p_1,0.06,0.75),(p_2,0.50,0.15),(p_3,0.03,0.80)\}\Big\}
\end{aligned}
$$

Now

$$
\begin{aligned}
(F_2,S)^c=\Big\{ F_2^c(e_1) &= \{(p_1,0.15,0.80),(p_2,0.75,0.20),(p_3,0.09,0.90)\}\\
F_1^c(e_2) &= \{(p_1,0.10,0.86),(p_2,0.70,0.05),(p_3,0.14,0.73)\}\\
F_1^c(e_3) &= \{(p_1,0.67,0.07),(p_2,0.05,0.86),(p_3,0.78,0.10)\}\\
F_1^c(e_4) &= \{(p_1,0.75,0.06),(p_2,0.15,0.50),(p_3,0.80,0.03)\}\Big\}
\end{aligned}
$$

This set represents the complement of the intuitionistic fuzzy soft set (F_2, S). Now we will represent the intuitionistic fuzzy soft set (F_2, S) and $(F_2, S)^c$ by the matrices N_1 and N_2 as follows

$$
N_1 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}
\begin{array}{c}
\begin{array}{cccc} \quad e_1 \quad & \quad e_2 \quad & \quad e_3 \quad & \quad e_4 \quad \end{array}\\
\begin{pmatrix}
(0.80,0.15) & (0.86,0.10) & (0.07,0.67) & (0.06,0.75)\\
(0.20,0.75) & (0.05,0.70) & (0.86,0.05) & (0.50,0.15)\\
(0.90,0.09) & (0.73,0.14) & (0.10,0.78) & (0.03,0.80)
\end{pmatrix}
\end{array}
$$

and

$$
N_2 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}
\begin{array}{c}
\begin{array}{cccc} \quad e_1 \quad & \quad e_2 \quad & \quad e_3 \quad & \quad e_4 \quad \end{array}\\
\begin{pmatrix}
(0.15,0.80) & (0.10,0.86) & (0.67,0.07) & (0.75,0.06)\\
(0.75,0.20) & (0.70,0.05) & (0.05,0.86) & (0.15,0.50)\\
(0.09,0.90) & (0.14,0.73) & (0.78,0.10) & (0.80,0.03)
\end{pmatrix}
\end{array}
$$

Therefore the product matrices T_1 and T_2 are

$$
T_1 = N_1M_1 \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}
\begin{array}{c}
\begin{array}{cc} \quad d_1 \quad & \quad d_2 \quad \end{array}\\
\begin{pmatrix}
(0.45,0.50) & (0.86,0.10)\\
(0.86,0.05) & (0.20,0.70)\\
(0.45,0.50) & (0.73,0.14)
\end{pmatrix}
\end{array}
$$

and

$$
T_2 = N_2M_2 \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}
\begin{array}{c}
\begin{array}{cc} \quad d_1 \quad & \quad d_2 \quad \end{array}\\
\begin{pmatrix}
(0.30,0.80) & (0.75,0.15)\\
(0.70,0.20) & (0.25,0.50)\\
(0.30,0.73) & (0.80,0.03)
\end{pmatrix}
\end{array}
$$

Now we find the membership value matrix and non membership value matrix T_1^μ and T_1^ν respectively of the product matrix T_1

$$T_1^\mu = \begin{array}{c c} & \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} & \begin{pmatrix} 0.45 & 0.86 \\ 0.86 & 0.20 \\ 0.45 & 0.73 \end{pmatrix} \end{array} \text{ and } T_1^\nu = \begin{array}{c c} & \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} & \begin{pmatrix} 0.50 & 0.10 \\ 0.05 & 0.70 \\ 0.50 & 0.14 \end{pmatrix} \end{array}$$

We observe that $T_1^\mu(d_1) \leq T_1^\mu(d_2)$ for patients p_1 and p_3 and $T_1^\mu(d_1) \geq T_1^\mu(d_2)$ for patients p_2 and $T_1^\nu(d_1) \geq T_1^\nu(d_2)$ for patients p_1 and p_3 and $T_1^\nu(d_1) \leq T_1^\nu(d_2)$ for patients p_2. The inference that we have drawn is that patients p_1 and p_3 is more likely to be suffering from disease d_1 i.e. jaundice and patient p_2 is more likely to be suffering from disease d_2 i.e. typhoid. Again we find the membership value matrix and non membership value matrix T_2^μ and T_2^ν respectively of the product matrix T_2.

$$T_2^\mu = \begin{array}{c c} & \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} & \begin{pmatrix} 0.30 & 0.75 \\ 0.70 & 0.25 \\ 0.30 & 0.80 \end{pmatrix} \end{array} \text{ and } T_2^\nu = \begin{array}{c c} & \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} & \begin{pmatrix} 0.80 & 0.15 \\ 0.20 & 0.50 \\ 0.73 & 0.03 \end{pmatrix} \end{array}$$

We observe that $T_2^\mu(d_1) \leq T_2^\mu(d_2)$ for patients p_1 and p_3 and $T_2^\mu(d_1) \geq T_2^\mu(d_2)$ for patients p_2 and $T_2^\nu(d_1) \geq T_2^\nu(d_2)$ for patients $p_1$and $p_3$and $T_1^\nu(d_1) \leq T_1^\nu(d_2)$ for patients p_2. The inference that we have drawn is that patients p_1 and p_3 is more likely to be suffering from disease d_1 i.e. jaundice and patient p_2 is more likely to be suffering from disease d_2 i.e. typhoid.Thus by using product of matrix representation of intuitionistic fuzzy soft set and matrix representation of complement of the same intuitionistic fuzzy soft set we get the same results i.e. patient p_1 and p_3 is more likely to be suffering from disease jaundice and patient p_2 is more likely to be suffering from disease typhoid.

5. Conclusions

In our work we have defined different types of intuitionistic fuzzy soft matrix. Operations of addition, multiplication and complement of intuitionistic fuzzy soft matrix are defined with examples. Some related propositions with proof is also worked out. Further our work is supported by a decision making problem in medical diagnosis.

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