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Interval valued intuitionistic fuzzy soft multisets and their relations

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Abstract. In this paper we introduce the concept of interval valued intuitionistic fuzzy soft multisets and study their properties and operations. Then the concept of interval valued intuitionistic fuzzy soft multiset relations (IVIFSMS-relations for short) is proposed. The basic properties of the IVIFSMS-relations are also discussed. Finally various types of IVIFSMS-relations are presented.

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1. Introduction

Most of the problems in engineering, medical science, economics, environments etc have various uncertainties. Molodtsov [9] initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties. Research works on soft set theory are progressing rapidly. Maji et al.[6] defined several operations on soft set theory. Based on the analysis of several operations on soft sets introduced in [6], Ali et al.[1] presented some new algebraic operations for soft sets. Combining soft sets with fuzzy sets [10] and intuitionistic fuzzy sets[4], Maji et al. [7, 8] defined fuzzy soft sets and intuitionistic fuzzy soft sets which are rich potentials for solving decision making problems. The notion of the interval-valued intuitionistic fuzzy set was introduced by Atanassov and Gargov [5]. Alkhazaleh et al. [2] as a generalization of Molodtsov soft set, presented the definition of a soft multiset and its basic operations such as complement, union, and intersection etc. In 2012 Alkhazaleh and Salleh [3] introduced the concept of fuzzy soft multiset theory and studied the application of these sets.

In this paper we introduce the concept of interval valued intuitionistic fuzzy soft multisets and study its properties and operations. Also the concept of interval valued intuitionistic fuzzy soft multiset relations (IVIFSMS-relations for short) is proposed. The basic properties of the IVIFSMS-relations are discussed. Also various types of IVIFSMS-relations are presented.

2. Preliminaries

Definition 2.1 ([10])**.** Let X be a non empty set. Then a fuzzy set (FS for short) A is a set having the form A={(x, μ_A(x)): x ∈ X} where the function μ_A: X→[0,1] is called the membership function and μ_A(x) is called the degree of membership of each element x∈X.
After the introduction of the concept of fuzzy set by Zadeh[10], several researchers were conducted on the generalization of the notion of a fuzzy set. The idea of "intuitionistic fuzzy set" was first published by Atanassov [4].

Definition 2.2 ([4])**.** Let U be a non empty set. Then an intuitionistic fuzzy set (IFS for short) A on U is a set having the form A={⟨x, μ_A(x), γ_A(x)⟩: x ∈ U} where the functions μ_A: U→[0,1] and γ_A: U→[0,1] represents the degree of membership and the degree of non-membership respectively of each element x∈U and 0≤μ_A(x)+ γ_A(x)≤1 for each x∈U.

Definition 2.3 ([5])**.** An interval valued intuitionistic fuzzy set A over an universe set U is defined as the object of the form A={<x, μ_A(x), γ_A(x)>: x∈U}, where μ_A: U→ Int([0,1]) and γ_A: U→ Int([0, 1]) are functions such that the condition: ∀x∈U, sup μ_A(x)+ supγ_A(x)≤1 is satisfied(where Int([0, 1]) is the set of all closed sub-intervals of [0,1]).

The class of all interval valued intuitionistic fuzzy sets on U is denoted by IV-IFS(U).

Definition 2.4 ([9])**.** Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A⊆ E. Then the pair (F, A) is called a soft set over U, where F is a mapping given by F: A→ P(U).

Definition 2.5 ([2])**.** Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} P(U_i)$ where $P(U_i)$ denotes the power set of U_i ,$E = \prod_{i\in I} E_{U_i}$ and A⊆ E. A pair (F, A) is called a soft multiset over U, where F is a mapping given by F: A→ U.

Definition 2.6 ([3])**.** Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} FS(U_i)$ where $FS(U_i)$ denotes the set of all fuzzy subsets of U_i ,$E = \prod_{i\in I} E_{U_i}$ and A⊆ E . A pair (F, A) is called a fuzzy soft multiset over U, where F is a mapping given by F: A→ U .

Definition 2.7 ([3])**.** A fuzzy soft multiset (F, A) over U is called a fuzzy soft multisubset of a fuzzy soft multiset (G, B) if

(a) $A \subseteq B$ and

(b) $\forall e_{U_i,j} \in a_k, (e_{U_i,j}, F_{e_{U_i,j}})$ is a fuzzy subset of $(e_{U_i,j}, G_{e_{U_i,j}})$ where $a_k \in A$, $k \in \{1, 2, 3, ..., n\}$, $i \in \{1, 2, 3, ..., m\}$ and $j \in \{1, 2, 3, ..., r\}$. This relationship is denoted by $(F, A)\tilde{\subseteq}(G, B)$.

Definition 2.8 ([3]). The complement of a fuzzy soft multiset (F, A) over U is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to U$ is a mapping given by $F^c(\alpha) = c(F(\alpha)), \forall \alpha \in A$ where c is the fuzzy complement.

Definition 2.9 ([3]). A fuzzy soft multiset (F, A) over U is called a semi-null fuzzy soft multiset, denoted by $(F, A)_{\approx\phi}$, if at least one of a fuzzy soft multiset parts of (F, A) equals ϕ.

Definition 2.10 ([3]). A fuzzy soft multiset (F, A) over U is called a null fuzzy soft multiset, denoted by $(F, A)_{\phi}$, if all the fuzzy soft multiset parts of (F, A) equals ϕ.

Definition 2.11 ([3]). A fuzzy soft multiset (F, A) over U is called a semi-absolute fuzzy soft multiset, denoted by $(F, A)_{\approx U_i}$, if $(e_{U_i,j}, F_{e_{U_i,j}}) = U_i$ for at least one i, $a_k \in A, k = \{1, 2, 3, ..., n\}, i = \{1, 2, 3, ..., m\}$ *and* $j = \{1, 2, 3, ..., r\}$.

Definition 2.12 ([3]). A fuzzy soft multiset (F, A) over U is called an absolute fuzzy soft multiset, denoted by $(F, A)_U$, if $(e_{U_i,j}, F_{e_{U_i,j}}) = U_i \ \forall i$.

Definition 2.13 ([3]). The union of two fuzzy soft multisets (F, A) and (G, B) over U is a fuzzy soft multiset (H, D) where $D = A \cup B$ and $\forall e \in D$,

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ \bigcup(F(e), G(e)) & if\ e \in A \cap B \end{cases}$$

where $\bigcup(F(e), G(e)) = s(F_{e_{U_i,j}}, G_{e_{U_i,j}}) \ \ \forall i \in \{1, 2, ..., m\}$ and $j \in \{1, 2, ..., r\}$ with 's' as an s-norm and is written as $(F, A)\tilde{\cup}(G, B) = (H, D)$.

Definition 2.14 ([3]). The intersection of two fuzzy soft multisets (F, A) and (G, B) over U is a fuzzy soft multiset (H, D) where $D = A \cup B$ and $\forall e \in D$,

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ \bigcap(F(e), G(e)) & if\ e \in A \cap B \end{cases}$$

where $\bigcap(F(e), G(e)) = t(F_{e_{U_i,j}}, G_{e_{U_i,j}}) \ \ \forall i \in \{1, 2, ..., m\}$ and $j \in \{1, 2, ..., r\}$ with 't' as an t-norm and is written as $(F, A)\tilde{\cap}(G, B) = (H, D)$.

3. Interval valued intuitionistic fuzzy soft multisets

In this section, we introduce the definition of an interval valued intuitionistic fuzzy soft multiset, and its basic operations such as complement, union, and intersection etc. We give examples for these concepts. Basic properties of the operations are also given.

Definition 3.1. Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} IVIFS(U_i)$ where $IVIFS(U_i)$ denotes the set of all interval valued intuitionistic fuzzy subsets of U_i ,$E = \prod_{i\in I} E_{U_i}$ and A$\subseteq$ E.Then the pair (F, A) is called an interval valued intuitionistic fuzzy soft multiset over U, where F is a mapping given by $F : A \to U$.

Remark 3.2. It is to be noted that if $U = \prod_{i\in I} IFS(U_i)$, then the pair (F, A) is called an intuitionistic fuzzy soft multi set over U. To illustrate this let us consider the following example:

Let us consider there are three universes U_1, U_2 and U_3. Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\}$, $E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\}$, $E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}$. Let $U = \prod_{i=1}^{3} IFS(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and A$\subseteq$ E, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}.$$

Let $F : A \to U$ be defined by
$F(a_1) = (\{\langle h_1, 0.3, 0.5\rangle, \langle h_2, 0.4, 0.6\rangle, \langle h_3, 0.9, 0.1\rangle\}, \{\langle c_1, 0.9, 0\rangle, \langle c_2, 0.6, 0.3\rangle,$
$\langle c_3, 0.5, 0.4\rangle\}, \{\langle v_1, 0.9, 0\rangle, \langle v_2, 0.8, 0.2\rangle\}),$
$F(a_2) = (\{\langle h_1, 0.4, 0.4\rangle, \langle h_2, 0.7, 0.3\rangle, \langle h_3, 0.6, 0.4\rangle\}, \{\langle c_1, 0.6, 0.3\rangle, \langle c_2, 0.4, 0.4\rangle,$
$\langle c_3, 0.7, 0.2\rangle\}, \{\langle v_1, 0.6, 0.3\rangle, \langle v_2, 0.5, 0.3\rangle\}).$
Then
$(F, A) = \{(a_1, (\{\langle h_1, 0.3, 0.5\rangle, \langle h_2, 0.4, 0.6\rangle, \langle h_3, 0.9, 0.1\rangle\}, \{\langle c_1, 0.9, 0\rangle, \langle c_2, 0.6, 0.3\rangle,$
$\langle c_3, 0.5, 0.4\rangle\}, \{\langle v_1, 0.9, 0\rangle, \langle v_2, 0.8, 0.2\rangle\})), (a_2, (\{\langle h_1, 0.4, 0.4\rangle,$
$\langle h_2, 0.7, 0.3\rangle, \langle h_3, 0.6, 0.4\rangle\}, \{\langle c_1, 0.6, 0.3\rangle, \langle c_2, 0.4, 0.4\rangle, \langle c_3, 0.7, 0.2\rangle\},$
$\{\langle v_1, 0.6, 0.3\rangle, \langle v_2, 0.5, 0.3\rangle\}))\}.$

Here (F, A) is an intuitionistic fuzzy soft multiset over U.

Example 3.3. Let us consider there are three universes U_1 , U_2 and U_3. Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\}$,$E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\}$, $E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}$.
Let $U = \prod_{i=1}^{3} IVIFS(U_i)$,$E = \prod_{i=1}^{3} E_{U_i}$ and A$\subseteq$ E, such that
$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$
Let $F : A \to U$ be defined by
$F(a_1) = (\{\langle h_1, [0.1, 0.3], [0.3, 0.5]\rangle, \langle h_2, [0.3, 0.4], [0.4, 0.6]\rangle, \langle h_3, [0.7, 0.9], [0.0, 0.1]\rangle\},$
$\{\langle c_1, [0.7, 0.9], [0, 0]\rangle, \langle c_2, [0.4, 0.6], [0.2, 0.3]\rangle, \langle c_3, [0.3, 0.5], [0.2, 0.4]\rangle\},$
$\{\langle v_1, [0.7, 0.9], [0, 0]\rangle, \langle v_2, [0.6, 0.8], [0, 0.2]\rangle\}),$
$F(a_2) = (\{\langle h_1, [0.2, 0.4], [0.3, 0.4]\rangle, \langle h_2, [0.6, 0.7], [0.2, 0.3]\rangle, \langle h_3, [0.5, 0.6], [0.3, 0.4]\rangle\},$
$\{\langle c_1, [0.3, 0.6], [0.2, 0.3]\rangle, \langle c_2, [0.1, 0.4], [0.3, 0.4]\rangle, \langle c_3, [0.5, 0.7], [0.1, 0.2]\rangle\},$
$\{\langle v_1, [0.5, 0.6], [0.2, 0.3]\rangle, \langle v_2, [0.2, 0.5], [0.1, 0.3]\rangle\}).$
Then(F, A)=
$\{(a_1, (\{\langle h_1, [0.1, 0.3], [0.3, 0.5]\rangle, \langle h_2, [0.3, 0.4], [0.4, 0.6]\rangle, \langle h_3, [0.7, 0.9], [0.0, 0.1]\rangle\},$
$\{\langle c_1, [0.7, 0.9], [0, 0]\rangle, \langle c_2, [0.4, 0.6], [0.2, 0.3]\rangle, \langle c_3, [0.3, 0.5], [0.2, 0.4]\rangle\},$
$\{\langle v_1, [0.7, 0.9], [0, 0]\rangle, \langle v_2, [0.6, 0.8], [0, 0.2]\rangle\})), (a_2, (\{\langle h_1, [0.2, 0.4], [0.3, 0.4]\rangle,$
$\langle h_2, [0.6, 0.7], [0.2, 0.3]\rangle, \langle h_3, [0.5, 0.6], [0.3, 0.4]\rangle\}, \{\langle c_1, [0.3, 0.6], [0.2, 0.3]\rangle,$
$\langle c_2, [0.1, 0.4], [0.3, 0.4]\rangle, \langle c_3, [0.5, 0.7], [0.1, 0.2]\rangle\}, \{\langle v_1, [0.5, 0.6], [0.2, 0.3]\rangle,$
$\langle v_2, [0.2, 0.5], [0.1, 0.3]\rangle\}))\}.$
Here (F, A) is an interval valued intuitionistic fuzzy soft multiset over U.

Definition 3.4. Let (F, A) and (G, B) be two interval valued intuitionistic fuzzy soft multisets over U. Then (F, A) is called an interval valued intuitionistic fuzzy soft multi subset of (G, B) if

(a) $A \subseteq B$ and

(b) $\forall e_{U_i,j} \in a_k, (e_{U_i,j}, F_{e_{U_i,j}})$ is an interval valued intuitionistic fuzzy subset of $(e_{U_i,j}, G_{e_{U_i,j}})$ where $a_k \in A$, $k \in \{1,2,3,....,n\}$, $i \in \{1,2,3,....,m\}$ and $j \in \{1,2,3,....,r\}$. This relationship is denoted by $(F,A)\tilde{\subseteq}(G,B)$.

Example 3.5. Let us consider Example 3.3. Let

$B = \{b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), b_3 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,2})\}$

Clearly A$\subseteq$ B. Let (G, B) be an interval valued intuitionistic fuzzy soft multiset over U, such that

(G,B)=

$\{(b_1, (\{\langle h_1, [0.2, 0.4], [0.3, 0.5]\rangle, \langle h_2, [0.4, 0.6], [0.2, 0.4]\rangle, \langle h_3, [0.7, 0.9], [0, 0.1]\rangle\},$
$\{\langle c_1, [0.7, 0.9], [0, 0]\rangle, \langle c_2, [0.6, 0.8], [0.1, 0.2]\rangle, \langle c_3, [0.4, 0.6], [0.2, 0.4]\rangle\},$
$\{\langle v_1, [0.7, 0.9], [0, 0]\rangle, \langle v_2, [0.7, 0.9], [0, 0.1]\rangle\})), (b_2, (\{\langle h_1, [0.3, 0.5], [0.2, 0.3]\rangle,$
$\langle h_2, [0.6, 0.8], [0.1, 0.2\rangle, \langle h_3, [0.7, 0.9], [0, 0.1]\rangle\}, \{\langle c_1, [0.4, 0.7], [0.1, 0.2]\rangle,$
$\langle c_2, [0.4, 0.6], [0.2, 0.3]\rangle, \langle c_3, [0.7, 0.9], [0, 0]\rangle\}, \{\langle v_1, [0.5, 0.7], [0.2, 0.3]\rangle,$
$\langle v_2, [0.3, 0.5], [0.1, 0.3]\rangle\})), (b_3, (\{\langle h_1, [0.6, 0.8], [0.1, 0.2]\rangle,$
$\langle h_2, [0.6, 0.8], [0.1, 0.2]\rangle, \langle h_3, [0, 0.2], [0.5, 0.7]\rangle\}, \{\langle c_1, [0.7, 0.9], [0, 0.1]\rangle,$
$\langle c_2, [0.5, 0.7], [0.1, 0.3]\rangle, \langle c_3, [0.2, 0.4], [0.3, 0.5]\rangle\}, \{\langle v_1, [0.4, 0.6], [0.2, 0.4]\rangle,$
$\langle v_2, [0.3, 0.5], [0.2, 0.4]\rangle\}))\}.$

Therefore $(F,A)\tilde{\subseteq}(G,B)$.

Definition 3.6. The complement of an interval valued intuitionistic fuzzy soft multiset (F, A) over U is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, A)$, where $F^c : A \to U$ is a mapping given by $F^c(\alpha) = c(F(\alpha))$,$\forall\alpha \in A$ where c is the interval valued intuitionistic fuzzy complement.

Example 3.7. Let us consider Example 3.3. Then $(F,A)^c$=

$\{(a_1, (\{\langle h_1, [0.3, 0.5], [0.1, 0.3]\rangle, \langle h_2, [0.4, 0.6], [0.3, 0.4]\rangle, \langle h_3, [0, 0.1], [0.7, 0.9]\rangle\},$
$\{\langle c_1, [0, 0], [0.7, 0.9]\rangle, \langle c_2, [0.2, 0.3], [0.4, 0.6]\rangle, \langle c_3, [0.2, 0.4], [0.3, 0.5]\rangle\},$
$\{\langle v_1, [0, 0], [0.7, 0.9]\rangle, \langle v_2, [0, 0.2], [0.6, 0.8]\rangle\})), (a_2, (\{\langle h_1, [0.3, 0.4], [0.2, 0.4]\rangle,$
$\langle h_2, [0.2, 0.3], [0.6, 0.7]\rangle, \langle h_3, [0.3, 0.4], [0.5, 0.6]\rangle\}, \{\langle c_1, [0.2, 0.3], [0.3, 0.6]\rangle,$
$\langle c_2, [0.3, 0.4], [0.1, 0.4]\rangle, \langle c_3, [0.1, 0.2], [0.5, 0.7]\rangle\}, \{\langle v_1, [0.2, 0.3], [0.5, 0.6]\rangle,$
$\langle v_2, [0.1, 0.3], [0.2, 0.5]\rangle\}))\}.$

Definition 3.8. An interval valued intuitionistic fuzzy soft multiset (F, A) over U is called a semi-null interval valued intuitionistic fuzzy soft multiset, denoted by $(F,A)_{\approx\phi}$, if at least one of the interval valued intuitionistic fuzzy soft multiset parts of (F, A) equals ϕ.

Example 3.9. Let us consider there are three universes U_1 , U_2 and U_3. Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\}$, $E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\}$, $E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}$. Let $U = \prod_{i=1}^{3} IVIFS(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and A$\subseteq$ E, such that

$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$.

Then a semi-null interval valued intuitionistic fuzzy soft multiset $(F, A)_{\approx\phi}$ is given by
$(F, A)_{\approx\phi}=$
$\{(a_1, (\{\langle h_1, [0, 0], [1, 1]\rangle, \langle h_2, [0, 0], [1, 1]\rangle, \langle h_3, [0, 0], [1, 1]\rangle\},$
$\{\langle c_1, [0.7, 0.9], [0, 0]\rangle, \langle c_2, [0.4, 0.6], [0.3, 0.4]\rangle, \langle c_3, [0.3, 0.5], [0.2, 0.4]\rangle\},$
$\{\langle v_1, [0.7, 0.9], [0, 0]\rangle, \langle v_2, [0.6, 0.8], [0, 0.2]\rangle\})), (a_2, (\{\langle h_1, [0, 0], [1, 1]\rangle,$
$\langle h_2, [0, 0], [1, 1]\rangle, \langle h_3, [0, 0], [1, 1]\rangle\}, \{\langle c_1, [0.3, 0.5], [0.2, 0.4]\rangle,$
$\langle c_2, [0.4, 0.5], [0.3, 0.5]\rangle, \langle c_3, [0.7, 0.9], [0, 0]\rangle\}, \{\langle v_1, [0.3, 0.5], [0.2, 0.4]\rangle,$
$\langle v_2, [0.3, 0.5], [0.2, 0.4]\rangle\}))\}.$

Definition 3.10. An interval valued intuitionistic fuzzy soft multiset (F, A) over U is called a null interval valued intuitionistic fuzzy soft multiset, denoted by $(F, A)_\phi$, if all the interval valued intuitionistic fuzzy soft multiset parts of (F, A) equals ϕ.

Example 3.11. Let us consider there are three universes U_1 , U_2 and U_3.Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\}$,$E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\}$,$E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}$. Let $U = \prod_{i=1}^{3} IVIFS(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and A$\subseteq$ E, such that
$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$.
Then a null interval valued intuitionistic fuzzy soft multiset $(F, A)_\phi$ is given by
$(F, A)_\phi =$
$\{(a_1, (\{\langle h_1, [0, 0], [1, 1]\rangle, \langle h_2, [0, 0], [1, 1]\rangle, \langle h_3, [0, 0], [1, 1]\rangle\},$
$\{\langle c_1, [0, 0], [1, 1]\rangle, \langle c_2, [0, 0], [1, 1]\rangle, \langle c_3, [0, 0], [1, 1]\rangle\},$
$\{\langle v_1, [0, 0], [1, 1]\rangle, \langle v_2, [0, 0], [1, 1]\rangle\})), (a_2, (\{\langle h_1, [0, 0], [1, 1]\rangle,$
$\langle h_2, [0, 0], [1, 1]\rangle, \langle h_3, [0, 0], [1, 1]\rangle\}, \{\langle c_1, [0, 0], [1, 1]\rangle,$
$\langle c_2, [0, 0], [1, 1]\rangle, \langle c_3, [0, 0], [1, 1]\rangle\}, \{\langle v_1, [0, 0], [1, 1]\rangle,$
$\langle v_2, [0, 0], [1, 1]\rangle\}))\}.$

Definition 3.12. An interval valued intuitionistic fuzzy soft multiset (F, A) over U is called a semi-absolute interval valued intuitionistic fuzzy soft multiset, denoted by $(F, A)_{\approx U_i}$, if $(e_{U_i,j}, F_{e_{U_i,j}}) = U_i$ for at least one i, $a_k \in A, k \in \{1, 2, 3, ..., n\}, i \in \{1, 2, 3, ..., m\}$ *and* $j \in \{1, 2, 3, ..., r\}$.

Example 3.13. Let us consider there are three universes U_1 , U_2 and U_3.Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\}$, $E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\}$, $E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}$. Let $U = \prod_{i=1}^{3} IVIFS(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and A$\subseteq$ E, such that
$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$.
Then a semi-absolute interval valued intuitionistic fuzzy soft multiset $(F, A)_{\approx U_i}$ is given by
$(F, A)_{\approx U_i} =$
$\{(a_1, (\{\langle h_1, [1, 1], [0, 0]\rangle, \langle h_2, [1, 1], [0, 0]\rangle, \langle h_3, [1, 1], [0, 0]\rangle\},$
$\{\langle c_1, [0.7, 0.9], [0, 0]\rangle, \langle c_2, [0.4, 0.5], [0.3, 0.5]\rangle, \langle c_3, [0.3, 0.5], [0.2, 0.4]\rangle\},$
$\{\langle v_1, [0.7, 0.9], [0, 0]\rangle, \langle v_2, [0.6, 0.8], [0, 0.2]\rangle\})), (a_2, (\{\langle h_1, [1, 1], [0, 0]\rangle,$

$\langle h_2, [1,1], [0,0]\rangle, \langle h_3, [1,1], [0,0]\rangle\}, \{\langle c_1, [0.3, 0.5], 0.2, 0.4]\rangle,$
$\langle c_2, [0.4, 0.6], [0.3, 0.4]\rangle, \langle c_3, [0.7, 0.9], [0,0]\rangle\}, \{\langle v_1, [0.3, 0.5], [0.2, 0.4]\rangle,$
$\langle v_2, [0.3, 0.5], [0.2, 0.4]\rangle\}))\}.$

Definition 3.14. An interval valued intuitionistic fuzzy soft multiset (F, A) over U is called an absolute interval valued intuitionistic fuzzy soft multiset, denoted by $(F, A)_U$, if $(e_{U_i,j}, F_{e_{U_i,j}}) = U_i\ \forall i$.

Example 3.15. Let us consider there are three universes U_1, U_2 and U_3. Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\}$, $E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\}$, $E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}$. Let $U = \prod_{i=1}^{3} IVIFS(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and A$\subseteq$ E, such that
$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$.
Then an absolute interval valued intuitionistic fuzzy soft multiset $(F, A)_U$ is given by
$(F, A)_U =$
$\{(a_1, (\{\langle h_1, [1,1], [0,0]\rangle, \langle h_2, [1,1], [0,0]\rangle, \langle h_3, [1,1], [0,0]\rangle\},$
$\{\langle c_1, [[1,1], [0,0]\rangle, \langle c_2, [1,1], [0,0]\rangle, \langle c_3, [1,1], [0,0]\rangle\},$
$\{\langle v_1, [1,1], [0,0]\rangle, \langle v_2, [1,1], [0,0]\rangle\})), (a_2, (\{\langle h_1, [1,1], [0,0]\rangle,$
$\langle h_2, [1,1], [0,0]\rangle, \langle h_3, [1,1], [0,0]\rangle\}, \{\langle c_1, [1,1], [0,0]\rangle,$
$\langle c_2, [1,1], [0,0]\rangle, \langle c_3, [1,1], [0,0]\rangle\}, \{\langle v_1, [1,1], [0,0]\rangle,$
$\langle v_2, [1,1], [0,0]\rangle\}))\}.$

Proposition 3.16. *For an interval valued intuitionistic fuzzy soft multiset (F, A) over U,*

(a) $((F, A)^c)^c$=*(F, A)*
(b) $(F, A)^c_{\approx\phi_i} = (F, A)_{\approx U_i}$
(c) $(F, A)^c_{\approx\phi} = (F, A)_U$
(d) $(F, A)^c_{\approx U_i} = (F, A)_{\approx\phi_i}$
(e) $(F, A)^c_U = (F, A)_\phi$

Proof. Straightforward. □

Definition 3.17. The union of two interval valued intuitionistic fuzzy soft multisets (F, A) and (G, B) over U is an interval valued intuitionistic fuzzy soft multiset (H, D) where $D = A \cup B$ and $\forall e \in D$,

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ \bigcup(F(e), G(e)) & if\ e \in A \cap B \end{cases}$$

where $\bigcup(F(e), G(e)) = F_{e_{U_i,j}} \cup G_{e_{U_i,j}}\ \ \forall i \in \{1, 2, ..., m\}$ with $\cup$ as an interval valued intuitionistic fuzzy union and is written as $(F, A)\tilde{\cup}(G, B) = (H, D)$.

Example 3.18. Let us consider there are three universes U_1, U_2 and U_3.Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\},\ E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\},\ E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}.$$

Let $U = \prod_{i=1}^{3} IVIFS(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and A,B$\subseteq$ E, such that

$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), a_3 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,1})\}$,

$B = \{b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), b_3 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,2})\}$

Let (F, A) and (G, B) be two interval valued intuitionistic fuzzy soft multiset over U, such that

(F, A)=
$\{(a_1, (\{\langle h_1, [0.1, 0.3], [0.3, 0.5]\rangle, \langle h_2, [0.3, 0.4], [0.4, 0.6]\rangle, \langle h_3, [0.7, 0.9], [0.0, 0.1]\rangle\},$
$\{\langle c_1, [0.7, 0.9], [0, 0]\rangle, \langle c_2, [0.4, 0.6], [0.3, 0.4]\rangle, \langle c_3, [0.3, 0.5], [0.2, 0.4]\rangle\},$
$\{\langle v_1, [0.7, 0.9], [0, 0]\rangle, \langle v_2, [0.6, 0.8], [0, 0.2]\rangle\})), (a_2, (\{\langle h_1, [0.1, 0.3], [0.3, 0.5]\rangle,$
$\langle h_2, [0.3, 0.5], [0.4, 0.5]\rangle, \langle h_3, [0.7, 0.9], [0, 0.1]\rangle\}, \{\langle c_1, [0.3, 0.5], [0.2, 0.4]\rangle,$
$\langle c_2, [0.4, 0.6], [0.3, 0.4]\rangle, \langle c_3, [0.7, 0.9], [0, 0]\rangle\}, \{\langle v_1, [0.3, 0.5], [0.2, 0.4]\rangle,$
$\langle v_2, [0.3, 0.5], [0.2, 0.4]\rangle\})), (a_3, (\{\langle h_1, [0.6, 0.8], [0.1, 0.2]\rangle,$
$\langle h_2, [0.6, 0.8], [0.1, 0.2]\rangle, \langle h_3, [0, 0.2], [0.5, 0.7]\rangle\}, \{\langle c_1, [0.7, 0.9], [0, 0.1]\rangle,$
$\langle c_2, [0.5, 0.7], [0.1, 0.3]\rangle, \langle c_3, [0.2, 0.4], [0.3, 0.5]\rangle\}, \{\langle v_1, [0.4, 0.6], [0.2, 0.4]\rangle,$
$\langle v_2, [0.3, 0.5], [0.2, 0.4]\rangle\}))\}.$

(G, B)=
$\{(b_1, (\{\langle h_1, [0.2, 0.4], [0.3, 0.5]\rangle, \langle h_2, [0.4, 0.6], [0.2, 0.4]\rangle, \langle h_3, [0.7, 0.9], [0, 0.1]\rangle\},$
$\{\langle c_1, [0.7, 0.9], [0, 0]\rangle, \langle c_2, [0.6, 0.8], [0.1, 0.2]\rangle, \langle c_3, [0.4, 0.6], [0.2, 0.4]\rangle\},$
$\{\langle v_1, [0.7, 0.9], [0, 0]\rangle, \langle v_2, [0.7, 0.9], [0, 0.1]\rangle\})), (b_2, (\{\langle h_1, [0.3, 0.5], [0.2, 0.3]\rangle,$
$\langle h_2, [0.3, 0.5], [0.2, 0.4]\rangle, \langle h_3, [0.7, 0.9], [0, 0.1]\rangle\}, \{\langle c_1, [0.3, 0.5], [0.2, 0.4]\rangle,$
$\langle c_2, [0.4, 0.6], [0.3, 0.4]\rangle, \langle c_3, [0.7, 0.9], [0, 0]\rangle\}, \{\langle v_1, [0.5, 0.7], [0.2, 0.3]\rangle,$
$\langle v_2, [0.3, 0.5], [0.2, 0.4]\rangle\})), (b_3, (\{\langle h_1, [0.6, 0.8], [0.1, 0.2]\rangle,$
$\langle h_2, [0.6, 0.8], [0.1, 0.2]\rangle, \langle h_3, [0, 0.2], [0.5, 0.7]\rangle\}, \{\langle c_1, [0.7, 0.9], [0, 0.1]\rangle,$
$\langle c_2, [0.5, 0.7], [0.1, 0.3]\rangle, \langle c_3, [0.2, 0.4], [0.3, 0.5]\rangle\}, \{\langle v_1, [0.4, 0.6], [0.2, 0.4]\rangle,$
$\langle v_2, [0.3, 0.5], [0.2, 0.4]\rangle\}))\}.$

Then $(F, A)\tilde{\cup}(G, B) = (H, D)$=
$\{(d_1, (\{\langle h_1, [0.2, 0.4], [0.3, 0.5]\rangle, \langle h_2, [0.4, 0.6], [0.2, 0.4]\rangle, \langle h_3, [0.7, 0.9], [0, 0.1]\rangle\},$
$\{\langle c_1, [0.7, 0.9], [0, 0]\rangle, \langle c_2, [0.6, 0.8], [0.1, 0.2]\rangle, \langle c_3, [0.4, 0.6], [0.2, 0.4]\rangle\},$
$\{\langle v_1, [0.7, 0.9], [0, 0]\rangle, \langle v_2, [0.7, 0.9], [0, 0.1]\rangle\})), (d_2, (\{\langle h_1, [0.3, 0.5], [0.2, 0.3]\rangle,$
$\langle h_2, [0.3, 0.5], [0.2, 0.4]\rangle, \langle h_3, [0.7, 0.9], [0, 0.1]\rangle\}, \{\langle c_1, [0.3, 0.5], [0.2, 0.4]\rangle,$
$\langle c_2, [0.4, 0.6], [0.3, 0.4]\rangle, \langle c_3, [0.7, 0.9], [0, 0]\rangle\}, \{\langle v_1, [0.5, 0.7], [0.2, 0.3]\rangle,$
$\langle v_2, [0.3, 0.5], [0.2, 0.4]\rangle\})), (d_3, (\{\langle h_1, [0.6, 0.8], [0.1, 0.2]\rangle,$
$\langle h_2, [0.6, 0.8], [0.1, 0.2]\rangle, \langle h_3, [0, 0.2], [0.5, 0.7]\rangle\}, \{\langle c_1, [0.7, 0.9], [0, 0.1]\rangle,$
$\langle c_2, [0.5, 0.7], [0.1, 0.3]\rangle, \langle c_3, [0.2, 0.4], [0.3, 0.5]\rangle\}, \{\langle v_1, [0.4, 0.6], [0.2, 0.4]\rangle,$
$\langle v_2, [0.3, 0.5], [0.2, 0.4]\rangle\})), (d_4, (\{\langle h_1, [0.6, 0.8], [0.1, 0.2]\rangle,$
$\langle h_2, [0.6, 0.8], [0.1, 0.2]\rangle, \langle h_3, [0, 0.2], [0.5, 0.7]\rangle\}, \{\langle c_1, [0.7, 0.9], [0, 0.1]\rangle,$
$\langle c_2, [0.5, 0.7], [0.1, 0.3]\rangle, \langle c_3, [0.2, 0.4], [0.3, 0.5]\rangle\}, \{\langle v_1, [0.4, 0.6], [0.2, 0.4]\rangle,$
$\langle v_2, [0.3, 0.5], [0.2, 0.4]\rangle\}))\},$
where D=$\{d_1 = a_1 = b_1, d_2 = a_2 = b_2, d_3 = a_3, d_4 = b_3\}$.

Proposition 3.19. *If (F, A), (G, B) and (H, C) are three interval valued intuitionistic fuzzy soft multisets over U, then*

(a) $(F, A)\tilde{\cup}((G, B)\tilde{\cup}(H, C)) = ((F, A)\tilde{\cup}(G, B))\tilde{\cup}(H, C)$
(b) $(F, A)\tilde{\cup}(F, A) = (F, A)$
(c) $(F, A)\tilde{\cup}(G, A)_{\approx\phi_i} = (R, A)$, *where R is defined by* 3.16
(d) $(F, A)\tilde{\cup}(G, A)_{\phi} = (F, A)$

(e) $(F,A)\tilde{\cup}(G,B)_{\approx\phi_i} = (R,D),\ where\ D = A\cup B\ and\ R\ is\ defined\ by$ 3.16

(f) $(F,A)\tilde{\cup}(G,B)_{\phi} = \begin{cases} (F,A), & if\ A=B \\ (R,D), & otherwise \end{cases}$

where $D = A\cup B$

(g) $(F,A)\tilde{\cup}(G,A)_{U_i} = (R,A)_{\approx U_i}$

(h) $(F,A)\tilde{\cup}(G,A)_{U} = (G,A)_U$

(i) $(F,A)\tilde{\cup}(G,B)_{U_i} = \begin{cases} (R,D)_{U_i}, & if\ A=B \\ (R,D), & otherwise \end{cases}$

where $D = A\cup B$

(j) $(F,A)\tilde{\cup}(G,B)_{U} = \begin{cases} (G,B)_{U}, & if\ A=B \\ (R,D), & otherwise \end{cases}$

where $D = A\cup B$

Proof. Straightforward. □

Definition 3.20. The intersection of two interval valued intuitionistic fuzzy soft multisets (F, A) and (G, B) over U is an interval valued intuitionistic fuzzy soft multiset (H, D) where $D = A\cup B$ and $\forall e\in D$,

$$H(e) = \begin{cases} F(e), & if\ e\in A-B \\ G(e), & if\ e\in B-A \\ \bigcap(F(e),G(e)) & if\ e\in A\cap B \end{cases}$$

where $\bigcap(F(e),G(e)) = F_{e_{U_i,j}} \cap G_{e_{U_i,j}}\quad \forall i\in\{1,2,...,m\}$ with $\cap$ as an interval valued intuitionistic fuzzy intersection and is written as $(F,A)\tilde{\cap}(G,B) = (H,D)$.

Example 3.21. Let us consider example 3.18. Then
$(F,A)\tilde{\cap}(G,B) = (H,D)$=
$\{(d_1,(\{\langle h_1,[0.1,0.3],[0.3,0.5]\rangle,\langle h_2,[0.3,0.4],[0.4,0.6]\rangle,\langle h_3,[0.7,0.9],[0,0.1]\rangle\},$
$\{\langle c_1,[0.7,0.9],[0,0]\rangle,\langle c_2,[0.4,0.6],[0.3,0.4]\rangle,\langle c_3,[0.3,0.5],[0.2,0.4]\rangle\},$
$\{\langle v_1,[0.7,0.9],[0,0]\rangle,\langle v_2,[0.6,0.8],[0,0.2]\rangle\})),(d_2,(\{\langle h_1,[0.1,0.3],[0.3,0.5]\rangle,$
$\langle h_2,[0.3,0.5],[0.4,0.5]\rangle,\langle h_3,[0.7,0.9],[0,0.1]\rangle\},\{\langle c_1,[0.3,0.5],[0.2,0.4]\rangle,$
$\langle c_2,[0.4,0.6],[0.3,0.4]\rangle,\langle c_3,[0.7,0.9],[0,0]\rangle\},\{\langle v_1,[0.3,0.5],[0.2,0.4]\rangle,$
$\langle v_2,[0.3,0.5],[0.2,0.4]\rangle\})),(d_3,(\{\langle h_1,[0.6,0.8],[0.1,0.2]\rangle,$
$\langle h_2,[0.6,0.8],[0.1,0.2]\rangle,\langle h_3,[0,0.2],[0.5,0.7]\rangle\},\{\langle c_1,[0.7,0.9],[0,0.1]\rangle,$
$\langle c_2,[0.5,0.7],[0.1,0.3]\rangle,\langle c_3,[0.2,0.4],[0.3,0.5]\rangle\},\{\langle v_1,[0.4,0.6],[0.2,0.4]\rangle,$
$\langle v_2,[0.3,0.5],[0.2,0.4]\rangle\})),(d_4,(\{\langle h_1,[0.6,0.8],[0.1,0.2]\rangle,$
$\langle h_2,[0.6,0.8],[0.1,0.2]\rangle,\langle h_3,[0,0.2],[0.5,0.7]\rangle\},\{\langle c_1,[0.7,0.9],[0,0.1]\rangle,$
$\langle c_2,[0.5,0.7],[0.1,0.3]\rangle,\langle c_3,[0.2,0.4],[0.3,0.5]\rangle\},\{\langle v_1,[0.4,0.6],[0.2,0.4]\rangle,$
$\langle v_2,[0.3,0.5],[0.2,0.4]\rangle\}))\},$
where D=$\{d_1 = a_1 = b_1, d_2 = a_2 = b_2, d_3 = a_3, d_4 = b_3\}$.

Proposition 3.22. *If (F, A), (G, B) and (H, C) are three interval valued intuitionistic fuzzy soft multisets over U, then*

(a) $(F,A)\tilde{\cap}((G,B)\tilde{\cap}(H,C)) = ((F,A)\tilde{\cap}(G,B))\tilde{\cap}(H,C)$

(b) $(F,A)\tilde{\cap}(F,A) = (F,A)$

(c) $(F,A)\tilde{\cap}(G,A)_{\approx\phi_i} = (R,A)_{\approx\phi_i},\ where\ R\ is\ defined\ by$ 3.19

(d) $(F,A)\tilde{\cap}(G,A)_{\phi} = (R,A)_{\phi}$

(e) $(F,A)\tilde{\cap}(G,B)_{\approx\phi_i} = \begin{cases} (R,D)_{\phi_i}, & if\ A \subseteq B \\ (R,D), & otherwise \end{cases}$
where $D = A \cup B$ and R is defined by 3.19
(f) $(F,A)\tilde{\cap}(G,B)_{\phi} = \begin{cases} (R,D)_{\phi}, & if\ A \subseteq B \\ (R,D), & otherwise \end{cases}$
where $D = A \cup B$ and R is defined by 3.19
(g) $(F,A)\tilde{\cap}(G,A)_{U_i} = (R,D)$ *where* $D = A \cup B$ *and* R *is defined by 3.19*
(h) $(F,A)\tilde{\cup}(G,A)_{U} = (F,A)$
(i) $(F,A)\tilde{\cap}(G,B)_{U_i} = (R,D)$ *where* $D = A \cup B$ *and* R *is defined by 3.19*
(j) $(F,A)\tilde{\cap}(G,B)_{U} = \begin{cases} (F,A), & if\ A \supseteq B \\ (R,D), & otherwise \end{cases}$
where $D = A \cup B$ and R is defined by 3.19.

Proof. Straightforward. □

4. Relations on interval valued intuitionistic fuzzy soft sets

The concept of interval valued intuitionistic fuzzy soft multiset relations (IVIFSMS-relations for short) is given in this section. Also the basic properties of the IVIFSMS-relations are presented in this section.

Definition 4.1. Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} IVIFS(U_i)$ where $IVIFS(U_i)$ denotes the set of all interval valued intuitionistic fuzzy subsets of U_i ,$E = \prod_{i\in I} E_{U_i}$ and A,B⊆ E. Let (F, A) and (G, B) be two interval valued intuitionistic fuzzy soft multisets over U, where F,G are mappings given by $F, G : A \to U$. Then a relation between them is defined as a pair (H, A×B), where H is mapping given by H: A×B→U. The collection of relations on interval valued intuitionistic fuzzy soft multisets on A×B over U is denoted by MSSR_U(A×B).

Example 4.2. Let us consider there are three universes U_1, U_2 and U_3. Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2\}$ and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\}$, $E_{U_2} = \{e_{U_2,1}, e_{U_2,2}\}$, $E_{U_3} = \{e_{U_3,1}, e_{U_3,2}\}$.
Let $U = \prod_{i=1}^{3} IVIFS(U_i)$,$E = \prod_{i=1}^{3} E_{U_i}$ and A,B⊆ E, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$$

and

$$B = \{b_1 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,1}), b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,2})\}.$$

Let the tabular representation of the interval valued intuitionistic fuzzy soft multiset (F, A) be given below:

	a_1	a_2
h_1	([.1, .3], [.3, .5])	([.2, .5], [.3, .4])
h_2	([.3, .4], [.4, .6])	([.1, .2], [.4, .7])
h_3	([.7, .9], [0, .1])	([.3, .6], [.2, .4])
c_1	([.7, .9], [0, 0])	([.3, .6], [.2, .4])
c_2	([.4, .6], [.3, .4])	([.4, .5], [.1, .3])
v_1	([.7, .9], [0, 0])	([.6, .8], [0, 0])
v_2	([.6, .8], [.1, .2])	([.4, .7], [.1, .2])

Let the tabular representation of the interval valued intuitionistic fuzzy soft multiset (G, B) be given below:

	b_1	b_2
h_1	([.2, .3], [.4, .6])	([.1, .2], [.2, .4])
h_2	([.4, .5], [.3, .4])	([.3, .4], [.1, .3])
h_3	([.6, .7], [.2, .3])	([.4, .6], [.2, .3])
c_1	([.8, .9], [0, .1])	([.5, .8], [0, 0])
c_2	([.3, .5], [.2, .4])	([.3, .7], [.1, .2])
v_1	([.4, .7], [.2, .3])	([.2, .4], [.2, .5])
v_2	([.5, .7], [.1, .3])	([.6, .8], [.1, .2])

Then a relation $R_1(= (H, A \times B), say)$ between them is given by

	(a_1, b_1)	(a_1, b_2)	(a_2, b_1)	(a_2, b_2)
h_1	([.2, .3], [.4, .6])	([.6, .8], [.1, .2])	([.2, .4], [.2, .4])	([.5, .7], [.1, .3])
h_2	([.4, .5], [.3, .4])	([.4, .7], [.2, .3])	([.6,.7], [.1, .2])	([.4, .5], [.3, .4])
h_3	([.4, .6], [.3, .4])	([.2, .4],[.2, .5])	([.2, .3], [.4, .6])	([.2, .4],[.4, .6])
c_1	([.2, .5], [.1, .3])	([.1, .4], [.4, .6])	([.5, .7], [.1, .3])	([.3, .6], [.1, .3])
c_2	([.5, .7],[.2, .3])	([.4, .5], [.3, .4])	([.1, .3], [.4,.5])	([.1, .2], [.4, .7])
v_1	([.3, .6],[.1, .3])	([.3, .7], [.1, .2])	([.6, .8], [.1,.2])	([.4, .7], [.2, .3])
v_2	([.6, .8],[.1, .2])	([.2, .3], [.4, .6])	([.2, .5], [.3,.4])	([.2, .3], [.4, .5])

Let a relation $R_2(= (J, A \times B), say)$ between them be given by

	(a_1, b_1)	(a_1, b_2)	(a_2, b_1)	(a_2, b_2)
h_1	([.4, .6], [.3, .4])	([.4, .5], [.2, .4])	([.3, .4], [.1, .3])	([.4, .7], [.1, .2])
h_2	([.1, .5], [.1, .4])	([.2, .3], [.6, .7])	([.5,.6], [.2, .3])	([.4, .5], [.3, .4])
h_3	([.3, .4], [.3, .5])	([.4, .7],[.1, .3])	([.2, .3], [.4, .6])	([.2, .4],[.4, .6])
c_1	([.2, .3], [.2, .4])	([.1, .2], [.4, .6])	([.3, .6], [.2, .4])	([.4, .6], [.1, .4])
c_2	([.5, .6],[.1, .3])	([.3, .6], [.2, .4])	([.4, .5], [.2,.3])	([.1, .2], [.4, .7])
v_1	([.4, .5],[.2, .3])	([.5, .7], [.1, .3])	([.4, .6], [.3,.4])	([.3, .7], [.1, .2])
v_2	([0, .1],[.7, .9])	([.2, .3], [.6, .7])	([.2, .4], [.5,.6])	([.2, .3], [.3, .6])

The tabular representations of R_1 and R_2 are called relational matrices for R_1 and R_2 respectively. From above we have, $\mu_{H(a_1,b_2)}(h_1)$=[0.6, 0.8] and $\gamma_{J(a_1,b_2)}(c_2)$=[0.2, 0.4] etc. But this intervals lie on the 1st row-2nd column and 5th row-2nd column respectively. So we denote $\mu_{H(a_1,b_2)}(h_1)|_{(1,2)}$=[0.6, 0.8] and $\gamma_{J(a_1,b_2)}(c_2)|_{(5,2)}$=[0.2, 0.4] etc to make the clear concept about what are the positions of the intervals in the relational matrices.

Remark 4.3. Let (F_1, A_1), (F_2, A_2),...,(F_n, A_n) be n numbers of interval valued intuitionistic fuzzy soft multisets over U. Then a relation R between them is defined as a pair (H, $A_1 \times A_2 \times ... \times A_n$), where H is mapping given by H: $A_1 \times A_2 \times .... \times A_n \rightarrow$ U.

Definition 4.4. Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} IVIFS(U_i)$ where $IVIFSFS(U_i)$ denotes the set of all interval valued intuitionistic fuzzy subsets of U_i ,$E = \prod_{i\in I} E_{U_i}$ and A,B⊆ E.Let (F, A) and (G, B) be two interval valued intuitionistic fuzzy soft multisets over U, where F,G are mappings given by $F, G : A \to U$. Let R be a relation between them. Then the order of the relational matrix is (α, β), where $\alpha = \sum_i n(U_i)$ and β=number of pairs of parameters considered in the relational matrix. In example 4.2, both the relational matrix for R_1 and R_2 are of order (7, 4). If α=β, then the relation matrix is called a square matrix.

Definition 4.5. Let $R_1, R_2 \in MSSR_U$(A×B), R_1=(H, A×B), R_2=(J, A×B) and the order of their relational matrices are same. Then we define

(i) $R_1 \bigvee R_2$=(H⧫J, A×B), where H⧫J: A×B→ U is defined as

(H⧫J)(a_i, b_j) =H(a_i, b_j)⋃J(a_j b_j)

for (a_i, b_j)∈A×B, where ⋃ denotes the interval valued intuitionistic fuzzy union.

(ii) $R_1 \bigwedge R_2$=(H•J, A×B), where H•J: A×B→ U is defined as

(H•J)(a_i, b_j)=H(a_i, b_j)⋂J(a_i, b_j)

for (a_i, b_j)∈A×B, where ⋂ denotes the interval valued intuitionistic fuzzy intersection.

(iii) R_1^C =(∼H, A×B), where ∼H: A×B→U is defined as

∼H(a_i, b_j)=[H(a_i, b_j)]$^\sharp$

for (a_i, b_j)∈ A×B, where ♯ denotes the interval valued intuitionistic fuzzy complement.

Example 4.6. Consider the interval valued intuitionistic fuzzy soft multi-sets (F, A) and (G, B) given in example 4.2. Then we get

$R_1 \bigvee R_2$:

	(a_1, b_1)	(a_1, b_2)	(a_2, b_1)	(a_2, b_2)
h_1	([.4, .6], [.3, .4])	([.6, .8], [.1, .2])	([.3, .4], [.1, .3])	([.5, .7], [.1, .2])
h_2	([.4, .5], [.1, .4])	([.4, .7], [.2, .3])	([.6,.7], [.1, .2])	([.4, .5], [.3, .4])
h_3	([.4, .6], [.3, .4])	([.4, .7],[.1, .3])	([.2, .3], [.4, .6])	([.2, .4],[.4, .6])
c_1	([.2, .5], [.1, .3])	([.1, .4], [.4, .6])	([.5, .7], [.1, .3])	([.4, .6], [.1, .3])
c_2	([.5, .7],[.1, .3])	([.4, .6], [.2, .4])	([.4, .5], [.2,.3])	([.1, .2], [.4, .7])
v_1	([.4, .6],[.1, .3])	([.5, .7], [.1, .2])	([.6, .8], [.1,.2])	([.4, .7], [.1, .2])
v_2	([.6, .8],[.1, .2])	([.2, .3], [.4, .6])	([.2, .5], [.3,.4])	([.2, .3], [.3, .5])

$R_1 \bigwedge R_2$:

	(a_1, b_1)	(a_1, b_2)	(a_2, b_1)	(a_2, b_2)
h_1	([.2, .3], [.4, .6])	([.4, .5], [.2, .4])	([.2, .4], [.2, .4])	([.4, .7], [.1, .3])
h_2	([.1, .5], [.3, .4])	([.2, .3], [.6, .7])	([.5,.6], [.2, .3])	([.4, .5], [.3, .4])
h_3	([.3, .4], [.3, .5])	([.2, .4],[.2, .5])	([.2, .3], [.4, .6])	([.2, .4],[.4, .6])
c_1	([.2, .3], [.2, .4])	([.1, .2], [.4, .6])	([.3, .6], [.2, .4])	([.3, .6], [.1, .4])
c_2	([.5, .6],[.2, .3])	([.3, .5], [.3, .4])	([.1, .3], [.4,.5])	([.1, .2], [.4, .7])
v_1	([.3, .5],[.2, .3])	([.3, .7], [.1, .3])	([.4, .6], [.3,.4])	([.3, .7], [.2, .3])
v_2	([0, .1],[.7, .9])	([.2, .3], [.6, .7])	([.2, .4], [.5,.6])	([.2, .3], [.4, .5])

(iii) $R_1{}^C$:

	(a_1, b_1)	(a_1, b_2)	(a_2, b_1)	(a_2, b_2)
h_1	([.4, .6], [.2, .3])	([.1, .2,[.6, .8])	([.2, .4], [.2, .4])	([.1, .3], [.5, .7])
h_2	([.3, .4], [.4, .5])	([.2, .3], [.4, .7])	([.1,.2], [.6, .7])	([.3, .4], [.4, .5])
h_3	([.3, .4], [.4, .6])	([.2, .5],[.2, .4])	([.4, .6], [.2, .3])	([.4, .6],[.2, .4])
c_1	([.1, .3], [.2, .5])	([.4, .6], [.1, .4])	([.1, .3], [.5, .7])	([.1, .3], [.3, .6])
c_2	([.2, .3],[.5, .7])	([.3, .4], [.4, .5])	([.4, .5], [.1,.3])	([.4, .7], [.1, .2])
v_1	([.1, .3],[.3, .6])	([.1, .2], [.3, .7])	([.1, .2], [.6,.8])	([.2, .3], [.4, .7])
v_2	([.1, .2],[.6, .8])	([.4, .6], [.2, .3])	([.3, .4], [.2,.5])	([.4, .5], [.2, .3])

Theorem 4.7. *Let $R_1, R_2, R_3 \in MSSR_U(A \times B)$ and the order of their relational matrices are same. Then the following properties hold:*

(a) $(R_1 \bigvee R_2)^C = R_1^C \bigwedge R_2^C$
(b) $(R_1 \bigwedge R_2)^C = R_1^C \bigvee R_2^C$
(c) $R_1 \bigvee (R_2 \bigvee R_3) = (R_1 \bigvee R_2) \bigvee R_3$
(d) $R_1 \bigwedge (R_2 \bigwedge R_3) = (R_1 \bigwedge R_2) \bigwedge R_3$
(e) $R_1 \bigwedge (R_2 \bigvee R_3) = (R_1 \bigwedge R_2) \bigvee (R_1 \bigwedge R_3)$
(f) $R_1 \bigvee (R_2 \bigwedge R_3) = (R_1 \bigvee R_2) \bigwedge (R_1 \bigvee R_3)$

Proof. Straightforward. □

Definition 4.8. Let $R_1, R_2 \in MSSR_U(A \times B)$ and the order of their relational matrices are same. Then $R_1 \leq R_2$ iff H(a_i, b_j)⊆J(a_i, b_j) for (a_i, b_j)∈ $A \times B$ where R_1=(H, A×B) and R_2= (J, A×B).

Example 4.9. Consider the interval valued intuitionistic fuzzy soft multi-sets (F, A) and (G, B) given in example 4.2.Let $R_1, R_2 \in MSSR_U(A \times B)$ be defined as follows:

R_1 :

	(a_1, b_1)	(a_1, b_2)	(a_2, b_1)	(a_2, b_2)
h_1	([.2, .3], [.4, .6])	([.6, .8], [.1, .2])	([.2, .4], [.2, .4])	([.5, .7], [.1, .3])
h_2	([.4, .5], [.3, .4])	([.4, .7], [.2, .3])	([.6,.7], [.1, .2])	([.4, .5], [.3, .4])
h_3	([.4, .6], [.3, .4])	([.2, .4],[.2, .5])	([.2, .3], [.4, .6])	([.2, .4],[.4, .6])
c_1	([.2, .5], [.1, .3])	([.1, .4], [.4, .6])	([.5, .7], [.1, .3])	([.3, .6], [.1, .3])
c_2	([.5, .7],[.2, .3])	([.4, .5], [.3, .4])	([.1, .3], [.4,.5])	([.1, .2], [.4, .7])
v_1	([.3, .6],[.1, .3])	([.3, .7], [.1, .3])	([.6, .8], [.1,.2])	([.4, .7], [.2, .3])
v_2	([.6, .8],[.1, .2])	([.2, .3], [.4, .6])	([.2, .5], [.3,.4])	([.2, .3], [.4, .5])

R_2 :

	(a_1, b_1)	(a_1, b_2)	(a_2, b_1)	(a_2, b_2)
h_1	([.2, .4], [.3, .4])	([.6, .8], [.1, .2])	([.3, .5], [.1, .3])	([.5, .8], [.1, .2])
h_2	([.4, .6], [.1, .4])	([.6, .7], [.2, .3])	([.6,.8], [.1, .2])	([.3, .6], [.3, .4])
h_3	([.4, .6], [.3, .4])	([.2, .4],[.2, .4])	([.2, .4], [.4, .5])	([.2, .5],[.4, .5])
c_1	([.2, .5], [.1, .2])	([.1, .5], [.3, .4])	([.5, .7], [.1, .2])	([.3, .6], [.1, .2])
c_2	([.6, .7],[.1, .2])	([.4, .5], [.3, .4])	([.2, .4], [.4,.5])	([.2, .3], [.3, .6])
v_1	([.4, .8],[.1, .2])	([.3, .8], [.1, .2])	([.6, .8], [.1,.2])	([.4, .7], [.2, .3])
v_2	([.6, .9],[0, .1])	([.2, .3], [.4, .5])	([.5, .6], [.2,.3])	([.3, .4], [.4, .5])

Then $R_1 \leq R_2$.

Definition 4.10. Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} IVIFS(U_i)$ where $IVIFS(U_i)$ denotes the set of all interval valued intuitionistic fuzzy subsets of U_i, $E = \prod_{i\in I} E_{U_i}$ and A,B⊆ E. Let (F, A) and (G, B) be two interval valued intuitionistic fuzzy soft multisets over U, where F,G are mappings given by $F, G : A \to U$.

(i) Then a null relation between them is denoted by O_U and is defined as $O_U = (H, A \times B)_\phi$

(ii) Then an absolute relation between them is denoted by I_U and is defined as $I_U = (H, A \times B)_U$

Example 4.11. Consider the interval valued intuitionistic fuzzy soft multisets (F, A) and (G, B) given in example 4.2.
Then a null relation between them is given by:

	(a_1, b_1)	(a_1, b_2)	(a_2, b_1)	(a_2, b_2)
h_1	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])
h_2	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])
h_3	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])
c_1	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])
c_2	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])
v_1	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])
v_2	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])	([0, 0], [1, 1])

An absulate relation between them is given by:

	(a_1, b_1)	(a_1, b_2)	(a_2, b_1)	(a_2, b_2)
h_1	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])
h_2	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])
h_3	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])
c_1	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])
c_2	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])
v_1	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])
v_2	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])	([1, 1], [0, 0])

Remark 4.12. For any R∈$MSSR_U$(A×B), we have

(i) $R \bigvee O_U = R$

(ii) $R \bigwedge O_U = O_U$

(iii) $R \bigvee I_U = I_U$

(iv) $R \bigwedge I_U = R$.

5. Various types of interval valued intuitionistic fuzzy soft multiset relations

Various types of IVIFSMS-relations are presented in this section.

Definition 5.1. Let $\{U_\lambda : \lambda \in I\}$ be a collection of universes such that $\bigcap_{\lambda\in I} U_\lambda = \phi$ and let $\{E_{U_\lambda} : \lambda \in I\}$ be a collection of sets of parameters. Let $U = \prod_{\lambda\in I} IVIFS(U_\lambda)$

where $IVIFS(U_\lambda)$ denotes the set of all interval valued intuitionistic fuzzy subsets of U_λ ,$E = \prod_{\lambda\in I} E_{U_\lambda}$ and A,B⊆ E.Let (F, A) and (G, B) be two interval valued intuitionistic fuzzy soft multisets over U. Let R∈$MSSR_U$(A×B) and R=(H, A×B) whose relational matrix is a square matrix. Then R is called a reflexive IVIFSMS-relation if for (a_i, b_j)∈ $A \times B$ and $h^*_k \in U_\lambda$, we have, $\mu_{H(a_i,b_j)}(h^*_k)|_{(m,n)}$=[1, 1] and $\gamma_{H(a_i,b_j)}(h^*_k)|_{(m,n)}$=[0, 0] for m=n=k.

Example 5.2. Let us consider there are three universes U_1 , U_2 and U_3. Let

$$U_1 = \{h_1, h_2\} \text{ , } U_2 = \{c_1\} \text{ and } U_3 = \{v_1\}.$$

Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\},\ E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\},\ E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}.$$

Let $U = \prod_{\lambda=1}^{3} IVIFS(U_i)$, $E = \prod_{\lambda=1}^{3} E_{U_i}$ and A,B⊆ E, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$$

and

$$B = \{b_1 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,1}), b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,2})\}.$$

Then a reflexive IVIFSS-relation between them is

	(a_1, b_1)	(a_1, b_2)	(a_2, b_1)	(a_2, b_2)
h^*_1	([1, 1], [0, 0])	([.6, .8], [.1, .2])	([.3, .5], [.1, .3])	([.5, .8], [.1, .2])
h^*_2	([.4, .6], [.1, .4])	([1, 1], [0, 0])	([.6,.8], [.1, .2])	([.3, .6], [.3, .4])
h^*_3	([.4, .6], [.3, .4])	([.2, .4],[.2, .4])	([1, 1], [0, 0])	([.2, .5],[.4, .5])
h^*_4	([.2, .5], [.1, .2])	([.1, .5], [.3, .4])	([.5, .7], [.1, .2])	([1, 1], [0, 0])

where $h^*_1 = h_1$,$h^*_2 = h_2$,$h^*_3 = c_1$,$h^*_4 = v_1$.

Definition 5.3. Let $\{U_\lambda : \lambda \in I\}$ be a collection of universes such that $\bigcap_{\lambda\in I} U_\lambda = \phi$ and let $\{E_{U_\lambda} : \lambda \in I\}$ be a collection of sets of parameters. Let $U = \prod_{\lambda\in I} IVIFS(U_\lambda)$ where $IVIFS(U_\lambda)$ denotes the set of all interval valued intuitionistic fuzzy subsets of U_λ ,$E = \prod_{\lambda\in I} E_{U_\lambda}$ and A,B⊆ E.Let (F, A) and (G, B) be two interval valued intuitionistic fuzzy soft multisets over U. Let R∈$MSSR_U$(A×B) and R=(H, A×B) whose relational matrix is a square matrix. Then R is called a symmetric IVIFSMS-relation if for each $(a_i, b_j) \in A \times B$ and $h^*_k \in U_\lambda$, $\exists (a_p, a_q) \in A \times B$ and $h^*_l \in U_\lambda$ such that $\mu_{H(a_i,b_j)}(h^*_k)|_{(m,n)} = \mu_{H(a_p,b_q)}(h^*_l)|_{(n,m)}$ and $\gamma_{H(a_i,b_j)}(h^*_k)|_{(m,n)} = \gamma_{H(a_p,b_q)}(h^*_l)|_{(n,m)}$.

Example 5.4. Let us consider there are three universes U_1 , U_2 and U_3.Let $U_1 = \{h_1, h_2\}$, $U_2 = \{c_1\}$ and $U_3 = \{v_1\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\},\ E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\},\ E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}.$$

Let $U = \prod_{\lambda=1}^{3} IVIFS(U_\lambda)$, $E = \prod_{\lambda=1}^{3} E_{U_\lambda}$ and A,B⊆ E, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$$

and

$$B = \{b_1 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,1}), b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,2})\}.$$

Then a symmetric IVIFSMS-relation between them is

	(a$_1$, b$_1$)	(a$_1$, b$_2$)	(a$_2$, b$_1$)	(a$_2$, b$_2$)
h_1^*	([0, .2], [.4, .6])	([.6, .8], [.1, .2])	([.3, .5], [.1, .3])	([.2, .5], [.1, .2])
h_2^*	([.6, .8], [.1, .2])	([.3, .4], [.5, .6])	([.2,.4], [.2, .4])	([.3, .6], [.3, .4])
h_3^*	([.3, .5], [.1, .3])	([.2, .4],[.2, .4])	([0, 0], [1, 1])	([.2, .5],[.4, .5])
h_4^*	([.2, .5], [.1, .2])	([.3, .6], [.3, .4])	([.2, .5], [.4, .5])	([.4, .6], [.2, .3])

where $h_1^* = h_1$,$h_2^* = h_2$,$h_3^* = c_1$,$h_4^* = v_1$.

Definition 5.5. Let $\{U_\lambda : \lambda \in I\}$ be a collection of universes such that $\bigcap_{\lambda\in I} U_\lambda = \phi$ and let $\{E_{U_\lambda} : \lambda \in I\}$ be a collection of sets of parameters. Let $U = \prod_{\lambda\in I} IVIFS(U_\lambda)$ where $IVIFS(U_\lambda)$ denotes the set of all interval valued intuitionistic fuzzy subsets of U_λ ,$E = \prod_{\lambda\in I} E_{U_\lambda}$ and A⊆ E.Let (F, A) and (G, A) be two interval valued intuitionistic fuzzy soft multisets over U. Let $R_1, R_2 \in MSSR_U(A\times A)$ and R_1=(H, A×A), R_2=(J, A×A) and the order of their relational matrices are same. Then the composition of R_1 and R_2, denoted by $R_1 \circ R_2$, is defined by $R_1 \circ R_2$=(H∘ J, A×A) where H∘J: A×A→ U is defined as

$$(H \circ J)(a_i, a_j) = (\{\langle h_k^*, \mu_{(H\circ J)(a_i,a_j)}(h_k^*), \gamma_{(H\circ J)(a_i,a_j)}(h_k)\rangle : h_k^* \in U_\lambda\} : \lambda \in I),$$

where

$$\mu_{(H\circ J)(a_i,a_j)}(h_k^*) = [max_l(min(inf\mu_{H(a_i,a_l)}(h_k^*), inf\mu_{J(a_l,a_j)}(h_k^*))),\\ max_l(min(sup\mu_{H(a_i,a_l)}(h_k^*), sup\mu_{J(a_l,a_j)}(h_k^*)))]$$

and

$$\gamma_{(H\circ J)(a_i,a_j)}(h_k^*) = [min_l(max(inf\gamma_{H(a_i,a_l)}(h_k^*), inf\gamma_{J(a_l,a_j)}(h_k^*))),\\ min_l(max(sup\gamma_{H(a_i,a_l)}(h_k^*), sup\gamma_{J(a_l,a_j)}(h_k^*)))]$$

for $(a_i, a_j) \in A \times A$.

Example 5.6. Let us consider there are three universes U_1, U_2 and U_3. Let

$$U_1 = \{h_1, h_2\}\ , U_2 = \{c_1\} \text{ and } U_3 = \{v_1\}.$$

Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\},\ E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\},\ E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}.$$

Let $U = \prod_{\lambda=1}^3 IVIFS(U_\lambda)$, $E = \prod_{\lambda=1}^3 E_{U_\lambda}$ and A⊆ E, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}.$$

Let $R_1, R_2 \in MSSR_U(A \times A)$ be defined by

R_1 :

	(a$_1$, a$_1$)	(a$_1$, a$_2$)	(a$_2$, a$_1$)	(a$_2$, a$_2$)
h_1^*	([.3, .4], [.3, .4])	([.2, .4], [.3, .5])	([.2, .5], [.3, .4])	([.2, .3], [.3, .6])
h_2^*	([1, 1], [0, 0])	([.1, .2], [0, 0])	([.4, .5], [.1, .3])	([.4, .7], [.1, .3])
h_3^*	([.2, .6], [.1, .4])	([.2, .6],[.1, .3])	([.2, .3], [.4, .6])	([.2, .5],[.2, .3])
h_4^*	([.2, .4], [.3, .5])	([.3, .4], [.4, .5])	([.3, .4], [.2, .3])	([0, .2], [.4, .5])

R_2 :

	(a$_1$, a$_1$)	(a$_1$, a$_2$)	(a$_2$, a$_1$)	(a$_2$, a$_2$)
h_1^*	([.5, .8], [.1, .2])	([.2, .3], [.3, .6])	([.1, .4], [.3, .5])	([.2, .4], [.2, .3])
h_2^*	([.4, .5], [.2, .4])	([.4, .6], [.2, .3])	([.1,.5], [.4, .5])	([.4, .5], [.1, .2])
h_3^*	([.2, .3], [.5, .6])	([.3, .4],[.4, .5])	([.7, .8], [.1, .2])	([.3, .5],[.3, .4])
h_4^*	([.3, .5], [.3, .4])	([.3, .5], [.2, .4])	([.2, .4], [.2, .3])	([.3, .7], [.1, .3])

where $h_1^* = h_1$,$h_2^* = h_2$,$h_3^* = c_1$,$h_4^* = v_1$. Then, $R_1 \circ R_2$:

U	(a_1, a_1)	(a_1, a_2)	(a_2, a_1)	(a_2, a_2)
h_1^*	([.3, .4], [.3, .4])	([.2, .4], [.3, .5])	([.2, .5], [.3, .4])	([.2, .3], [.3, .6])
h_2^*	([.4, .5], [.2, .4])	([.1, .6], [.1, .2])	([.4, .5], [.2, .4])	([.4, .5], [.1, .3])
h_3^*	([.2, .6], [.1, .3])	([.2, .5], [.3, .4])	([.2, .5], [.2, .3])	([.2, .5], [.3, .4])
h_4^*	([.2, .4], [.3, .5])	([.3, .4], [.3, .5])	([.3, .4], [.3, .4])	([.3, .4], [.2, .4])

Definition 5.7. Let $\{U_\lambda : \lambda \in I\}$ be a collection of universes such that $\bigcap_{\lambda \in I} U_\lambda = \phi$ and let $\{E_{U_\lambda} : \lambda \in I\}$ be a collection of sets of parameters. Let $U = \prod_{\lambda \in I} IVIFS(U_\lambda)$ where $IVIFS(U_\lambda)$ denotes the set of all interval valued intuitionistic fuzzy subsets of U_λ ,$E = \prod_{\lambda \in I} E_{U_\lambda}$ and A⊆ E.Let (F, A) and (G, A) be two interval valued intuitionistic fuzzy soft multisets over U.Let $R \in MSSR_U(A \times A)$. Then R is called a transitive IVIFSMS- relation if $R \circ R \subseteq R$.

Example 5.8. Let us consider there are three universes U_1, U_2 and U_3. Let

$$U_1 = \{h_1, h_2\},\ U_2 = \{c_1\} \text{ and } U_3 = \{v_1\}.$$

Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\},\ E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\},\ E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}.$$

Let $U = \prod_{\lambda=1}^{3} IVIFS(U_\lambda)$, $E = \prod_{\lambda=1}^{3} E_{U_\lambda}$ and A⊆ E, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}.$$

Let R :

	(a_1, a_1)	(a_1, a_2)	(a_2, a_1)	(a_2, a_2)
h_1^*	([.3, .4], [.3, .4])	([.2, .4], [.3, .6])	([.2, .5], [.3, .4])	([.2, .4], [.3, .6])
h_2^*	([1, 1], [0, 0])	([.1, .2], [0, 0])	([.4, .5], [.1, .3])	([.4, .7], [.1, .3])
h_3^*	([.2, .6], [.1, .4])	([.2, .6], [.1, .3])	([.2, .3], [.4, .6])	([.2, .5], [.2, .3])
h_4^*	([.3, .4], [.3, .4])	([.2, .4], [.3, .5])	([.2, .5], [.3, .4])	([.2, .4], [.3, .5])

Then $R \circ R$:

	(a_1, a_1)	(a_1, a_2)	(a_2, a_1)	(a_2, a_2)
h_1^*	([.3, .4], [.3, .4])	([.2, .4], [.3, .6])	([.2, .4], [.3, .4])	([.2, .4], [.3, .6])
h_2^*	([1, 1], [0, 0])	([.1, .2], [0, 0])	([.4, .5], [.1, .3])	([.4, .7], [.1, .3])
h_3^*	([.2, .6], [.1, .4])	([.2, .6], [.1, .3])	([.2, .3], [.4, .6])	([.2, .5], [.2, .3])
h_4^*	([.3, .4], [.3, .4])	([.2, .4], [.3, .5])	([.2, .4], [.3, .4])	([.2, .4], [.3, .5])

where $h_1^* = h_1$,$h_2^* = h_2$,$h_3^* = c_1$,$h_4^* = v_1$. Then clearly, $R \circ R \subseteq R$ and so R is a transitive IVIFSMS- relation.

6. Conclusions

In 1999 Molodtsov [9] introduced the concept of soft set theory as a general mathematical tool for dealing with uncertainties. Alkhazaleh et al. [2] in 2011 introduced the definition of soft multiset as a generalisation of Molodtsov soft set. In 2012 Alkhazaleh and Salleh [3] introduced the concept of fuzzy soft multiset theory. In this paper we have introduced the concept of interval valued intuitionistic fuzzy soft multisets and studied some of its properties and operations. Also we have defined interval valued intuitionistic fuzzy soft multiset relations. The basic properties of these relations are discussed. Also various types of these relations have been discussed in this paper.

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