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Intuitionistic fuzzy *lim* extremally disconnected spaces

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Abstract. In this paper we introduce the concept of a new class of an intuitionistic fuzzy convergence topological spaces. Besides giving some interesting properties of these spaces. The concept of an intuitionistic fuzzy *lim* extremally disconnected space is introduced and interesting properties are discussed.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh [11]. Since then the concept has invaded nearly all branches of Mathematics. Chang [2] introduced and developed the theory of fuzzy topological spaces and since then various notions in classical topology have been extended to fuzzy topological spaces. Atanassov [1] generalised fuzzy sets to intuitionistic fuzzy sets. Cocker [3,4] introduced the notions of an intuitionistic fuzzy topological space. Lowen et al.[6] defined the notion of an fuzzy convergence space. Tomasz Kubiak [8,9] studied L-Fuzzy normal spaces and Tietze extension Theorem and extending continuous L-Real functions. E.Roja, M.K.Uma and G.Balasubramanian [10] discussed Tietze extention theorem for ordered fuzzy pre extremally disconnected spaces. In this paper, the concepts of an intuitionistic fuzzy convergence space on the basis of an intuitionistic fuzzy filter is introduced and developed. The concepts of an intuitionistic fuzzy convergence topological space, intuitionistic fuzzy *lim* open set,intuitionistic fuzzy *lim* closed set, intuitionistic fuzzy *lim* extremally disconnected spaces are introduced and studied.

Some interesting properties and characterizations are discussed. Tietze extension theorem for intuitionistic fuzzy *lim* extremally disconnected spaces is estabilished.

2. Preliminaries

Definition 2.1 ([1])**.** Let X be a nonempty fixed set and I be the closed interval [0,1]. An intuitionistic fuzzy set(IFS) A is an object of the following form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, where the mappings $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form, $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.
For a given non empty set X, denote the family of all intuitionistic fuzzy sets in X by the symbol ζ^X.

Definition 2.2 ([1])**.** Let X be a nonempty set and the IFSs A and B in the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(i) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;
(ii) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;
(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$.

Definition 2.3 ([1])**.** The IFSs $0_\sim$ and $1_\sim$ are defined by $0_\sim$=$\{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim$=$\{\langle x, 1, 0\rangle : x \in X\}$.

Definition 2.4 ([3])**.** An intuitionistic fuzzy topology (IFT) in Coker's sense on a non empty set X is a family τ of IFSs in X satisfying the following axioms.

(T_1) $0_\sim$,$1_\sim \in \tau$
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(T_3) $\cup G_i \in \tau$ for arbitrary family $\{G_i/i \in I\} \subseteq \tau$

In this paper by (X, τ) or simply by X we will denote the Cocker's intuitionistic fuzzy topological space (IFTS). Each IFSs in τ is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.5 ([7])**.** Let a and b be two real numbers in [0,1] satisfying the inequality $a + b \leq 1$. Then the pair $\langle a, b\rangle$ is called an intuitionistic fuzzy pair. Let $\langle a_1, b_1\rangle$,$\langle a_2, b_2\rangle$ be any two intuitionistic fuzzy pairs. Then define

(1) $\langle a_1, b_1\rangle \leq \langle a_2, b_2\rangle$ if and only if $a_1 \leq a_2$ and $b_1 \geq b_2$.
(2) $\langle a_1, b_1\rangle = \langle a_2, b_2\rangle$ if and only if $a_1 = a_2$ and $b_1 = b_2$.
(3) If $\{\langle a_i, b_i/i \in J\rangle\}$ is a family of intuitionistic fuzzy pairs,then $\vee\langle a_i, b_i\rangle = \langle \vee a_i, \wedge b_i\rangle$ and $\wedge\langle a_i, b_i\rangle = \langle \wedge a_i, \vee b_i\rangle$.
(4) The complement of an intuitionistic fuzzy pair $\langle a, b\rangle$ is the intuitionistic fuzzy pair defined by $\overline{\langle a, b\rangle} = \langle b, a\rangle$
(5) $1^\sim = \langle 1, 0\rangle$ and $0^\sim = \langle 0, 1\rangle$.

Definition 2.6 ([4]). Let X be a non empty set and $x \in X$ a fixed element in X. If $r \in I_0$,$s \in I_1$ are fixed real numbers such that $r + s \leq 1$, then the intuitionistic fuzzy set $x_{r,s} = \langle x, x_r, 1 - x_{1-s}\rangle$is called an intuitionistic fuzzy point (IFP) in X, where r denotes the degree of membership of $x_{r,s}$ and $x \in X$ the support of $x_{r,s}$.

The IFP $x_{r,s}$ is contained in the IFS $A(x_{r,s} \in A)$ if and only if $r < \mu_A(x)$ and $s > \gamma_A(x)$.

Definition 2.7 ([5]). An fuzzy topological space X is said to be fuzzy extremally disconnected if the closure of every fuzzy open set in X is fuzzy open in X.

3. Intuitionistic fuzzy *lim* extremally disconnected spaces

Notation 3.1. $\mathcal{F}, \mathcal{G}$: filters on X
$\mathfrak{F}, \mathfrak{G}$: intuitionistic fuzzy filters on X
$U(X)$: set of all ultrafilters on X
$\mathbb{IF}(X)$: set of all intuitionistic fuzzy filter on X
$\mathbb{IF}_p(X)$: set of all intuitionistic fuzzy prime filters on X
$\mathbb{IF}_m(\mathfrak{F})$: set of all minimal prime fuzzy filters finer than $\mathfrak{F}$
ζ^X : set of all intuitionistic fuzzy sets on X
$lim\mathfrak{F}, lim\mathfrak{G}, lim\mathfrak{H}, lim\mathfrak{J}, lim\mathfrak{K}, lim\mathfrak{L}, lim\mathfrak{V}$: intuitionistic fuzzy sets on X
$c(\mathfrak{F})$: characterisitic* set of $\mathfrak{F}$
$\psi_{lim\mathfrak{F}}$: intuitionistic fuzzy characteristic function of $lim\mathfrak{F}$

Definition 3.2. A nonempty collection $\mathfrak{F}$ of elements ζ^X is called an intuitionistic fuzzy filter on X provided

(i) $0_\sim \notin \mathfrak{F}$,
(ii) $A, B \in \mathfrak{F}$ implies $A \cap B \in \mathfrak{F}$,
(iii) $C \in \mathfrak{F}$ when $C \supseteq A$ and $A \in \mathfrak{F}$.

Definition 3.3. An intuitionistic fuzzy filter $\mathfrak{F}$ is said to be intuitionistic fuzzy prime filter (or) prime intuitionistic fuzzy filter whenever, $A \cup B \in \mathfrak{F}$ implies $A \in \mathfrak{F}$ or $B \in \mathfrak{F}$.

Definition 3.4. A base for an intuitionistic fuzzy filter is a nonempty subset $\mathfrak{B}$ of ζ^X obeying

(i) $0_\sim \notin \mathfrak{B}$,
(ii) $A, B \in \mathfrak{B}$ implies $A \cap B \supseteq C$ for some $C \in \mathfrak{B}$.

The intuitionistic fuzzy filter generated by $\mathfrak{B}$ is denoted by $[\mathfrak{B}] = \{B \in \zeta^X : B \supseteq A, A \in \mathfrak{B}\}$.

Definition 3.5. Let X be a nonempty set. Let $A_i = \langle x, \mu_{A_i}, \gamma_{A_i}\rangle$ be collection of an intuitionistic fuzzy sets on X. The characteristic* set of $\mathfrak{F} \in \mathbb{IF}(X)$ is defined by

$$c(\mathfrak{F}) = \bigcup_{A_i \in \mathfrak{F}} A_i$$

That is, $c(\mathfrak{F}) = c(f\mathfrak{F})$.

Definition 3.6. Let $x_{r,s}$ be an intuitionistic fuzzy point. An intuitionistic fuzzy characteristic function of $x_{r,s}$ is denoted by $1_{x_{r,s}} = \langle x, \mu_{x_{r,s}}, \gamma_{x_{r,s}}\rangle$.

Definition 3.7. An intuitionistic fuzzy set $\alpha 1_{x_{r,s}}$ is of the form

$$\alpha 1_{x_{r,s}} = \langle y, \alpha\mu_{x_{r,s}}(y), \alpha\gamma_{x_{r,s}}(y)\rangle, \text{where } r+s \leq 1 \text{ and } \alpha \in (0,1)$$

Note 3.8. The collection of all intuitionistic fuzzy set of the form $\alpha 1_{x_{r,s}}$ is denoted by $\mathfrak{E}$.

Definition 3.9. An intuitionistic fuzzy filter generated by $\mathfrak{E}$ is denoted and defined by $[\mathfrak{E}] = \{B \in \zeta^X : B \supseteq A, A \in \mathfrak{E}\}$, however $[\mathfrak{E}]$ is written as $\alpha\dot{1}_{x_{r,s}}$.

Definition 3.10. Given a set X, the pair (X, lim) is called an intuitionistic fuzzy convergence space, where $lim : \mathbb{IF}(X) \to \zeta^X$, provided:

(i) For every $\mathfrak{F} \in \mathbb{IF}(X), lim\mathfrak{F} = \bigcap_{\mathfrak{G}\in\mathbb{IF}_m(\mathfrak{F})} lim\mathfrak{G}$,
(ii) For every $\mathfrak{F} \in \mathbb{IF}_p(X), lim\mathfrak{F} \subseteq c(\mathfrak{F})$,
(iii) For every $\mathfrak{F}, \mathfrak{G} \in \mathbb{IF}_p(X), \mathfrak{F} \subseteq \mathfrak{G} \Rightarrow lim\mathfrak{G} \subseteq lim\mathfrak{F}$,
(iv) For every $\alpha \in (0,1)$ and $\alpha 1_{x_{r,s}} \in \mathfrak{E}$,$lim(\alpha\dot{1}_{x_{r,s}}) \supseteq \alpha 1_{x_{r,s}}$.

Definition 3.11. Let (X, lim) be an intuitionistic fuzzy convergence space. Then the operator $int : \zeta^X \to \zeta^X$ is defined by

$$int(lim\mathfrak{F}) = \bigcup\{lim\mathfrak{G} : lim\mathfrak{G} \subseteq lim\mathfrak{F}, \text{for every} \mathfrak{G} \in U(X) and \mathfrak{F} \in \mathbb{IF}(X)\}$$

Definition 3.12. Let (X, lim) be an intuitionistic fuzzy convergence space and

$$T_{lim} = \{lim\mathfrak{F} : int(lim\mathfrak{F}) = lim(\mathfrak{F}), \text{for every} \mathfrak{F} \in \mathbb{IF}(X)\}$$

The pair (X, T_{lim}) is called an intuitionistic fuzzy convergence topological space and the members of T_{lim} are known as intuitionistic fuzzy *lim* open sets (inshort, IFLOS)

Note 3.13. The complement of an intuitionistic fuzzy lim open sets are intuitionistic fuzzy *lim* closed sets (inshort, IFLCS).

Definition 3.14. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. An intuitionistic fuzzy set $lim\mathfrak{F}$ in (X, T_{lim}) which is both intuitionistic fuzzy *lim* open and intuitionistic fuzzy *lim* closed is defined by intuitionistic fuzzy *lim* clopen set.

Definition 3.15. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. The intuitionistic fuzzy *lim* closure of $lim\mathfrak{F}$ is denoted and defined by

$$IFLcl(lim\mathfrak{F}) = \bigcap\{lim\mathfrak{G} : lim\mathfrak{G} \text{ is an intuitionistic fuzzy } lim \text{ closed set and } lim\mathfrak{G} \supseteq lim\mathfrak{F}, \text{ for every } \mathfrak{G}, \mathfrak{F} \in \mathbb{IF}(X)\}$$

Definition 3.16. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. The intuitionistic fuzzy *lim* interior of $lim\mathfrak{F}$ is denoted and defined by

$$IFLint(lim\mathfrak{F}) = \bigcup\{lim\mathfrak{G} : lim\mathfrak{G} \text{ is an intuitionistic fuzzy } lim \text{ open set and } lim\mathfrak{G} \subseteq lim\mathfrak{F}, \text{ for every } \mathfrak{G}, \mathfrak{F} \in \mathbb{IF}(X)\}$$

Remark 3.17. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. For any intuitionistic fuzzy set $lim\mathfrak{F}$ in X, we have

(i) $IFLint(lim\mathfrak{F}) \subseteq lim\mathfrak{F} \subseteq IFLcl(lim\mathfrak{F})$, for every $\mathfrak{F} \in \mathbb{IF}(X)$
(ii) $IFLcl(\overline{lim\mathfrak{F}}) = \overline{IFLint(lim\mathfrak{F})}$, for every $\mathfrak{F} \in \mathbb{IF}(X)$
(iiI) $IFLint(\overline{lim\mathfrak{F}}) = \overline{IFLcl(lim\mathfrak{F})}$, for every $\mathfrak{F} \in \mathbb{IF}(X)$

Definition 3.18. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. Then (X, T_{lim}) is said to be an intuitionistic fuzzy lim extremally disconnected space if the intuitionistic fuzzy lim closure of an intuitionistic fuzzy lim open set $lim\mathfrak{F}$ is an intuitionistic fuzzy lim open set.

Proposition 3.19. *Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. Then the following statements are equivalent*

(i) (X, T_{lim}) *is an intuitionistic fuzzy lim extremally disconnected space*
(ii) *For each intuitionistic fuzzy lim closed set $lim\mathfrak{F}$, we have $IFLint(lim\mathfrak{F})$ is an intuitionistic fuzzy lim closed set*
(iii) *For each intuitionistic fuzzy lim open set $lim\mathfrak{F}$, we have $IFLcl$ $(IFLint(\overline{lim\mathfrak{F}})) = \overline{IFLcl(lim\mathfrak{F})}$*
(iv) *For each pair of an intuitionistic fuzzy lim open sets $lim\mathfrak{F}, lim\mathfrak{G}$ in (X, T_{lim}) with $IFLcl(lim\mathfrak{F}) = \overline{lim\mathfrak{G}}$, we have $IFLcl(lim\mathfrak{G}) = \overline{IFLcl(lim\mathfrak{F})}$, for every $\mathfrak{F}, \mathfrak{G} \in \mathbb{IF}(X)$*

Proof. (i)⇒(ii) Let $lim\mathfrak{F}$ be any intuitionistic fuzzy lim closed set. Then $\overline{lim\mathfrak{F}}$ is an intuitionistic fuzzy lim open set. By assumption (i) $IFLcl(\overline{lim\mathfrak{F}})$ is an intuitionistic fuzzy lim open set. Now, $IFLcl(\overline{lim\mathfrak{F}}) = \overline{IFLint(lim\mathfrak{F})}$ is an intuitionistic fuzzy lim open set. Hence $IFLint(lim\mathfrak{F})$ is an an intuitionistic fuzzy lim closed set.

(ii)⇒(iii) Let $lim\mathfrak{F}$ be any intuitionistic fuzzy lim open set. Then $\overline{lim\mathfrak{F}}$ is an intuitionistic fuzzy lim closed set. By assumption(ii) $IFLint(\overline{lim\mathfrak{F}})$ is an intuitionistic fuzzy lim closed set. Consider $IFLcl(IFLint(\overline{lim\mathfrak{F}})) = IFLint(\overline{lim\mathfrak{F}}) = \overline{IFLcl(lim\mathfrak{F})}$. This implies that, $IFLcl(IFLint(\overline{lim\mathfrak{F}})) = \overline{IFLcl(lim\mathfrak{F})}$.

(iii)⇒ (iv) Let $lim\mathfrak{F}$ and $lim\mathfrak{G}$ be any two intuitionistic fuzzy lim open sets such that $IFLcl(lim\mathfrak{F}) = \overline{lim\mathfrak{G}}$. By (iii),

$$IFLcl(lim\mathfrak{G}) = IFLcl(\overline{IFLcl(lim\mathfrak{F})}) = \overline{IFLcl(lim\mathfrak{F})} = IFLint(\overline{lim\mathfrak{F}})$$

That is,

$$IFLcl(lim\mathfrak{G}) = IFLint(\overline{lim\mathfrak{F}})$$

(iv)⇒(i) Let $lim\mathfrak{F}$ be any intuitionistic fuzzy lim open set. Let $\overline{IFLcl(lim\mathfrak{F})} = lim\mathfrak{G}$. By (iv), it follow that $IFLcl(lim\mathfrak{G}) = \overline{IFLcl(lim\mathfrak{F})}$. That is, $\overline{IFLcl(lim\mathfrak{F})}$ is an intuitionistic fuzzy lim closed set. This implies that $IFLcl(lim\mathfrak{F})$ is an intuitionistic fuzzy lim open set. Hence, (X, T_{lim}) is an intuitionistic fuzzy lim extremally disconnected space. □

Proposition 3.20. *Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. Then (X, T_{lim}) is an intuitionistic fuzzy lim extremally disconnected space if and only if for each intuitionistic fuzzy lim open set $lim\mathfrak{F}$ and intuitionistic fuzzy lim closed set $lim\mathfrak{G}$ such that $lim\mathfrak{F} \subseteq lim\mathfrak{G}$, we have $IFLcl(lim\mathfrak{F}) \subseteq IFLint(lim\mathfrak{G})$.*

Proof. Let $lim\mathfrak{F}$ be an intuitionistic fuzzy lim open set and $lim\mathfrak{G}$ be an intuitionistic fuzzy lim closed set such that $lim\mathfrak{F} \subseteq lim\mathfrak{G}$. Then by (ii) of Proposition 3.19, $IFLint(lim\mathfrak{G})$ is an intuitionistic fuzzy lim closed set. Also, since $lim\mathfrak{F}$ is an intuitionistic fuzzy lim open set, $IFLcl(lim\mathfrak{F}) \subseteq IFLint(lim\mathfrak{G})$.

Conversely, let $lim\mathfrak{G}$ be any intuitiionistic fuzzy lim closed set. Then $IFLint(lim\mathfrak{G})$

is an intuitionistic fuzzy lim open set and $IFLint(lim\mathfrak{G}) \subseteq lim\mathfrak{G}$. By assumption, $IFLcl(IFLint(lim\mathfrak{G})) \subseteq IFLint(lim\mathfrak{G})$. Also we know that $IFLint(lim\mathfrak{G}) \subseteq IFLcl(IFLint(lim\mathfrak{G}))$. This implies that $IFLcl(IFLint(lim\mathfrak{G})) = IFLint(lim\mathfrak{G})$. Therefore, $IFLint(lim\mathfrak{G})$ is an intuitionistic fuzzy lim closed set. Hence by (ii) of Proposition 3.19, it follows that (X, T_{lim}) is an intuitionistic fuzzy lim extremally disconnected space. □

Remark 3.21. Let (X, T_{lim}) be an intuitionistic fuzzy lim extremally disconnected space. Let $\{lim\mathfrak{F}_i, \overline{lim\mathfrak{G}_j} \mid i, j \in N\}$ be collection such that $lim\mathfrak{F}_i$'s are intuitionistic fuzzy lim open sets and $lim\mathfrak{G}_j$'s are intuitionistic fuzzy lim closed sets and let $lim\mathfrak{F}$ and $lim\mathfrak{G}$ be intuitionistic fuzzy lim clopen sets. If $lim\mathfrak{F}_i \subseteq lim\mathfrak{F} \subseteq lim\mathfrak{G}_j$ and $lim\mathfrak{F}_i \subseteq lim\mathfrak{G} \subseteq lim\mathfrak{G}_j$ for all $i, j \in N$, then there exists an intuitionistic fuzzy lim clopen set $lim\mathfrak{H}$ such that $IFLcl(lim\mathfrak{F}_i) \subseteq lim\mathfrak{H} \subseteq IFLint(lim\mathfrak{G}_j)$ for all $i, j \in N$.

Proof. By Proposition 3.20, $IFLcl(lim\mathfrak{F}_i) \subseteq IFLcl(lim\mathfrak{F}) \cap IFLint(lim\mathfrak{G}) \subseteq IFLint(lim\mathfrak{G}_j)$ for all $i, j \in N$. Letting $lim\mathfrak{H} = IFLcl(lim\mathfrak{F}) \cap IFLint(lim\mathfrak{G})$ in the above, we have $lim\mathfrak{H}$ is an intuitionistic fuzzy lim clopen set satisfying the required conditions. □

Proposition 3.22. *Let (X, T_{lim}) be intuitionistic fuzzy lim extremally disconnected space. Let $\{lim\mathfrak{F}_q\}_{q \in Q}$ and $\{lim\mathfrak{G}_q\}_{q \in Q}$ be monotone increasing collections of an intuitionistic fuzzy lim open sets and intuitionistic fuzzy lim closed sets of (X, T_{lim}) respectively. Suppose that $lim\mathfrak{F}_{q_1} \subseteq lim\mathfrak{G}_{q_2}$ whenever $q_1 < q_2$ (Q is the set of all rational numbers). Then there exists a monotone increasing collection $\{lim\mathfrak{H}_q\}_{q \in Q}$ of an intuitionistic fuzzy lim clopen sets of (X, T_{lim}) such that $IFLcl(lim\mathfrak{F}_{q_1}) \subseteq lim\mathfrak{H}_{q_2}$ and $lim\mathfrak{H}_{q_1} \subseteq IFLint(lim\mathfrak{G}_{q_2})$ whenever $q_1 < q_2$.*

Proof. Let us arrange all rational numbers into a sequence $\{q_n\}$ (without repetitions). For every $n \geq 2$, we shall define inductively a collection $\{lim\mathfrak{H}_{q_i} / 1 \leq i < n\} \subset \zeta^X$ such that

$$IFLcl(lim\mathfrak{F}_q) \subseteq lim\mathfrak{H}_{q_i} \text{if } q < q_i,$$
$$lim\mathfrak{H}_{q_i} \subseteq IFLint(lim\mathfrak{G}_q) \text{if } q_i < q, \text{for all } i < n \qquad (S_n)$$

By Proposition 3.20, the countable collections $\{IFLcl(lim\mathfrak{F}_q)\}$ and $\{IFLint(lim\mathfrak{G}_q)\}$ satisfy $IFLcl(lim\mathfrak{F}_{q_1}) \subseteq IFLint(lim\mathfrak{G}_{q_2})$ if $q_1 < q_2$. By Remark 3.21, there exists an intuitionistic fuzzy lim clopen set $lim\mathfrak{K}_1$ such that

$$IFLcl(lim\mathfrak{F}_{q_1}) \subseteq lim\mathfrak{K}_1 \subseteq IFLint(lim\mathfrak{G}_{q_2})$$

Letting $lim\mathfrak{H}_{q_1} = lim\mathfrak{K}_1$, we get (S_2). Define $lim\mathfrak{L} = \cup\{lim\mathfrak{G}_{q_i} / i < n, q_i < q_n\} \cup lim\mathfrak{F}_{q_n}$ and $lim\mathfrak{M} = \cap\{lim\mathfrak{H}_{q_j} / j < n, q_j > q_n\} \cap lim\mathfrak{G}_{q_n}$. Then $IFLcl(lim\mathfrak{H}_{q_i}) \subseteq IFLcl(lim\mathfrak{L}) \subseteq IFLint(lim\mathfrak{H}_{q_j})$ and $IFLcl(lim\mathfrak{H}_{q_i}) \subseteq IFLint(lim\mathfrak{M}) \subseteq IFLint(lim\mathfrak{H}_{q_j})$ whenever $q_i < q_n < q_j (i, j < n)$, as well as $lim\mathfrak{F}_q \subseteq IFLcl(lim\mathfrak{K}) \subseteq lim\mathfrak{G}_{q'}$ and $lim\mathfrak{F}_q \subseteq IFLint(lim\mathfrak{L}) \subseteq lim\mathfrak{G}_{q'}$, $lim\mathfrak{F}_q \subseteq IFLint(lim\mathfrak{M}) \subseteq lim\mathfrak{G}_{q'}$ whenever $q < q_n < q'$. This shows that the countable collections $\{lim\mathfrak{H}_{q_i} / i < n, q_i < q_n\} \cup \{lim\mathfrak{F}_q | q < q_n\}$ and $\{lim\mathfrak{H}_{q_j} / j < n, q_j > q_n\} \cup \{lim\mathfrak{G}_q | q > q_n\}$ together with $lim\mathfrak{L}$ and $lim\mathfrak{M}$ fulfil the conditions of Remark 3.21. Hence, there exists an intuitionistic fuzzy lim clopen set $lim\mathfrak{J}_n$ such that $IFLcl(lim\mathfrak{J}_n) \subseteq lim\mathfrak{G}_q$ if $q_n < q$, $lim\mathfrak{F}_q \subseteq IFLint(lim\mathfrak{J}_n)$ if $q < q_n$, $IFLcl(lim\mathfrak{H}_{q_i}) \subseteq IFLint(lim\mathfrak{J}_n)$ if

$q_i < q_n$ $IFLcl(lim\mathfrak{I}_n) \subseteq IFLint(lim\mathfrak{H}_{q_i})$ if $q_n < q_j$ where $1 \leq i,\ j \leq n-1$. Letting $lim\mathfrak{H}_{q_n} = lim\mathfrak{I}_n$ we obtain an intuitionistic fuzzy sets

$$lim\mathfrak{H}_{q_1}, lim\mathfrak{H}_{q_2}, lim\mathfrak{H}_{q_3}, ..., lim\mathfrak{H}_{q_n}$$

that satisfy $(\mathbf{S_{n+1}})$. Therefore, the collection $\{lim\mathfrak{H}_{q_i}/i = 1, 2, ...\}$ has the required property. □

4. Properties and characterizations of intuitionistic fuzzy *lim* extremally disconnected spaces

In this section various properties and characterizations of intuitionistic fuzzy *lim* extremally disconnected spaces are discussed.

Definition 4.1. An intuitionistic fuzzy real line $\mathbb{R}_I(I)$ is the set of all monotone decreasing intuitionistic fuzzy set $lim\mathfrak{F} \in \zeta^R$ satisfying

$$\bigcup\{lim\mathfrak{F}(t) : t \in \mathbb{R}\} = 0^\sim \text{ and}$$
$$\bigcap\{lim\mathfrak{F}(t) : t \in \mathbb{R}\} = 1^\sim$$

After the identification of an intuitionistic fuzzy sets $lim\mathfrak{F}, lim\mathfrak{G} \in \mathbb{R}_I(I)$ if and only if $lim\mathfrak{F}(t^-) = lim\mathfrak{G}(t^-)$ and $lim\mathfrak{F}(t^+) = lim\mathfrak{G}(t^+)$ for all $t \in \mathbb{R}$, where

$$lim\mathfrak{F}(t^-) = \bigcap\{lim\mathfrak{F}(s) : s < t\} \text{ and}$$
$$lim\mathfrak{F}(t^+) = \bigcup\{lim\mathfrak{F}(s) : s > t\}$$

The natural intuitionistic fuzzy convergence topology on $\mathbb{R}_I(I)$ is generated from the subbasis $\{L_t^I, R_t^I : t \in \mathbb{R}\}$ where $L_t^I, R_t^I : \mathbb{R}_I(I) \to \mathbb{I}_I(I)$ are given by $L_t^I[lim\mathfrak{F}] = \overline{lim\mathfrak{F}(t^-)}$ and $R_t^I[lim\mathfrak{F}] = lim\mathfrak{F}(t^+)$.

The intuitionistic fuzzy unit interval $\mathbb{I}_I(I)$ is a subset of $\mathbb{R}_I(I)$ such that $[lim\mathfrak{F}] \in \mathbb{I}_I(I)$ if the member and non member of $lim\mathfrak{F}$ are defined by

$$\mu_{lim_\mathfrak{F}}(t) = \begin{cases} 0 & \text{if } t > 1 \\ 1 & \text{if } t < 0 \end{cases}$$

and

$$\gamma_{lim_\mathfrak{F}}(t) = \begin{cases} 1 & \text{if } t > 1 \\ 0 & \text{if } t < 1 \end{cases}$$

Definition 4.2. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. A function $f : X \to \mathbb{R}_I(I)$ is said be lower(resp. upper) intuitionistic fuzzy *lim* continuous function if $f^{-1}(R_t^I)(resp. f^{-1}(L_t^I))$ is an intuitionistic fuzzy *lim* open (resp. intuitionistic fuzzy *lim* closed) set, for each $t \in \mathbb{R}$.

Note 4.3. Let X be a non empty set and $lim\mathfrak{F} \in \zeta^X$. Then

$$lim\mathfrak{F}^\sim = \langle x, \mu_{lim\mathfrak{F}}(x), \gamma_{lim\mathfrak{F}}(x)\rangle$$

for every $x \in X$.

Proposition 4.4. *Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. Let $lim\mathfrak{F}$ be an intuitionistic fuzzy set in X, for each $\mathfrak{F} \in \mathbb{IF}(X)$, and let $f : X \to \mathbb{R}_I(I)$ be such that*

$$f(x)(t) = \begin{cases} 1^\sim & if\, t < 0 \\ lim\mathfrak{F}^\sim & if\, 0 \leq t \leq 1 \\ 0^\sim & if\, t > 1 \end{cases}$$

for all $x \in X$. Then f is an lower (resp. upper) intuitionistic fuzzy lim continuous function if and only if $lim\mathfrak{F}$ is an intuitionistic fuzzy lim open(resp.intuitionistic fuzzy lim closed) set.

Proof.

$$f^{-1}(R_t^I) = \begin{cases} 1_\sim & \text{if } t < 0 \\ lim\mathfrak{F} & \text{if } 0 \leq t \leq 1 \\ 0_\sim & \text{if } t > 1 \end{cases}$$

implies that f is an lower intuitionistic fuzzy lim continuous function if and only if $lim\mathfrak{F}$ is an intuitionistic fuzzy lim open set in X.

$$f^{-1}(\overline{L_t^I}) = \begin{cases} 1_\sim & \text{if } t < 0 \\ lim\mathfrak{F} & \text{if } 0 \leq t \leq 1 \\ 0_\sim & \text{if } t > 1 \end{cases}$$

implies that f is an upper intuitionistic fuzzy lim continuous function if and only if $lim\mathfrak{F}$ is an intuitionistic fuzzy lim closed set in X. □

Definition 4.5. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. An intuitionistic fuzzy* characteristic function of an intuitionistic fuzzy set $lim\mathfrak{F} \in X$ is a map $\psi_{lim\mathfrak{F}} : X \to \mathbb{I}_I(I)$ defined by $\psi_{lim\mathfrak{F}}(x) = lim\mathfrak{F}^\sim$ for each $x \in X$.

Proposition 4.6. *Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space, and let $lim\mathfrak{F}$ be an intuitionistic fuzzy set in X, for each $\mathfrak{F} \in \zeta^X$. Let $\psi_{lim\mathfrak{F}}$ be an intuitionistic fuzzy characteristic function of an intuitionistic fuzzy set $lim\mathfrak{F}$ in X. Then $\psi_{lim\mathfrak{F}}$ is an lower(resp. upper) intuitionistic fuzzy lim continuous function if and only if $lim\mathfrak{F}$ is an intuitionistic fuzzy lim open (resp. intuitionistic fuzzy lim closed)set.*

Proof. This follows from Proposition 4.4. □

Definition 4.7. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. A function $f : X \to \mathbb{I}_I(I)$ is said to be an strongly intuitionistic fuzzy lim continuous function if $f^{-1}(R_t^I)$ is an intuitionistic fuzzy lim open set and $f^{-1}(L_t^I)$ is both intuitionistic fuzzy lim open set and intuitionistic fuzzy lim closed set, for each $t \in \mathbb{R}$.

Proposition 4.8. *Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. Then the following conditions are equivalent:*

(i) *(X, T_{lim}) is an intuitionistic fuzzy lim extremally disconnected space.*
(ii) *If $g, h : X \to \mathbb{R}_I(I)$, g is an lower intuitionistic fuzzy lim continuous function, h is upper intuitionistic fuzzy lim continuous function and $g \subseteq h$, then there exists a strongly intuitionistic fuzzy lim continuous function such that $g \subseteq f \subseteq h$.*
(iii) *If $\overline{lim\mathfrak{F}}$ and $lim\mathfrak{G}$ are intuitionistic fuzzy lim open sets such that $lim\mathfrak{G} \subseteq lim\mathfrak{F}$, then there exists strongly intuitionistic fuzzy lim continuous function $f : X \to \mathbb{R}_I(I)$ such that $lim\mathfrak{G} \subseteq f^{-1}(\overline{L_1^I}) \subseteq f^{-1}(R_0^I) \subseteq lim\mathfrak{F}$.*

Proof. (i)⇒(ii) Define two mappings $lim\mathfrak{F}, lim\mathfrak{G} : Q \to \mathbb{R}_I(I)$ by $lim\mathfrak{F}_r = h^{-1}(L_r^I)$ and $lim\mathfrak{G}_r = g^{-1}(\overline{R_r^I})$, for all $r \in Q$ (Q is the set of all rationals). Clearly,

$\{lim\mathfrak{F}_r\}_{r\in Q}$ and $\{lim\mathfrak{G}_r\}_{r\in Q}$ are monotone increasing families of an intuitionistic fuzzy lim open sets and intuitionistic fuzzy lim closed sets of (X, T_{lim}). Moreover $lim\mathfrak{F}_r \subseteq lim\mathfrak{G}_s$ if $r < s$. By Proposition 3.22, there exists a monotone increasing family $\{lim\mathfrak{H}_r\}_{r\in Q}$ of an intuitionistic fuzzy lim clopen sets of (X, T_{lim}) such that $IFLcl(lim\mathfrak{F}_r) \subseteq lim\mathfrak{H}_s$ and $lim\mathfrak{H}_r \subseteq IFLint(lim\mathfrak{G}_s)$ whenever $r < s$ $(r, s \in Q)$. Letting $lim\mathfrak{V}_t = \bigcap_{r<t} \overline{lim\mathfrak{H}_r}$ for $t \in \mathbb{R}$, we define a monotone decreasing family $\{lim\mathfrak{V}_t \mid t \in \mathbb{R}\} \subseteq \mathbb{IF}(X)$. Moreover we have $IFLcl(lim\mathfrak{V}_t) \subseteq IFLint(lim\mathfrak{V}_s)$ whenever $s < t$. We have,

$$\bigcup_{t\in\mathbb{R}} lim\mathfrak{V}_t = \bigcup_{t\in\mathbb{R}}\bigcap_{r<t} \overline{lim\mathfrak{H}_r} \supseteq \bigcup_{t\in\mathbb{R}}\bigcap_{r<t} \overline{lim\mathfrak{G}_r} = \bigcup_{t\in\mathbb{R}}\bigcap_{r<t} g^{-1}(R_r^I)$$
$$= \bigcup_{t\in R} g^{-1}(\overline{L_t^I}) = g^{-1}(\bigcup_{t\in\mathbb{R}} L_t^I) = 1_\sim$$

Similarly, $\bigcap_{t\in\mathbb{R}} lim\mathfrak{V}_t = 0_\sim$. Now define a function $f : X \to \mathbb{R}_I(I)$ possessing required conditions. Let $f(x)(t) = lim\mathfrak{V}_t(x)$, for all $x \in X$ and $t \in \mathbb{R}$. By the above discussion, it follows that f is well defined. To prove f is an strongly intutionistic fuzzy lim continuous function. Observe that $\bigcup_{s>t} lim\mathfrak{V}_s = \bigcup_{s>t} IFLint(lim\mathfrak{V}_s)$ and $\bigcap_{s<t} lim\mathfrak{V}_s = \bigcap_{s<t} IFLint(lim\mathfrak{V}_s)$. Then

$$f^{-1}(R_t^I) = \textstyle\bigcup_{s>t} lim\mathfrak{V}_s = \bigcup_{s>t} IFLint(lim\mathfrak{V}_s)$$

is an intuitionistic fuzzy lim clopen set and

$$f^{-1}(\overline{L_t^I}) = \textstyle\bigcap_{s<t} lim\mathfrak{V}_s = \bigcap_{s<t} IFLcl(lim\mathfrak{V}_s)$$

is an intuitionistic fuzzy lim clopen set. Therefore, f is strongly intuitionistic fuzzy lim continuous function. To conclude the proof it remains to show that $g \subseteq f \subseteq h$. That is $g^{-1}(\overline{L_t^I}) \subseteq f^{-1}(\overline{L_t^I}) \subseteq h^{-1}(\overline{L_t^I})$ and $g^{-1}(R_t^I) \subseteq f^{-1}(R_t^I) \subseteq h^{-1}(R_t^I)$ for each $t \in \mathbb{R}$. We have,

$$g^{-1}(\overline{L_t^I}) = \bigcap_{s<t} g^{-1}(\overline{L_s^I}) = \bigcap_{s<t}\bigcap_{r<s} g^{-1}(R_r^I) = \bigcap_{s<t}\bigcap_{r<s} \overline{lim\mathfrak{G}_r}$$
$$\subseteq \bigcap_{s<t}\bigcap_{r<s} \overline{lim\mathfrak{H}_r} = \bigcap_{s<t} lim\mathfrak{V}_s = f^{-1}(\overline{L_t^I})$$

and

$$f^{-1}(\overline{L_t^I}) = \bigcap_{s<t} lim\mathfrak{V}_s = \bigcap_{s<t}\bigcap_{r<s} \overline{lim\mathfrak{H}_r} \subseteq \bigcap_{s<t}\bigcap_{r<s} \overline{lim\mathfrak{F}_r}$$
$$= \bigcap_{s<t}\bigcap_{r<s} h^{-1}(\overline{L_r^I}) = \bigcap_{s<t} h^{-1}(\overline{L_s^I}) = h^{-1}(\overline{L_t^I})$$

Similarly,

$$g^{-1}(R_t^I) = \bigcup_{s>t} g^{-1}(R_s^I) = \bigcup_{s>t}\bigcup_{r>s} g^{-1}(R_r^I) = \bigcup_{s>t}\bigcup_{r>s} \overline{lim\mathfrak{G}_r}$$
$$\subseteq \bigcup_{s>t}\bigcap_{r<s} \overline{lim\mathfrak{H}_r} = \bigcup_{s>t} lim\mathfrak{V}_s = f^{-1}(\overline{R_t^I})$$

and

$$f^{-1}(\overline{R_t^I}) = \bigcup_{s>t} lim\mathfrak{V}_s = \bigcup_{s>t}\bigcap_{r<s} \overline{lim\mathfrak{H}_r} \subseteq \bigcup_{s>t}\bigcup_{r>s} \overline{lim\mathfrak{F}_r}$$
$$= \bigcup_{s>t}\bigcup_{r>s} h^{-1}(\overline{L_r^I}) = \bigcup_{s>t} h^{-1}(R_s^I) = h^{-1}(R_t^I).$$

Hence, the condition (ii) is proved.

(ii)⇒(iii) Let $\overline{lim\mathfrak{F}}$ be an intuitionistic fuzzy *lim* open set and $lim\mathfrak{G}$ be an intuitionistic fuzzy *lim* open set such that $lim\mathfrak{G} \subseteq lim\mathfrak{F}$. Then $\psi_{lim\mathfrak{G}} \subseteq \psi_{lim\mathfrak{F}}$ where $\psi_{lim\mathfrak{F}}$, $\psi_{lim\mathfrak{G}}$ are lower and upper intuitionistic fuzzy *lim* continuous functions respectively. By (ii), there exists a strongly intuitionistic fuzzy *lim* continuous function $f : X \to \mathbb{R}_I(I)$ such that $\psi_{lim\mathfrak{G}} \subseteq f \subseteq \psi_{lim\mathfrak{F}}$. Clearly, $f(x) \in \mathbb{I}_I(I)$ for all $x \in X$ and $lim\mathfrak{G} = \psi^{-1}_{lim\mathfrak{G}}(\overline{L_1^I}) \subseteq f^{-1}(\overline{L_1^I}) \subseteq f^{-1}(R_0^I) \subseteq \psi^{-1}_{lim\mathfrak{F}}(R_0^I) = lim\mathfrak{F}$. Therefore, $lim\mathfrak{G} \subseteq f^{-1}(\overline{L_1^I}) \subseteq f^{-1}(R_0^I) \subseteq lim\mathfrak{F}$.

(iii)⇒(i) Since $f^{-1}(\overline{L_1^I})$ and $f^{-1}(R_0^I)$ are intuitionistic fuzzy *lim* clopen sets and by Proposition 3.20, (X, T_{lim}) is an intuitionistic fuzzy *lim* extremally disconnected space. □

5. Tietze extension theorem for intuitionistic fuzzy *lim* extremally disconnected spaces.

In this section Tietze extension theorem for intuitionistic fuzzy *lim* extremally disconnected spaces is studied.

Note 5.1. Let X be a non empty set and $A \subset X$. Then an intuitionistic fuzzy set χ_A^* is of the form $\langle x, \chi_A(x), 1 - \chi_A(x)\rangle$ where

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

Proposition 5.2. *Let (X, T_{lim}) be an intuitionistic fuzzy lim extremally disconnected space. Let $A \subset X$ be such that χ_A^* is an intuitionistic fuzzy lim open set in X. Let $f : (A, T_{lim}/A) \to \mathbb{I}_I(I)$ be an strongly intuitionistic fuzzy lim continuous function. Then f has an strongly intuitionistic fuzzy lim continuous extension over (X, T_{lim}).*

Proof. Let $g, h : X \to \mathbb{I}_I(I)$ be such that $g = f = h$ on $lim\mathfrak{F}$ and $g(x) = \langle 0, 1\rangle = 0^{\sim}$, $h(x) = \langle 1, 0\rangle = 1^{\sim}$ if $x \notin lim\mathfrak{F}$. For every $t \in \mathbb{R}$, We have,

$$g^{-1}(R_t^I) = \begin{cases} lim\mathfrak{G}_t \cap \chi_A^* & \text{if } t \geq 0\ , \\ 1_{\sim} & \text{if } t < 0, \end{cases}$$

where $lim\mathfrak{G}_t$ is an intuitionistic fuzzy *lim* clopen set such that $lim\mathfrak{G}_t/A = f^{-1}(R_t^I)$ and

$$h^{-1}(L_t^I) = \begin{cases} lim\mathfrak{H}_t \cap \chi_A^* & \text{if } t \leq 1\ , \\ 1_{\sim} & \text{if } t > 1, \end{cases}$$

where $lim\mathfrak{H}_t$ is an intuitionistic fuzzy *lim* clopen set such that $lim\mathfrak{H}_t/A = f^{-1}(L_t^I)$. Thus, g is lower intuitionistic fuzzy *lim* continuous function and h is upper intuitionistic fuzzy *lim* continuous function with $g \subseteq h$. By Proposition 4.6, there is an strongly intuitionistic fuzzy *lim* continuous function $F : X \to \mathbb{I}_I(I)$ such that $g \subseteq F \subseteq h$. Hence $F \equiv f$ on A. □

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