

## An application on soft $\tilde{L}$ -fuzzy convergence structure

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**ABSTRACT.** In this paper, the concept of soft  $\tilde{L}$ -fuzzy set is introduced. A new notion of soft  $\tilde{L}$ -fuzzy convergence structure on the basis of soft fuzzy filter is introduced. The main aim of this paper is to present an application on soft  $\tilde{L}$ -fuzzy convergence structure which has been discussed analogously as in the paper of Thomas Kubiak. Besides, characterization and several properties of soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence extremally disconnected space are established.

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**Keywords:** Soft fuzzy filter, Soft  $\tilde{L}$ -fuzzy set, Soft  $\tilde{L}$ -fuzzy real line, Soft  $\tilde{L}$ -fuzzy convergence structure, Soft  $\tilde{L}$ -fuzzy  $lim$ -convergence topology, Soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence extremally disconnected space, Characteristic\* function.

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### 1. INTRODUCTION

**Z**adeh [19] introduced the fundamental concepts of fuzzy sets in his classical paper. The fuzzy concept in various branches of Mathematics was developed by Rodabaugh [11, 12, 13]. Fuzzy sets have applications in many fields such as information [7] and control [9]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Fuzzy topological space was introduced by C.L.Chang[1]. Bruce Hutton[3] constructed an interesting L-fuzzy topological space, called L-fuzzy unit interval, which plays the same role in fuzzy topology that the unit interval plays in general topology. Thomas Kubiak[14] introduced and studied about the properties of L-sets. Thomas Kubiak[15] also investigated the Tietze Extension Theorem using L-fuzzy sets in normal spaces. Kent[4] introduced the concept of convergence space whereas Richardson.G [10] established the fuzzy convergence space on the basis of fuzzy filters.

The concept of soft set theory was introduced by D.Molodtsov [6]. The idea of fuzzy soft sets, introduced and developed by P.K.Maji, R.Biswas, A.R.Roy [5]. The concept of soft fuzzy set over a poset  $I$  was introduced by Ismail U. Tiriyaki [16]. A new idea of  $C$ -set in topological space was introduced and developed by E.Hatir, T.Noiri and S.Yuksel [2] in 1996. The concept of  $C$ -set in fuzzy topological space was introduced by M.K.Uma, E.Roja and G.Balasubramanian [17] in 2005. T.Yogalakshmi, E.Roja, M.K.Uma [18] introduced the notion of a soft fuzzy  $C$ -open set in the soft fuzzy topological space.

In this paper, the notion of soft  $\tilde{L}$ -fuzzy set is introduced. The concept of soft  $\tilde{L}$ -fuzzy convergence structure on the basis of soft fuzzy filter is introduced. Using Sostak's idea [8] on the structure, soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence topology is established. The concept of soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence extremally disconnected space and its characterizations are discussed. In this connection, Insertion and Extension Theorem are established, which is one of the most applicable one in digital space.

## 2. PRELIMINARIES

**Definition 2.1** ([4]). A *convergence space* is a pair  $(X, q)$ , where  $X$  is a set and  $q : F(X) \rightarrow 2^X$  satisfies the following:

- (CS1)  $x \in q(\dot{x})$ , for each  $x \in X$ , where  $\dot{x}$  is the ultrafilter containing  $\{x\}$ .
- (CS2)  $q(\mathcal{F}) \subseteq q(\mathcal{G})$ , when  $\mathcal{F} \subseteq \mathcal{G}$ .
- (CS3)  $q(\mathcal{F}) = \cap \{q(\mathcal{G}) : \mathcal{G} \text{ is an ultrafilter containing } \mathcal{F}\}$

**Definition 2.2** ([3]). Let  $X$  be a set and  $L$  be a complete lattice. An  $L$ -fuzzy set on  $X$  is a map from  $X$  into  $L$ . That is, if  $\lambda$  is a  $L$ -fuzzy subset of  $X$  then  $\lambda \in L^X$ , where  $L^X$  denotes the collection of all maps from  $X$  into  $L$ .

**Definition 2.3** ([10]). The pair  $(X, \lim)$  is called as a *fuzzy convergence space*, where  $\lim : \mathbb{F}(X) \rightarrow I^X$  provided :

- (FC1)  $\forall \mathfrak{F} \in \mathbb{F}(X), \lim \mathfrak{F} = \cap_{\mathfrak{G} \in \mathbb{F}_m(\mathfrak{F})} \lim \mathfrak{G}$ .
- (FC2)  $\forall \mathfrak{F} \in \mathbb{F}_p(X), \lim \mathfrak{F} \subseteq c(\mathfrak{F})$ .
- (FC3)  $\forall \mathfrak{F}, \mathfrak{G} \in \mathbb{F}_p(X)$ , when  $\mathfrak{F} \subseteq \mathfrak{G} \Rightarrow \lim \mathfrak{G} \subseteq \lim \mathfrak{F}$ .
- (FC4)  $\forall x \in X, 0 < \alpha \leq 1, \lim (\alpha \dot{1}_x) \supseteq (\alpha \dot{1}_x)$

**Definition 2.4** ([16, 18]). Let  $X$  be a nonempty set and  $I=[0,1]$  be the unit interval. Let  $\mu$  be a fuzzy subset of  $X$  such that  $\mu : X \rightarrow [0, 1]$  and  $M$  be any crisp subset of  $X$ . Then, the pair  $(\mu, M)$  is called as a *soft fuzzy set* in  $X$ . The family of all soft fuzzy subsets of  $X$ , will be denoted by **SF(X)**.

**Definition 2.5** ([18]). Let  $X$  be a non-empty set. Then the *complement* of a soft fuzzy set  $(\mu, M)$  is defined as  $(\mu, M)' = (1 - \mu, X \setminus M)$

**Definition 2.6** ([18]). Let  $(X, \tau)$  be a SFTS. A soft fuzzy set  $(\lambda, N)$  is said to be *soft fuzzy C-open*, if

$$(\lambda, N) = (\mu, M) \sqcap (\gamma, K)$$

where,  $(\mu, M)$  is a soft fuzzy open set and  $(\gamma, K)$  is a soft fuzzy  $\alpha^*$ -open set.

The complement of soft fuzzy  $C$ -open set ( in short. **SF $c$ OS**) is called as a *soft fuzzy C-closed set*. (in short. **SF $c$ CS**)

### 3. ON SOFT $\tilde{L}$ -FUZZY SET

Throughout this paper  $\langle \tilde{L}, \sqcup, \sqcap, ' \rangle$  is an infinitely distributive lattice with an order-reversing involution. Such a lattice being complete has a least element  $(0,0)$  and greatest element  $(1,1)$ .

**Definition 3.1.** Let  $X$  be a non-empty crisp set. A *soft fuzzy filter*,  $\mathfrak{F}$  is a non-empty collection of soft fuzzy sets in  $X$  provided:

- (1)  $(\lambda, N) \notin \mathfrak{F}$  such that either  $\lambda = 1_\phi$  or  $N = \phi$ .
- (2)  $(\mu, M), (\lambda, N) \in \mathfrak{F} \Rightarrow (\mu, M) \sqcap (\lambda, N) \in \mathfrak{F}$ .
- (3)  $(\lambda, N) \in \mathfrak{F}$  and  $(\lambda, N) \sqsubseteq (\mu, M) \Rightarrow (\mu, M) \in \mathfrak{F}$ .

**Note:** If  $A \subseteq X$ , then  $1_A$  denotes the characteristic function of  $A$ , and  $1_{\{x\}}$  is written simply as  $1_x$ .

**Definition 3.2.** A soft fuzzy filter  $\mathfrak{F}$  is said to be a *soft fuzzy prime filter*, whenever  $(\lambda, N) \sqcup (\mu, M) \in \mathfrak{F}$  with either  $\widehat{(\lambda, N)} \neq \phi$  or  $\widehat{(\mu, M)} \neq \phi \Rightarrow (\mu, M) \in \mathfrak{F}_c$  or  $(\lambda, M) \in \mathfrak{F}$

**Notation:**  $\widehat{(\lambda, N)} = \{x \in X : \lambda(x) > 0 \text{ and } x \in N\}$  and  $(\lambda, N)_0 = \{x \in X : \lambda(x) > 0 \text{ or } x \in N\}$ .

**Note :**

- The set of all soft fuzzy prime filters on  $X$  is denoted by  $\mathbb{F}_p(X)$ .
- The set of all soft fuzzy filters on  $X$  is denoted by  $\mathbb{F}(X)$ .
- The set of all soft fuzzy prime filters containing  $\mathfrak{F}$  on  $X$  is denoted by  $\mathbb{F}_p(\mathfrak{F})$ .
- The set of all filters on  $X$  is denoted as  $F(X)$ .
- The set  $\mathfrak{F} \in \mathbb{F}(X)$  of all soft fuzzy filters is ordered by set inclusion. For  $\mathfrak{F} \in \mathbb{F}(X)$ , the set  $\mathbb{F}_p(X)$  of all soft fuzzy prime filters finer than  $\mathfrak{F}$  is inductive and by Zorn's lemma, there exist minimal elements in  $\mathbb{F}_p(\mathfrak{F})$  on  $X$ , the set of which is denoted by  $\mathbb{P}_m(\mathfrak{F})$ . It follows that,

$$\mathfrak{F} = \sqcap_{\mathfrak{G} \in \mathbb{P}_m(\mathfrak{F})} \mathfrak{G}.$$

**Definition 3.3.** The *soft fuzzy principal filter*  $\widehat{(\mu, M)}$  is the soft fuzzy filter generated by singleton set  $\{(\mu, M)\}$ , that is,

$$\widehat{(\mu, M)} := [\{(\lambda, N) \in SF(X) : (\mu, M) \sqsubseteq (\lambda, N)\}]$$

**Definition 3.4.** Let  $X$  be a non-empty set and  $L$  be any lattice. Let  $N \subseteq X$ . Associated to each soft fuzzy set  $(\lambda, N)$ , a soft fuzzy  $\tilde{L}$ -fuzzy set is defined. A *soft  $\tilde{L}$ -fuzzy set*,  $(\lambda, \psi_N)$  in  $X$  is an element of the set of all functions from  $X$  to  $L \times L$ , where  $\lambda$  is  $L$ -fuzzy set on  $X$  and  $\psi_N : X \rightarrow L$  such that

$$\psi_N(x) = \begin{cases} 1, & \text{if } N = X \\ l \in L \ (0 < l < 1), & \text{if } x \in N \\ 0, & \text{if } N = \phi \end{cases}$$

The family of all soft  $\tilde{L}$ -fuzzy sets, is denoted by  $\tilde{L}^X$ .

**Definition 3.5.** Let  $X$  be a non-empty set and the soft  $\tilde{L}$ -fuzzy sets  $A$  and  $B$  be in the form,

$$A = \{(\mu, \psi_M) : \mu(x) \in L \text{ and } \psi_M(x) \in L, \forall x \in X\}$$

$$B = \{(\lambda, \psi_N) : \lambda(x) \in L \text{ and } \psi_N(x) \in L, \forall x \in X\}$$

Then,

- (1)  $A \sqsubseteq B \Leftrightarrow \mu(x) \leq \lambda(x) \text{ and } \psi_M(x) \leq \psi_N(x), \forall x \in X.$
- (2)  $A = B \Leftrightarrow \mu(x) = \lambda(x) \text{ and } \psi_M(x) = \psi_N(x), \forall x \in X.$
- (3)  $A \sqcap B \Leftrightarrow \mu(x) \wedge \lambda(x) \text{ and } \psi_M(x) \wedge \psi_N(x), \forall x \in X.$
- (4)  $A \sqcup B \Leftrightarrow \mu(x) \vee \lambda(x) \text{ and } \psi_M(x) \vee \psi_N(x), \forall x \in X.$

**Definition 3.6.** The pair  $(X, \lim)$  is called a *soft  $\tilde{L}$ -fuzzy convergence structure*, where  $\lim : \mathbb{F}(X) \rightarrow \tilde{L}^X$  provided :

- (SFC1)  $\forall \mathfrak{F} \in \mathbb{F}(X), \lim \mathfrak{F} = \bigcap_{\mathfrak{G} \in \mathbb{P}_m(\mathfrak{F})} \lim \mathfrak{G}.$
- (SFC2)  $\forall \mathfrak{F}, \mathfrak{G} \in \mathbb{F}_p(X), \text{ when } \mathfrak{F} \sqsubseteq \mathfrak{G} \Rightarrow \lim \mathfrak{G} \sqsubseteq \lim \mathfrak{F}.$
- (SFC3)  $\forall x \in X, 0 < \alpha < 1, \lim(\alpha 1_x, \{x\}) \supseteq (0_X, 0_X), \text{ where } \alpha 1_x = \alpha \wedge 1_x.$
- (SFC4)  $\lim(\widetilde{1_X}, X) = (1_X, 1_X)$

**Definition 3.7.** Let  $(X, \lim), (Y, \lim)$  be the soft  $\tilde{L}$ -fuzzy convergence structures. Let  $\mathfrak{F} \in \mathbb{F}(X)$ . If  $f$  is a function from  $X$  to  $Y$  and  $\lim \mathfrak{F} \in \tilde{L}^X$ , then the *image* of  $\lim \mathfrak{F}$ ,  $f(\lim \mathfrak{F})$  is the soft  $\tilde{L}$ -fuzzy set in  $Y$  defined by

$$f(\lim \mathfrak{F})(y) = \sup_{x \in f^{-1}(y)} \{\lim \mathfrak{F}(x)\}$$

**Definition 3.8.** Let  $(X, \lim), (Y, \lim)$  be the soft  $\tilde{L}$ -fuzzy convergence structures. Let  $\mathfrak{G} \in \mathbb{F}(Y)$ . If  $f$  is a function from  $X$  to  $Y$  and  $\lim \mathfrak{G} \in \tilde{L}^Y$ , then the *inverse image* of  $\lim \mathfrak{G}$ ,  $f^{-1}(\lim \mathfrak{G})$  is the soft  $\tilde{L}$ -fuzzy set in  $Y$  defined by

$$f^{-1}(\lim \mathfrak{G}) = \lim \mathfrak{G} \circ f$$

**Note:** Analogously,  $\sup_{i \in J} \{\lim \mathfrak{F}_i\}, \inf_{i \in J} \{\lim \mathfrak{F}_i\} \sqcup_{i \in J} \lim \mathfrak{F}_i, \bigcap_{i \in J} \lim \mathfrak{F}_i$  e.t.c., are defined in usual way as in  $L$ -fuzzy sets.

**Definition 3.9.** Let  $(X, \lim)$  be a soft  $\tilde{L}$ -fuzzy convergence structure. Then, the *interior operator*,  $\text{int} : \tilde{L}^X \rightarrow \tilde{L}^X$  is defined for  $\mathfrak{F} \in \mathbb{F}(X)$  as,

$$\text{int}(\lim \mathfrak{F}) = \sqcup \{\lim \mathfrak{G} \in \tilde{L}^X : \lim \mathfrak{F} \supseteq \lim \mathfrak{G}, \mathfrak{G} \in \mathbb{F}_p(X)\}$$

**Definition 3.10.** Let  $(X, \lim)$  be a soft  $\tilde{L}$ -fuzzy convergence structure. Let  $\mathfrak{F} \in \mathbb{F}(X)$ . Let  $\lim \mathfrak{F} \in \tilde{L}^X$ . Then, the *soft  $\tilde{L}$ -fuzzy lim-convergence topology* is defined by

$$\tau_{\lim} = \{\lim \mathfrak{F} : \text{int}(\lim \mathfrak{F}) = \lim \mathfrak{F}, \mathfrak{F} \in \mathbb{F}_p(X)\}$$

Now, the pair  $(X, \tau_{\lim})$  is said to be a *soft  $\tilde{L}$ -fuzzy lim-convergence topological space*.

The member of  $\tau_{\lim}$  is called as a soft  $\tilde{L}$ -fuzzy lim-open set and its complement is defined as

$$(\lim \mathfrak{F})' = \widetilde{\lim(1_X, X)} - \lim \mathfrak{F}$$

which is called as a *soft  $\tilde{L}$ -fuzzy lim-closed set*.

**Definition 3.11.** Let  $(X, \tau_{\lim})$  be a soft  $\tilde{L}$ -fuzzy lim-convergence topological space. Let  $\mathfrak{F} \in \mathbb{F}(X)$ . Let  $\lim \mathfrak{F} \in \tilde{L}^X$ . Then, the *soft  $\tilde{L}$ -fuzzy lim -closure* and *soft  $\tilde{L}$ -fuzzy lim-interior* of a soft  $\tilde{L}$ -fuzzy set,  $\lim \mathfrak{F}$  in  $X$  will be defined as

$$\begin{aligned} S\tilde{L}F-int(\lim \mathfrak{F}) &= \sqcup \{ \lim \mathfrak{G} \in \tilde{L}^X : \lim \mathfrak{G} \text{ is a soft } \tilde{L} \text{-fuzzy lim-open set,} \\ &\quad \lim \mathfrak{F} \sqsupseteq \lim \mathfrak{G}, \mathfrak{G} \in \mathbb{F}_p(X) \} \\ S\tilde{L}F-cl(\lim \mathfrak{F}) &= \sqcap \{ \lim \mathfrak{G} \in \tilde{L}^X : \lim \mathfrak{G} \text{ is a soft } \tilde{L} \text{-fuzzy lim-closed set,} \\ &\quad \lim \mathfrak{F} \sqsubseteq \lim \mathfrak{G}, \mathfrak{G} \in \mathbb{F}_p(X) \} \end{aligned}$$

**Proposition 3.12.** For any  $\mathfrak{F} \in \mathbb{F}(X)$ , we have

- (a)  $S\tilde{L}F-cl((\lim \mathfrak{F})') = (S\tilde{L}F-int(\lim \mathfrak{F}))'$   
(b)  $S\tilde{L}F-int((\lim \mathfrak{F})') = (S\tilde{L}F-cl(\lim \mathfrak{F}))'$

*Proof.* (a)  $S\tilde{L}F-cl((\lim \mathfrak{F})') = S\tilde{L}F-cl(\widetilde{\lim(1_X, X) - \lim \mathfrak{F}})$   
 $= \sqcap \{ \widetilde{\lim(1_X, X) - \lim \mathfrak{G}} \in \tilde{L}^X : \widetilde{\lim(1_X, X) - \lim \mathfrak{G}} \text{ is a soft } \tilde{L} \text{-fuzzy}$   
 $\text{lim-closed set } \widetilde{\lim(1_X, X) - \lim \mathfrak{F}} \sqsubseteq \widetilde{\lim(1_X, X) - \lim \mathfrak{G}}, \mathfrak{G} \in \mathbb{F}_p(X) \}$   
 $= \widetilde{\lim(1_X, X) - \sqcup \{ \lim \mathfrak{G} \in \tilde{L}^X : \lim \mathfrak{G} \text{ is a soft } \tilde{L} \text{-fuzzy lim-open set,}$   
 $\quad \lim \mathfrak{F} \sqsupseteq \lim \mathfrak{G}, \mathfrak{G} \in \mathbb{F}_p(X) \}}$   
 $= \widetilde{\lim(1_X, X) - S\tilde{L}F-int(\lim \mathfrak{F})}.$   
(b) Similar proof of (a).  $\square$

**Note:** If  $\mathfrak{F}, \mathfrak{G} \in \mathbb{F}(X)$ , and  $\lim \mathfrak{F}, \lim \mathfrak{G} \in \tilde{L}^X$ ,  $\lim \mathfrak{F} \sqsubseteq \lim \mathfrak{G}$  is defined analogously as in L-fuzzy set in usual way.

**Definition 3.13.** Let  $(X, \tau_{lim})$  be a soft  $\tilde{L}$ -fuzzy *lim*-convergence topological space and  $A$  be a subset of  $X$ . If  $\mathfrak{F} \in \mathbb{F}_p(X)$  and  $(\lim \mathfrak{F})/A \in \tilde{L}^A$ , then

$$\tau_{lim/A} = \{ (\lim \mathfrak{F})/A : \lim \mathfrak{F} \in \tau_{lim} \}$$

is called as a *soft  $\tilde{L}$ -fuzzy lim-convergence subspace topology*. Now, the pair  $(A, \tau_{lim/A})$  is called as a *soft  $\tilde{L}$ -fuzzy lim-convergence subspace* of  $(X, \tau_{lim})$ .

**Definition 3.14.** Let  $(X, \lim)$  be a soft  $\tilde{L}$ -fuzzy convergence structure. Let  $\mathfrak{F} \in \mathbb{F}(X)$ . Let  $\lim \mathfrak{F} \in \tilde{L}^X$ . Then, the *soft  $\tilde{L}$ -fuzzy real line  $\tilde{L}^{\mathbb{R}}$*  i.e.  $\mathbb{R}(L \times L)$  is the set of all monotone decreasing element  $\lim \mathfrak{F} \in \tilde{L}^{\mathbb{R}}$  satisfying

$$\begin{aligned} \sqcup \{ \lim \mathfrak{F}(t) : t \in \mathbb{R} \} &= (1, 1) \\ \sqcap \{ \lim \mathfrak{F}(t) : t \in \mathbb{R} \} &= (0, 0) \end{aligned}$$

after the identification of  $\lim \mathfrak{F}, \lim \mathfrak{G} \in \tilde{L}^{\mathbb{R}}$  and  $\mathfrak{F}, \mathfrak{G} \in \mathbb{F}(X)$  iff

$$\begin{aligned} \lim \mathfrak{F}(t-) &= \lim \mathfrak{G}(t-) \\ \lim \mathfrak{F}(t+) &= \lim \mathfrak{G}(t+) \end{aligned}$$

for all  $t \in \mathbb{R}$ , where,

$$\begin{aligned} \lim \mathfrak{F}(t-) &= \sqcap_{s < t} \lim \mathfrak{F}(s) = \lim_{s \rightarrow t-} \lim \mathfrak{F}(s). \\ \lim \mathfrak{F}(t+) &= \sqcup_{s > t} \lim \mathfrak{F}(s) = \lim_{s \rightarrow t+} \lim \mathfrak{F}(s). \end{aligned}$$

**Definition 3.15.** Let  $(X, \lim)$  be a soft  $\tilde{L}$ -fuzzy convergence structure. Let  $\mathfrak{F} \in \mathbb{F}(X)$ . Let  $\lim \mathfrak{F} \in \tilde{L}^X$ . The *natural soft  $\tilde{L}$ -fuzzy lim-convergence topology* on  $\mathbb{R}(L \times L)$  is generated from the sub-basis  $\{L_t, R_t : t \in \mathbb{R}\}$ , where,  $L_t, R_t : \mathbb{R} \rightarrow L \times L$  and  $L_t(\lim \mathfrak{F}) = (\lim \mathfrak{F}(t-))' = (1, 1) - \lim \mathfrak{F}(t-)$  and  $R_t(\lim \mathfrak{F}) = \lim \mathfrak{F}(t+)$ , for all  $\lim \mathfrak{F} \in \tilde{L}^{\mathbb{R}}$ . This topology is called as the *usual topology* for  $\mathbb{R}(L \times L)$ .

$\mathcal{L} = \{L_t : t \in \mathbb{R}\} \cup \{(0_X, 0_X), (1_X, 1_X)\}$  and  $\mathcal{R} = \{R_t : t \in \mathbb{R}\} \cap \{(0_X, 0_X), (1_X, 1_X)\}$  are called the *left and right hand I-topologies* respectively.

**Definition 3.16.** A partial order on  $\mathbb{R}(L \times L)$  is defined by  $[\lim \mathfrak{F}] \sqsubseteq [\lim \mathfrak{G}]$  iff  $\lim \mathfrak{F}(t-) \sqsubseteq \lim \mathfrak{G}(t-)$  and  $\lim \mathfrak{F}(t+) \sqsubseteq \lim \mathfrak{G}(t+)$ , for all  $t \in \mathbb{R}$  and  $\mathfrak{F}, \mathfrak{G} \in \mathbb{F}(X)$ .

**Definition 3.17.** Let  $(X, \lim)$  be a soft  $\tilde{L}$ -fuzzy convergence structure. The soft  $\tilde{L}$ -fuzzy unit interval  $I(L \times L)$  is a subset of  $\mathbb{R}(L \times L)$  such that  $[\lim \mathfrak{F}] \in I(L \times L)$  i.e.  $[\lim \mathfrak{F}] \in \tilde{L}^I$ , if

$$\begin{aligned} \lim \mathfrak{F}(t) &= (1, 1) \text{ for } t < 0, t \in \mathbb{R} \\ \lim \mathfrak{F}(t) &= (0, 0) \text{ for } t > 1, t \in \mathbb{R} \end{aligned}$$

It is equipped with the soft  $\tilde{L}$ -fuzzy  $\lim$ -convergence subspace topology.

**Definition 3.18.** Let  $(X, \tau_{\lim})$  be a soft  $\tilde{L}$ -fuzzy  $\lim$ -convergence topological space. A soft  $\tilde{L}$ -fuzzy set,  $\lim \mathfrak{F}$  is said to be a soft  $\tilde{L}$ -fuzzy  $\alpha^*$ -lim-open set (in short.,  $S\tilde{L}F\alpha^*$ -limOS.), if

$$S\tilde{L}F\text{-int}(\lim \mathfrak{F}) = S\tilde{L}F\text{-int}(S\tilde{L}F\text{-cl}(S\tilde{L}F\text{-int}(\lim \mathfrak{F}))), \text{ for each } \mathfrak{F} \in \mathbb{F}_p(X).$$

The complement of a soft  $\tilde{L}$ -fuzzy  $\alpha^*$ -lim-open set is called as a soft  $\tilde{L}$ -fuzzy  $\alpha^*$ -lim-closed set. It is denoted by  $S\tilde{L}F\alpha^*\text{-limCS}$ .

**Definition 3.19.** Let  $(X, \tau_{\lim})$  be a soft  $\tilde{L}$ -fuzzy  $\lim$ -convergence topological space. A soft  $\tilde{L}$ -fuzzy set,  $\lim \mathfrak{F}$  is said to be a soft  $\tilde{L}$ -fuzzy C-lim-open set (in short.,  $S\tilde{L}FC$ -limOS.), if

$$\lim \mathfrak{F} = \lim \mathfrak{G}_1 \sqcap \lim \mathfrak{G}_2, \mathfrak{F} \in \mathbb{F}(X), \mathfrak{G}_1, \mathfrak{G}_2 \in \mathbb{F}_p(X).$$

where,  $\lim \mathfrak{G}_1$  is a soft  $\tilde{L}$ -fuzzy  $\lim$ -open set and  $\lim \mathfrak{G}_2$  is a soft  $\tilde{L}$ -fuzzy  $\alpha^*$ -lim-open set

Now, the complement of a soft  $\tilde{L}$ -fuzzy C-lim-open set is called as a soft  $\tilde{L}$ -fuzzy C-lim-closed set. It is denoted by  $S\tilde{L}FC\text{-limCS}$ .

**Definition 3.20.** Let  $(X, \tau_{\lim})$  be a soft  $\tilde{L}$ -fuzzy  $\lim$ -convergence topological space. Let  $\mathfrak{F} \in \mathbb{F}(X)$ . Let  $\lim \mathfrak{F} \in \tilde{L}^X$ . Then, the soft  $\tilde{L}$ -fuzzy C-lim-closure and soft  $\tilde{L}$ -fuzzy C-lim-interior of a soft  $\tilde{L}$ -fuzzy set  $\lim \mathfrak{F}$  in  $X$  will be defined as

$$\begin{aligned} S\tilde{L}FC\text{-int}(\lim \mathfrak{F}) &= \sqcup \{ \lim \mathfrak{G} \in \tilde{L}^X : \lim \mathfrak{G} \text{ is a soft } \tilde{L}\text{-fuzzy C-lim-open set and} \\ &\quad \lim \mathfrak{F} \sqsupseteq \lim \mathfrak{G}, \mathfrak{G} \in \mathbb{F}_p(X) \} \\ S\tilde{L}FC\text{-cl}(\lim \mathfrak{F}) &= \sqcap \{ \lim \mathfrak{G} \in \tilde{L}^X : \lim \mathfrak{G} \text{ is a soft } \tilde{L}\text{-fuzzy C-lim-closed set and} \\ &\quad \lim \mathfrak{F} \sqsubseteq \lim \mathfrak{G}, \mathfrak{G} \in \mathbb{F}_p(X) \} \end{aligned}$$

**Remark 3.21.**

- (1)  $S\tilde{L}F\text{-int}(\lim \mathfrak{F}) \sqsubseteq \lim \mathfrak{F} \sqsubseteq S\tilde{L}F\text{-cl}(\lim \mathfrak{F})$
- (2)  $S\tilde{L}FC\text{-int}(\lim \mathfrak{F}) \sqsubseteq \lim \mathfrak{F} \sqsubseteq S\tilde{L}FC\text{-cl}(\lim \mathfrak{F})$

**Proposition 3.22.** For any  $\mathfrak{F} \in \mathbb{F}(X)$ , we have

- (a)  $S\tilde{L}FC\text{-cl}((\lim \mathfrak{F})') = (S\tilde{L}FC\text{-int}(\lim \mathfrak{F}))'$
- (b)  $S\tilde{L}FC\text{-int}((\lim \mathfrak{F})') = (S\tilde{L}FC\text{-cl}(\lim \mathfrak{F}))'$

*Proof.* (a)

$$\begin{aligned}
 S\tilde{L}FC-cl((\lim\mathfrak{F})') &= S\tilde{L}FC-cl(\widetilde{\lim(1_X, X)} - \lim\mathfrak{F}) \\
 &= \sqcap\{\widetilde{\lim(1_X, X)} - \lim\mathfrak{G} \in \tilde{L}^X : \widetilde{\lim(1_X, X)} - \lim\mathfrak{G} \text{ is a soft } \tilde{L}\text{-fuzzy} \\
 &\quad \text{C-lim-closed set, } \widetilde{\lim(1_X, X)} - \lim\mathfrak{F} \sqsubseteq \widetilde{\lim(1_X, X)} - \lim\mathfrak{G}, \mathfrak{G} \in \mathbb{F}_p(X)\} \\
 &= \widetilde{\lim(1_X, X)} - \sqcup\{\lim\mathfrak{G} \in \tilde{L}^X : \lim\mathfrak{G} \text{ is a soft } \tilde{L}\text{-fuzzy C-lim-open set,} \\
 &\quad \lim\mathfrak{F} \supseteq \lim\mathfrak{G}, \mathfrak{G} \in \mathbb{F}_p(X)\} \\
 &= \widetilde{\lim(1_X, X)} - S\tilde{L}FC-int(\lim\mathfrak{F}).
 \end{aligned}$$

(b) Similar proof of (a). □

#### 4. SOFT $\tilde{L}$ -FUZZY C-*lim*-CONVERGENCE EXTREMALLY DISCONNECTED SPACE

**Definition 4.1.** Let  $(X, \tau_{lim})$  be a soft  $\tilde{L}$ -fuzzy *lim*-convergence topological space. If the soft  $\tilde{L}$ -fuzzy C-*lim*-closure of a soft  $\tilde{L}$ -fuzzy C-*lim*-open set is soft  $\tilde{L}$ -fuzzy C-*lim*-open, then  $(X, \tau_{lim})$  is said to be soft  $\tilde{L}$ -fuzzy C-*lim*-convergence extremally disconnected space.

**Proposition 4.2.** For any soft  $\tilde{L}$ -fuzzy *lim*-convergence topological space  $(X, \tau_{lim})$ , the following are equivalent:

- (a)  $(X, \tau_{lim})$  is a soft  $\tilde{L}$ -fuzzy C-*lim*-convergence extremally disconnected space.
- (b) For each soft  $\tilde{L}$ -fuzzy C-*lim*-closed set  $\lim\mathfrak{F}$ ,  $S\tilde{L}FC-int(\lim\mathfrak{F})$  is a soft  $\tilde{L}$ -fuzzy C-*lim*-closed set.
- (c) For each soft  $\tilde{L}$ -fuzzy C-*lim*-open set  $\lim\mathfrak{F}$ , we have  $S\tilde{L}FC-int(S\tilde{L}FC-cl(\lim\mathfrak{F})) = S\tilde{L}FC-cl(\lim\mathfrak{F})$ .
- (d) For each pair of soft  $\tilde{L}$ -fuzzy C-*lim*-open sets  $\lim\mathfrak{F}$  and  $\lim\mathfrak{G}$  in  $(X, \tau_{lim})$  with  $S\tilde{L}FC-int(\widetilde{\lim(1_X, X)} - \lim\mathfrak{F}) = \lim\mathfrak{G}$ , we have  $\widetilde{\lim(1_X, X)} - S\tilde{L}FC-cl(\lim\mathfrak{F}) = S\tilde{L}FC-cl(\lim\mathfrak{G})$ .

*Proof.* (a)  $\Rightarrow$  (b): Let  $\lim\mathfrak{F}$  be a soft  $\tilde{L}$ -fuzzy C-*lim*-closed set. Now,  $(\lim\mathfrak{F})'$  is a soft  $\tilde{L}$ -fuzzy C-*lim*-open set. By (a),  $S\tilde{L}FC-cl((\lim\mathfrak{F})')$  is a soft  $\tilde{L}$ -fuzzy C-*lim*-open set. Now,

$$\begin{aligned}
 S\tilde{L}FC-cl((\lim\mathfrak{F})') &= S\tilde{L}FC-cl(\widetilde{\lim(1_X, X)} - \lim\mathfrak{F}) \\
 &= \widetilde{\lim(1_X, X)} - S\tilde{L}FC-int(\lim\mathfrak{F}).
 \end{aligned}$$

This implies that,  $S\tilde{L}FC-int(\lim\mathfrak{F})$  is a soft  $\tilde{L}$ -fuzzy C-*lim*-closed set.

(b)  $\Rightarrow$  (c): Let  $\lim\mathfrak{F}$  be a soft  $\tilde{L}$ -fuzzy C-*lim*-open set. Then,  $(\lim\mathfrak{F})'$  is a soft  $\tilde{L}$ -fuzzy C-*lim*-closed set. By (b),  $S\tilde{L}FC-int((\lim\mathfrak{F})')$  is a soft  $\tilde{L}$ -fuzzy C-*lim*-closed

set. Now,

$$\begin{aligned} & \lim(\widetilde{1_X, X}) - \tilde{S\tilde{L}FC-int}(\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F})) \\ &= \tilde{S\tilde{L}FC-cl}(\tilde{S\tilde{L}FC-int}(\lim(\widetilde{1_X, X}) - \lim\mathfrak{F})) \\ &= \tilde{S\tilde{L}FC-int}(\lim(\widetilde{1_X, X}) - \lim\mathfrak{F}) \\ &= \lim(\widetilde{1_X, X}) - \tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F}) \end{aligned}$$

Hence,  $\tilde{S\tilde{L}FC-int}(\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F})) = \tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F})$ .

(c)  $\Rightarrow$  (d): Let  $\lim\mathfrak{F}$ ,  $\lim\mathfrak{G}$  be two soft  $\tilde{L}$ -fuzzy C- $\lim$ -open sets such that

$$\tilde{S\tilde{L}FC-int}(\lim(\widetilde{1_X, X}) - \lim\mathfrak{F}) = \lim\mathfrak{G}.$$

By (c), we have  $\tilde{S\tilde{L}FC-int}(\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F})) = \tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F})$ . Now,

$$\begin{aligned} & \lim(\widetilde{1_X, X}) - \tilde{S\tilde{L}FC-int}(\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F})) = \lim(\widetilde{1_X, X}) - \tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F}) \\ &= \tilde{S\tilde{L}FC-int}((\lim\mathfrak{F})') = \lim\mathfrak{G}. \end{aligned}$$

This implies that  $\tilde{S\tilde{L}FC-cl}(\tilde{S\tilde{L}FC-int}(\lim(\widetilde{1_X, X}) - \lim\mathfrak{F})) = \lim\mathfrak{G}$ . It follows that  $\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{G}) = \lim\mathfrak{G}$ . This implies that

$$\begin{aligned} & \lim(\widetilde{1_X, X}) - \tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F}) = \tilde{S\tilde{L}FC-int}(\lim(\widetilde{1_X, X}) - \lim\mathfrak{F}) \\ &= \lim\mathfrak{G} = \tilde{S\tilde{L}FC-cl}(\lim\mathfrak{G}). \end{aligned}$$

(d)  $\Rightarrow$  (a): Let  $\lim\mathfrak{F}$  be any soft  $\tilde{L}$ -fuzzy C- $\lim$ -open set. Consider another soft  $\tilde{L}$ -fuzzy C- $\lim$ -open set  $\lim\mathfrak{G}$ , with  $\tilde{S\tilde{L}FC-int}(\lim(\widetilde{1_X, X}) - \lim\mathfrak{F}) = \lim\mathfrak{G}$ . By (d),  $(\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F}))' = \tilde{S\tilde{L}FC-cl}(\lim\mathfrak{G})$ . This implies that  $\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F}) = (\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{G}))'$ . It follows that  $\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F})$  is a soft  $\tilde{L}$ -fuzzy C- $\lim$ -open set. Hence,  $(X, \tau_{\lim})$  is soft  $\tilde{L}$ -fuzzy C- $\lim$ -convergence extremally disconnected space.  $\square$

**Proposition 4.3.** *Let  $(X, \tau_{\lim})$  be a soft  $\tilde{L}$ -fuzzy  $\lim$ -convergence topological space. Then,  $(X, \tau_{\lim})$  is soft  $\tilde{L}$ -fuzzy C- $\lim$ -convergence extremally disconnected space iff for each soft  $\tilde{L}$ -fuzzy C- $\lim$ -open set,  $\lim\mathfrak{F}$  and soft  $\tilde{L}$ -fuzzy C- $\lim$ -closed set,  $\lim\mathfrak{G}$  such that  $\lim\mathfrak{F} \subseteq \lim\mathfrak{G}$ , we have  $\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F}) \subseteq \tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})$ .*

*Proof.* Suppose  $(X, \tau_{\lim})$  is a soft  $\tilde{L}$ -fuzzy C- $\lim$ -convergence extremally disconnected space. Let  $\lim\mathfrak{F}$  be any soft  $\tilde{L}$ -fuzzy C- $\lim$ -open set and  $\lim\mathfrak{G}$  be any soft  $\tilde{L}$ -fuzzy C- $\lim$ -closed set. Then, by (b) of Proposition 4.2,  $\tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})$  is soft  $\tilde{L}$ -fuzzy C- $\lim$ -closed. Since  $\lim\mathfrak{F}$  is a soft  $\tilde{L}$ -fuzzy C- $\lim$ -open set and  $\lim\mathfrak{F} \subseteq \lim\mathfrak{G}$ , it follows that,  $\lim\mathfrak{F} \subseteq \tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})$ . Thus, it follows that,  $\tilde{S\tilde{L}FC-cl}(\lim\mathfrak{F}) \subseteq \tilde{S\tilde{L}FC-cl}(\tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})) = \tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})$ .

Conversely, let  $\lim\mathfrak{G}$  be a soft  $\tilde{L}$ -fuzzy C- $\lim$ -closed set. Then,  $\tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})$  is a  $\tilde{S\tilde{L}FC}$ - $\lim$  open set. Also,  $\tilde{S\tilde{L}FC-int}(\lim\mathfrak{G}) \subseteq \lim\mathfrak{G}$ . Now, by assumption, it follows that,  $\tilde{S\tilde{L}FC-cl}(\tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})) \subseteq \tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})$ . This implies that,  $\tilde{S\tilde{L}FC-cl}(\tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})) = \tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})$ . Thus,  $\tilde{S\tilde{L}FC-int}(\lim\mathfrak{G})$  is a soft



$\tilde{L}$ -fuzzy  $C$ -lim-closed set. Hence, by the Proposition 4.2, we have  $(X, \tau_{lim})$  is a soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence extremally disconnected space.  $\square$

**Definition 4.4.** A soft  $\tilde{L}$ -fuzzy set which is both soft  $\tilde{L}$ -fuzzy  $C$ -lim-open and soft  $\tilde{L}$ -fuzzy  $C$ -lim-closed set is called as a soft  $\tilde{L}$ -fuzzy  $C$ -lim-clopen set. It is denoted by  **$\tilde{S}\tilde{L}FC$ -limCOS**.

**Definition 4.5.** A soft  $\tilde{L}$ -fuzzy  $lim$ -convergence topological space,  $(X, \tau_{lim})$  is said to have *property*  $\nabla$ , if the union of any family of  $C$ -lim-open sets is  $C$ -lim-open.

**Remark 4.6.** Let  $(X, \tau_{lim})$  be a soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence extremally disconnected space. Let  $(X, \tau_{lim})$  possess the property  $\nabla$ . Let  $\{lim\mathfrak{F}_i, lim\mathfrak{G}_j : i, j \in \mathbb{N}\}$  be a collection such that each  $lim\mathfrak{F}_i$ 's are  $\tilde{S}\tilde{L}FC$ -limOS and each  $lim\mathfrak{G}_j$ 's are  $\tilde{S}\tilde{L}FC$ -limCS. Let  $lim\mathfrak{F}, lim\mathfrak{G}$  be the  $\tilde{S}\tilde{L}FC$ -lim open set and  $\tilde{S}\tilde{L}FC$ -lim closed set respectively. If  $lim\mathfrak{F}_i \subseteq lim\mathfrak{F} \subseteq lim\mathfrak{G}_j$  and  $lim\mathfrak{F}_i \subseteq lim\mathfrak{G} \subseteq lim\mathfrak{G}_j$ , for all  $i, j \in \mathbb{N}$ , then there exists a soft  $\tilde{L}$ -fuzzy  $C$ -lim-clopen set,  $lim\mathfrak{M}$  such that  $\tilde{S}\tilde{L}FC-cl(lim\mathfrak{F}_i) \subseteq lim\mathfrak{M} \subseteq \tilde{S}\tilde{L}FC-int(lim\mathfrak{G}_j)$ , for all  $i, j \in \mathbb{N}$ .

*Proof.* By the Proposition 4.3,

$$\begin{aligned} \tilde{S}\tilde{L}FC-cl(lim\mathfrak{F}_i) &\subseteq \tilde{S}\tilde{L}FC-cl(lim\mathfrak{F}) \cap \tilde{S}\tilde{L}FC-int(lim\mathfrak{G}) \\ &\subseteq \tilde{S}\tilde{L}FC-int(lim\mathfrak{G}_j) \end{aligned}$$

for all  $i, j \in \mathbb{N}$ . Therefore, using the hypothesis,

$$lim\mathfrak{M} = \tilde{S}\tilde{L}FC-cl(lim\mathfrak{F}) \cap \tilde{S}\tilde{L}FC-int(lim\mathfrak{G})$$

is a soft  $\tilde{L}$ -fuzzy  $C$ -lim-clopen set satisfying the required condition.  $\square$

**Proposition 4.7.** Let  $(X, \tau_{lim})$  be a soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence extremally disconnected space. Let  $(X, \tau_{lim})$  possess the property  $\nabla$ . Let  $\{lim\mathfrak{F}_q\}_{q \in \mathbb{Q}}$  and  $\{lim\mathfrak{G}_q\}_{q \in \mathbb{Q}}$  be the monotone increasing collections of soft  $\tilde{L}$ -fuzzy  $C$ -lim-open sets and soft  $\tilde{L}$ -fuzzy  $C$ -lim-closed sets of  $(X, \tau_{lim})$ . ( $\mathbb{Q}$  is the set of all rational numbers). If  $lim\mathfrak{F}_{q_1} \subseteq lim\mathfrak{G}_{q_2}$ , whenever  $q_1 < q_2$  ( $q_1, q_2 \in \mathbb{Q}$ ), then, there exists a monotone increasing collection  $\{lim\mathfrak{H}_q\}_{q \in \mathbb{Q}}$  of soft  $\tilde{L}$ -fuzzy  $C$ -lim-clopen sets of  $(X, \tau_{lim})$  such that  $\tilde{S}\tilde{L}FC-cl(lim\mathfrak{F}_{q_1}) \subseteq lim\mathfrak{H}_{q_2}$  and  $lim\mathfrak{H}_{q_1} \subseteq \tilde{S}\tilde{L}FC-int(lim\mathfrak{G}_{q_2})$  whenever  $q_1 < q_2$ .

*Proof.* Let us arrange into a sequence  $\{q_n\}$  of rational numbers without repetitions. For every  $n \geq 2$ , define inductively a collection  $\{lim\mathfrak{H}_{q_i} : 1 \leq i < n\} \subseteq \tilde{L}^X$  such that By Proposition 4.3, the countable collections  $\{\tilde{S}\tilde{L}FC-cl(lim\mathfrak{F}_{q_i})\}_{i \in \mathbb{N}}$  and  $\{\tilde{S}\tilde{L}FC-int(lim\mathfrak{G}_{q_i})\}_{i \in \mathbb{N}}$  satisfying  $\tilde{S}\tilde{L}FC-cl(lim\mathfrak{F}_{q_1}) \subseteq \tilde{S}\tilde{L}FC-int(lim\mathfrak{G}_{q_2})$ , if  $q_1 < q_2$  ( $q_1, q_2 \in \mathbb{Q}$ ). By the Remark:4.6, there exists  $\tilde{S}\tilde{L}FC$ -lim clopen set,  $lim\mathfrak{M}$  such that  $\tilde{S}\tilde{L}FC-cl(lim\mathfrak{F}_{q_1}) \subseteq lim\mathfrak{M} \subseteq \tilde{S}\tilde{L}FC-int(lim\mathfrak{G}_{q_2})$ . By setting  $lim\mathfrak{H}_{q_1} = lim\mathfrak{M}$ , we get  $(S_2)$ .

Assume that soft  $\tilde{L}$ -fuzzy sets  $lim\mathfrak{H}_{q_i}$  (already defined), for  $i < n$  and satisfy  $(S_n)$ . Define

$$\begin{aligned} \Phi &= \sqcup \{lim\mathfrak{H}_{q_i} : i < n, q_i < q_n\} \sqcup lim\mathfrak{F}_{q_n} \\ \Omega &= \cap \{lim\mathfrak{H}_{q_j} : j < n, q_j > q_n\} \cap lim\mathfrak{G}_{q_n} \end{aligned}$$

Then, we have,  $S\tilde{L}FC-cl(\lim\mathfrak{H}_{q_i}) \subseteq S\tilde{L}FC-cl(\Phi) \subseteq S\tilde{L}FC-int(\lim\mathfrak{H}_{q_j})$  and  $S\tilde{L}FC-cl(\lim\mathfrak{H}_{q_i}) \subseteq S\tilde{L}FC-int(\Omega) \subseteq S\tilde{L}FC-int(\lim\mathfrak{H}_{q_j})$ , whenever  $q_i < q_n < q_j$  ( $i, j < n$ ), as well as  $\lim\mathfrak{F}_q \subseteq S\tilde{L}FC-cl(\Phi) \subseteq \lim\mathfrak{G}_{q'}$  and  $\lim\mathfrak{F}_q \subseteq S\tilde{L}FC-int(\Omega) \subseteq \lim\mathfrak{G}_{q'}$ , whenever  $q < q_n < q'$ . This shows that the countable collections  $\{\lim\mathfrak{H}_{q_i} : i < n, q_i < q_n\} \cup \{\lim\mathfrak{F}_q : q < q_n\}$  and  $\{\lim\mathfrak{H}_{q_j} : j < n, q_j > q_n\} \cup \{\lim\mathfrak{G}_{q'} : q' > q_n\}$  together with  $\Phi$  and  $\Omega$ , fulfil all the conditions of the Remark:4.6. Hence, there exists a  $S\tilde{L}FC-lim$  clopen set,  $\lim\mathfrak{M}_n$  such that  $S\tilde{L}FC-cl(\lim\mathfrak{M}_n) \subseteq \lim\mathfrak{G}_q$  if  $q_n < q$ , and  $\lim\mathfrak{F}_q \subseteq S\tilde{L}FC-int(\lim\mathfrak{M}_n)$  if  $q < q_n$ . Also,  $S\tilde{L}FC-cl(\lim\mathfrak{H}_{q_i}) \subseteq S\tilde{L}FC-int(\lim\mathfrak{M}_n)$  if  $q_i < q_n$  and  $S\tilde{L}FC-cl(\lim\mathfrak{M}_n) \subseteq S\tilde{L}FC-int(\lim\mathfrak{H}_{q_j})$  if  $q_n < q_j$ , where  $1 \leq i, j \leq n-1$ . Now setting  $\lim\mathfrak{H}_{q_n} = \lim\mathfrak{M}_n$ , we obtain the soft  $\tilde{L}$ -fuzzy sets  $\lim\mathfrak{H}_{q_1}, \lim\mathfrak{H}_{q_2}, \dots, \lim\mathfrak{H}_{q_n}$  that satisfy  $(S_{n+1})$ . Therefore, the collection  $\{\lim\mathfrak{H}_{q_i} : i = 1, 2, 3, \dots\}$  has the required property. This completes the proof.  $\square$

## 5. CHARACTERIZATIONS OF SOFT $\tilde{L}$ -FUZZY C- $lim$ -CONVERGENCE EXTREMALLY DISCONNECTED SPACE

**Definition 5.1.** Let  $(X, \tau_{lim})$  and  $(Y, \sigma_{lim})$  be two soft  $\tilde{L}$ -fuzzy  $lim$ -convergence topological spaces. Let  $f : (X, \tau_{lim}) \rightarrow (Y, \sigma_{lim})$  be a function. Let  $\mathfrak{F} \in \mathbb{F}_p(Y)$ . Then,  $f$  is said to be *soft  $\tilde{L}$ -fuzzy C- $lim$ -convergence continuous function* if, for every soft  $\tilde{L}$ -fuzzy  $lim$ -open set,  $\lim\mathfrak{F}$  in  $(Y, \sigma_{lim})$ , there exists a soft  $\tilde{L}$ -fuzzy C- $lim$ -open set  $f^{-1}(\lim\mathfrak{F})$  of  $(X, \tau_{lim})$ .

Equivalently, if, for every soft  $\tilde{L}$ -fuzzy  $lim$ -closed set,  $\lim\mathfrak{F}$  in  $(Y, \sigma_{lim})$ , there exists a soft  $\tilde{L}$ -fuzzy C- $lim$ -closed set  $f^{-1}(\lim\mathfrak{F})$  of  $(X, \tau_{lim})$ .

**Proposition 5.2.** Let  $(X, \tau_{lim})$  and  $(Y, \sigma_{lim})$  be two soft  $\tilde{L}$ -fuzzy  $lim$ -convergence topological spaces. A function  $f : (X, \tau_{lim}) \rightarrow (Y, \sigma_{lim})$  is a soft  $\tilde{L}$ -fuzzy C- $lim$ -convergence continuous function iff

$$f(S\tilde{L}FC-cl(\lim\mathfrak{F})) \subseteq S\tilde{L}Fcl(f(\lim\mathfrak{F}))$$

for all  $\lim\mathfrak{F} \in \tilde{L}^X$ ,  $\mathfrak{F} \in \mathbb{F}(X)$ .

*Proof.* Suppose that  $f$  is a soft  $\tilde{L}$ -fuzzy C- $lim$ -convergence continuous function. Let  $\mathfrak{F} \in \mathbb{F}(X)$ . Let  $\lim\mathfrak{F} \in \tilde{L}^X$ . Then  $f(\lim\mathfrak{F}) \in \tilde{L}^Y$ . By the hypothesis,

$$f^{-1}(S\tilde{L}Fcl(f(\lim\mathfrak{F})))$$

is a soft  $\tilde{L}$ -fuzzy C- $lim$ -closed set in  $X$ . Also,

$$\lim\mathfrak{F} \subseteq f^{-1}(f(\lim\mathfrak{F})) \subseteq f^{-1}(S\tilde{L}Fcl(f(\lim\mathfrak{F}))).$$

Now, by the definition of soft  $\tilde{L}$ -fuzzy C- $lim$ -closure,

$$S\tilde{L}FC-cl(\lim\mathfrak{F}) \subseteq f^{-1}(S\tilde{L}Fcl(f(\lim\mathfrak{F}))).$$

This implies that,  $f(S\tilde{L}FC-cl(\lim\mathfrak{F})) \subseteq S\tilde{L}Fcl(f(\lim\mathfrak{F}))$ .

Conversely, let  $\mathfrak{F} \in \mathbb{F}_p(Y)$ . Suppose that  $\lim\mathfrak{F} \in \tilde{L}^Y$  is soft  $\tilde{L}$ -fuzzy  $lim$ -closed. Then,  $f^{-1}(\lim\mathfrak{F}) \in \tilde{L}^X$ . By hypothesis,

$$f(S\tilde{L}FC-cl(f^{-1}(\lim\mathfrak{F}))) \subseteq S\tilde{L}Fcl(f(f^{-1}(\lim\mathfrak{F}))) \subseteq S\tilde{L}Fcl(\lim\mathfrak{F}) = \lim\mathfrak{F}.$$

This implies that,  $S\tilde{L}FC-cl(f^{-1}(lim\mathfrak{F})) \sqsubseteq f^{-1}(lim\mathfrak{F})$ . It follows that,  $f^{-1}(lim\mathfrak{F})$  is soft  $\tilde{L}$ -fuzzy C-*lim*-closed. Hence,  $f$  is a soft  $\tilde{L}$ -fuzzy C-*lim*-convergence continuous function.  $\square$

**Proposition 5.3.** *Let  $(X, \tau_{lim})$  be a soft  $\tilde{L}$ -fuzzy C-*lim*-convergence extremally disconnected space and  $(Y, \sigma_{lim})$  be a soft  $\tilde{L}$ -fuzzy *lim*-convergence topological space. A function  $f : (X, \tau_{lim}) \rightarrow (Y, \sigma_{lim})$  is a soft  $\tilde{L}$ -fuzzy C-*lim*-convergence continuous function. Then, for each  $lim\mathfrak{F} \in \tilde{L}^X$ ,  $S\tilde{L}FC-cl(S\tilde{L}Fint(lim\mathfrak{F})) \sqsubseteq S\tilde{L}FC-int(f^{-1}(S\tilde{L}Fcl(f(S\tilde{L}Fint(lim\mathfrak{F}))))$ .*

*Proof.* Let  $\mathfrak{F} \in \mathbb{F}(X)$ . Let  $lim\mathfrak{F} \in \tilde{L}^X$ . Then,  $S\tilde{L}Fint(lim\mathfrak{F})$  is a soft  $\tilde{L}$ -fuzzy *lim*-open set in  $X$ . By the hypothesis,

$$f(S\tilde{L}FC-cl(S\tilde{L}Fint(lim\mathfrak{F}))) \sqsubseteq S\tilde{L}Fcl(f(S\tilde{L}Fint(lim\mathfrak{F}))).$$

This implies that,  $S\tilde{L}FC-cl(S\tilde{L}Fint(lim\mathfrak{F})) \sqsubseteq f^{-1}(S\tilde{L}Fcl(f(S\tilde{L}Fint(lim\mathfrak{F}))))$ . Since  $(X, \tau_{lim})$  is a soft  $\tilde{L}$ -fuzzy C-*lim*-convergence extremally disconnected space, it follows that,  $S\tilde{L}FC-cl(S\tilde{L}Fint(lim\mathfrak{F})) \sqsubseteq S\tilde{L}FC-int(f^{-1}(S\tilde{L}Fcl(f(S\tilde{L}Fint(lim\mathfrak{F}))))$ .  $\square$

**Definition 5.4.** Let  $(X, \tau_{lim})$  be a soft  $\tilde{L}$ -fuzzy *lim*-convergence topological space. A mapping  $f : X \rightarrow \mathbb{R}(L \times L)$  is called as the *lower (upper) soft  $\tilde{L}$ -fuzzy C-*lim*-convergence continuous function*, if  $f^{-1}R_t$  (resp.  $f^{-1}L_t$ ) is a soft  $\tilde{L}$ -fuzzy C-*lim*-open set (soft  $\tilde{L}$ -fuzzy C-*lim*-closed set), for each  $t \in \mathbb{R}$ .

**Proposition 5.5.** *Let  $(X, \tau_{lim})$  be a soft  $\tilde{L}$ -fuzzy *lim*-convergence topological space. Let  $lim\mathfrak{F} \in \tilde{L}^X$ . Let  $f : X \rightarrow \mathbb{R}(L \times L)$  be such that*

$$f(x)(t) = \begin{cases} (1, 1), & \text{if } t < 0 \\ lim\mathfrak{F}(x), & \text{if } t \in [0, 1] \\ (0, 0), & \text{if } t > 1 \end{cases}$$

*for all  $x \in X$ . Then,  $f$  is lower (resp. upper) soft  $\tilde{L}$ -fuzzy C-*lim*-convergence continuous function iff  $lim\mathfrak{F}$  is soft  $\tilde{L}$ -fuzzy C-*lim*-open (resp. C-*lim*-closed).*

*Proof.* Let

$$f^{-1}R_t = \begin{cases} (1_X, 1_X), & \text{if } t < 0 \\ lim\mathfrak{F}, & \text{if } t \in [0, 1] \\ (0_X, 0_X), & \text{if } t > 1 \end{cases}$$

This implies that,  $f$  is lower soft  $\tilde{L}$ -fuzzy C-*lim*-convergence continuous function iff  $lim\mathfrak{F}$  is soft  $\tilde{L}$ -fuzzy C-*lim*-open. Let

$$f^{-1}L_t = \begin{cases} (1_X, 1_X), & \text{if } t < 0 \\ lim\mathfrak{F}, & \text{if } t \in [0, 1] \\ (0_X, 0_X), & \text{if } t > 1 \end{cases}$$

This implies that,  $f$  is upper soft  $\tilde{L}$ -fuzzy C-*lim*-convergence continuous function iff  $lim\mathfrak{F}$  is soft  $\tilde{L}$ -fuzzy C-*lim*-closed.  $\square$

**Definition 5.6.** The *lim-characteristic function* of  $\lim\mathfrak{F} \in \tilde{L}^X$  is the map  $\chi_{\lim\mathfrak{F}} : X \rightarrow \mathbb{R}(L \times L)$  defined by

$$\chi_{\lim\mathfrak{F}}(x) = \begin{cases} (1, 1), & \text{if } t < 0 \\ \lim\mathfrak{F}(x), & \text{if } t \in [0, 1] \\ (0, 0), & \text{if } t > 1 \end{cases}$$

for all  $x \in X$ .

**Proposition 5.7.** Let  $(X, \tau_{\lim})$  be a soft  $\tilde{L}$ -fuzzy *lim-convergence* topological space. Let  $\mathfrak{F} \in \mathbb{F}(X)$ . Let  $\lim\mathfrak{F} \in \tilde{L}^X$ . Then,  $\chi_{\lim\mathfrak{F}}$  is lower (upper) soft  $\tilde{L}$ -fuzzy *C-lim-convergence continuous* function iff  $\lim\mathfrak{F}$  is soft  $\tilde{L}$ -fuzzy *C-lim-open* (*C-lim-closed*).

*Proof.* It follows from the Proposition 5.5.  $\square$

**Proposition 5.8.** Let  $(X, \tau_{\lim})$  be a soft  $\tilde{L}$ -fuzzy *lim-convergence* topological space. Let  $(X, \tau_{\lim})$  possess the property  $\nabla$ . Then, the following are equivalent.

- (a)  $(X, \tau_{\lim})$  be a soft  $\tilde{L}$ -fuzzy *C-lim-convergence* extremally disconnected space.
- (b) (Soft  $\tilde{L}$ -fuzzy Insertion Theorem:) Let  $g, h : X \rightarrow \mathbb{R}(L \times L)$ . If  $g$  is lower soft  $\tilde{L}$ -fuzzy *C-lim-convergence continuous* function and  $h$  is upper soft  $\tilde{L}$ -fuzzy *C-lim-convergence continuous* function with  $g \sqsubseteq h$ , then there exists a soft  $\tilde{L}$ -fuzzy *C-lim-convergence continuous* function,  $f : X \rightarrow \mathbb{R}(L \times L)$  such that  $g \sqsubseteq f \sqsubseteq h$ .
- (c) (Soft  $\tilde{L}$ -fuzzy Urysohn Lemma:) Let  $\mathfrak{F}, \mathfrak{G} \in \mathbb{F}_p(X)$ . If  $(\lim\mathfrak{F})', \lim\mathfrak{G}$  are soft  $\tilde{L}$ -fuzzy *C-lim-open* sets such that  $\lim\mathfrak{G} \sqsubseteq \lim\mathfrak{F}$ , then there exists a soft  $\tilde{L}$ -fuzzy *C-lim-convergent continuous* function,  $f : X \rightarrow [0, 1](L \times L)$  such that  $\lim\mathfrak{G} \sqsubseteq (L_1)'f \sqsubseteq R_0f \sqsubseteq \lim\mathfrak{F}$ .

*Proof.* (a)  $\Rightarrow$  (b): Define  $\lim\mathfrak{H}_r = L_r h$  and  $\lim\mathfrak{G}_r = R_r' g$ ,  $r \in \mathbb{Q}$ . Then, we have two monotone increasing families of soft  $\tilde{L}$ -fuzzy *C-lim-open* and soft  $\tilde{L}$ -fuzzy *C-lim-closed* sets of  $(X, \tau_{\lim})$ . Moreover,  $\lim\mathfrak{H}_r \sqsubseteq \lim\mathfrak{G}_r$ , if  $r < s$ . By Proposition: 4.7, there exists a monotone increasing family  $\{\lim\mathfrak{F}_r\}_{r \in \mathbb{Q}}$  of soft  $\tilde{L}$ -fuzzy *C-lim-clopen* sets of  $(X, \tau_{\lim})$  such that  $S\tilde{L}FC-cl(\lim\mathfrak{H}_r) \sqsubseteq \lim\mathfrak{F}_s$  and  $\lim\mathfrak{F}_r \sqsubseteq S\tilde{L}FC-int(\lim\mathfrak{G}_s)$ , whenever  $r < s$ . Let  $\lim\mathfrak{U}_t = \sqcap_{r < t} (\lim\mathfrak{F}_r)'$ , for all  $t \in \mathbb{R}$ , we define monotone decreasing family  $\{\lim\mathfrak{U}_t : t \in \mathbb{R}\} \subseteq \tilde{L}^X$ . Moreover, we have  $S\tilde{L}FC-cl(\lim\mathfrak{U}_t) \sqsubseteq S\tilde{L}FC-int(\lim\mathfrak{U}_s)$ , whenever  $s < t$ . Now,

$$\begin{aligned} \sqcup_{t \in \mathbb{R}} \lim\mathfrak{U}_t &= \sqcup_{t \in \mathbb{R}} \sqcap_{r < t} (\lim\mathfrak{F}_r)' \\ &\supseteq \sqcup_{t \in \mathbb{R}} \sqcap_{r < t} (\lim\mathfrak{G}_r)' \\ &= \sqcup_{t \in \mathbb{R}} \sqcap_{r < t} g^{-1} R_r \\ &= \sqcup_{t \in \mathbb{R}} g^{-1} R_t \\ &= g^{-1} (\sqcup_{t \in \mathbb{R}} R_t) \\ &= (1_X, 1_X) \end{aligned}$$

Similarly

$$\sqcap_{t \in \mathbb{R}} \lim\mathfrak{U}_t = (0_X, 0_X).$$

We now define a function  $f : X \rightarrow \mathbb{R}(L \times L)$  possessing the required properties. Let  $f(x)(t) = \lim\mathfrak{U}_t(x)$ , for all  $x \in X, t \in \mathbb{R}$ . By the above discussion, it follows that  $f$

is well defined . To prove  $f$  is soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence continuous function, we observe that,

$$\sqcup_{s>t} \lim \mathfrak{U}_s = \sqcup_{s>t} S\tilde{L}FC-int(\lim \mathfrak{U}_s)$$

and

$$\sqcap_{s<t} \lim \mathfrak{U}_s = \sqcap_{s<t} S\tilde{L}FC-cl(\lim \mathfrak{U}_s)$$

Then,  $f^{-1}R_t = \sqcup_{s>t} \lim \mathfrak{U}_s = \sqcup_{s>t} S\tilde{L}FC-int(\lim \mathfrak{U}_s)$  is soft  $\tilde{L}$ -fuzzy  $C$ -lim-open set and also,  $f^{-1}(L'_t) = \sqcap_{s<t} \lim \mathfrak{U}_s = \sqcap_{s<t} S\tilde{L}FC-cl(\lim \mathfrak{U}_s)$  is soft  $\tilde{L}$ -fuzzy  $C$ -lim-closed set. Therefore,  $f$  is a soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence continuous function. To conclude the proof, it remains to show that  $g \sqsubseteq f \sqsubseteq h$ . It is enough to show that,  $g^{-1}(\widetilde{\lim(1_X, X)} - L_t) \sqsubseteq f^{-1}(L'_t) \sqsubseteq h^{-1}(L'_t)$  and  $g^{-1}R_t \sqsubseteq f^{-1}R_t \sqsubseteq h^{-1}R_t$ , for each  $t \in \mathbb{R}$ . Now, we have

$$\begin{aligned} g^{-1}(\widetilde{\lim(1_X, X)} - L_t) &= g^{-1}(L'_t) \\ &= \sqcap_{s<t} g^{-1}(L'_s) \\ &= \sqcap_{s<t} \sqcap_{r<s} g^{-1}R_r \\ &= \sqcap_{s<t} \sqcap_{r<s} (\lim \mathfrak{G}_r)' \\ &\sqsubseteq \sqcap_{s<t} \sqcap_{r<s} (\widetilde{\lim(1_X, X)} - \lim \mathfrak{F}_r) \\ &= \sqcap_{s<t} \lim \mathfrak{U}_s \\ &= f^{-1}(\widetilde{\lim(1_X, X)} - L_t) \end{aligned}$$

Now,

$$\begin{aligned} f^{-1}(L'_t) &= \sqcap_{s<t} \lim \mathfrak{U}_s \\ &= \sqcap_{s<t} \sqcap_{r<s} (\widetilde{\lim(1_X, X)} - \lim \mathfrak{F}_r) \\ &\sqsubseteq \sqcap_{s<t} \sqcap_{r<s} (\widetilde{\lim(1_X, X)} - \lim \mathfrak{H}_r) \\ &= \sqcap_{s<t} \sqcap_{r<s} h^{-1}(\widetilde{\lim(1_X, X)} - L_r) \\ &= \sqcap_{s<t} h^{-1}(L'_s) \\ &= h^{-1}(L'_t) \end{aligned}$$

Similarly, we obtain,

$$\begin{aligned} g^{-1}R_t &= \sqcup_{s>t} g^{-1}R_s \\ &= \sqcup_{s>t} \sqcup_{r>s} g^{-1}R_r \\ &= \sqcup_{s>t} \sqcup_{r>s} (\widetilde{\lim(1_X, X)} - \lim \mathfrak{G}_r) \\ &\sqsubseteq \sqcup_{s>t} \sqcap_{r<s} (\widetilde{\lim(1_X, X)} - \lim \mathfrak{F}_r) \\ &= \sqcup_{s>t} \lim \mathfrak{U}_s \\ &= f^{-1}R_t \end{aligned}$$

Now,

$$\begin{aligned}
 f^{-1}R_t &= \sqcup_{s>t} \lim \mathfrak{A}_s \\
 &= \sqcup_{s>t} \sqcap_{r<s} (\widetilde{\lim(1_X, X)} - \lim \mathfrak{F}_r) \\
 &\sqsubseteq \sqcup_{s>t} \sqcup_{r>s} (\widetilde{\lim(1_X, X)} - \lim \mathfrak{H}_r) \\
 &= \sqcup_{s>t} \sqcup_{r>s} h^{-1}(L'_r) \\
 &= \sqcup_{s>t} h^{-1}R_s \\
 &= h^{-1}R_t
 \end{aligned}$$

Thus, (b) is proved.

(b)  $\Rightarrow$  (c): Suppose that  $\lim \mathfrak{F}$  is soft  $\tilde{L}$ -fuzzy C- $\lim$ -closed set and  $\lim \mathfrak{G}$  is soft  $\tilde{L}$ -fuzzy C- $\lim$ -open set such that  $\lim \mathfrak{G} \sqsubseteq \lim \mathfrak{F}$ . Then, by the Proposition: 5.7,  $\chi_{\lim \mathfrak{G}} \sqsubseteq \chi_{\lim \mathfrak{F}}$ , where  $\chi_{\lim \mathfrak{G}}$  and  $\chi_{\lim \mathfrak{F}}$  are the lower and upper soft  $\tilde{L}$ -fuzzy C- $\lim$ -convergence continuous functions respectively. Hence, by (b), there exists a soft  $\tilde{L}$ -fuzzy C- $\lim$ -convergence continuous function,  $f : X \rightarrow \mathbb{R}(L \times L)$  such that  $\chi_{\lim \mathfrak{G}} \sqsubseteq f \sqsubseteq \chi_{\lim \mathfrak{F}}$ . Clearly,  $f(x) \in \tilde{L}^{\mathbb{R}}$ , for all  $x \in \mathbb{R}$  and

$$\lim \mathfrak{G} = L'_1 \chi_{\lim \mathfrak{G}} \sqsubseteq L'_1 f \sqsubseteq R_0 f \sqsubseteq R_0 \chi_{\lim \mathfrak{F}} = \lim \mathfrak{F}.$$

Therefore,  $\lim \mathfrak{G} \sqsubseteq L'_1 f \sqsubseteq R_0 f \sqsubseteq \lim \mathfrak{F}$ .

(c)  $\Rightarrow$  (a): Let  $\lim \mathfrak{F}$  be a soft  $\tilde{L}$ -fuzzy C- $\lim$ -closed set and  $\lim \mathfrak{G}$  be a soft  $\tilde{L}$ -fuzzy C- $\lim$ -open set such that  $\lim \mathfrak{G} \sqsubseteq \lim \mathfrak{F}$ . Then, by the hypothesis there exists a soft  $\tilde{L}$ -fuzzy C- $\lim$ -convergence continuous function,  $f : X \rightarrow [0, 1](L \times L)$  such that  $L'_1 f \sqsubseteq R_0 f$ . In fact that,  $L'_1$  is a soft  $\tilde{L}$ -fuzzy  $\lim$ -closed set and  $R_0$  is a soft  $\tilde{L}$ -fuzzy  $\lim$ -open set. Since  $\lim \mathfrak{G} \sqsubseteq L'_1 f \sqsubseteq R_0 f \sqsubseteq \lim \mathfrak{F}$ , it follows that,  $S\tilde{L}FC-cl(\lim \mathfrak{G}) \sqsubseteq S\tilde{L}FC-cl((L'_1)f) = (L'_1)f$ . Similarly,  $R_0 f = S\tilde{L}FC-int(R_0 f) \sqsubseteq S\tilde{L}FC-int(\lim \mathfrak{F})$ . This implies that,  $S\tilde{L}FC-cl(\lim \mathfrak{G}) \sqsubseteq S\tilde{L}FC-int(\lim \mathfrak{F})$ . By the Proposition: 4.3,  $(X, \tau_{\lim})$  is a soft  $\tilde{L}$ -fuzzy C- $\lim$ -convergence extremally disconnected space.  $\square$

## 6. TIETZE EXTENSION THEOREM ON SOFT $\tilde{L}$ -FUZZY C- $\lim$ -CONVERGENCE EXTREMALLY DISCONNECTED SPACE

**Definition 6.1.** Let  $X$  be any non-empty crisp set. Let  $A$  be any subset of  $X$  and  $\chi_A^* : X \rightarrow \{(1_X, 1_X), (0_X, 0_X)\}$ . Then, the *characteristic\* function* of  $A$ ,  $\chi_A^*$  is defined as

$$\chi_A^*(x) = \begin{cases} (1_X, 1_X), & \text{if } x \in A \\ (0_X, 0_X), & \text{if } x \notin A \end{cases}$$

for all  $x \in X$ .

**Proposition 6.2.** Let  $(X, \tau_{\lim})$  be a soft  $\tilde{L}$ -fuzzy C- $\lim$ -convergence extremally disconnected space. Let  $(X, \tau_{\lim})$  possess the property  $\nabla$ . Let  $A \subseteq X$  such that  $\chi_A^*$  is a soft  $\tilde{L}$ -fuzzy C- $\lim$ -open set in  $(X, \tau_{\lim})$ . Let  $f : (A, \tau_{\lim/A}) \rightarrow [0, 1](L \times L)$  be a soft  $\tilde{L}$ -fuzzy C- $\lim$ -convergence continuous function. Then,  $f$  has a soft  $\tilde{L}$ -fuzzy C- $\lim$ -convergence continuous extension over  $(X, \tau_{\lim})$ .

*Proof.* Let  $g, h : X \rightarrow [0, 1](L \times L)$  be such that  $g = f = h$  on  $A$  and  $g(x) = (0_X, 0_X)$ , if  $x \notin A$  and  $h(x) = (1_X, 1_X)$ , if  $x \notin A$ . We now have,

$$R_t g = \begin{cases} \lim \mathfrak{G}_t \sqcap \chi_A^*, & \text{if } t \geq 0 \\ (1_X, 1_X), & \text{if } t < 0 \end{cases}$$

for all  $t \in \mathbb{R}$ , where  $\lim \mathfrak{G}_t$  is a soft  $\tilde{L}$ -fuzzy  $C$ -lim-open set such that  $\lim \mathfrak{G}_t / A = R_t g$  and

$$L_t h = \begin{cases} \lim \mathfrak{H}_t \sqcap \chi_A^*, & \text{if } t \leq 1 \\ (0_X, 0_X), & \text{if } t > 1 \end{cases}$$

for all  $t \in \mathbb{R}$ , where  $\lim \mathfrak{H}_t$  is a soft  $\tilde{L}$ -fuzzy  $C$ -lim-closed set such that  $\lim \mathfrak{H}_t / A = L_t h$ . Thus,  $g$  is a lower soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence continuous function and  $h$  is an upper soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence continuous function with  $g \sqsubseteq h$ . Now, by the Proposition: 5.8, there exists a soft  $\tilde{L}$ -fuzzy  $C$ -lim-convergence continuous function,  $\mathcal{F} : X \rightarrow [0, 1](L \times L)$  such that  $g \leq \mathcal{F} \leq h$ . Hence,  $\mathcal{F} \equiv f$  on  $A$ .  $\square$

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