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Interval valued intuitionistic fuzzy soft topological spaces

Anjan Mukherjee, Ajoy Kanti Das, Abhijit Saha

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Abstract. In this paper the concept of interval valued intuitionistic fuzzy soft topological space(IVIFS topological space) together with interval valued intuitionistic fuzzy soft open sets(IVIFS open sets) and interval valued intuitionistic fuzzy soft closed sets(IVIFS closed sets) are introduced. We define neighbourhood of an IVIFS set, interior IVIFS set, interior of an IVIFS set, exterior IVIFS set, exterior of an IVIFS set, closure of an IVIFS set, IVIFS basis, IVIFS subspace. Some examples and theorems regarding these concepts are presented.

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Corresponding Author: Anjan Mukherjee (anjan2002_m@yahoo.co.in)

1. Introduction

The vagueness or the representation of imperfect knowledge has been a problem for a long time for the mathematicians. There are many mathematical tools for dealing with uncertainties; some of them are fuzzy set theory [14] and soft set theory [9]. Maji et al. [6] defined several operations on soft set theory. Based on the analysis of several operations on soft sets introduced in [6], Ali et al. [1] presented some new algebraic operations for soft sets and proved that certain De Morgan's law holds in soft set theory with respect to these new definitions. Combining soft sets [9] with fuzzy sets [14] and intuitionistic fuzzy sets[2], Maji et al. [7, 8] defined fuzzy soft sets and intuitionistic fuzzy soft sets which are rich potential for solving decision making problems. As a generalization of fuzzy soft set theory, intuitionistic fuzzy soft set theory makes description of the objective more realistic, more practical and accurate in some cases, making it more promising. In 2011 Shabir and Naz [11] defined soft topology by using soft sets and presented basic properties in their paper. Tanay and Kandemir [13] defined fuzzy soft topology on a fuzzy soft set over an initial universe. Later on Simseklar and Yuksel [12] introduced fuzzy soft topology over a fuzzy soft set with a fixed parameter set. Li and Cui [5] defined the topological structure of intuitionistic fuzzy soft sets taking the whole parameter set E. In this paper we introduce the concepts of interval valued intuitionistic fuzzy soft topological space (IVIFS topological space) together with interval valued intuitionistic fuzzy soft open sets (IVIFS open sets) and interval valued intuitionistic fuzzy soft closed sets(IVIFS closed sets) .Then we define neighbourhood of an IVIFS set, interior IVIFS set, interior of an IVIFS set, exterior IVIFS set, exterior of an IVIFS set, closure of an IVIFS set, IVIFS basis, IVIFS subspace. Some examples and theorems regarding these concepts are presented.

2. Preliminaries

Definition 2.1 ([14])**.** Let X be a non empty set. Then a fuzzy set(FS for short) A is a set having the form A={(x, μ_A(x)): x $\in$ X}, where the function μ_A:X$\rightarrow$[0,1] is called the membership function and μ_A(x) is called the degree of membership of each element x$\in$ X.

Definition 2.2 ([9])**.** Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A$\subseteq$ E. Then the pair (F, A) is called a soft set over U, where F is a mapping given by F: A$\rightarrow$ P(U).

Definition 2.3 ([2])**.** Let X be a non empty set. Then an intuitionistic fuzzy set(IFS for short) A is a set having the form A={$\langle$x, μ_A(x), γ_A(x)$\rangle$: x $\in$ X} where the functions μ_A:X$\rightarrow$[0,1] and γ_A:X$\rightarrow$[0,1] represents the degree of membership and the degree of non-membership respectively of each element x$\in$U and 0$\leq\mu_A$(x)+ γ_A(x)$\leq$1 for each x$\in$X.

Definition 2.4 ([7])**.** Let U be an initial universe and E be a set of parameters. Let F(U) be the set of all fuzzy subsets of U and A$\subseteq$E. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A $\rightarrow$ F(U).

Definition 2.5 ([8])**.** Let U be an initial universe and E be a set of parameters. Let IF(U) be the set of all intuitionistic fuzzy subsets of U and A$\subseteq$ E. Then the pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by F: A $\rightarrow$ IF(U).

Definition 2.6 ([3])**.** An interval valued intuitionistic fuzzy set A over a universe set U is defined as the object of the form A={<x, μ_A(x), γ_A(x)>: x$\in$U}, where μ_A:U$\rightarrow$ Int([0,1]) and γ_A:U$\rightarrow$ Int([0, 1]) are functions such that the condition: $\forall$x$\in$U, supμ_A(x)+ supγ_A(x)$\leq$1 is satisfied.

The class of all interval valued intuitionistic fuzzy sets on U is denoted by IV-IFS(U).

Definition 2.7 ([4])**.** Let U be an initial universe and E be a set of parameters. Let IVIFS(U) be the set of all interval valued intuitionistic fuzzy sets on U and A $\subseteq$E. Then the pair (F, A) is called an interval valued intuitionistic fuzzy soft set (IVIFSS for short) over U, where F is a mapping given by F: A $\rightarrow$ IVIFS(U).

For any parameter $e \in A$, $F(e) = \{(x, \mu_{F(e)}(x), \gamma_{F(e)}(x)) : x \in U\}$, where $\mu_{F(e)}(x)$ is the interval valued fuzzy membership degree that object x holds on parameter e and $\gamma_{F(e)}(x)$ is the interval valued fuzzy membership degree that object x does not hold on parameter e.

Definition 2.8 ([4])**.** The union of two interval valued intuitionistic fuzzy soft sets (f, A) and (g, B) over (U, E) is an interval valued intuitionistic fuzzy soft set (h, C), where C= A∪B and ∀e∊C,

$$h(e) = \begin{cases} f(e), & if\ e \in A - B \\ g(e), & if\ e \in B - A \\ f(e)\tilde{\cup}g(e) & if\ e \in A \cap B \end{cases}$$

and is written as (f,A)∪(g,B)=(h,C) (here $\tilde{\cup}$ represents the interval valued intuitionistic fuzzy union)

Definition 2.9 ([4])**.** The intersection of two interval valued intuitionistic fuzzy soft sets (f, A) and (g, B) over (U, E) is an interval valued intuitionistic fuzzy soft set (h, C), where C= A∪B and ∀e∊C,

$$h(e) = \begin{cases} f(e), & if\ e \in A - B \\ g(e), & if\ e \in B - A \\ f(e)\tilde{\cap}g(e) & if\ e \in A \cap B \end{cases}$$

and is written as (f,A)∩(g,B)=(h,E) (here $\tilde{\cap}$ represents the interval valued intuitionistic fuzzy intersection).

Definition 2.10 ([4])**.** For two interval valued intuitionistic fuzzy soft sets (f, A) and (g, B) over (U, E). Then we say that (f, A) is an interval valued intuitionistic fuzzy soft subset of (g, B), if

(i) A⊆B

(ii) for all e∈A, f(e)⊆g(e).

Definition 2.11 ([13])**.** Let (f_A,E) be a fuzzy soft set on (U,E) and τ_f be the collection of fuzzy soft subsets of (f_A,E), then τ_f is said to be fuzzy soft topology on (f_A,E) if the following conditions hold:

[O_1]. ϕ, (f_A,E)∈τ_f (where ϕ has been defined in [10])

[O_2]. $\{(f_A^k,E)|\ k\in K\}\subseteq\tau_f \Rightarrow \bigcup_{k\in K}(f_A^k,E)\in\tau_f$

[O_3]. If (g_A,E), (h_A,E)∈τ_f, then (g_A,E)∩(h_A,E)∈τ_f,

The triplet ((f_A,E,τ_f) is called a fuzzy soft topology over (f_A,E).

Definition 2.12 ([5])**.** Let (f_E,E) be an intuitionistic fuzzy soft set on (U,E) and τ_f be the collection of all intuitionistic fuzzy soft subsets of (f_E,E), then is said to be an intuitionistic fuzzy soft topology on (f_E,E) if the following conditions hold:

[O_1]. ϕ_E, (f_E,E)∈τ_f (where ϕ_E has been defined in [10])

[O_2].$\{(f_E^k,E)|\ k\in K\}\subseteq\tau_f \Rightarrow \bigcup_{k\in K}(f_E^k,E)\in\tau_f$

[O_3]. If (g_E,E), (h_E,E)∈τ_f, then (g_E,E)∩(h_E,E)∈τ_f,

The triplet ((f_E,E,τ_f) is called an intuitionistic fuzzy soft topology over (f_E,E).

3. Interval valued intuitionistic fuzzy soft topological spaces

Let U be the initial universe, E be the set of parameters, P(U) be the set of all sub sets of U, IVIFset(U) be the set of all interval valued intuitionistic fuzzy sets in U and IVIFS(U; E) be the family of all interval valued intuitionistic fuzzy soft sets over U via parameters in E. Simseklar and Yuksel [[12]] introduced fuzzy soft topology over a fuzzy soft set with a fixed parameter set A⊆E. Li and Cui [[5]] defined the topological structured of intuitionistic fuzzy soft sets taking the whole parameter set E. In this section we introduce the concept of interval valued intuitionistic fuzzy soft topological spaces with a fixed parameter set A⊆E, which is the extension of fuzzy soft topological spaces introduced by Simseklar and Yuksel [[12]] as well as intuitionistic fuzzy soft topological spaces introduced by Li and Cui [[5]]

Definition 3.1. Let (ξ_A,E) be an element of IVIFS(U; E), $P(\xi_A,E)$ be the collection of all IVIFS subsets of (ξ_A,E). A sub family τ of $P(\xi_A,E)$ is called an interval valued intuitionistic fuzzy soft topology (in short IVIFS-topology) on (ξ_A,E) if the following axioms are satisfied:

$[O_1]$.(ϕ_{ξ_A},E), $(\xi_A,E)\in\tau$

$[O_2]$.$\{(f^k_A,E)|\ k\in K\}\subseteq\tau \Rightarrow \bigcup_{k\in K}(f^k_A,E)\in\tau$

$[O_3]$. If (f_A,E), $(g_A,E)\in\tau$, then $(f_A,E)\cap(g_A,E)\in\tau$,

Then the pair $((\xi_A,E),\tau)$ is called an interval valued intuitionistic fuzzy soft topological space (IVIFS-topological space for short). The members of τ are called τ-open IVIFS sets (or simply open sets). (where ϕ_{ξ_A} : A→IVIFS(U) is defined as $\phi_{\xi_A}(e)=\{\langle x,[0,0],[1,1]\rangle: x\in U\}$, ∀e∈A)

Example 3.2. Let $U=\{u^1, u^2, u^3\}$, $E=\{e_1, e_2, e_3, e_4\}$, $A=\{e_1, e_2, e_3\}$ and

$(\xi_A,E)=\{e_1=\{u^1_{([1,1],[0,0])}, u^2_{([0.7,0.8],[0,0])}, u^3_{([1,1],[0,0])}\}$,
$e_2=\{u^1_{([0.4,0.5],[0,0])}, u^2_{([1,1],[0,0])}, u^3_{([0.4,0.5],[0.2,0.3])}\}$,
$e_3=\{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0.4,0.5],[0.1,0.2])}\}\}$,

$(\phi_{\xi_A},E)=\{e_1=\{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0,0],[1,1])}\}$,
$e_2=\{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0,0],[1,1])}\}$,
$e_3=\{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0,0],[1,1])}\}\}$,

$(f^1_A,E)=\{e_1=\{u^1_{([0.5,0.6],[0.2,0.3])}, u^2_{([0.4,0.5],[0,0.1])}, u^3_{([1,1],[0,0])}\}$,
$e_2=\{u^1_{([0.4,0.5],[0.2,0.3])}, u^2_{([0.4,0.5],[0,0])}, u^3_{([0,0],[1,1])}\}$,
$e_3=\{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0,0],[1,1])}\}\}$,

$(f^2_A,E)=\{e_1=\{u^1_{([0.3,0.4],[0.1,0.2])}, u^2_{([0.6,0.7],[0.2,0.3])}, u^3_{([1,1],[0,0])}\}$,
$e_2=\{u^1_{([0.2,0.3],[0,0.1])}, u^2_{([1,1],[0,0])}, u^3_{([0,0],[1,1])}\}$,
$e_3=\{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0,0],[1,1])}\}\}$,

$(f^3_A,E)=(f^1_A,E)\bigcap(f^2_A,E)$
$=\{e_1=\{u^1_{([0.3,0.4],[0.2,0.3])}, u^2_{([0.4,0.5],[0.2,0.3])}, u^3_{([1,1],[0,0])}\}$,
$e_2=\{u^1_{([0.2,0.3],[0.2,0.3])}, u^2_{([0.4,0.5],[0,0])}, u^3_{([0,0],[1,1])}\}$,

$e_3=\{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0,0],[1,1])}\}\}$,

$(f^4_A,E)=(f^1_A,E)\bigcup(f^2_A,E)$
$=\{e_1=\{u^1_{([0.5,0.6],[0.1,0.2])}, u^2_{([0.6,0.7],[0,0.1])}, u^3_{([1,1],[0,0])}\}$,
$e_2=\{u^1_{([0.4,0.5],[0,0.1])}, u^2_{([1,1],[0,0])}, u^3_{([0,0],[1,1])}\}$,
$e_3=\{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0,0],[1,1])}\}\}$.

Then we observe that the sub family $\tau_1=\{(\phi_{\xi_A},E), (\xi_A,E), (f^1_A,E), (f^2_A,E), (f^3_A,E), (f^4_A,E)\}$ of $P(\xi_A,E)$ is a IVIFS-topology on (ξ_A,E) since it satisfies the necessary three axioms $[O_1]$, $[O_2]$ and $[O_3]$ and $((\xi_A,E),\tau_1)$ is an interval valued intuitionistic fuzzy soft topological space. But the sub family $\tau_2=\{(\phi_{\xi_A},E), (\xi_A,E), (f^1_A,E), (f^2_A,E)\}$ of $P(\xi_A,E)$ is not an IVIFS-topology on (ξ_A,E) since the union $(f^1_A,E)\bigcup(f^2_A,E)=(f^4_A,E)$ which does not belong to τ_2.

Definition 3.3. As every IVIFS-topology on (ξ_A,E) must contain the sets (ϕ_{ξ_A},E) and (ξ_A,E), so the family $\vartheta=\{(\phi_{\xi_A},E), (\xi_A,E)\}$, forms an IVIFS-topology on (ξ_A,E). This topology is called indiscrete IVIFS-topology and the pair $((\xi_A,E),\vartheta)$ is called an indiscrete interval valued intuitionistic fuzzy soft topological space (or simply indiscrete IVIFS-topological space).

Definition 3.4. Let D denote family of all IVIFS subsets of (ξ_A,E). Then we observe that D satisfies all the axioms for topology on (ξ_A,E). This topology is called discrete IVIFS-topology and the pair $((\xi_A,E),D)$ is called a discrete interval valued intuitionistic fuzzy soft topological space (or simply discrete IVIFS-topological space).

Theorem 3.5. *Let $\{\tau_i$: i∈I$\}$ be any collection of IVIFS-topology on (ξ_A,E). Then their intersection $\bigcap_{i\in I}\tau_i$ is also a topology on (ξ_A,E).*

O_1. . Since $(\phi_{\xi_A},E), (\xi_A,E)\in\tau_i$, for each i∈I, hence $(\phi_{\xi_A},E), (\xi_A,E)\in\bigcap_{i\in I}\tau_i$.

$[O_2]$. Let $\{(f^k_A,E)|\ k\in K\}$ be an arbitrary family of interval valued intuitionistic fuzzy soft sets where $(f^k_A,E)\in\bigcap_{i\in I}\tau_i$ for each k∈K. Then for each i∈I, $(f^k_A,E)\in\tau_i$ for k∈K and since for each i∈I, τ_i is an topology, therefore $\bigcup_{k\in K}(f^k_A,E)\in\tau_i$, for each i∈I. Hence $\bigcup_{k\in K}(f^k_A,E)\in\bigcap_{i\in I}\tau_i$.

$[O_3]$. Let (f_A,E) and $(g_A,E)\in\bigcap_{i\in I}\tau_i$, then (f_A,E) and $(g_A,E)\in\tau_i$, for each i∈I and since τ_i for each i∈I is an topology, therefore $(f_A,E)\bigcap(g_A,E)\in\tau_i$ for each i∈I. Hence $(f_A,E)\bigcap(g_A,E)\in\bigcap_{i\in I}\tau_i$.

Thus $\bigcap_{i\in I}\tau_i$ satisfies all the axioms of topology. Hence $\bigcap_{i\in I}\tau_i$ forms a topology. But union of topologies need not be a topology; we can show this with following example. □

Remark 3.6. The union of two IVIFS-topology may not be a IVIFS-topology. If we consider the example 3.2, then the sub families $\tau_3=\{(\phi_{\xi_A},E), (\xi_A,E), (f^1_A,E)\}$ and $\tau_4=\{(\phi_{\xi_A},E), (\xi_A,E), (f^2_A,E)\}$ are the topologies in (ξ_A,E). But their union $\tau_3\cup\tau_4=\{(\phi_{\xi_A},E), (\xi_A,E), (f^1_A,E), (f^2_A,E)\}=\tau_2$ which is not a topology on (ξ_A,E).

Definition 3.7. Let $((\xi_A,E),\tau)$ be an IVIFS-topological space over (ξ_A,E). An IVIFS subset (f_A,E) of (ξ_A,E) is called interval valued intuitionistic fuzzy soft closed(in short IVIFS closed) if its complement $(f_A,E)^c$ is a member of τ.

Example 3.8. Let us consider example: 3.2 , then the IVIFS closed sets in $((\xi_A,E),\tau_1)$ are:

$(\phi_{\xi_A},E)^c = (U;E) = \{e_1=\{u^1_{([1,1],[0,0])}, u^2_{([1,1],[0,0])}, u^3_{([1,1],[0,0])}\},$
$e_2=\{u^1_{([1,1],[0,0])}, u^2_{([1,1],[0,0])}, u^3_{([1,1],[0,0])}\},$
$e_3=\{u^1_{([1,1],[0,0])}, u^2_{([1,1],[0,0])}, u^3_{([1,1],[0,0])}\}\},$

$(\xi_A,E)^c=\{e_1=\{u^1_{([0,0],[1,1])}, u^2_{([0,0],[0.7,0.8])}, u^3_{([0,0],[1,1])}\},$
$e_2=\{u^1_{([0,0],[0.4,0.5])}, u^2_{([0,0],[1,1])}, u^3_{([0.2,0.3],[0.4,0.5])}\},$
$e_3=\{u^1_{([1,1],[0,0])}, u^2_{([1,1],[0,0])}, u^3_{([0.1,0.2],[0.4,0.5])}\}\},$

$(f^1_A,E)^c=\{e_1=\{u^1_{([0.2,0.3],[0.5,0.6])}, u^2_{([0,0.1],[0.4,0.5])}, u^3_{([0,0],[1,1])}\},$
$e_2=\{u^1_{([0.2,0.3],[0.4,0.5])}, u^2_{([0,0],[0.4,0.5])}, u^3_{([1,1],[0,0])}\},$
$e_3=\{u^1_{([1,1],[0,0])}, u^2_{([1,1],[0,0])}, u^3_{([1,1],[0,0])}\}\},$

$(f^2_A,E)^c=\{e_1=\{u^1_{([0.1,0.2],[0.3,0.4])}, u^2_{([0.2,0.3],[0.6,0.7])}, u^3_{([0,0],[1,1])}\},$
$e_2=\{u^1_{([0,0.1],[0.2,0.3])}, u^2_{([0,0],[1,1])}, u^3_{([1,1],[0,0])}\},$
$e_3=\{u^1_{([1,1],[0,0])}, u^2_{([1,1],[0,0])}, u^3_{([1,1],[0,0])}\}\},$

$(f^3_A,E)^c=\{e_1=\{u^1_{([0.2,0.3],[0.3,0.4])}, u^2_{([0.2,0.3],[0.4,0.5])}, u^3_{([0,0],[1,1])}\},$
$e_2=\{u^1_{([0.2,0.3],[0.2,0.3])}, u^2_{([0,0],[0.4,0.5])}, u^3_{([1,1],[0,0])}\},$
$e_3=\{u^1_{([1,1],[0,0])}, u^2_{([1,1],[0,0])}, u^3_{([1,1],[0,0])}\}\},$

$(f^4_A,E)^c=\{e_1=\{u^1_{([0.1,0.2],[0.5,0.6])}, u^2_{([0,0.1],[0.6,0.7])}, u^3_{([0,0],[1,1])}\},$
$e_2=\{u^1_{([0,0.1],[0.4,0.5])}, u^2_{([0,0],[1,1])}, u^3_{([1,1],[0,0])}\},$
$e_3=\{u^1_{([1,1],[0,0])}, u^2_{([1,1],[0,0])}, u^3_{([1,1],[0,0])}\}\}$
are the intuitionistic fuzzy soft closed sets in $((\xi_A,E),\tau_1)$.

Theorem 3.9. *Let $((\xi_A,E),\tau)$ be an interval valued intuitionistic fuzzy soft topological space over (ξ_A,E). Then,*

(1) $(\phi_{\xi_A},E)^c$ and $(\xi_A,E)^c$ are interval valued intuitionistic fuzzy soft closed sets,

(2) The arbitrary intersection of interval valued intuitionistic fuzzy soft closed sets is interval valued intuitionistic fuzzy soft closed,

(3) The union of two interval valued intuitionistic fuzzy soft closed sets is an interval valued intuitionistic fuzzy soft closed set.

Proof. (1) Since (ϕ_{ξ_A},E), $(\xi_A,E)\in\tau$ implies $(\phi_{\xi_A},E)^c$ and $(\xi_A,E)^c$ are closed.

(2) Let $\{(f^k_A,E)|\ k\in K\}$ be an arbitrary family of IVIFS closed sets in $((\xi_A,E),\tau)$ and let $(f_A,E)= \bigcap_{k\in K}(f^k_A,E)$. Now since $(f_A,E)^c= (\bigcap_{k\in K}(f^k_A,E))^c=\bigcup_{k\in K}(f^k_A,E)^c$ and $(f^k_A,E)^c\in\tau$, for each $k\in K$, so $\bigcup_{k\in K}(f^k_A,E)^c\in\tau$. Hence $(f_A,E)^c\in\tau$. Thus (f_A,E) is IVIFS closed set.

(3) Let $\{(f^i_A,E)|\ i=1,2,3,....,n\}$ be a finite family of IVIFS closed sets in $((\xi_A,E),\tau)$ and let $(g_A,E)= \bigcup_{i=1}^n(f^i_A,E)$. Now since $(g_A,E)^c= (\bigcup_{i=1}^n(f^i_A,E))^c = \bigcap_{i=1}^n(f^i_A,E)^c$ and $(f^i_A,E)^c\in\tau$. So $\bigcap_{i=1}^n(f^i_A,E)^c\in\tau$. Hence $(g_A,E)^c\in\tau$. Thus (g_A,E) is an IVIFS-closed set. □

Remark 3.10. The intersection of an arbitrary family of IVIFS-open set may not be an IVIFS-open and the union of an arbitrary family of IVIFS-closed set may not be an IVIFS-closed. Let us consider U={u^1, u^2, u^3}, E={e_1, e_2, e_3 ,e_4}, A={e_1, e_2, e_3} and let

(ξ_A,E)={e_1={$u^1_{([1,1],[0,0])}$, $u^2_{([1,1],[0,0])}$, $u^3_{([0,0],[1,1])}$},
e_2={$u^1_{([1,1],[0,0])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$},
e_3={$u^1_{([0,0],[1,1])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$}},

(ϕ_{ξ_A},E)={e_1={$u^1_{([0,0],[1,1])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$},
e_2={$u^1_{([0,0],[1,1])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$},
e_3={$u^1_{([0,0],[1,1])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$}}.

For each n∈N, we define
(f^n_A,E)={e_1={$u^1_{([\frac{1}{4n},\frac{1}{2n}],[\frac{1}{5}-\frac{1}{2n},\frac{1}{2}-\frac{1}{3n}])}$, $u^2_{([1,1],[0,0])}$, $u^3_{([0,0],[1,1])}$},
e_2={$u^1_{([\frac{1}{3n},\frac{1}{2n}],[\frac{1}{3}-\frac{1}{3n},\frac{1}{3}-\frac{1}{4n}])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$},
e_3={$u^1_{([0,0],[1,1])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$}}.

Let us consider the sub family τ of P(ξ_A,E), such that (ϕ_{ξ_A},E), (ξ_A,E)∈τ and (f^n_A,E)∈τ, (for n=1,2,3,.....).

Then we observe that τ is a IVIFS-topology on (ξ_A,E).

But $\bigcap_{n=1}^{\infty}$(f^n_A,E)= {e_1={$u^1_{([0,0],[0.2,0.5])}$, $u^2_{([1,1],[0,0])}$, $u^3_{([0,0],[1,1])}$},
e_2={$u^1_{([0,0],[0.33,0.5])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$},
e_3={$u^1_{([0,0],[1,1])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$}}$\notin\tau$.

The IVIFS-closed sets in the IVIFS-topological space ((ξ_A,E),τ)
are: $(\phi_{\xi_A},E)^c$, $(\xi_A,E)^c$, $(f^n_A,E)^c$, (for n=1,2,3,........).

But $\bigcup_{n=1}^{\infty}(f^n_A,E)^c$ = {e_1={$u^1_{([0.2,0.5],[0,0])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([1,1],[0,0])}$},
e_2={$u^1_{([0.33,0.5],[0,0])}$, $u^2_{([1,1],[0,0])}$, $u^3_{([1,1],[0,0])}$},
e_3={$u^1_{([1,1],[0,0])}$, $u^2_{([1,1],[0,0])}$, $u^3_{([1,1],[0,0])}$}} is not an IVIFS-closed set in IVIFS-topological space ((ξ_A,E),τ), since $(\bigcup_{n=1}^{\infty}(f^n_A,E)^c)^c\notin\tau$.

Definition 3.11. Let ((ξ_A,E),τ_1) and ((ξ_A,E),τ_2) be two IVIFS-topological spaces. If each (f_A,E)∈τ_1 ⇒ (f_A,E)∈τ_2, then τ_2 is called interval valued intuitionistic fuzzy soft finer topology than τ_1 and τ_1 is called interval valued intuitionistic fuzzy soft coarser topology than τ_2.

Example 3.12. If we consider the topologies τ_1={(ϕ_{ξ_A},E), (ξ_A,E), (f^1_A,E), (f^2_A,E), (f^3_A,E), (f^4_A,E)} as in the example: 3.2 and τ_5={(ϕ_{ξ_A},E), (ξ_A,E), (f^3_A,E)} on (ξ_A,E). Then τ_1 is interval valued intuitionistic fuzzy soft finer topology than τ_5 and τ_5 is interval valued intuitionistic fuzzy soft coarser topology than τ_1.

Definition 3.13. Let ((ξ_A,E),τ) be an IVIFS-topological space on (ξ_A,E) and B be a subfamily of τ. If every element of τ can be express as the arbitrary interval valued intuitionistic fuzzy soft union of some element of B, then B is called an interval valued intuitionistic fuzzy soft basis for the interval valued intuitionistic fuzzy soft topology τ.

Example 3.14. In the example: 3.2 for the topology τ_1={(ϕ_{ξ_A},E), (ξ_A,E), (f^1_A,E), (f^2_A,E), (f^3_A,E), (f^4_A,E)} the sub family B={(ϕ_{ξ_A},E), (ξ_A,E), (f^1_A,E), (f^2_A,E), (f^3_A,E)} of P(ξ_A,E) is a basis for the topology τ_1.

4. Neighbourhoods and neighbourhood systems

In this section we introduce neighbourhood of a IVIFS-set and the neighbourhood systems for future discussion.

Definition 4.1. Let τ be the IVIFS-topology on (ξ_A,E)∈IVIFS(U;E) and (f_A,E) be an IVIFS-set in P(ξ_A,E). A IVIFS-set (f_A,E) in P(ξ_A,E) is a neighbourhood of a IVIFS-set (g_A,E) if and only if there exists an τ-open IVIFS-set (h_A,E) i.e. (h_A,E)∈τ such that (g_A,E)⊆(h_A,E)⊆(f_A,E).

Example 4.2. In an IVIFS-topology τ={(ϕ_{ξ_A},E), (ξ_A,E), {e_1={$u^1_{([0.4,0.5],[0.4,0.5])}$, $u^2_{([0.3,0.4],[0.5,0.6])}$, $u^3_{([0.4,0.5],[0.1,0.2])}$}},
where (ξ_A,E)={e_1={$u^1_{([1,1],[0,0])}$, $u^2_{([0.7,0.8],[0,0])}$, $u^3_{([1,1],[0,0])}$},
e_2={$u^1_{([0.4,0.5],[0,0])}$, $u^2_{([1,1],[0,0])}$, $u^3_{([0.4,0.5],[0.2,0.3])}$},
e_3={$u^1_{([0,0],[1,1])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0.4,0.5],[0.1,0.2])}$}},
and (ϕ_{ξ_A},E)={e_1={$u^1_{([0,0],[1,1])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$},
e_2={$u^1_{([0,0],[1,1])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$},
e_3={$u^1_{([0,0],[1,1])}$, $u^2_{([0,0],[1,1])}$, $u^3_{([0,0],[1,1])}$}},
the IVIFS-set
(f_A,E)= { {e_1={$u^1_{([0.5,0.6],[0.2,0.3])}$, $u^2_{([0.3,0.4],[0.5,0.6])}$, $u^3_{([0.4,0.5],[0,0.1])}$}}
is a neighbourhood of the IVIFS-set
(g_A,E)= { {e_1={$u^1_{([0.3,0.4],[0.4,0.5])}$, $u^2_{([0.1,0.2],[0.6,0.7])}$, $u^3_{([0.4,0.5],[0.3,0.4])}$}},
because there exists an τ-open IVIFS-set
(h_A,E)={e_1={$u^1_{([0.4,0.5],[0.4,0.5])}$, $u^2_{([0.3,0.4],[0.5,0.6])}$, $u^3_{([0.4,0.5],[0.1,0.2])}$}}∈ τ
such that (g_A,E)⊆(h_A,E)⊆(f_A,E).

Theorem 4.3. *A IVIFS-set (f_A,E) in P(ξ_A,E) is an open IVIFS-set if and only if (f_A,E) is a neighbourhood of each IVIFS-set (g_A,E) contained in (f_A,E).*

Proof. Let (f_A,E) be an open IVIFS-set and (g_A,E) be any IVIFS-set contained in (f_A,E). Since we have (g_A,E)⊆(f_A,E)⊆(f_A,E), it follows that (f_A,E) is a neighbourhood of (g_A,E).

Conversely let (f_A,E) be a neighbourhood for every IVIFS-sets contained it. Since (f_A,E)⊆(f_A,E) there exist an open IVIFS-set (h_A,E) such that (f_A,E)⊆(h_A,E)⊆(f_A,E). Hence (f_A,E) = (h_A,E) and (f_A,E) is open. □

Definition 4.4. Let ((ξ_A,E),τ) be an interval valued intuitionistic fuzzy soft topological space on (ξ_A,E) and (f_A,E) be a IVIFS-set in P(ξ_A,E). The family of all neighbourhoods of (f_A,E) is called the neighbourhood system of (f_A,E) up to topology and is denoted by $N_{(f_A,E)}$.

Theorem 4.5. *Let ((ξ_A,E),τ) be an interval valued intuitionistic fuzzy soft topological space. If $N_{(f_A,E)}$ is the neighbourhood system of an interval valued intuitionistic fuzzy soft set (f_A,E). Then,* *1. $N_{(f_A,E)}$ is non empty and (f_A,E) belong to the each member of $N_{(f_A,E)}$.*

2. The intersection of any two members of $N_{(f_A,E)}$ belong to $N_{(f_A,E)}$.

3. Each interval valued intuitionistic fuzzy soft set which contains a member of $N_{(f_A,E)}$ belong to $N_{(f_A,E)}$.

Proof. 1. If $(h_A,E) \in N_{(f_A,E)}$, then there exist an IVIFS open set $(g_A,E) \in \tau$ such that $(f_A,E) \subseteq (g_A,E) \subseteq (h_A,E)$; hence $(f_A,E) \subseteq (h_A,E)$. Note $(\xi_A,E) \in N_{(f_A,E)}$ and since (ξ_A,E) is an open set containing (f_A,E); so $N_{(f_A,E)}$ is non empty.

2. Let (g_A,E) and (h_A,E) are two neighbourhoods of (f_A,E), so there exist two open sets (g^*_A,E), (h^*_A,E), such that

$(f_A,E) \subseteq (g^*_A,E) \subseteq (g_A,E)$ and $(f_A,E) \subseteq (h^*_A,E) \subseteq (h_A,E)$.

Hence $(f_A,E) \subseteq (g^*_A,E) \bigcap (h^*_A,E) \subseteq (g_A,E) \bigcap (h_A,E)$ and $(g^*_A,E) \bigcap (h^*_A,E)$ is open. Thus $(g_A,E) \bigcap (h_A,E)$ is a neighbourhoods of (f_A,E).

3. Let (g_A,E) is a neighbourhood of (f_A,E) and $(g_A,E) \subseteq (h_A,E)$, so there exist an open set (g^*_A,E), such that $(f_A,E) \subseteq (g^*_A,E) \subseteq (g_A,E)$. By hypothesis $(g_A,E) \subseteq (h_A,E)$, so $(f_A,E) \subseteq (g^*_A,E) \subseteq (g_A,E) \subseteq (h_A,E)$, which implies that $(f_A,E) \subseteq (g^*_A,E) \subseteq (h_A,E)$ and hence (h_A,E) is a neighbourhood of (f_A,E). □

5. Interior, exterior and closure

In this section we introduce interior IVIFS set, interior of an IVIFS set, exterior IVIFS set, exterior of an IVIFS set.

Definition 5.1. Let $((\xi_A,E),\tau)$ be an interval valued intuitionistic fuzzy soft topological space on (ξ_A,E) and (f_A,E), (g_A,E) be IVIFS-sets in $P(\xi_A,E)$ such that $(g_A,E) \subseteq (f_A,E)$. Then (g_A,E) is called an interior IVIFS-set of (f_A,E) if and only if (f_A,E) is a neighbourhood of (g_A,E).

Definition 5.2. Let $((\xi_A,E),\tau)$ be an interval valued intuitionistic fuzzy soft topological space on (ξ_A,E) and (f_A,E) be an IVIFS-set in $P(\xi_A,E)$. Then the union of all interior IVIFS-set of (f_A,E) is called the interior of (f_A,E) and is denoted by $int(f_A,E)$ and defined by $int(f_A,E) = \bigcup\{(g_A,E) |\ (f_A,E)$ is a neighbourhood of $(g_A,E)\}$.

Or equivalently $int(f_A,E) = \bigcup\{(g_A,E) |\ (g_A,E)$ is an IVIFS-open set contained in $(f_A,E)\}$.

Example 5.3. : Let us consider the IVIFS-topology $\tau_1 = \{(\phi_{\xi_A},E), (\xi_A,E), (f^1_A,E), (f^2_A,E), (f^3_A,E), (f^4_A,E)\}$ as in the example: 3.2 and let

$(f_A,E) = \{e_1 = \{u^1_{([0.1,0.5],[0.1,0.2])}, u^2_{([0.6,0.7],[0.2,0.3])}, u^3_{([1,1],[0,0])}\}$,
$e_2 = \{u^1_{([0.3,0.4],[0,0.1])}, u^2_{([1,1],[0,0])}, u^3_{([0,0],[1,1])}\}$,
$e_3 = \{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0,0],[1,1])}\}\}$ be an IVIFS-set,
then
$int(f_A,E) = \bigcup\{(g_A,E) |\ (g_A,E)$ is an IVIFS-open set contained in $(f_A,E)\}$
$= (f^2_A,E) \bigcup (f^3_A,E)$
$= (f^2_A,E)$
$= \{e_1 = \{u^1_{([0.3,0.4],[0.1,0.2])}, u^2_{([0.6,0.7],[0.2,0.3])}, u^3_{([1,1],[0,0])}\}$,
$e_2 = \{u^1_{([0.2,0.3],[0,0.1])}, u^2_{([1,1],[0,0])}, u^3_{([0,0],[1,1])}\}$,
$e_3 = \{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0,0],[1,1])}\}\}$.
Since $(f^2_A,E) \subseteq (f_A,E)$ and $(f^3_A,E) \subseteq (f_A,E)$.

Theorem 5.4. *Let $((\xi_A,E),\tau)$ be an interval valued intuitionistic fuzzy soft topological space on (ξ_A,E) and (f_A,E) be an IVIFS-set in $P(\xi_A,E)$. Then*

1. $int(f_A,E)$ is an open and $int(f_A,E)$ is the largest open IVIFS set contained in (f_A,E).

2. The IVIFS set (f_A,E) is open if and only if $(f_A,E)=int(f_A,E)$.

Proof. 1. Since $\text{int}(f_A,E)=\bigcup\{(g_A,E)|\ (f_A,E)$ is a neighbourhood of $(g_A,E)\}$., we have that (f_A,E) is itself an interior IVIFS-set of (f_A,E). Then there exists an open IVIFS set (h_A,E) such that $\text{int}(f_A,E)\subseteq(h_A,E)\subseteq(f_A,E)$. But (h_A,E) is an interior IVIFS-set of (f_A,E), hence $(h_A,E)\subseteq\text{int}(f_A,E)$. Hence $(h_A,E)=\text{int}(f_A,E)$. Thus $\text{int}(f_A,E)$ is open and $\text{int}(f_A,E)$ is the largest open IVIFS set contained in (f_A,E).

2. Let (f_A,E) be an open IVIFS set. Since $\text{int}(f_A,E)$ is an interior IVIFS-set of (f_A,E) we have $(f_A,E)= \text{int}(f_A,E)$. Conversely, if $(f_A,E)= \text{int}(f_A,E)$ then (f_A,E) is obviously open. □

Proposition 5.5. *for any two IVIFS-sets (f_A,E) and (g_A,E) in an interval valued intuitionistic fuzzy soft topological space $((\xi_A,E),\tau)$ on $P(\xi_A,E)$, then*

(i)$(g_A,E) \subseteq (f_A,E) \Rightarrow int(g_A,E) \subseteq int(f_A,E)$

(ii)$int(\phi_{\xi_A},E)=(\phi_{\xi_A},E)$ and $int(\xi_A,E)= (\xi_A,E)$

(iii)$int(int(f_A,E))=int(f_A,E)$

(iv)$int((g_A,E)\cap(f_A,E)) =int(g_A,E)\cap int(f_A,E)$

(v)$int((g_A,E)\cup(f_A,E)) \supseteq int(g_A,E)\cup int(f_A,E)$

Proof. (i) Since $(g_A,E) \subseteq(f_A,E)$, implies all the IVIFS-open set contained in (g_A,E) also contained in (f_A,E). Therefore

$\{(g^*_A,E)|\ (g^*_A,E)$ is an IVIFS-open set contained in $(g_A,E)\} \subseteq\{(f^*_A,E)|\ (f^*_A,E)$ is an IVIFS-open set contained in $(f_A,E)\}$ This implies

$\bigcup\{(g^*_A,E)|\ (g^*_A,E)$ is an IVIFS-open set contained in $(g_A,E)\} \subseteq\bigcup\{(f^*_A,E)|\ (f^*_A,E)$ is an IVIFS-open set contained in $(f_A,E)\}$

So $\text{int}(g_A,E) \subseteq\text{int}(f_A,E)$.

(ii) Straight forward.

(iii) $\text{int}(\text{int}(f_A,E))= \bigcup\{(g_A,E)|\ (g_A,E)$ is an IVIFS-open set contained in $\text{int}(f_A,E)\}$ and since $\text{int}(f_A,E)$ is the largest open IVIFS set contained in $\text{int}(f_A,E)$, therefore $\text{int}(\text{int}(f_A,E))=\text{int}(f_A,E)$.

(iv) Since $\text{int}(g_A,E)\subseteq(g_A,E)$ and $\text{int}(f_A,E)\subseteq(f_A,E)$, we have

$\text{int}(g_A,E)\cap\text{int}(f_A,E)\subseteq(g_A,E)\cap(f_A,E)$.

hence $\text{int}(g_A,E)\cap\text{int}(f_A,E)\subseteq\text{int}((g_A,E)\cap(f_A,E))$.............(1)

Again since$(g_A,E)\cap(f_A,E)\subseteq(g_A,E)$ and $(g_A,E)\cap(f_A,E)\subseteq(f_A,E)$,

we have $\text{int}((g_A,E)\cap(f_A,E))\subseteq\text{int}(g_A,E)$ and $\text{int}((g_A,E)\cap(f_A,E))\subseteq\text{int}(f_A,E)$.

So $\text{int}((g_A,E)\cap(f_A,E))\subseteq\text{int}(g_A,E)\cap\text{int}(f_A,E)$............(2)

Using (1) and (2) we get,

$\text{int}((g_A,E)\cap(f_A,E))=\text{int}(g_A,E)\cap\text{int}(f_A,E)$.

(v) Since $(g_A,E) \subseteq (g_A,E)\cup(f_A,E)$ and $(f_A,E) \subseteq (g_A,E)\cup(f_A,E)$,

so $\text{int}(g_A,E) \subseteq \text{int}((g_A,E)\cup(f_A,E))$ and $\text{int}(f_A,E) \subseteq \text{int}((g_A,E)\cup(f_A,E))$.

Hence $\text{int}(g_A,E)\cup\text{int}(f_A,E) \subseteq \text{int}((g_A,E)\cup(f_A,E))$. □

Definition 5.6. Let $((\xi_A,E),\tau)$ be an interval valued intuitionistic fuzzy soft topological space on (ξ_A,E) and let (f_A,E), (g_A,E) be two IVIFS-sets in $P(\xi_A,E)$. Then

(g_A,E) is called an exterior IVIFS-set of (f_A,E) if and only if (g_A,E) is an interior IVIFS-set of the complement of (f_A,E).

Definition 5.7. Let $((\xi_A,E),\tau)$ be an interval valued intuitionistic fuzzy soft topological space on (ξ_A,E) and (f_A,E) be an IVIFS-set in P(ξ_A,E). Then the union of all exterior IVIFS-set of (f_A,E) is called the exterior of (f_A,E) and is denoted by ext(f_A,E) and is defined by ext(f_A,E)= $\bigcup\{(g_A,E)|(f_A,E)^c$ is a neighbourhood of $(g_A,E)\}$.

Clearly from definition ext(f_A,E)=int($(f_A,E)^c$)

Proposition 5.8. *for any two IVIFS-sets (f_A,E) and (g_A,E) in an interval valued intuitionistic fuzzy soft topological space $((\xi_A,E),\tau)$ on P(ξ_A,E), then*

(i) ext(f_A,E) is open and is the largest open set contained in $(f_A,E)^c$.

(ii) $(f_A,E)^c$ is open if and only if $(f_A,E)^c$=ext(f_A,E).

(iii) $(g_A,E)\subseteq(f_A,E)\Rightarrow ext(f_A,E)\subseteq ext(g_A,E)$

(iv) $ext((g_A,E)\cap(f_A,E))\supseteq ext(g_A,E)\cup ext(f_A,E)$

(v) $ext((g_A,E)\cup(f_A,E))= ext(g_A,E)\cap ext(f_A,E)$

Proof. Straight forward. □

Definition 5.9. Let $((\xi_A,E),\tau)$ be an interval valued intuitionistic fuzzy soft topological space on (ξ_A,E) and (f_A,E) be an IVIFS-set in P(ξ_A,E). Then the intersection of all closed IVIFS-set containing (f_A,E) is called the closure of (f_A,E) and is denoted by cl(f_A,E) and defined by cl(f_A,E)=$\bigcap\{(g_A,E)|(g_A,E)$ is a IVIFS-closed set containing $(f_A,E)\}$.

Observe first that cl(f_A,E) is an IVIFS-closed set, since it is the intersection of IVIFS-closed sets. Furthermore, cl(f_A,E) is the smallest IVIFS-closed set containing (f_A,E).

Example 5.10. Let us consider an interval valued intuitionistic fuzzy soft topology $\tau_1=\{(\phi_{\xi_A},E), (\xi_A,E), (f^1_A,E), (f^2_A,E), (f^3_A,E), (f^4_A,E)\}$ as in the example: 3.2 and let

$(f_A,E)=\{e_1=\{u^1_{([0.2,0.3],[0.6,0.7])}, u^2_{([0,0.1],[0.4,0.5])}, u^3_{([0,0],[1,1])}\},$

$e_2=\{u^1_{([0.1,0.2],[0.5,0.6])}, u^2_{([0,0],[0.4,0.5])}, u^3_{([1,1],[0,0])}\},$

$e_3=\{u^1_{([0,0],[1,1])}, u^2_{([0,0],[1,1])}, u^3_{([0,0],[1,1])}\}\}$

be an IVIFS-set, then

cl(f_A,E)=$\bigcap\{(g_A,E)|(g_A,E)$ is a IVIFS-closed set containing $(f_A,E)\}$

$=(f^1_A,E)^c\bigcap(f^4_A,E)^c$

$=(f^1_A,E)^c$

$=\{e_1=\{u^1_{([0.2,0.3],[0.5,0.6])}, u^2_{([0,0.1],[0.4,0.5])}, u^3_{([0,0],[1,1])}\},$

$e_2=\{u^1_{([0.2,0.3],[0.4,0.5])}, u^2_{([0,0],[0.4,0.5])}, u^3_{([1,1],[0,0])}\},$

$e_3=\{u^1_{([1,1],[0,0])}, u^2_{([1,1],[0,0])}, u^3_{([1,1],[0,0])}\}\},$

since $(f_A,E)\subseteq(f^1_A,E)^c$ and $(f_A,E)\subseteq(f^4_A,E)^c$

Proposition 5.11. *for any two IVIFS-sets (f_A,E) and (g_A,E) in an interval valued intuitionistic fuzzy soft topological space$((\xi_A,E),\tau)$ on P(ξ_A,E), then*

(i) cl(f_A,E) is the smallest IVIFS-closed set containing (f_A,E).

(ii) (f_A,E) is IVIFS-closed if and only if (f_A,E)=cl(f_A,E).

(iii) $(g_A,E)\subseteq(f_A,E)\Rightarrow cl(g_A,E)\subseteq cl(f_A,E)$

(iv) cl(cl(f_A,E))=cl(f_A,E)
(v) cl(ϕ_{ξ_A},E)=(ϕ_{ξ_A},E) and cl(ξ_A,E)= (ξ_A,E)
(vi) cl((g_A,E)∪(f_A,E))= cl(g_A,E)∪cl(f_A,E)
(Vii) cl((g_A,E)∩(f_A,E))⊆ cl(g_A,E)∩cl(f_A,E)

Proof. (i) and (ii) can be proved from definition.

(iii) Since (g_A,E)⊆(f_A,E), implies all the closed set containing (f_A,E) also contained (g_A,E). Therefore
$\bigcap\{(g^*_A,E)|\ (g^*_A,E)$ is an IVIFS-closed set containing $(g_A,E)\} \subseteq \bigcap\{(f^*_A,E)|\ (f^*_A,E)$ is an IVIFS-closed set containing $(f_A,E)\}$

So cl(g_A,E)⊆cl(f_A,E).

(iv) cl(cl(f_A,E))= $\bigcap\{(g_A,E)|\ (g_A,E)$ is an IVIFS-closed set containing cl$(f_A,E)\}$
and since cl(f_A,E) is the largest closed IVIFS set containing cl(f_A,E),
therefore cl(cl(f_A,E))=cl(f_A,E).

(v) Straight forward.

(vi) Since cl(g_A,E)⊇ (g_A,E) and cl(f_A,E)⊇ (f_A,E), we have
cl(g_A,E)∪cl(f_A,E)⊇ (g_A,E)∪(f_A,E)
This implies cl(g_A,E)∪cl(f_A,E)⊇ cl((g_A,E)∪(f_A,E)).........(1)
Again since (g_A,E)∪(f_A,E)⊇ (g_A,E) and (g_A,E)∪(f_A,E)⊇ (f_A,E),
so cl((g_A,E)∪(f_A,E))⊇ cl(g_A,E) and cl((g_A,E)∪(f_A,E))⊇ cl(f_A,E),
Therefore cl((g_A,E)∪(f_A,E))⊇ cl(g_A,E)∪cl(f_A,E)..........(2)
Using (1) and (2) we get, cl((g_A,E)∪(f_A,E))= cl(g_A,E)∪cl(f_A,E).

(vii) Since (g_A,E)⊇ (g_A,E)∩(f_A,E) and (f_A,E)⊇ (g_A,E)∩(f_A,E),
so cl(g_A,E)⊇ cl((g_A,E)∩(f_A,E)) and cl(f_A,E)⊇ cl((g_A,E)∩(f_A,E))
Hence cl(g_A,E)∩cl(f_A,E)⊇ cl((g_A,E)∩(f_A,E)) □

6. Interval valued intuitionistic fuzzy soft subspace topology

Theorem 6.1. *Let ((ξ_A,E),τ) be an interval valued intuitionistic fuzzy soft topological space on (ξ_A,E) and (f_A,E) be an IVIFS-set in P(ξ_A,E). Then the collection $\tau_{(f_A,E)}= \{(f_A,E)\bigcap(g_A,E)|(g_A,E)\in\tau\}$ is an interval valued intuitionistic fuzzy soft topology on the interval valued intuitionistic fuzzy soft set (f_A,E).*

Proof. (i) Since (ϕ_{ξ_A},E), (ξ_A,E)∈τ, (f_A,E)=(f_A,E)$\bigcap$(ξ_A,E)
and (ϕ_{f_A},E)=(f_A,E)$\bigcap$(ϕ_{f_ξ},E), therefore (ϕ_{f_A},E), (f_A,E)∈τ.

(ii) Let $\{(f^i_A,E)|$ i=1,2,3,...,n$\}$ be a finite subfamily of intuitionistic fuzzy soft open sets in $\tau_{(f_A,E)}$, then for each i=1,2,3,....,n, there exist (g^i_A,E)∈τ such that (f^i_A,E)=(f_A,E)$\bigcap$(g^i_A,E).
Now $\bigcap_{i=1}^n(f^i_A,E)=\bigcap_{i=1}^n((f_A,E)\bigcap(g^i_A,E))$
$=(f_A,E)\bigcap(\bigcap_{i=1}^n(g^i_A,E))$ and since $\bigcap_{i=1}^n(g^i_A,E)\in\tau$,
so $\bigcap_{i=1}^n(f^i_A,E)\in\tau_{(f_A,E)}$.

(iii) Let $\{(f^k_A,E)|$ k∈K$\}$ be an arbitrary family of interval valued intuitionistic fuzzy soft open sets in $\tau_{(f_A,E)}$, then for each k∈K, there exist (g^k_A,E)∈τ such that (f^k_A,E)=(f_A,E)$\bigcap$(g^k_A,E).
Now $\bigcup_{k\in K}(f^k_A,E)=\bigcup_{k\in K}((f_A,E)\bigcap(g^k_A,E))$
$=(f_A,E)\bigcap(\bigcup_{k\in K}(g^k_A,E))$ and since $\bigcup_{k\in K}(g^k_A,E)\in\tau$,
so $\bigcup_{k\in K}(f^k_A,E)\in\tau_{(f_A,E)}$. □

Definition 6.2. Let $((\xi_A,\mathrm{E}),\tau)$ be an IVIFS-topological space on (ξ_A,E) and $(\mathrm{f}_A,\mathrm{E})$ be an IVIFS-set in $\mathrm{P}(\xi_A,\mathrm{E})$. Then the IVIFS-topology

$\tau_{(f_A,E)}=\{(\mathrm{f}_A,\mathrm{E})\bigcap(\mathrm{g}_A,\mathrm{E})|(\mathrm{g}_A,\mathrm{E})\in\tau\}$ is called interval valued intuitionistic fuzzy soft subspace topology (in short IVIFS-topological subspace) and $((\mathrm{f}_A,\mathrm{E}),\tau_{(f_A,E)})$ is called interval valued intuitionistic fuzzy soft subspace of $((\xi_A,\mathrm{E}),\tau)$.

Example 6.3. Let us consider the interval valued intuitionistic fuzzy soft topology $\tau_1=\{(\phi_{\xi_A},\mathrm{E}), (\xi_A,\mathrm{E}), (\mathrm{f}^1_A,\mathrm{E}), (\mathrm{f}^2_A,\mathrm{E}), (\mathrm{f}^3_A,\mathrm{E}), (\mathrm{f}^4_A,\mathrm{E})\}$ as in the example: 3.2 and an IVIFS set

$(\mathrm{f}_A,\mathrm{E})=\{\mathrm{e}_1=\{\mathrm{u}^1_{([0.2,0.3],[0,0.1])}, \mathrm{u}^2_{([0.5,0.6],[0.1,0.2])}, \mathrm{u}^3_{([0.2,0.3],[0.6,0.7])}\},$
$\mathrm{e}_2=\{\mathrm{u}^1_{([0.3,0.4],[0.1,0.2])}, \mathrm{u}^2_{([0.4,0.5],[0.2,0.3])}, \mathrm{u}^3_{([0.4,0.5],[0.2,0.3])}\},$
$\mathrm{e}_3=\{\mathrm{u}^1_{([0,0],[1,1])}, \mathrm{u}^2_{([0,0],[1,1])}, \mathrm{u}^3_{([0,0],[1,1])}\}\}\in\mathrm{P}(\xi_A,\mathrm{E})$
Then
$(\phi_{f_A},\mathrm{E})=(\mathrm{f}_A,\mathrm{E})\bigcap(\phi_{\xi_A},\mathrm{E})=\{\mathrm{e}_1=\{\mathrm{u}^1_{([0,0],[1,1])}, \mathrm{u}^2_{([0,0],[1,1])}, \mathrm{u}^3_{([0,0],[1,1])}\},$
$\mathrm{e}_2=\{\mathrm{u}^1_{([0,0],[1,1])}, \mathrm{u}^2_{([0,0],[1,1])}, \mathrm{u}^3_{([0,0],[1,1])}\},$
$\mathrm{e}_3=\{\mathrm{u}^1_{([0,0],[1,1])}, \mathrm{u}^2_{([0,0],[1,1])}, \mathrm{u}^3_{([0,0],[1,1])}\}\},$

$(\mathrm{g}^1_A,\mathrm{E})=(\mathrm{f}_A,\mathrm{E})\bigcap(\mathrm{f}^1_A,\mathrm{E})$
$=\{\mathrm{e}_1=\{\mathrm{u}^1_{([0.2,0.3],[0.2,0.3])}, \mathrm{u}^2_{([0.4,0.5],[0.1,0.2])}, \mathrm{u}^3_{([0.2,0.3],[0.6,0.7])}\},$
$\mathrm{e}_2=\{\mathrm{u}^1_{([0.3,0.4],[0.2,0.3])}, \mathrm{u}^2_{([0.4,0.5],[0.2,0.3])}, \mathrm{u}^3_{([0,0],[1,1])}\},$
$\mathrm{e}_3=\{\mathrm{u}^1_{([0,0],[1,1])}, \mathrm{u}^2_{([0,0],[1,1])}, \mathrm{u}^3_{([0,0],[1,1])}\}\},$

$(\mathrm{g}^2_A,\mathrm{E})=(\mathrm{f}_A,\mathrm{E})\bigcap(\mathrm{f}^2_A,\mathrm{E})$
$=\{\mathrm{e}_1=\{\mathrm{u}^1_{([0.2,0.3],[0.1,0.2])}, \mathrm{u}^2_{([0.5,0.6],[0.2,0.3])}, \mathrm{u}^3_{([0.2,0.3],[0.6,0.7])}\},$
$\mathrm{e}_2=\{\mathrm{u}^1_{([0.2,0.3],[0.1,0.2])}, \mathrm{u}^2_{([0.4,0.5],[0.2,0.3])}, \mathrm{u}^3_{([0,0],[1,1])}\},$
$\mathrm{e}_3=\{\mathrm{u}^1_{([0,0],[1,1])}, \mathrm{u}^2_{([0,0],[1,1])}, \mathrm{u}^3_{([0,0],[1,1])}\}\},$

$(\mathrm{g}^3_A,\mathrm{E})=(\mathrm{f}_A,\mathrm{E})\bigcap(\mathrm{f}^3_A,\mathrm{E})$
$=\{\mathrm{e}_1=\{\mathrm{u}^1_{([0.2,0.3],[0.2,0.3])}, \mathrm{u}^2_{([0.4,0.5],[0.2,0.3])}, \mathrm{u}^3_{([0.2,0.3],[0.6,0.7])}\},$
$\mathrm{e}_2=\{\mathrm{u}^1_{([0.2,0.3],[0.2,0.3])}, \mathrm{u}^2_{([0.4,0.5],[0.2,0.3])}, \mathrm{u}^3_{([0,0],[1,1])}\},$
$\mathrm{e}_3=\{\mathrm{u}^1_{([0,0],[1,1])}, \mathrm{u}^2_{([0,0],[1,1])}, \mathrm{u}^3_{([0,0],[1,1])}\}\},$

$(\mathrm{g}^4_A,\mathrm{E})=(\mathrm{f}_A,\mathrm{E})\bigcap(\mathrm{f}^4_A,\mathrm{E})$
$=\{\mathrm{e}_1=\{\mathrm{u}^1_{([0.2,0.3],[0.1,0.2])}, \mathrm{u}^2_{([0.5,0.6],[0.1,0.2])}, \mathrm{u}^3_{([0.2,0.3],[0.6,0.7])}\},$
$\mathrm{e}_2=\{\mathrm{u}^1_{([0.3,0.4],[0.1,0.2])}, \mathrm{u}^2_{([0.4,0.5],[0.2,0.3])}, \mathrm{u}^3_{([0,0],[1,1])}\},$
$\mathrm{e}_3=\{\mathrm{u}^1_{([0,0],[1,1])}, \mathrm{u}^2_{([0,0],[1,1])}, \mathrm{u}^3_{([0,0],[1,1])}\}\}.$

Then $\tau_{(f_A,E)}=\{(\phi_{f_A},\mathrm{E}), (\mathrm{f}_A,\mathrm{E}), (\mathrm{g}^1_A,\mathrm{E}), (\mathrm{g}^2_A,\mathrm{E}), (\mathrm{g}^3_A,\mathrm{E}), (\mathrm{g}^4_A,\mathrm{E})\}$ is an interval valued intuitionistic fuzzy soft subspace topology for τ and $((\mathrm{f}_A,\mathrm{E}),\tau_{(f_A,E)})$ is called interval valued intuitionistic fuzzy soft subspace of $((\xi_A,\mathrm{E}),\tau_1)$.

Theorem 6.4. *Let $((\xi_A,E),\tau)$ be an interval valued intuitionistic fuzzy soft topological space on (ξ_A,E), B be an interval valued intuitionistic fuzzy soft basis for τ and (f_A,E) be an IVIFS-set in $P(\xi_A,E)$. Then the family*

$B_{(f_A,E)}$= $\{(f_A,E)\bigcap(g_A,E)|(g_A,E)\in B\}$ is an interval valued intuitionistic fuzzy soft basis for subspace topology $\tau_{(f_A,E)}$.

Proof. Let $(h_A,E)\in\tau_{(f_A,E)}$, then there exists an IVIFS set $(g_A,E)\in\tau$, such that $(h_A,E)=(f_A,E)\bigcap(g_A,E)$.

Since B is a base for τ, so there exists sub collection $\{(\chi^i_A,E)|i\in I\}$ of B, such that $(g_A,E)=\bigcup_{i\in I}(\chi^i_A,E)$.

Therefore $(h_A,E)=(f_A,E)\bigcap(g_A,E)=(f_A,E)\bigcap(\bigcup_{i\in I}(\chi^i_A,E))=\bigcup_{i\in I}((f_A,E)\bigcap(\chi^i_A,E))$. Since $(f_A,E)\bigcap(\chi^i_A,E)\in B_{(f_A,E)}$, which implies that $B_{(f_A,E)}$ is an IVIFS basis for the IVIFS subspace topology $\tau_{(f_A,E)}$. □

Theorem 6.5. *Let $((\xi_A,E),\tau)$ be an IVIFS-topological subspace of $((\eta_A,E),\tau^*)$ and let $((\eta_A,E),\tau^*)$ be an IVIFS topological subspace of $((\Im_A,E),\tau^{**})$. Then $((\xi_A,E),\tau)$ is also an IVIFS- topological subspace of $((\Im_A,E),\tau^{**})$.*

Proof. Since $(\xi_A,E)\subseteq(\eta_A,E)\subseteq(\Im_A,E)$, $((\xi_A,E),\tau)$ is an interval valued intuitionistic fuzzy soft topological subspace of $((\Im_A,E),\tau^{**})$ if and only if $\tau^{**}_{(\xi_A,E)}=\tau$. Let $(f_A,E)\in\tau$, now since $((\xi_A,E),\tau)$ be an interval valued intuitionistic fuzzy soft topological subspace of $((\eta_A,E),\tau^*)$ i.e. $\tau^*_{(\eta_A,E)}=\tau$, so there exist $(f_A{}^*,E)\in\tau^*$ such that $(f_A,E)=(\xi_A,E)\bigcap(f^*_A,E)$. But $((\eta_A,E),\tau^*)$ be an interval valued intuitionistic fuzzy soft topological subspace of $((\Im_A,E),\tau^{**})$,

so there exist $(f^{**}_A,E)\in\tau^{**}$ such that $(f^*_A,E)=(\eta_A,E)\bigcap(f^{**}_A,E)$.

Thus $(f_A,E)=(\xi_A,E)\bigcap(f^*_A,E)=(\xi_A,E)\bigcap(\eta_A,E)\bigcap(f^{**}_A,E)$
$=(\xi_A,E)\bigcap(f^{**}_A,E)$, since $(\xi_A,E)\subseteq(\eta_A,E)$;
so $(f_A,E)\in\tau^{**}_{(\xi_A,E)}$. Accordingly, $\tau\subseteq\tau^{**}_{(\xi_A,E)}$.
Now assume $(g_A,E)\in\tau^{**}_{(\xi_A,E)}$, i.e. there exist $(h_A,E)\in\tau^{**}$
such that $(g_A,E)=(\xi_A,E)\bigcap(h_A,E)$. But $(\eta_A,E)\bigcap(h_A,E)\in\tau^{**}_{(\eta_A,E)}=\tau^*$,
so $(\xi_A,E)\bigcap((\eta_A,E)\bigcap(h_A,E))\in\tau^*_{(\xi_A,E)}=\tau$.
Since $(\xi_A,E)\bigcap((\eta_A,E)\bigcap(h_A,E))=(\xi_A,E)\bigcap(h_A,E)=(g_A,E)$, we have $(g_A,E)\in\tau$.
Accordingly, $\tau^{**}_{(\xi_A,E)}\subseteq\tau$ and thus the theorem is proved. □

7. Conclusions

Soft sets, fuzzy soft sets, intuitionistic fuzzy soft sets, interval valued intuitionistic fuzzy soft sets are all mathematical tools for dealing with uncertainties. In this paper we have introduced the concept of interval valued intuitionistic fuzzy soft topological spaces together with some basic concepts over a fixed parameter set, which is the extension of fuzzy soft topological spaces introduced by Simsekler and Yuksel [12] as well as intuitionistic fuzzy soft topological spaces introduced by Li and Cui [5].

References

[1] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57 (2009) 1547–1553.

[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.

[3] K. Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31 (1989) 343–349.

[4] Y. Jiang, Y. Tang, Q. Chen, H. Liu and J. Tung, Interval -valued intuitionistic fuzzy soft sets and their properties, Comput. Math. Appl. 60 (2010) 906–918.

[5] Z. Li and R. Cui, On the topological structure of fuzzy soft sets, Ann. Fuzzy Math. Inform. 5(1) (2013) 229–239.
[6] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[7] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (2001) 589–602.
[8] P. K. Maji, R. Biswas and A. R. Roy, Intuitionistic fuzzy soft sets, J. Fuzzy Math. 12 (2004) 669–683.
[9] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37 (1999) 19–31.
[10] S. Roy and T. K. Samanta, A note on fuzzy soft topological space, Ann. Fuzzy Math. Inform. 3(2) (2012) 305–311.
[11] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786–1799.
[12] T. Simsekler and S. Yuksel, Fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 5(1) (2013) 87–96.
[13] B. Tanay and M. B. Kandemir, Topological structure of fuzzy soft sets, Comput. Math. Appl. 61 (2011) 2952–2957.
[14] L. A. Zadeh, Fuzzy sets, Information and control 8 (1965) 338–353 .

Anjan Mukherjee (anjan2002_m@yahoo.co.in)
Department of Mathematics, Tripura University, Suryamaninagar, Agartala-799022, Tripura, India

Ajoy Kanti Das (ajoykantidas@gmail.com)
Department of Mathematics, Tripura University, Suryamaninagar, Agartala-799022, Tripura, India

Abhijit Saha (abhijit84_mt@yahoo.in)
Department of Mathematics, Tripura University, Suryamaninagar, Agartala-799022, Tripura, India