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Compactification via intuitionistic fuzzy centred net and open filter

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Abstract. In this paper,the concept of intuitionistic fuzzy centred structure space is introduced.The concepts of intuitionistic fuzzy centred net and intuitionistic fuzzy centred open filter are studied.In this connection compactification of an intuitionistic fuzzy centred structure space is established.

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1. Introduction

The method of centred systems in fuzzy topological spaces was introduced by M. K. Uma, E. Roja and G. Balasubramanian [1]. A process of compactification via net and open-filter was introduced by H. J. Wu and W. Wu[2]. In this paper,the concept of intuitionistic fuzzy centred structure space is introduced. The concepts of intuitionistic fuzzy centred net and intuitionistic fuzzy centred open filter are studied. In this connection compactification of an intuitionistic fuzzy centred structure space is established.

Definition 1.1 ([1])**.** Let R be a fuzzy Hausdorff space. A system $p = \{\lambda_\alpha\}$ of fuzzy open sets of R is called fuzzy centred if any finite collection of fuzzy sets of the system has a non-zero intersection. The system p is called maximal fuzzy centred system or a fuzzy end if it cannot be included in any larger fuzzy centred system of fuzzy open sets.

Definition 1.2. A set Λ is a directed set iff there is a relation $\leq$ on Λ satisfying:

(1) $\lambda \leq \lambda$,for each $\lambda \in \Lambda$.

(2) if $\lambda_1 \leq \lambda_2$ and $\lambda_2 \leq \lambda_3$ then $\lambda_1 \leq \lambda_3$.
(3) if $\lambda_1, \lambda_2 \in \Lambda$ then there is some $\lambda_3 \in \Lambda$ with $\lambda_1 \leq \lambda_3$,$\lambda_2 \leq \lambda_3$.

Definition 1.3. A net in a set X is a function $P : \Lambda \to X$,where Λ is directed set,the point $P(\lambda)$ is usually denoted x_λ, and we often speak of the net $(x_\lambda)_{\lambda\in\Lambda}$ or the net (x_λ) if this can cause no confusion.

Definition 1.4. A function $f : X \to R$ is called continuous if $f^{-1}(V)$ is open in X,for every $V \in R$.

Definition 1.5. A function $f : X \to R$ is called bounded if there exists a real number $m > 0$ such that $\mid f(x) \mid\leq m$ for every $x \in X$.

2. Compactification via intuitionistic fuzzy centred net and intuitionistic fuzzy centred filter

Definition 2.1. Let (X, T) be an intuitionistic fuzzy Hausdorff space. The system $P = \{A_i\}_{i\in\Lambda}$ of all intuitionistic fuzzy open sets of (X, T) is said to be intuitionistic fuzzy centred if any finite collections of intuitionistic fuzzy sets $\{A_i\}_{i=1}^n$ such that $\bigcap_{i=1}^n A_i \neq 0_\sim$. The system P is called maximal intuitionistic fuzzy centred system or intuitionistic fuzzy end if it cannot be included in any larger intuitionistic fuzzy centred system.

Definition 2.2. Let $\mathbb{P}_\mathbb{X} = \{P_i : i \in \Lambda\}$ where $P_i's$ are intuitionistic fuzzy centred systems in (X, T). They are also called intuitionistic fuzzy centred points. Then the family $\mathbb{T}_P$ is said to be an intuitionistic fuzzy centred structure if it satisfying the following axioms:

(1) $\varnothing, \mathbb{P}_\mathbb{X} \in \mathbb{T}_P$.
(2) The union of the elements of any subcollections of $\mathbb{T}_P$ is in $\mathbb{T}_P$.
(3) The intersection of the elements of any finite subcollections of $\mathbb{T}_P$ is in $\mathbb{T}_P$.

The pair $(\mathbb{P}_X, \mathbb{T}_P)$ is called intuitionistic fuzzy centred structure space. The member of $(\mathbb{P}_X, \mathbb{T}_P)$ are called intuitionistic fuzzy centred(in shortly IFC_P)open. The complement of an intuitionistic fuzzy centred open set is intuitionistic fuzzy centred closed.

Definition 2.3. Let $(\mathbb{P}_X, \mathbb{T}_P)$ be an intuitionistic fuzzy centred structure space and an intuitionistic fuzzy centred subset $A \subseteq \mathbb{P}_X$. Then intuitionistic fuzzy centred closure and intuitionistic fuzzy centred interior of A are defined by

$IFC_P cl(A) = \cap\{B : B$ is an intuitionistic fuzzy centred closed and $A \subseteq B\}$.
$IFC_P int(A) = \cup\{B : B$ is an intuitionistic fuzzy centred open and $A \supseteq B\}$.

Definition 2.4. An collection $\{A_i : i \in \Lambda\}$ of intuitionistic fuzzy centred open sets in an intuitionistic fuzzy centred structure space $(\mathbb{P}_X, \mathbb{T}_P)$ is called an intuitionistic fuzzy centred open cover of intuitionistic fuzzy centred subset B of $\mathbb{P}_X$ if $B \subset \bigcup\{A_i : i \in \Lambda\}$ holds

Definition 2.5. An intuitionistic fuzzy centred structure space $(\mathbb{P}_X, \mathbb{T}_P)$ is said to be intuitionistic fuzzy centred compact if every intuitionistic fuzzy centred open cover of $\mathbb{P}_X$ has a finite subcover.

Definition 2.6. Let $(\mathbb{P}_X, \mathbb{T}_P)$ and $(\mathbb{P}_Y, \mathbb{S}_P)$ be any two intuitionistic fuzzy centred structure space. Then $f : (\mathbb{P}_X, \mathbb{T}_P) \to (\mathbb{P}_Y, \mathbb{S}_P)$ is an intuitionistic fuzzy centred continuous function if $f^{-1}(V)$ is an intuitionistic fuzzy centred open in $(\mathbb{P}_X, \mathbb{T}_P)$ for each intuitionistic fuzzy centred open set V in $(\mathbb{P}_Y, \mathbb{S}_P)$.

Definition 2.7. Let $(\mathbb{P}_X, \mathbb{T}_P)$ be an intuitionistic fuzzy centred structure space and let $P \in \mathbb{P}_X$. Then an intuitionistic fuzzy centred subset $N \subseteq \mathbb{P}_X$ is said to be intuitionistic fuzzy centred neighbourhood if there exists an intuitionistic fuzzy centred open set G such that $P \in G \subseteq N$.

Definition 2.8. An intuitionistic fuzzy centred subset A of $\mathbb{P}_X$ is said to be an intuitionistic fuzzy centred dense if $IFC_P cl(A) = \mathbb{P}_X$.

Definition 2.9. Let $\mathbb{P}_X$ be an intuitionistic fuzzy centred system. Then a nonempty family $\mathbb{F}$ of subsets of $\mathbb{P}_X$is called intuitionistic fuzzy centred filter on $\mathbb{P}_X$ iff the following axioms are satisfied:

(1) $\varnothing \notin \mathbb{F}$,
(2) if $F \in \mathbb{F}$ and $F \subseteq H$,then $H \in \mathbb{F}$,
(3) if $F_1, F_2 \in \mathbb{F}$ then $F_1 \cap F_2 \in \mathbb{F}$.

Definition 2.10. An intuitionistic fuzzy centred filter base on $\mathbb{P}_X$ is a nonempty family $\mathbb{B}$ of subsets of $\mathbb{P}_X$ such that

(1) $\varnothing \notin \mathbb{B}$
(2) if $F_1 \in \mathbb{B}$ and $F_2 \in \mathbb{B}$ then there exists $F \in \mathbb{B}$ such that $F \subseteq F_1 \cap F_2$.

Definition 2.11. An intuitionsitic fuzzy centred filter $\mathbb{F}$ on $\mathbb{P}_X$ is said to be an intuitionistic fuzzy centred ultrafilter iff $\mathbb{F}$ is not properly contained in any other intuitionistic fuzzy centred filter on $\mathbb{P}_X$.

Definition 2.12. An intuitionistic fuzzy centred net in an intuitionistic fuzzy centred structure space $(\mathbb{P}_X, \mathbb{T}_P)$ is a mapping from a directed set Δ into $\mathbb{P}_X$; denoted $\{P_\alpha\}_{\alpha\in\Delta}$.

Definition 2.13. Let $\{P_\alpha\}_{\alpha\in\Delta}$ be an intuitionistic fuzzy centred net in an intuitionistic fuzzy centred structure $\mathbb{P}_X$ and let G be an intuitionistic fuzzy centred subset of $\mathbb{P}_X$. Then the intuitionistic fuzzy centred net is said to be

(1) in G iff $\{P_\alpha\} \in G, \forall\alpha \in \Delta$
(2) eventually in G iff there exists $\beta \in \Delta$ for all $\alpha \in \Delta$,$\alpha \geq \beta$ such that $\{P_\alpha\}$ in G.
(3) frequently in G iff for every $\beta \in \Delta$,there exists $\alpha \in \Delta, \alpha \geq \beta$ and $\{P_\alpha\}$ in G.

Definition 2.14. If $\{P_\alpha\}$ is an intuitionistic fuzzy centred net in the intuitionisitic fuzzy centred structure $\mathbb{P}_X$ and P is an intuitionistic fuzzy centred element of $\mathbb{P}_X$,we say that the intuitionistic fuzzy centred net converges towards P iff for every intutionistic fuzzy centred neighbourhood U of P,$\{P_\alpha\}$is eventually in U.

Definition 2.15. An intuitionistic fuzzy centred point P_1 in $\mathbb{P}_X$ is said to be an intuitionistic fuzzy centred accumulation point or cluster point of an intuitionistic fuzzy centred net iff for every intuitionistic fuzzy centred neighbourhood U of P_1,the intuitionistic fuzzy centred net is frequently in U.

Definition 2.16. An intuitionistic fuzzy centred net $\{P\}$ in a set $\mathbb{P}_X$ is called an intuitionistic fuzzy centred universal or intuitionistic fuzzy centred ultranet if every intuitionistic fuzzy centred subset A of $\mathbb{P}_X$,either $\{P\}$ is eventually in A or $\{P\}$ is eventually in $\mathbb{P}_X - A$.

Definition 2.17. Let Φ be a family of intuitionistic fuzzy centred continuous functions on an intuitionistic fuzzy centred structure space $\mathbb{P}_X$. An intuitionistic fuzzy centred net $\{P_i\}$ in $\mathbb{P}_X$ will be called an intuitionistic fuzzy centred Φ net,if $\{f(P_i)\}$ converges for each $f \in \Phi$.

Definition 2.18. Let Φ be a family of bounded real-valued continuous functions on $\mathbb{P}_X$. Let $\mathbb{P}_X$ is an intuitionistic fuzzy centred compact if every Φ net has an intuitionistic fuzzy centred cluster point in $\mathbb{P}_X$.

Definition 2.19. Let $\mathbb{P}_X$be any arbitrary intuitionistic fuzzy centred structure space, $C^*(\mathbb{P}_X) = \{f_\alpha | \alpha \in \Lambda\}$ the family of all bounded real-valued continuous function on $\mathbb{P}_X$. For a $C^*(\mathbb{P}_X)$ net $\{P_i\}$,let $\mathbb{F}_{\{P_i\}} = \{U | U$ is intuitionistic fuzzy centred open in $\mathbb{P}_X$ and $\{P_i\}$ is eventually in $U\}$. It is clear that $\mathbb{F}_{\{P_i\}}$ is an intuitionsitic fuzzy centred open filter,and for any $f_\alpha \in C^*(\mathbb{P}_X)$,any $\epsilon > 0, f_\alpha^{-1}((r_\alpha - \epsilon, r_\alpha + \epsilon)) \in \mathbb{F}_{\{P_i\}}$,where $r_\alpha = \lim\{f_\alpha(P_i)\}$. We will called $\mathbb{F}_{\{P_i\}}$ the intuitionistic fuzzy centred open filter on $\mathbb{P}_X$ induced by $\{P_i\}$.

Definition 2.20. If $\mathbb{F}$ is an intuitionistic fuzzy centred filter on $\mathbb{P}_X$,let $\Delta_\mathbb{F} = \{(P, F) | P \in F \in \mathbb{F}\}$. Then $\Delta_\mathbb{F}$ is directed by the relation $(P_1, F_1) \leq (P_2, F_2)$iff $F_2 \subset F_1$,so the map $M : \Delta_\mathbb{F} \to \mathbb{P}_X$ defined by $M(P, F_1) = P$ is an intuitionistic fuzzy centred net in $\mathbb{P}_X$. It is called the intuitionistic fuzzy centred net based on $\mathbb{F}$.

Definition 2.21. An intuitionistic fuzzy centred filter $\mathbb{F}_P$ converges to P in $\mathbb{P}_X$ if the intuitionistic fuzzy centred net based on $\mathbb{F}_P$ converges to P.

Definition 2.22. Let $\mathcal{P}$ be an intuitionsitic fuzzy centred open filter on $\mathbb{P}_X$,$\{P_i\}$ the intuitionistic fuzzy centred net based on $\mathcal{P}$,and $\mathcal{I} = \{U | U$ is intuitionistic fuzzy centred open in $\mathbb{P}_X$ and $\{P_i\}$ is eventually in $U\}$. Then $\mathcal{I} = \mathcal{P}$.

For each $C^*(\mathbb{P}_X)$ net $\{P_i\}$ in $\mathbb{P}_X$,let $\{w_k^{P_i}\}$ be the intuitionistic fuzzy centred net based on the intuitionistic fuzzy centred open filter $\mathbb{F}_{\{P_i\}}$ induced by $\{P_i\}$.

(a) $\{w_k^{P_i}\}$ is uniquely determined by $\mathbb{F}_{\{P_i\}}$ and $\mathbb{F}_{\{P_i\}} = \mathbb{F}_{\{P_j\}}$ iff $\{w_k^{P_i}\} = \{w_k^{P_j}\}$
(b) $\mathbb{F}_{\{P_i\}} = \mathbb{F}_{\{w_k^{P_i}\}} = \{O | O$ is intuitionistic fuzzy centred open in $\mathbb{P}_X$ and $\{w_k^{P_i}\}$ is eventually in $O\}$
(c) $\{w_k^{P_i}\}$ is a $C^*(\mathbb{P}_X)$ net and $\lim\{f_\alpha(w_k^{P_i})\} = \lim\{f_\alpha(P_i)\}$ for all f_α in $C^*(\mathbb{P}_X)$.
(d) The following are equivalent
  (i) $\{w_k^{P_i}\}$ converges to P
  (ii) $\{P_i\}$ converges to P and
  (iii) $\mathbb{F}_{\{P_i\}}$ converges to P

To avoid the confusion between $\{w_k^{P_i}\}$ as an intuitionistic fuzzy centred net in $\mathbb{P}_X$ and $\{w_k^{P_i}\}$ is an intuitionistic fuzzy centred point in a set,we will use $\{w_k^{P_i}\}^*$ to represent $\{w_k^{P_i}\}$ when it is regraded as an intuitionistic fuzzy centred point in a set. Let $Y_P = \{\{w_k^{P_i}\}^*|\{P_i\}$ is an $C^*(\mathbb{P}_X)$ net the does not converges in $\mathbb{P}_X$,$\{w_k^{P_i}\}$ is the intuitionistic fuzzy centred net based on intuitionistic fuzzy centred filter $\mathbb{F}_{\{P_i\}}\}$, $\mathbb{P}_X^* = \mathbb{P}_X \cup Y_P$,the disjoint union of $\mathbb{P}_X$ and Y_P. For each intuitionistic fuzzy centred open set $U \subset \mathbb{P}_X$,define $U^* \subset \mathbb{P}_X^*$ to be the set $U^* = U \cup \{\{w_k^{P_i}\}^*|\{w_k^{P_i}\}^* \in Y_P$ and $\{w_k^{P_i}\}$ is eventually in $U\}$. It is clear that if $U \subset V$,then $U^* \subset V^*$.

Proposition 2.23. *For any two intuitionistic fuzzy centred open sets U and V in $\mathbb{P}_X$,$(U \cap V)^* = U^* \cap V^*$.*

Proof. Let $P_2 \in (U \cap V)^* \cap Y_P$,then $P_2 = \{w_k^{P_i}\}^*$ and $\{w_k^{P_i}\}$ is eventually in $U \cap V$. This implies that $\{w_k^{P_i}\}$ is eventually in U and in V,thus $\{w_k^{P_i}\}^*$ is in $U^* \cap V^*$. For the other direction,if $P_2 \in (U^* \cap V^*) \cap Y_P$,then $P_2 = \{w_k^{P_i}\}^*$ and $\{w_k^{P_i}\}$ is eventually in U and in V,so $\{w_k^{P_i}\}$ is eventually in $U \cap V$. Thus P_2 is in $(U \cap V)^*$. □

Proposition 2.24. *Let $\mathfrak{B} = \{U^*|U$ be an intuitionistic fuzzy centred open in $\mathbb{P}_X\}$. Then $\mathfrak{B}$ is an intuitionistic fuzzy centred base for an intuitionistic fuzzy centred structure on $\mathbb{P}_X^*$ if*

(a) $\mathbb{P}_X^* = \{U^*|U^* \in \mathfrak{B}\}$
(b) *whenever $U^*, V^* \in \mathfrak{B}$ with $P_2 \in U^* \cap V^*$,there is some $W^* = (U \cap V)^* \in \mathfrak{B}, P_2 \in W^* \subset U^* \cap V^*$.*

Proof. (a) $\mathbb{P}_X^* = \{U^*|U^* \in \mathfrak{B}\}$,let $P_2 \in Y_P$,then $P_2 = \{w_k^{P_i}\}^*$. For any $f_\alpha \in C^*(\mathbb{P}_X)$,let $r_\alpha = \lim\{f_\alpha(w_k^{P_i})\}$,then $\{w_k^{P_i}\}$ is eventually in $f_\alpha^{-1}((r_\alpha - \epsilon, r_\alpha + \epsilon))$ for any $\epsilon > 0$,that is $\{w_k^{P_i}\}^*$ is in $f_\alpha^{-1}((r_\alpha - \epsilon, r_\alpha + \epsilon))$,for any $\epsilon > 0$,thus $Y_P \subset \cup\{U^*|U^* \in \mathfrak{B}\}$,therefore $\mathbb{P}_X^* \subset \cup\{U^*|U^* \in \mathfrak{B}\}$. For $\{U^*|U^* \in \mathfrak{B}\} \subset \mathbb{P}_X^*$ is clear.
(b) if $P_2 \in U^* \cap V^*$ for any U^* and V^* in $\mathfrak{B}$,since $(U \cap V)^*$ is in $\mathfrak{B}$ and $(U \cap V)^* = U^* \cap V^*$,thus $P_2 \in (U \cap V)^* \subset U^* \cap V^*$.

□

Provide $\mathbb{P}_X^*$ with the intuitionistic fuzzy structure induced by the intuitionsitic fuzzy centred base $\mathfrak{B}$. For each f_α in $C^*(\mathbb{P}_X)$,define $f_\alpha^* : \mathbb{P}_X^* \to R$ by setting that $f_\alpha^*(P_1) = f_\alpha(P_1)$ if $P_1 \in \mathbb{P}_{\mathbb{X}}$; $f_\alpha^*(\{w_k^{P_i}\}^*) = \lim\{f_\alpha(w_k^{P_i})\}$ for $\{w_k^{P_i}\}^*$ in Y_P. It is clear that f_α^* is well defined and is a bounded real valued function on $\mathbb{P}_X^*$

Proposition 2.25. *For any f_α in $C^*(\mathbb{P}_X)$,f_α^* is a bounded real valued continuous function on $\mathbb{P}_X^*$.*

Proof. To show the continuity of f_α^* at any P_3 in $\mathbb{P}_X^*$,let $t_\alpha = f_\alpha^*(P_3)$. It will be shown that for any $\epsilon > 0$,there is an intuitionistic fuzzy centred open set $U^* \in \mathfrak{B}$ such that $P_3 \in U^* \subset f_\alpha^{*-1}((t_\alpha - \epsilon, t_\alpha + \epsilon))$. Let $U = f_\alpha^{-1}((t_\alpha - \frac{\epsilon}{2}, t_\alpha + \frac{\epsilon}{2}))$. If $P_3 \in \mathbb{P}_X$,since $f_\alpha(P_3) = f_\alpha^*(P_3) = t_\alpha$,thus $P_3 \in f_\alpha^{-1}((t_\alpha - \frac{\epsilon}{2}, t_\alpha + \frac{\epsilon}{2})) \subset (f_\alpha^{-1}((t_\alpha - \frac{\epsilon}{2}, t_\alpha + \frac{\epsilon}{2})))^*$. If $P_3 \in Y_P$,then $P_3 = \{w_k^{P_i}\}^*$. Since $t_\alpha = f_\alpha^*(P_3) = \lim\{f_\alpha(w_k^{P_i})\}$,so $\{w_k^{P_i}\}$ is eventually in $f_\alpha^{-1}((t_\alpha - \frac{\epsilon}{2}, t_\alpha + \frac{\epsilon}{2}))$; that is $P_3 = \{w_k^{P_i}\}^* \in (f_\alpha^{-1}((t_\alpha - \frac{\epsilon}{2}, t_\alpha + \frac{\epsilon}{2})))^*$.

Finally,we show that $(f_\alpha^{-1}((t_\alpha - \frac{\epsilon}{2}, t_\alpha + \frac{\epsilon}{2})))^* \subset f_\alpha^{*-1}((t_\alpha - \epsilon, t_\alpha + \epsilon))$. If P_1 is in $\mathbb{P}_X \cap (f_\alpha^{-1}((t_\alpha - \frac{\epsilon}{2}, t_\alpha + \frac{\epsilon}{2})))^*$,then $P_1 \in f_\alpha^{-1}((t_\alpha - \frac{\epsilon}{2}, t_\alpha + \frac{\epsilon}{2})))$;that is $f_\alpha^*(P_1) = f_\alpha(P_1) \in (t_\alpha - \epsilon, t_\alpha + \epsilon)$. So $P_1 \in f_\alpha^{*-1}((t_\alpha - \epsilon, t_\alpha + \epsilon))$. If $P_2 \in (f_\alpha^{-1}((t_\alpha - \frac{\epsilon}{2}, t_\alpha +$

$\frac{\epsilon}{2})))^* \cap Y_P$,then $P_2 = \{w_k^{P_i}\}^*$, and $\{w_k^{P_i}\}$ is eventually in $f_\alpha^{-1}((t_\alpha - \frac{\epsilon}{2}, t_\alpha + \frac{\epsilon}{2}))$,thus $f_\alpha^*(P_2) = \lim\{f_\alpha(w_k^{P_i})\} \in [t_\alpha - \frac{\epsilon}{2}, t_\alpha + \frac{\epsilon}{2}] \subset (t_\alpha - \epsilon, t_\alpha + \epsilon)$. that is $P_2 \in f_\alpha^{*-1}((t_\alpha - \epsilon, t_\alpha + \epsilon))$. □

Corollary 2.26. *For any* $f_\alpha \in C^*(\mathbb{P}_X)$,*let* $t_\alpha \in IFC_Pcl(f(\mathbb{P}_X))$,*then for any* ϵ, δ *with* $0 < \delta < \epsilon$,$(f_\alpha^{-1}((t_\alpha - \epsilon, t_\alpha + \epsilon)))^* \subset f_\alpha^{*-1}((t_\alpha - \epsilon, t_\alpha + \epsilon))$.

Proposition 2.27. *Let* $k : \mathbb{P}_X \to \mathbb{P}_X^*$ *be defined by* $k(P_1) = P_1$. *Then* k *is an intuitionistic fuzzy centred continuous function from* $\mathbb{P}_X$ *into* $\mathbb{P}_X^*$.

Proof. For any intuitionistic fuzzy centred open set U^* in $\mathfrak{B}$,$k^{-1}(U^*) = U$ is intuitionistic fuzzy centred open in $\mathbb{P}_X$,so k is intuitionistic fuzzy centred continuous on $\mathbb{P}_X$. □

Corollary 2.28. *For any* f_α *in* $C^*(\mathbb{P}_X)$, $f_\alpha^* \circ k = f_\alpha$.

Proposition 2.29. For any P_2 in $\mathbb{P}_X^* - \mathbb{P}_X$ with $P_2 = \{w_k^{P_i}\}^*$,$\{k(w_k^{P_i})\}$ converges to $P_2 = \{(w_k^{P_i})\}^*$

Proof. Let U^* be any intuitionistic fuzzy centred open set in $\mathfrak{B}$ containing P_2,then $\{(w_k^{P_i})\}$ is eventually in U in $\mathbb{P}_X$. This implies that $\{k(w_k^{P_i})\}$ is eventually in U^*,thus $\{k(w_k^{P_i})\}$ converges to $P_2 = \{(w_k^{P_i})\}^*$. □

Proposition 2.30. $k(\mathbb{P}_X)$ *is intuitionistic fuzzy centred dense in* $\mathbb{P}_X^*$

Proof. For any P_2 in $\mathbb{P}_X^* - \mathbb{P}_X$,$P_2 = \{w_k^{P_i}\}^*$. Proposition 2.29 implies that $\{k(w_k^{P_i})\}$ converges to $P_2 = \{w_k^{P_i}\}^*$. Thus $IFC_Pcl(k(\mathbb{P}_X)) = \mathbb{P}_X^*$. □

For convenience,let $\mathfrak{C} = \{f_\alpha^* | \alpha \in \Lambda\}$ represent $\{f_\alpha^* | f_\alpha \in C^*(\mathbb{P}_X)\}$. For any $\mathfrak{C}$ net $\{P_i\}$ in $\mathbb{P}_X^*$,let $\mathcal{E} = \{O | O$ is intuitionistic fuzzy centred open in $\mathbb{P}_X^*$ and $\{P_i\}$ is eventually in $O\}$ and $\mathcal{L} = \{U | U$ is intuitionistic fuzzy centred open in $\mathbb{P}_X$ and $U^* \in \mathcal{E}\}$.

Proposition 2.31. *For a* $\mathfrak{C}$ *net* $\{P_i\}$ *in* $\mathbb{P}_X^*$,*let* $r_\alpha = \lim\{f_\alpha^*(P_i)\}$ *for each* $f_\alpha^* \in \mathfrak{C}$. *Then for any* $\epsilon > 0$,$f_\alpha^{*-1}((r_\alpha - \epsilon, r_\alpha + \epsilon)) \subset (f_\alpha^{-1}((r_\alpha - \epsilon, r_\alpha + \epsilon)))^*$

Proof. Let $P_3 \in f_\alpha^{*-1}((r_\alpha - \epsilon, r_\alpha + \epsilon))$,then $f_\alpha^*(P_3) \in (r_\alpha - \epsilon, r_\alpha + \epsilon)$. If $P_3 = k(P_1) = P_1$ for all P_1 in $\mathbb{P}_X$. Since $f_\alpha(P_1) = f_\alpha^*(P_3)$,so P_1 is in $f_\alpha^{-1}((r_\alpha - \epsilon, r_\alpha + \epsilon))$,thus $P_3 \in (f_\alpha^{-1}(r_\alpha - \epsilon, r_\alpha + \epsilon))^*$. If $P_3 = \{w_k^{P_i}\}^*$ in Y_P,then $\lim\{f_\alpha(w_k^{P_i})\} = f_\alpha^*(P_3) \in (r_\alpha - \epsilon, r_\alpha + \epsilon)$. This implies that $\{w_k^{P_i}\}$ is eventually in $f_\alpha^{-1}((r_\alpha - \epsilon, r_\alpha + \epsilon))$,thus $\{w_k^{P_i}\}^*$ is in $(f_\alpha^{-1}(r_\alpha - \epsilon, r_\alpha + \epsilon))^*$. □

Corollary 2.32. *For a* $\mathfrak{C}$ *net* $\{P_i\}$ *in* $\mathbb{P}_X^*$,*let* $r_\alpha = \lim\{f_\alpha^*(P_i)\}$ *for each* $f_\alpha^* \in \mathfrak{C}$. *Then for any* $\epsilon > 0$,$f_\alpha^{*-1}((r_\alpha - \epsilon, r_\alpha + \epsilon)) \in \mathcal{E}$ *and* $f_\alpha^{-1}((r_\alpha - \epsilon, r_\alpha + \epsilon)) \in \mathcal{L}$

Proof. It is clear that $f_\alpha^{*-1}((r_\alpha - \epsilon, r_\alpha + \epsilon)) \in \mathcal{E}$. By Proposition 2.31,$\{P_i\}$ is eventually in $(f_\alpha^{-1}(r_\alpha - \epsilon, r_\alpha + \epsilon))^*$,thus ,$f_\alpha^{-1}((r_\alpha - \epsilon, r_\alpha + \epsilon)) \in \mathcal{L}$. □

Proposition 2.33. $\mathcal{E}$ *and* $\mathcal{L}$ *are intuitionistic fuzzy centred open filter on* $\mathbb{P}_X^*$ *and* $\mathbb{P}_X$ *respectively.*

Proof. From the proof of Proposition 2.23 and the Corollary 2.32,it is clear that $\mathcal{E}$ is an intuitionistic fuzzy centred filter on $\mathbb{P}_X^*$. By the corollary 2.32,$\mathcal{L} \neq \varnothing$. If U and V are intuitionsitic fuzzy centred open sets in $\mathcal{L}$,then U^* and V^* are in $\mathcal{E}$. Since $(U \cap V)^* = U^* \cap V^*$ and $U^* \cap V^* \in \mathcal{E}$,thus $U \cap V \in \mathcal{L}$. If W is an intuitionsitic fuzzy centred open and $W \supset O$,then $W^* \supset O^*$. This implies $W^* \in \mathcal{E}$ and therefore,$W \in \mathcal{L}$. □

Proposition 2.34. *The $\mathfrak{C}$ net $\{P_i\}$ converges in $\mathbb{P}_X^*$.*

Proof. Let $\{w_k\}$ be the intuitionistic fuzzy centred net based on $\mathcal{L}$. Since for any $\alpha \in \Lambda$ and $\epsilon > 0, f_\alpha^{-1}((r_\alpha - \epsilon, r_\alpha + \epsilon))$ is in $\mathcal{L}$,where $r_\alpha = \lim\{f_\alpha^*(P_i)\}$. Thus, $\{f_\alpha(w_k)\}$ converges to r_α for all $\alpha \in \Lambda$;that is $\{w_k\}$ is a $C^*(\mathbb{P}_X)$ net. Since the intuitionistic fuzzy centred open filter $\mathbb{F}_{\{w_k\}}$ induced by the $C^*(\mathbb{P}_X)$ net $\{w_k\}$ is exactly $\mathcal{L}$,so if $\{w_k^{P_k}\}$ is the intuitionistic fuzzy centred net based on $\mathbb{F}_{\{w_k\}}$,then $\{w_k\} = \{w_k^{P_k}\}$.

Case 1: If $\{w_k\}$ converges to an intuitionistic fuzzy centred point P in $\mathbb{P}_X$. Let U^* be an intuitionistic fuzzy open set in $\mathfrak{B}$ containing $k(P)$,then P is in U and U is an intuitionistic fuzzy centred open in $\mathbb{P}_X$. Since $\{w_k\}$ converges to P,By Definition 2.21 implies that U is in $\mathcal{L}$ and therefore,U^* is in $\mathcal{E}$. This implies that $\{P_i\}$ converges to $k(P)$ in $\mathbb{P}_X^*$.

Case 2: If $\{w_k\}$ does not converges in $\mathbb{P}_X$,then $\{w_k\}^* = \{w_k^{P_k}\}^*$ is in Y_P. For any U^* in $\mathfrak{B}$ containing $\{w_k^{P_k}\}$,$\{w_k^{P_k}\}$ is in eventually U in $\mathbb{P}_X$,the Definition 2.22 implies that U is in $\mathcal{L}$ and therefore U^* is in $\mathcal{E}$. Thus $\{P_i\}$ converges to $\{w_k^{P_k}\}^* = \{w_k\}^*$ in $\mathbb{P}_X^*$.

□

Proposition 2.35. $\left(\mathbb{P}_X^*, k\right)$ *is an intuitionsitic fuzzy centred compactification of* $\mathbb{P}_X$

Proof. Since $\mathfrak{C}$ is a collection of bounded real valued continuous functions on $\mathbb{P}_X^*$ and every $\mathfrak{C}$ net $\{P_i\}$ converges in $\mathbb{P}_X^*$. By Definition 2.18 implies that $\mathbb{P}_X^*$ is intuitionistic fuzzy centred compact. Hence by Proposition 2.30 implies that $\left(\mathbb{P}_X^*, k\right)$ is an intuitionsitic fuzzy centred compactification of $\mathbb{P}_X$. □

Proposition 2.36. *Let $C\left(\mathbb{P}_X^*\right)$ be the set of all real continuous functions on $\mathbb{P}_X^*$. Then $C\left(\mathbb{P}_X^*\right) = \mathfrak{C} = \{f_\alpha^* | f_\alpha \in C^*(\mathbb{P}_X)\}$.*

Proof. Let $g \in C\left(\mathbb{P}_X^*\right)$. Since $\mathbb{P}_X^*$ is intuitionistic fuzzy centred compact, $g \circ k \in C\left(\mathbb{P}_X^*\right)$. By Proposition 2.27,2.29 and the continuity of g, we have $(g \circ k)^*(\{w_k^{P_i}\}^*) = \lim\{(g \circ k)(w_k^{P_i})\} = \lim\{g(k(w_k^{P_i}))\} = g(\lim\{k(w_k^{P_i})\}) = g(\{w_k^{P_i}\}^*)$ for all $\{w_k^{P_i}\}$ in Y_P and $(g \circ k)^*(k(P)) = (g \circ k)^*(P) = g(k(P))$ for all P in $\mathbb{P}_X$,hence $C\left(\mathbb{P}_X^*\right) \subset \mathfrak{C} = \{f_\alpha^* | f_\alpha \in C^*(\mathbb{P}_X)\}$. □

References

[1] M. K. Uma, E. Roja and G. Balasubramanian, The method of centred systems in fuzzy topological spaces, J. Fuzzy Math. 15(4) (2007) 783–789.

[2] H. J. Wu and W. Wu, A net and open-filter process of compactification and the Stone Čech, Wallman compactifications, Topology Appl. 153 (2006) 1291–1301.

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