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Rough sets and its applications in a computer network

A. M. Kozae, S. A. El-Sheikh, E. H. Aly, M. Hosny

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Abstract. The main aim of rough set is reducing the boundary region by increasing the lower approximation and decreasing the upper approximation. So in this paper, four different new methods are proposed to reduce the boundary region. Moreover, the applied examples in network connectivity devices, network cables, network topologies and viruses are introduced by applying the current methods to illustrate the concepts in a friendly way.

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Corresponding Author: M. Hosny (moona_hosny@yahoo.com)

1. Introduction

Rough set theory had been proposed by Pawlak [16] in the early of 1982. Rough set theory has achieved a large amount of applications in various real-life fields, like economics, medical diagnosis, biochemistry, environmental science, biology, chemistry, psychology, conflict analysis, medicine, pharmacology, banking, market research, engineering, speech recognition, material science, information analysis, data analysis, data mining, linguistics, networking and other fields can be found in [9, 15, 22].

The standard rough set theory starts from an equivalence relation. The theory is a new mathematical tool to deal with vagueness and imperfect knowledge. It is dealing with vagueness(ambiguous) of the set by using the concept of the lower and upper approximations [16]. The set has the same lower and upper approximations, called crisp (exact) set, otherwise known as rough (inexact) set. Therefore, the boundary region is defined as the difference between the upper and lower approximations, and then the accuracy of the set or ambiguous depending on the boundary region is empty or not respectively. Nonempty boundary region of a set means that our knowledge about the set is not sufficient to define the set precisely. The main aim of rough set is reducing the boundary region by increasing the lower approximation and decreasing the upper approximation.

In this paper, we present some definitions in Section 2. The aim of Section 3 is to use the notion of the after composed set and after -c composed set [1] to define the lower and upper approximations of any set with respect to any relation. The present method reduces the boundary region by increasing the lower approximation and decreasing the upper approximation with the comparison of Abo Khadra's method [1]. The lower and upper approximations satisfies the properties of Pawalk's spaces, and that can be considered as one of the differences between the present generalization and the other generalizations such as [2, 7] and Yao's space [18, 19]. Although they used general binary relation but they added some conditions to satisfy the properties of Pawlak's space. Moreover, an application of rough sets theory in network medium is presented. The goal of Section 4 is to use the intersection of the after set and the fore set to generalize Yao's approximations [20]. Moreover, an applied example of rough sets theory in network cable is introduced. In Section 5 a new method is used to define the lower and upper approximations of any set with respect to any relation. The present method is compared to Abo–Tabl's method [2] and shown to be more general. Furthermore, an application of rough sets theory in network topologies is presented. The main purpose of Section 6 is to use a subbase of filter to define the lower and upper approximations of any set with respect to any relation. The current approximations are better than Kozae's approximation [8] and Yao's approximation [21] because it decreases the boundary region by increasing the lower approximation and decreasing the upper approximation. Moreover, an application of rough sets theory in viruses is introduced.

2. Preliminaries

In this section, definitions of the lower (upper) approximations, boundary region, after set and fore set are presented.

Definition 2.1 ([1, 2, 4])**.** If R is a binary relation on X, then
(1) The after set of $x \in X$ denoted by xR,where $xR = \{y : xRy\}$.
(2) The fore set of $x \in X$ denoted byRx, where $Rx = \{y : yRx\}$.
(3) The class $S = \{xR : x \in X\}$ is a subbase for the topology τ.
(4) The after-fore set of $x \in X$ denoted by RxR, where $RxR = Rx \cap xR$
(5) The minimal right set denoted by $< x > R$, where $< x > R = \cap\{pR : x \in pR\}$
(6) The minimal left set denoted by $R < x >$, where $R < x >= \cap\{Rp : x \in Rp\}$.

Definition 2.2 ([13])**.** A class $\{A_i\}$ of sets is said to have the finite intersection property if every subclass $A_{i_1}, ..., A_{i_m}$ has a non-empty intersection, i.e. $A_{i_1} \cap ... \cap A_{i_m} \neq \varnothing$.

Definition 2.3 ([13])**.** A subfamily $\mathfrak{F}$ of $P(X)$ is called a filter on X if:
(1) $\varnothing \notin \mathfrak{F}$
(2) If $A_1, A_2 \in \mathfrak{F}$, then $A_1 \cap A_2 \in \mathfrak{F}$
(3) If $A \in \mathfrak{F}$ and $A \subseteq B \subseteq X$, then $B \in \mathfrak{F}$.
The first two properties imply that a filter has the finite intersection property.

Definition 2.4 ([13])**.** A subset $\mathfrak{B}$ of $P(X)$ is called a filter base if:
(1) $\varnothing \notin \mathfrak{B}$
(2) If $B_1, B_2 \in \mathfrak{B}$, then $\exists B_3 \subseteq \mathfrak{B} : B_3 \subseteq B_1 \cap B_2$.
A filter base can be turned into a filter by including all sets of $P(X)$ which contains a set of,i.e. $\mathfrak{F} = \{A \in P(X) : A \supseteq B, B \in \mathfrak{B}\}$.

Definition 2.5 ([13])**.** Let $\xi \subseteq P(X)$. Then ξ is called a filter subbasis on X if it satisfies the finite intersection property.

The original rough set theory introduced by Pawlak was based on an equivalence relation R on a finite universe X. In the approximation space (X, R), he considered two operators, the lower and upper approximations of subsets. Let $A \subseteq X$

$$\underline{apr}(A) = \{x \in X : [x]_R \subseteq A\}$$
$$\overline{apr}(A) = \{x \in X : [x]_R \cap A \neq \varnothing\}.$$

Boundary, positive and negative regions are also defined:

$$BN_R(A) = \overline{R}(A) - \underline{R}(A)$$
$$POS_R(A) = \underline{R}(A)$$
$$NEG_R(A) = X - \overline{R}(A).$$

There have been some extensions on Pawlak's original concept. One extension is to replace the equivalence relation by an arbitrary binary relation [1, 2, 8, 21]. If R is a binary general relation on X, then the pair (X, R) is called a generalized approximation space in briefly "GAS" [1]. For example of this extension:
Yao [21] introduced and investigated the notion of generalized approximation space by using the after sets concepts as follows:

$$\underline{R}(A) = \cup_{xR \subseteq A}\, xR \tag{2.1}$$
$$\overline{R}(A) = \underline{R}(A^{'})^{'}$$

Yao [20] introduced and investigated another notion of generalized approximation space by using the after sets concept as follows:

$$\underline{R}(A) = \{x \in X : xR \subseteq A\}, \tag{2.2}$$
$$\overline{R}(A) = \{x \in X : xR \cap A \neq \varnothing\}.$$

Obviously, if R is an equivalence relation, then $xR = [x]_R$. In addition, these definitions are equivalent to the original Pawlak's definitions.
Abo-Tabl [2] introduced and investigated another notion of generalized approximation space by using the after set concept as follows:

$$R_*(A) = \{x \in X :< x > R \subseteq A\}, \tag{2.3}$$
$$R^*(A) = \{x \in X :< x > R \cap A \neq \varnothing\}.$$

Obviously, if R is an equivalence relation, then $< x > R = [x]_R$. In addition, these definitions are equivalent to the original Pawlak's definitions. The other direction is to study rough set via topological method [1, 3, 6, 8]. For example
Kozae et al. [8], calculate the lower and the upper approximation by using the subbase $\{xR : x \in X\}$ of the topology τ as follows:

$$\underline{R}_\tau(A) = \cup\{G \in \tau : G \subseteq A\} \tag{2.4}$$

$$\overline{R}_{\tau}(A) = \cap\{H \in \tau' : A \subseteq H\}.$$

Abo Khadra et al. [1], introduced and investigated another notion of generalized approximation space by using the after composed set and after -c composed set. Let (X, R) be a "GAS" and $A \subseteq X$, then the lower (respectively upper) approximation is given by:

$$\underline{R}_{\tau_R}(A) = \cup\{G \in \tau_R : G \subseteq A\}; \tag{2.5}$$

$$\overline{R}_{\tau'_R}(A) = \cap\{F \in \tau'_R : A \subseteq F\}$$

where $\tau_R = \{A \subseteq X : \forall x \in A, xR \subseteq A\}(resp.\tau'_R = \{A \subseteq X : \forall x \in A, Rx \subseteq A\})$ is the class of all after composed (respectively after-c composed) sets.

Proposition 2.6 ([1])**.** *Let R be a binary relation on X. Then the class τ_R (respectively τ'_R) forms a topology on X.*

Theorem 2.7 ([1])**.** *Let R be a binary relation on X. Then τ_R is the complement topology of τ'_R and vice versa.*

3. Rough sets and common network connectivity devices

The aim of this section is to propose a new method to define the lower and upper approximations of any set with respect to any relation by using the after-fore composed set. The current method reduces the boundary region by increasing the lower approximation and decreasing the upper approximation with the comparison of Abo Khadra's method [1]. At the end of this section an applied example of the current method in the network connectivity devices is introduced.

Definition 3.1. Let R be a binary relation on $X, A \subseteq X$. Then A is called the after-fore composed set if A contains all the after-fore sets for all its elements, i.e. $\forall x \in A, RxR \subseteq A$.
The class of all after-fore composed sets is defined by the class

$$\tau^*_R = \{A \subseteq X : \forall x \in A, RxR \subseteq A\}.$$

Proposition 3.2. *Let R be a binary relation on $X, A \subseteq X$. Then the class τ^*_R forms a topology on X.*

Proof. (1) Clearly X and $\varnothing$ are after-fore composed sets, then X and $\varnothing \in \tau^*_R$.

(2) Let $A, B \in \tau^*_R$, and let $x \in A \cap B$. Then $x \in A$ and $x \in B$
$\Rightarrow RxR \subseteq A$ *and* $RxR \subseteq B$
$\Rightarrow RxR \subseteq A \cap B$
$\Rightarrow A \cap B \in \tau^*_R$.

(3) Let $A_i \in \tau^*_R$ $\forall i \in I$ and $x \in \cup_{i\in I} A_i$. Then $\exists i_0 \in I$ such that $x \in A_{i_0} \subseteq \cup_{i\in I} A_i$
$\Rightarrow RxR \subseteq A_{i_0} \subseteq \cup_{i\in I} A_i$
$\Rightarrow \cup_{i\in I} A_i \in \tau^*_R$.
From 1), 2) and 3) τ^*_R is a topology on X. □

Proposition 3.3. *Let R be a binary relation on $X, A \subseteq X$. Then if $A \in \tau^*_R$, then $A' \in \tau^*_R$*

Proof. Let $A \in \tau_R^*$. Then $RaR \subseteq A\ \forall a \in A$.
Let $b \in A^{'}$. Then there are two different cases :

(1) If $RbR \cap A \neq \varnothing \Rightarrow \exists c \in A$ and $c \in RbR$
$\Rightarrow \exists c \in A$ and $b \in RcR$ such that $b \notin A$ which is a contradiction.

(2) If $RbR \subseteq A^{'}$, then $A^{'} \in \tau_R^*$. □

Definition 3.4. Let R be a binary relation on $X, A \subseteq X$. Then the lower (respectively upper) approximations is given by:
$\underline{R}_{\tau_R^*}(A) = \cup\{G \in \tau_R^* : G \subseteq A\}$;
$\overline{R}_{\tau_R^*}(A) = \cap\{F \in \tau_R^* : A \subseteq F\}$.

It is easy to notice that the lower $\underline{R}_{\tau_R^*}(A)$ (respectively upper $\overline{R}_{\tau_R^*}(A)$) approximations of a subset A is exactly the interior A^0 (respectively closure $\overline{A}$) of A in the topology $\tau_R^*(A)$.

Proposition 3.5. *Let R be a binary relation on $X, A, B \subseteq X$. Then*

(1) $\underline{R}_{\tau_R^*}(A) \subseteq A \subseteq \overline{R}_{\tau_R^*}(A)$
(2) $\underline{R}_{\tau_R^*}(X) = X$ *and* $\underline{R}_{\tau_R^*}(\varnothing) = \varnothing$
(3) $A \subseteq B \Rightarrow \underline{R}_{\tau_R^*}(A) \subseteq \underline{R}_{\tau_R^*}(B)$
(4) $\underline{R}_{\tau_R^*}(A \cap B) = \underline{R}_{\tau_R^*}(A) \cap \underline{R}_{\tau_R^*}(B)$
(5) $\underline{R}_{\tau_R^*}(A \cup B) \supseteq \underline{R}_{\tau_R^*}(A) \cup \underline{R}_{\tau_R^*}(B)$
(6) $\underline{R}_{\tau_R^*}(A) = \underline{R}_{\tau_R^*}(\underline{R}_{\tau_R^*}(A))$
(7) $\underline{R}_{\tau_R^*}(\underline{R}_{\tau_R^*}(A)) \subseteq \overline{R}_{\tau_R^*}(\underline{R}_{\tau_R^*}(A))$ *and* $\underline{R}_{\tau_R^*}(\overline{R}_{\tau_R^*}(A)) \subseteq \overline{R}_{(\tau_R^*)}(A)$
(8) $\underline{R}_{\tau_R^*}(A) = [\overline{R}_{\tau_R^*}(A^{'})]^{'}, A^{'}$ *is the complement of* A

Proof. The proof is straightforward from Definition 3.4. □

It should be noted that in the present generalization the above properties satisfies without any condition on binary relation R, but in the other generalizations such as [7, 23] and Yao space [18, 19] this properties satisfied with adding some conditions on binary relation R.

The relation between the topology which was generated by Abo Khadra's method [1] and topology which is generated by the current method is presented in following proposition.

Proposition 3.6. *Let R be a binary relation on X. Then the topology τ_R^* is finer than the both topology $\tau_R, \tau_R^{'}$.*

Proof. Let $A \in \tau_R$. Then $xR \subseteq A\ \forall x \in A$
$\Rightarrow RxR \subseteq xR \subseteq A\ \forall x \in A$
$\Rightarrow A \in \tau_R^*$. Hence $\tau_R \subseteq \tau_R^*$. Similarly we can prove that $\tau_R^{'} \subseteq \tau_R^*$ and hence the topology τ_R^* is finer than the both topology $\tau_R, \tau_R^{'}$. □

Proposition 3.7. *Let R be a binary relation on $X, A \subseteq X$. Then*

(1) $\underline{R}_{\tau_R}(A) \subseteq \underline{R}_{\tau_R^*}(A)$
(2) $\overline{R}_{\tau_R^*}(A) \subseteq \overline{R}_{\tau_R^{'}}(A)$
(3) $BN_{\tau_R^*}(A) \subseteq BN_{\tau_R}(A)$

Proof. (1) Let $y \in \underline{R}_{\tau_R}(A) = \cup\{G \in \tau_R : G \subseteq A\}$. Then $\exists\, G \in \tau_R$ such that $y \in G \subseteq A$, and since $\tau_R \subseteq \tau_R^*$
$\Rightarrow G \in \tau_R^*$ such that $y \in G \subseteq A$
$\Rightarrow y \in \underline{R}_{\tau_R^*}(A)$ and hence $\underline{R}_{\tau_R}(A) \subseteq \underline{R}_{\tau_R^*}(A)$

(2) Let $y \notin \overline{R}_{\tau_R'}(A)$. Then $\exists\, O_y \in \tau_R'$ such that $O_y \cap A = \varnothing$, and since $\tau_R' \subseteq \tau_R^*$
$\Rightarrow \exists\, O_y \in \tau_R^*$ such that $O_y \cap A = \varnothing$
$\Rightarrow y \notin \overline{R}_{\tau_R^*}(A)$ and hence $\overline{R}_{\tau_R^*}(A) \subseteq \overline{R}_{\tau_R'}(A)$

(3) $BN_{\tau_R^*}(A) = \overline{R}_{\tau_R^*}(A) - \underline{R}_{\tau_R^*}(A)$
$= \overline{R}_{\tau_R^*}(A) \cap (\underline{R}_{\tau_R^*}(A))'$
$= \overline{R}_{\tau_R^*}(A) \cap (\overline{R}_{\tau_R^*}(A'))$
$\subseteq \overline{R}_{\tau_R'}(A) \cap \overline{R}_{\tau_R'}(A')$
$= \overline{R}_{\tau_R'}(A) - (\overline{R}_{\tau_R'}(A'))' = \overline{R}_{\tau_R'}(A) - \underline{R}_{\tau_R}(A)$.
Hence $BN_{\tau_R^*}(A) \subseteq BN_{\tau_R}(A)$. □

It is noted from Proposition 3.7 that the Definition 3.4 reduces the boundary region by increasing the lower approximation and decreasing the upper approximation with the comparison of Abo Khadra's method [1].

Networks deploy many different hardware components, known as devices. They are connected as resources for network users. We are already familiar with PCs, printers and other office equipment. Other equipment that we may not be familiar with are network interface card, hubs, switches, repeaters, bridges, routers and gateways [5, 11]. An applied example of the current method in the network connectivity devices is introduced.

Example 3.8. Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of four network connectivity devices and $A = \{A_1, A_2, A_3\}$ be the attributes of network connectivity devices, where
x_1 is a hub, x_2 switch, x_3 bridge, x_4 router,
A_1 = Connection $=\{a_1, b_1, c_1, d_1, e_1, f_1, g_1\}$, where
a_1= Connect all computers on each side of the network,
b_1= Connect two segments of the same LAN,
c_1= Connect two local area networks (LANs),
d_1= Connect two similar network,
e_1= Connect two dissimilar network,
f_1= Divide a busy network into two segments and
g_1= Select the best path to route a message based on the destination address and origin.
A_2 = Read or cannot read the addresses$=\{a_2, b_2, c_2, d_2\}$, where
a_2= Read the addresses of all computers on each side of the network,
b_2= Cannot read the addresses of any computer in the network,
c_2= Read the addresses of bridges on the network and
e_2= Read the addresses of other routers on the network.
A_3 = Cost $=\{a_3, b_3, c_3\}$, where a_3 = 100 L.E., b_3 = 300 L.E., c_3 = 500 L.E. and d_3 = 800 L.E.

Table 1. Attributes network connectivity devices

X/A	A_1	A_2	A_3
$\{x_1\}$	$\{a_1, b_1\}$	$\{b_2\}$	$\{a_3\}$
$\{x_2\}$	$\{a_1, b_1\}$	$\{a_2\}$	$\{b_3\}$
$\{x_3\}$	$\{a_1, b_1, c_1, d_1, f_1\}$	$\{a_2\}$	$\{c_3\}$
$\{x_4\}$	$\{a_1, b_1, c_1, d_1, e_1, g_1\}$	$\{a_2, c_2, d_2\}$	$\{d_3\}$

Let R be a general relation as follows:
$x_1 R x_2 \Leftrightarrow A(x_1) \subseteq A(x_2), x_1, x_2 \in X$.
For the first attribute A_1, we get
$R_{A_1} = \triangle \cup \{(x_1, x_2), (x_1, x_3), (x_1, x_4), (x_2, x_1), (x_2, x_3), (x_2, x_4)\}$;
$x_1 R_{A_1} = x_2 R_{A_1} = X, x_3 R_{A_1} = \{x_3\}, x_4 R_{A_1} = \{x_4\}$,
$R_{A_1} x_1 = R_{A_1} x_2 = \{x_1, x_2\}, R_{A_1} x_3 = \{x_1, x_2, x_3\}, R_{A_1} x_4 = \{x_1, x_2, x_4\}$,
$R_{A_1} x_1 R_{A_1} = R_{A_1} x_2 R_{A_1} = \{x_1, x_2\}, R_{A_1} x_3 R_{A_1} = \{x_3\}, R_{A_1} x_4 R_{A_1} = \{x_4\}$.
$\tau_R = \{X, \varnothing, \{x_3\}, x_4, \{x_3, x_4\}, \{x_3, x_4\}\}$,
$\tau_R^{'} = \{X, \varnothing, \{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}\}$,
$\tau_R^{*} = \{X, \varnothing, \{x_3\}, \{x_4\}, \{x_1, x_2\}, \{x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}\}$.

Table 2. Comparison between Abo Khadra's [1] method and the present method

A	Abo Khadra's [1] method			The present method		
	$\underline{R}_{\tau_R}(A)$	$\overline{R}_{\tau'_R}(A)$	$BN_{\tau_R}(A)$	$\underline{R}_{\tau^*_R}(A)$	$\overline{R}_{\tau^*_R}(A)$	$BN_{\tau^*_R}(A)$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{x_1\}$	$\emptyset$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\emptyset$	$\{x_1, x_2\}$	$\{x_1, x_2\}$
$\{x_2\}$	$\emptyset$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\emptyset$	$\{x_1, x_2\}$	$\{x_1, x_2\}$
$\{x_3\}$	$\{x_3\}$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2\}$	$\{x_3\}$	$\{x_3\}$	$\emptyset$
$\{x_4\}$	$\{x_4\}$	$\{x_4\}$	$\emptyset$	$\{x_4\}$	$\{x_4\}$	$\emptyset$
$\{x_1, x_2\}$	$\emptyset$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\emptyset$
$\{x_1, x_3\}$	$\{x_3\}$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2\}$	$\{x_3\}$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2\}$
$\{x_1, x_4\}$	$\{x_4\}$	$\{x_1, x_2, x_4\}$	$\{x_1, x_2\}$	$\{x_4\}$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2\}$
$\{x_2, x_3\}$	$\{x_3\}$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2\}$	$\{x_3\}$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2\}$
$\{x_2, x_4\}$	$\{x_4\}$	$\{x_1, x_2, x_4\}$	$\{x_1, x_2\}$	$\{x_4\}$	$\{x_1, x_2, x_4\}$	$\{x_1, x_2\}$
$\{x_3, x_4\}$	$\{x_3, x_4\}$	X	$\{x_1, x_2\}$	$\{x_3, x_4\}$	$\{x_3, x_4\}$	$\emptyset$
$\{x_1, x_2, x_3\}$	$\{x_3\}$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2\}$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2, x_3\}$	$\emptyset$
$\{x_1, x_2, x_4\}$	$\{x_4\}$	$\{x_1, x_2, x_4\}$	$\{x_1, x_2\}$	$\{x_1, x_2, x_4\}$	$\{x_1, x_2, x_4\}$	$\emptyset$
$\{x_1, x_3, x_4\}$	$\{x_3, x_4\}$	X	$\{x_1, x_2\}$	$\{x_3, x_4\}$	X	$\{x_1, x_2\}$
$\{x_2, x_3, x_4\}$	$\{x_3, x_4\}$	X	$\{x_1, x_2\}$	$\{x_3, x_4\}$	X	$\{x_1, x_2\}$
X	X	X	$\emptyset$	X	X	$\emptyset$

The current approximations are then better than Abo Khadra's approximation [1] because the current approximation decreases the boundary region by increasing the lower approximation and decreasing the upper approximation.

4. Rough sets and cables network medium

Yao [20] defined distinct approximation operators by using the after sets and fore sets concepts as follows:

$$\underline{R}(A) = \{x \in X : xR \subseteq A\} \tag{4.1}$$

$$\overline{R}(A) = \{x \in X : xR \cap A \neq \phi\}$$

$$\underline{R}(A) = \{x \in X : Rx \subseteq A\} \tag{4.2}$$

$$\overline{R}(A) = \{x \in X : Rx \cap A \neq \phi\}$$

$$\underline{R}(A) = \{x \in X : (xR \cup Rx) \subseteq A\} \tag{4.3}$$

$$\overline{R}(A) = \{x \in X : (xR \cup Rx) \cap A \neq \phi\}$$

$$\underline{R}(A) = \{x \in X : RxR \subseteq A\} \tag{4.4}$$

$$\overline{R}(A) = \{x \in X : RxR \cap A \neq \phi\},$$

where $RxR = xR \cap Rx$.

In this section, the definition (4.4) is used to show that this definition reduces the boundary region by increasing the lower approximation and decreasing the upper approximation with the comparison of the methods (4.1), (4.2) and (4.3). This definition satisfies some properties in analogues of Pawalk's properties. Furthermore, the necessary and sufficient condition that can be stated on a binary relation R is proposed in order to have the pair of lower (upper) approximations satisfying the Kuratowski's axioms of interior(closure) operators. Moreover, an application of rough sets in network cable is introduced.

Proposition 4.1. *Let R be any binary relation on a non-empty set X and $A, B \subseteq X$. Then the following conditions hold:*

(1) $\underline{R}(A) = [\overline{R}(A')]'$
(2) $\underline{R}(X) = X$
(3) $A \subseteq B \Rightarrow \underline{R}(A) \subseteq \underline{R}(B)$
(4) $\underline{R}(A \cap B) = \underline{R}(A) \cap \underline{R}(B)$
(5) $\underline{R}(A \cup B) \supseteq \underline{R}(A) \cup \underline{R}(B)$
(6) $A \subseteq \underline{R}(\overline{R}(A))$

Proof. (1) $[\overline{R}(A')]' = (\{x \in X : RxR \subseteq A'\})'$
$= \{x \in X : RxR \cap A' = \phi\}$
$= \{x \in X : RxR \subseteq A\}$
$= \underline{R}(A)$

(2) Let $x \in X$. Then $RxR \subseteq X$
$\Rightarrow x \in \underline{R}(X)$.
$\Rightarrow X \subseteq \underline{R}(X)$
and since $\underline{R}(X) \subseteq X$, then $\underline{R}(X) = X$

(3) Let $x \in \underline{R}(A)$. Then $RxR \subseteq A$. Also, $A \subseteq B$
$\Rightarrow RxR \subseteq B$
$\Rightarrow x \in \underline{R}(B)$. Hence $\underline{R}(A) \subseteq \underline{R}(B)$

(4) $\underline{R}(A\cap B)\subseteq \underline{R}(A)\cap\underline{R}(B)$(by 3). Let $x\in\underline{R}(A)\cap\underline{R}(B)$. Then $RxR\subseteq A,B$
$\Rightarrow RxR\subseteq A\cap B$
$\Rightarrow x\in\underline{R}(A\cap B)$
and hence $\underline{R}(A\cap B)=\underline{R}(A)\cap\underline{R}(B)$
(5) The proof is immediately follows from (3)
(6) Let $x\in A$. *Since* $\forall y\in RxR$, then $x\in RyR$
$\Rightarrow x\in RyR\cap A\Rightarrow y\in\overline{R}(A), i.e.\forall y\in RxR$
$\Rightarrow y\in\overline{R}(A)$
$\Rightarrow y\in\overline{R}(A)$
$\Rightarrow x\in\underline{R}(\overline{R}(A))$. *Hence* $A\subseteq\underline{R}(\overline{R}(A))$.

□

The following example shows that the following conditions do not hold generally $\forall\ A\subseteq X$

(1) $\underline{R}(\phi)=\phi$
(2) $\underline{R}(A)\subseteq A$
(3) $\underline{R}(\underline{R}(A))=\underline{R}(A)$

Example 4.2. Let $X=\{a,b,c,d\}$ *and* $R=\{(a,b),(a,d),(b,a),(b,c),(c,d),(d,a)\}$. Then
$aR=\{b,d\}, bR=\{a,c\}, cR=\{d\}, dR=\{a\}, Ra=\{b,d\}, Rb=\{a\}, Rc=\{b\}, Rd=\{a,c\}, RaR=\{b,d\}, RbR=\{a\}, RcR=\varnothing$ and $RdR=\{a\}$.
If we take $A=\phi$, *then* $\underline{R}(\varnothing)=\{c\}$. Hence $\underline{R}(\phi)\neq\phi$.
Also, if we take $A=\{a\}$, then $\underline{R}(A)=\{b,c,d\},\underline{R}(\underline{R}(A))=\{a,c\}$.
Hence $\underline{R}(A)\nsubseteq A$ and $\underline{R}(A)\neq\underline{R}(\underline{R}(A)$.

Proposition 4.3. *For any reflexive relation R on X the following conditions hold.*

(1) $\underline{R}(\varnothing)=\varnothing$
(2) $\underline{R}(A)\subseteq A$

Proof. (1) Since R is a reflexive relation on X, then $x\in RxR\ \forall\ x\in X$. Then $\nexists\, x\in X\ s.t\ RxR\subseteq\varnothing$ hence $\underline{R}(\varnothing)=\varnothing$
(2) Let $x\in\underline{R}(A)$. Then $RxR\subseteq A$. Since R is a reflexive relation on X, then $x\in RxR\ \forall x\in X$. Thus $x\in A$ and so $\underline{R}(A)\subseteq A$.

□

Proposition 4.4. *Let R be a preorder relation on X. Then $\underline{R}(\underline{R}(A))=\underline{R}(A)$.*

Proof. $\underline{R}(\underline{R}(A))\subseteq\underline{R}(A)$ by Proposition 4.3. Let $x\notin\underline{R}(\underline{R}(A))$. Then $RxR\nsubseteq\underline{R}(A)$
$\Rightarrow\ \exists y\in RxR, y\notin\underline{R}(A)$
$\Rightarrow\exists\ y\in RxR, RyR\nsubseteq A$
$\Rightarrow\exists\ y\in RxR\ \exists\ z\in RyR, z\notin A$
$\Rightarrow z\in RxR$(since R is transitive),$z\notin A$
$\Rightarrow RxR\nsubseteq A$. Hence $x\notin\underline{R}(A)$. Then $\underline{R}(\underline{R}(A))\supseteq\underline{R}(A)$ and consequently $\underline{R}(\underline{R}(A))=\underline{R}(A)$. □

Theorem 4.5. *For any preorder relation R on X. The pair of lower and upper approximations (4.4) is a pair of interior and closure operators satisfying Kuratowski's axioms.*

Proof. The proof follows immediately from Propositions 4.1, 4.3 and 4.4. □

The main network medium is network cable. Cable is the medium through which information usually moves from one network device to another. There are several types of cable such as twisted-pair cable, coaxial cable, and optical fiber [12]. An application of rough sets in network cable is introduced in the following example.

Example 4.6. Let $X = \{x_1, x_2, x_3\}$ be a set of three different cables and $A = \{A_1, A_2, A_3, A_4\}$ be the attributes of cables, where
x_1 is a twisted-pair cable, x_2 a coaxial cable, x_3 an optical fiber,
A_1 = The maximum segment length={Less than 185 m, 200-500 m, 80 km} = $\{a_1, b_1, c_1\}$
A_2 = The transmission speed range $=\{10Mb/s - 100Mb/s, 10Mb/s, 40Gb/s\} = \{a_2, b_2, c_2\}$
A_3 = Cost $=\{3L.E./m, 5L.E./m, 100L.E./m\} = \{a_3, b_3, c_3\}$
A_4 = Time of install ={1 minute, 5 minutes, 40 minutes} = $\{a_4, b_4, c_4\}$.

Table 3. Attributes of cables

X/A	A_1	A_2	A_3	A_4
$\{x_1\}$	$\{a_1\}$	$\{a_2\}$	$\{a_3\}$	$\{a_4\}$
$\{x_2\}$	$\{b_1\}$	$\{b_2\}$	$\{b_3\}$	$\{b_4\}$
$\{x_3\}$	$\{c_1\}$	$\{c_2\}$	$\{c_3\}$	$\{c_4\}$

Let R be a general relation as follows:
$x_1 R x_2 \Leftrightarrow A(x_1) \subseteq A(x_2), x_1, x_2 \in X$.
For the first attribute A_1, we get
$R_{A_1} = \triangle \cup \{(x_1, x_2), (x_1, x_3), (x_2, x_3)\}$;
$x_1 R_{A_1} = X, x_2 R_{A_1} = \{x_2, x_3\}, x_3 R_{A_1} = \{x_3\}$,
$R_{A_1} x_1 = x_1, R_{A_1} x_2 = \{x_1, x_2\}, R_{A_1} x_3 = X$,
$x_1 R_{A_1} x_1 = x_1, x_2 R_{A_1} x_2 = \{x_1, x_2\}, x_3 R_{A_1} x_3 = X$.

Table 4. Comparison between the method (4.1) and the method (4.4)

	The method (4.1)			The method (4.4)		
A	$\underline{R}(A)$	$\overline{R}(A)$	$BN(A)$	$\underline{R}(A)$	$\overline{R}(A)$	$BN(A)$
$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
$\{x_1\}$	$\varnothing$	$\{x_1\}$	$\{x_1\}$	$\{x_1\}$	$\{x_1\}$	$\varnothing$
$\{x_2\}$	$\varnothing$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{x_2\}$	$\{x_2\}$	$\varnothing$
$\{x_3\}$	$\{x_3\}$	X	$\{x_1, x_2\}$	$\{x_3\}$	$\{x_3\}$	$\varnothing$
$\{x_1, x_2\}$	$\varnothing$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\varnothing$
$\{x_1, x_3\}$	$\{x_3\}$	X	$\{x_1, x_2\}$	$\{x_1, x_3\}$	$\{x_1, x_3\}$	$\varnothing$
$\{x_2, x_3\}$	$\{x_2, x_3\}$	X	$\{x_1\}$	$\{x_2, x_3\}$	$\{x_2, x_3\}$	$\varnothing$
X	X	X	$\varnothing$	X	X	$\varnothing$

The approximations (4.4) are better than the approximations (4.1) because the approximation (4.4) decreases the boundary region by increasing the lower approximation and decreasing the upper approximation.

5. Rough sets and network topologies

The goal of this section is to introduce a new method to define the lower and upper approximations of any set with respect to any relation. The present method reduces the boundary region by increasing the lower approximation and decreasing the upper approximation with the comparison of Abo-Tabl's method [2]. At the end of this section an applied example of rough sets in the main types of physical topologies is presented.

We denote, $< x > R \cap R < x >$ by $R < x > R$, i.e
$< x > R \cap R < x >= R < x > R$

Definition 5.1. Let R be any binary relation on X. The lower and upper approximations on X according to R are defined as:

$$R_{**}(A) = \{x \in X : R < x > R \subseteq A\} \text{“\textit{Lower approximation}”} \tag{5.1}$$

$$R^{**}(A) = \{x \in X : R < x > R \cap A \neq \varnothing\} \text{“Upper approximation”}$$

$$BN_{**}(A) = \overline{R}(A) - \underline{R}(A) \text{“Boundary region”}$$

Proposition 5.2. *Let R be any binary relation on a non-empty set X and $A, B \subseteq X$. Then the following conditions hold:*

(1) $R_{**}(A) = [R^{**}(A')]'$
(2) $R_{**}(X) = X$
(3) $A \subseteq B \Rightarrow R_{**}(A) \subseteq R_{**}(B)$
(4) $R_{**}(A \cap B) = R_{**}(A) \cap R_{**}(B)$
(5) $R_{**}R(A \cup B) \supseteq R_{**}(A) \cup R_{**}(B)$
(6) $R_{**}(R_{**}R(A)) \subseteq R_{**}(A)$

Proof. The proof of 1, 2, 3, 4 and 5 are the same as in Proposition 4.1.
(6) Let $x \in R_{**}(A)$. Then $R < x > R \subseteq A$. We want to prove $x \in R_{**}(R_{**}(A))$, i.e.,

$$R < x > R \subseteq R_{**}(A).$$

So, let $y \in R < x > R$. Then $R < y > R \subseteq R < x > R$
$\Rightarrow R < y > R \subseteq A$
$\Rightarrow y \in R_{**}(A)$. Hence $R_{**}(R_{**}(A)) \subseteq R_{**}(A)$. □

It should be noted that the equality in Proposition 5.2 (5) and (6) is not hold in general, also the following conditions do not hold generally $\forall A \subseteq X$

(1) $R_{**}(\phi) = \phi$
(2) $R_{**}(A) \subseteq A$
(3) $A \subseteq R_{**}(R^{**}(A))$
(4) $R^{**}(A) \subseteq R_{**}(R^{**}(A))$

Proposition 5.3. *For any reflexive relation R on X the following conditions hold.*

(1) $R_{**}(\varnothing) = \varnothing$
(2) $R_{**}(A) \subseteq A$

Proof. The proof is the same as in the Proposition 4.3 □

Theorem 5.4. *For any reflexive relation R on X. The pair of lower and upper approximations is the pair of interior and closure operators satisfying Kuratowski's axioms.*

Proof. The proof follows immediately from Propositions 5.2 and 5.3. □

Proposition 5.5. *Let R be any binary relation on a non-empty set X and $A \subseteq X$, then $A \subseteq R_{**}(R^{**}(A))$ in the following cases:*

(1) *If R and R^{-1} are functional relations on X.*
(2) *If R is an irreflexive, symmetric and transitive relation*
(3) *If $R \backslash \Delta = \varnothing$, where Δ is an identity relation*
(4) *If R is an anti-identity relation*

Proof. We prove part 1 and the other parts are similar.

(1) Let $x \in A$. Since R and R^{-1} are functional relation, then
$R < x > R = \{x\}$ *or* $\varnothing \ \forall x \in X$.
If $R < x > R = \{x\}$, then $R < x > R \cap A \neq \varnothing$.
$\Rightarrow x \in R^{**}(A)$
$\Rightarrow \{x\} = R < x > R \subseteq R^{**}(A)$
$\Rightarrow x \in R_{**}(R^{**}(A))$,
and if $R < x > R = \varnothing$, then $R < x > R \subseteq R^{**}(A)$
$\Rightarrow x \in R_{**}(R^{**}(A))$.

□

Lemma 5.6. *If R is an anti-identity relation and $A \subseteq X$, then A is an exact set.*

Proof. Since R is an anti-identity relation, then $R < x > R = \{x\} \forall x \in X$
$\Rightarrow R_{**}(A) = R^{**}(A) = A$
$\Rightarrow BN_{**}(A) = \varnothing$. Hence A is an exact set. □

The relation between Abo–Tabl's approximations [2] and the current approximations is presented in the following proposition.

Proposition 5.7. *Let R be any binary relation on a non-empty set X and $A \subseteq X$. Then*

(1) $R_*(A) \subseteq R_{**}(A)$
(2) $R^{**}(A) \subseteq R^*(A)$
(3) $BN_{**}(A) \subseteq BN_*(A)$

Proof. Straightforward. □

In Computer networking "Topology" refers to the layout or design of the connected devices. Network Topologies can be physical or logical. The physical topology of a network refers to the configuration or the layout of cables, computers and other peripherals. It means the physical design of a network including the devices, location and cable installation. Physical topology should not be confused with logical topology which is the method used to pass the information between the computers. Logical Topology refers to the fact that how data actually transfers in a network as opposed to its design. The main types of physical topologies are bus topology, ring topology, star topology, and mesh topology [5, 11, 24, 25]. An applied example of rough sets in the main types of physical topologies is presented.

Example 5.8. Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of four different network topology and $A = \{A_1, A_2, A_3\}$ be the attributes network topology, where
x_1 is a bus topology, x_2 a ring topology, x_3 a star topology, x_3 a mesh topology
A_1 = The method of transfer data={broadcast, multicast, unicast} $= \{a_1, b_1, c_1\}$,
A_2 =Cable type={twisted pair cable, thin coaxial cable, thick coaxial cable, fiber optic cable} $=\{a_2, b_2, c_2, d_2\}$ and
A_3 = Bandwidth capacity $=\{10Mbit/s, 10 - 100Mbit/s, 10Mbit/s - 40Gbit/s\}$
$= \{a_3, b_3, c_3\}$.

Table 5. Attributes of network topologies

X/A	A_1	A_2	A_3
$\{x_1\}$	$\{a_1\}$	$\{b_2, c_2\}$	$\{a_3\}$
$\{x_2\}$	$\{a_1\}$	$\{c_2, d_2\}$	$\{b_3\}$
$\{x_3\}$	$\{a_1, b_1, c_1\}$	$\{a_2, b_2, c_2, d_2\}$	$\{c_3\}$
$\{x_4\}$	$\{c_1\}$	$\{a_2, b_2, c_2, d_2\}$	$\{c_3\}$

Let R be a general relation as follows:
$x_1 R x_2 \Leftrightarrow A(x_1) \subseteq A(x_2), x_1, x_2 \in X$.
For the first attribute A_1, we get
$R_{A_1} = \triangle \cup \{(x_1, x_2), (x_1, x_3), (x_2, x_1), (x_2, x_3), (x_4, x_3)\}$;
$x_1 R_{A_1} = \{x_1, x_2, x_3\}, x_2 R_{A_1} = \{x_1, x_2, x_3\}, x_3 R_{A_1} = \{x_3\}, x_4 R_{A_1} = \{x_3, x_4\}$,
$R_{A_1} x_1 = \{x_1, x_2\}, R_{A_1} x_2 = \{x_1, x_2\}, R_{A_1} x_3 = X, R_{A_1} x_4 = \{x_4\}$,
$< x_1 > R_{A_1} =< x_2 > R_{A_1} = \{x_1, x_2, x_3\}, < x_3 > R_{A_1} = \{x_3\}$,
$< x_4 > R_{A_1} = \{x_3, x_4\}$,
$R_{A_1} < x_1 >= R_{A_1} < x_2 >= \{x_1, x_2\}, R_{A_1} < x_3 >= X, R_{A_1} < x_4 >= \{x_4\}$,
$R_{A_1} < x_1 > R_{A_1} = R_{A_1} < x_2 > R_{A_1} = \{x_1, x_2\}, R_{A_1} < x_3 > R_{A_1} = \{x_3\}$,
$R_{A_1} < x_4 > R_{A_1} = \{x_4\}$.

TABLE 6. Comparison between Abo–Tabl's method [2] and the present method

A	Abo-Tabl's method [2]			The present method		
	$R_*(A)$	$R^*(A)$	$BN_*(A)$	$R_{**}(A)$	$R^{**}(A)$	$BN_{**}(A)$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{x_1\}$	$\emptyset$	$\{x_1,x_2\}$	$\{x_1,x_2\}$	$\emptyset$	$\{x_1,x_2\}$	$\{x_1,x_2\}$
$\{x_2\}$	$\emptyset$	$\{x_1,x_2\}$	$\{x_1,x_2\}$	$\emptyset$	$\{x_1,x_2\}$	$\{x_1,x_2\}$
$\{x_3\}$	$\{x_3\}$	X	$\{x_1,x_2,x_4\}$	$\{x_3\}$	$\{x_3\}$	$\emptyset$
$\{x_4\}$	$\emptyset$	$\{x_4\}$	$\{x_4\}$	$\{x_4\}$	$\{x_4\}$	$\emptyset$
$\{x_1,x_2\}$	$\emptyset$	$\{x_1,x_2\}$	$\{x_1,x_2\}$	$\{x_1,x_2\}$	$\{x_1,x_2\}$	$\emptyset$
$\{x_1,x_3\}$	$\{x_3\}$	X	$\{x_1,x_2,x_4\}$	$\{x_3\}$	$\{x_1,x_2,x_3\}$	$\{x_1,x_2\}$
$\{x_1,x_4\}$	$\emptyset$	$\{x_1,x_2,x_4\}$	$\{x_1,x_2,x_4\}$	$\{x_4\}$	$\{x_1,x_2,x_3\}$	$\{x_1,x_2\}$
$\{x_2,x_3\}$	$\{x_3\}$	X	$\{x_1,x_2,x_4\}$	$\{x_3\}$	$\{x_1,x_2,x_3\}$	$\{x_1,x_2\}$
$\{x_2,x_4\}$	$\emptyset$	$\{x_1,x_2,x_4\}$	$\{x_1,x_2,x_4\}$	$\{x_4\}$	$\{x_1,x_2,x_4\}$	$\{x_1,x_2\}$
$\{x_3,x_4\}$	$\{x_3,x_4\}$	X	$\{x_1,x_2\}$	$\{x_3,x_4\}$	$\{x_3,x_4\}$	$\emptyset$
$\{x_1,x_2,x_3\}$	$\{x_1,x_2,x_3\}$	X	$\{x_4\}$	$\{x_1,x_2,x_3\}$	$\{x_1,x_2,x_3\}$	$\emptyset$
$\{x_1,x_2,x_4\}$	$\emptyset$	$\{x_1,x_2,x_4\}$	$\{x_1,x_2,x_4\}$	$\{x_1,x_2,x_4\}$	$\{x_1,x_2,x_4\}$	$\emptyset$
$\{x_1,x_3,x_4\}$	$\{x_3,x_4\}$	X	$\{x_1,x_2\}$	$\{x_3,x_4\}$	X	$\{x_1,x_2\}$
$\{x_2,x_3,x_4\}$	$\{x_3,x_4\}$	X	$\{x_1,x_2\}$	$\{x_3,x_4\}$	X	$\{x_1,x_2\}$
X	X	X	$\emptyset$	X	X	$\emptyset$

The current approximations are better than Abo-Tabl's approximation [2] because the current approximation decreases the boundary region by increasing the lower approximation and decreasing the upper approximation.

6. Rough sets and viruses

Kozae et al. [8] introduced and investigated another notion of generalized approximation space by using the subbase $\{xR : x \in X\}$ of the topology τ. They proved that their approximations are better than Yao's approximations [21] because their approximations decrease the boundary region by increasing the lower approximation and decreasing the upper approximation. The goal of this section is to investigate a new definition of the lower and upper operators for any binary relation by using the set $\{xR : x \in X\}$ as a filter subbase. The current approximations decrease the boundary region by increasing the lower approximation and decreasing the upper approximation so it is better than Kozae's approximation [8] and Yao's approximations [21]. We consider the filter is generated by the after sets which has a nonempty finite intersection. To construct the filter $\mathfrak{F}$, let $\xi = \{xR : x \in X\}$ be a subbase of a filter $\mathfrak{F}$.

Definition 6.1. Let R be a binary relation on X and $A \subseteq X$. Then the lower and upper approximation on X according to R are defined as:

$$\underline{R}_{\mathfrak{F}}(A) = \cup\{G \in \mathfrak{F} : G \subseteq A\} \tag{6.1}$$

$$\overline{R}_{\mathfrak{F}}(A) = \{^{\cap\{H \in \mathfrak{F}' : A \subseteq H\}}_{X} \qquad _{if \nexists H \in \mathfrak{F}' : A \subseteq H}\}$$

Proposition 6.2. *Let R be a binary relation on a non-empty set X and $A, B \subseteq X$. Then the following conditions hold:*

(1) $\underline{R}_{\mathfrak{F}}(A) \subseteq A \subseteq \overline{R}_{\mathfrak{F}}(A)$
(2) $\underline{R}_{\mathfrak{F}}(X) = X, \underline{R}_{\mathfrak{F}}(\varnothing) = \varnothing$
(3) $A \subseteq B \Rightarrow \underline{R}_{\mathfrak{F}}(A) \cap \underline{R}_{\mathfrak{F}}(B)$
(4) $\underline{R}_{\mathfrak{F}}(A \cap B) = \underline{R}_{\mathfrak{F}}(A) \cap \underline{R}_{\mathfrak{F}}(B)$
(5) $\underline{R}_{\mathfrak{F}}(A \cup B) \supseteq \underline{R}_{\mathfrak{F}}(A) \cup \underline{R}_{\mathfrak{F}}(B)$
(6) $\underline{R}_{\mathfrak{F}}(A) = \underline{R}_{\mathfrak{F}}(\underline{R}_{\mathfrak{F}}(A))$
(7) $\underline{R}_{\mathfrak{F}}(\underline{R}_{\mathfrak{F}}(A)) \subseteq \overline{R}_{\mathfrak{F}}(\underline{R}_{\mathfrak{F}}(A))$ *and* $\underline{R}_{\mathfrak{F}}(\overline{R}_{\mathfrak{F}}(A)) \subseteq \overline{R}_{\mathfrak{F}}(A)$
(8) $\underline{R}_{\mathfrak{F}}(A) = [\overline{R}_{\mathfrak{F}}(A')]'$.

Proof. The proof follows immediately from the Definition 6.1. □

The relation between the topology τ generated by the subbase $\xi = \{xR : x \in X\}$ and the filter $\mathfrak{F}$ generated by the same subbase is given in the following proposition.

Proposition 6.3. *Let R be a binary relation on X. Then $\tau \setminus \varnothing \subseteq \mathfrak{F}$, where τ is the topology generated by the subbase $\xi = \{xR : x \in X\}$ and $\mathfrak{F}$ is a filter generated by the same subbase.*

Proof. Let $A \in \tau \setminus \varnothing$. Then $A = \cup B, B \in \mathfrak{B}$, where $\mathfrak{B}$ is a base for τ and $\mathfrak{F}$.
Since $B \subseteq \cup B = A, B \in \mathfrak{B}$, then $A \in \mathfrak{F}$ and hence $\tau \setminus \varnothing \subseteq \mathfrak{F}$. □

The following proposition shows that the current approximations are better than Kozae's approximation [8] because the current approximations decrease the boundary region by increasing the lower approximation and decreasing the upper approximation.

Proposition 6.4. *Let R be any binary relation on X. Then*

(1) $\underline{R}_\tau(A) \subseteq \underline{R}_{\mathfrak{F}}(A)$
(2) $\overline{R}_{\mathfrak{F}}(A) \subseteq \overline{R}_\tau(A)$
(3) $BN_{\mathfrak{F}}(A) \subseteq BN_\tau(A)$

Proof. The proof is straightforward from Proposition 6.2. □

A computer virus is a piece of programming code that alters the way our computer works without our knowledge or permission. Computer viruses do not generate by itself [17]. They must be written by someone and with a specific purpose. In the mid–1980s two brothers in Pakistan discovered that people were pirating their software. They responded by writing the first computer virus, a program that would put a copy of itself and a copyright message on any floppy disk copies their customers made. From these simple beginnings, an entire virus counter-culture has emerged. Computer viruses can be transmitted via a number of ways like programs and documents, internet, email, CDs and floppies [14]. There are different types of viruses such as file viruses, boot sector viruses, multipartite viruses and typically macro viruses [10]. An application of rough sets in different types of viruses is introduced in the following example.

Example 6.5. Let $V = \{v_1, v_2, v_3, v_4\}$ be set of four viruses and $A = \{A_1, A_2, A_3\}$ be the attributes viruses, where
A_1 = Disable computer={slow computer, run unwanted software, data theft} = $\{a_1, b_1, c_1\}$,
A_2 = Place of injury ={boot sector, executive programs, multipartite viruses} = $\{a_2, b_2, c_2\}$,
A_3 = Destroy files ={desktop files, files acrobatics, compressed files, executive files}= $\{a_3, b_3, c_3, d_3\}$.

Table 7. Attributes of viruses

X/A	A_1	A_2	A_3
$\{v_1\}$	$\{a_1, b_1\}$	$\{a_2, b_2\}$	$\{a_3\}$
$\{v_2\}$	$\{a_1\}$	$\{b_2, c_2\}$	$\{b_3\}$
$\{v_3\}$	$\{a_1, c_1\}$	$\{a_2\}$	$\{b_3, c_3\}$
$\{v_4\}$	$\{c_1\}$	$\{b_2, c_2\}$	$\{a_3, c_3\}$

Let R be a general relation as follows:
$v_1 R v_2 \Leftrightarrow A(v_1) \cap A(v_2) \neq \varnothing, v_1, v_2 \in V$. For the first attribute A_1, we get
$R_{A_1} = \triangle \cup \{(v_1, v_2), (v_1, v_3), (v_2, v_1), (v_2, v_3), (v_3, v_1), (v_3, v_2), (v_3, v_4), (v_4, v_3)\}$,
$v_1 R_{A_1} = v_2 R_{A_1} = \{v_1, v_2, v_3\}, v_3 R_{A_1} = V, v_4 R_{A_1} = \{v_3, v_4\}$,
$\xi = \{V, \{v_3, v_4\}, \{v_1, v_2, v_3\}\}$,
$\beta = \{V, \{v_3\}, \{v_3, v_4\}, \{v_1, v_2, v_3\}\}$,
$\tau = \{V, \varnothing, \{v_3\}, \{v_3, v_4\}, \{v_1, v_2, v_3\}\}$,
$\tau' = \{V, \varnothing, \{v_4\}, \{v_1, v_2\}, \{v_1, v_2, v_4\}\}$,
$\mathfrak{F} = \{V, \{v_3\}, \{v_1, v_3\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4\}\}$,
and $\mathfrak{F}' = \{\varnothing, \{v_1\}, \{v_2\}, \{v_4\}, \{v_1, v_2\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_1, v_2, v_4\}\}$.

TABLE 8. Comparison between Yao's method [21], Kozae's method [8] and the present method

A	Yao's method [21]			Kozae's method [8]			The present method		
	$\underline{R}(A)$	$\overline{R}(A)$	$BN_R(A)$	$\underline{R}_\tau(A)$	$\overline{R}_\tau(A)$	$BN_\tau(A)$	$\underline{R}_{\mathfrak{F}}(A)$	$\overline{R}_{\mathfrak{F}}(A)$	$BN_{\mathfrak{F}}(A)$
$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
$\{v_1\}$	$\varnothing$	$\{v_1, v_2\}$	$\{v_1, v_2\}$	$\varnothing$	$\{v_1, v_2\}$	$\{v_1, v_2\}$	$\varnothing$	$\{v_1\}$	$\{v_1\}$
$\{v_2\}$	$\varnothing$	$\{v_1, v_2\}$	$\{v_1, v_2\}$	$\varnothing$	$\{v_1, v_2\}$	$\{v_1, v_2\}$	$\varnothing$	$\{v_2\}$	$\{v_2\}$
$\{v_3\}$	$\varnothing$	V	V	$\{v_3\}$	V	$\{v_1, v_2, v_4\}$	$\{v_3\}$	V	$\{v_1, v_2, v_4\}$
$\{v_4\}$	$\varnothing$	$\{v_4\}$	$\{v_4\}$	$\varnothing$	$\{v_4\}$	$\{v_4\}$	$\varnothing$	$\{v_4\}$	$\{v_4\}$
$\{v_1, v_2\}$	$\varnothing$	$\{v_1, v_2\}$	$\{v_1, v_2\}$	$\varnothing$	$\{v_1, v_2\}$	$\{v_1, v_2\}$	$\varnothing$	$\{v_1, v_2\}$	$\{v_1, v_2\}$
$\{v_1, v_3\}$	$\varnothing$	V	V	$\{v_3\}$	V	$\{v_1, v_2, v_4\}$	$\{v_1, v_3\}$	V	$\{v_2, v_4\}$
$\{v_1, v_4\}$	$\varnothing$	V	V	$\{v_1, v_2, v_4\}$	$\{v_1, v_2, v_4\}$	$\varnothing$	$\varnothing$	$\{v_1, v_4\}$	$\{v_1, v_4\}$
$\{v_2, v_3\}$	$\varnothing$	V	V	$\{v_3\}$	V	$\{v_1, v_2, v_4\}$	$\{v_2, v_3\}$	V	$\{v_1, v_4\}$
$\{v_2, v_4\}$	$\varnothing$	V	V	$\varnothing$	$\{v_1, v_2, v_4\}$	$\{v_1, v_2, v_4\}$	$\varnothing$	$\{v_2, v_4\}$	$\{v_2, v_4\}$
$\{v_3, v_4\}$	$\{v_3, v_4\}$	V	$\{v_1, v_2\}$	$\{v_3, v_4\}$	V	$\{v_1, v_2\}$	$\{v_3, v_4\}$	V	$\{v_1, v_2\}$
$\{v_1, v_2, v_3\}$	$\{v_1, v_2, v_3\}$	V	$\{v_4\}$	$\{v_1, v_2, v_3\}$	V	$\{v_4\}$	$\{v_1, v_2, v_3\}$	V	$\{v_4\}$
$\{v_1, v_2, v_4\}$	$\varnothing$	V	V	$\varnothing$	$\{v_1, v_2, v_4\}$	$\{v_1, v_2, v_4\}$	$\varnothing$	$\{v_1, v_2, v_4\}$	$\{v_1, v_2, v_4\}$
$\{v_1, v_3, v_4\}$	$\{v_3, v_4\}$	V	$\{v_1, v_2\}$	$\{v_3, v_4\}$	V	$\{v_1, v_2\}$	$\{v_1, v_3, v_4\}$	V	$\{v_2\}$
$\{v_2, v_3, v_4\}$	$\{v_3, v_4\}$	V	$\{v_1, v_2\}$	$\{v_3, v_4\}$	V	$\{v_1, v_2\}$	$\{v_2, v_3, v_4\}$	V	$\{v_1\}$
V	V	V	$\varnothing$	V	V	$\varnothing$	V	V	$\varnothing$

The present approximations are better than Kozae's approximation [8] and Yao's approximations [21] because the present approximations decrease the boundary region by increasing the lower approximation and decreasing the upper approximation.

7. Conclusion

The main aim of rough set is reducing the boundary region by increasing the lower approximation and decreasing the upper approximation. In this paper, four different methods are proposed to achieve this main aim. Comparison between the current approximation and previous approximation [1, 2, 8, 20, 21] is presented. The current approximations are better than the previous approximation [1, 2, 8, 20, 21] because it reduces the boundary region by increasing the lower approximation and decreasing the upper approximation with comparison to the previous approximation [1, 2, 8, 20, 21]. Moreover, applications of rough sets theory in network connectivity devices, network cables, network topologies and viruses are introduced by applying the current methods.

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A.M. Kozae (akozae55@yahoo.com)
Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt

S.A. El-Sheikh (sobhyelsheikh@yahoo.com)
Department of Mathematics, Faculty of Education, Ain Shams University, Roxy 11757, Cairo, Egypt

E.H. Aly (emad-aly@hotmail.com)
* Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
* Department of Mathematics, Faculty of Education, Ain Shams University, Roxy 11757, Cairo, Egypt

M. Hosny (moona_hosny@yahoo.com)
Department of Mathematics, Faculty of Education, Ain Shams University, Roxy 11757, Cairo, Egypt.