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Somewhat soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous function in soft fuzzy quasi uniform topological space

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Abstract. In this paper the concept of soft fuzzy quasi uniform space and the topology $T_{\mathcal{U}}$ generated by the uniformity is introduced. Soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set, soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous, somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous, soft fuzzy G_δ pre $T_{\mathcal{U}}$ open, somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ open, soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous and somewhat soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous functions are discussed and the characterizations are established. Interrelations among them are discussed with counter examples.

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1. Introduction

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper [8]. Fuzzy sets have applications in many fields such as information [5] and control [6]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Chang [3] introduced and developed the concept of fuzzy topological spaces. The concept of soft fuzzy topological space is introduced by Ismail U.Tiryaki [7]. G.Balasubramanian [1] introduced the concept of fuzzy G_δ set. The concept of fuzzy pre open set was introduced by A.S.Bin Shahana [2]. The concept of a fuzzy quasi uniformity which was introduced by Hutton [4].

In this paper soft fuzzy quasi uniform space and the topology $T_{\mathcal{U}}$ generated by the soft fuzzy quasi uniformity is introduced. Soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set, soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous, somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous, soft fuzzy G_δ pre $T_{\mathcal{U}}$ open, somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ open, soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous and somewhat soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous functions are discussed and their characterizations are established. Interrelations are discussed with counter examples wherever necessary.

2. Preliminaries

Definition 2.1 ([1]). Let (X,T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \mu_i$,where each μ_i is fuzzy open set. The complement of a fuzzy G_δ set is fuzzy F_σ.

Definition 2.2 ([2]). Let (X,T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy pre open set if $\lambda \leq int(cl(\lambda))$. The complement of a pre open set is pre closed.

Definition 2.3 ([7]). Let X be a set, μ be a fuzzy subset of X and M $\subseteq$ X. Then,the pair (μ, M) will be called a soft fuzzy subset of X. The set of all soft fuzzy subsets of X will be denoted by SF(X).

Definition 2.4 ([7]). The relation $\sqsubseteq$ on SF(X) is given by $(\mu, M) \sqsubseteq (\gamma, N) \Leftrightarrow (\mu(x) < \gamma(x)) or (\mu(x) = \gamma(x) and x \notin M/N), \forall\, x \in X$ and for all $(\mu, M), (\gamma, N) \in SF(X)$.

Proposition 2.5 ([7]). *If $(\mu_j, M_j)_{j\in J} \in SF(X)$,then the family $\{(\mu_j, M_j)|\, j \in J\}$ has a meet,that is greatest lower bound, in $(SF(X), \sqsubseteq)$, denoted by*

$$\sqcap_{j\in J}(\mu_j, M_j) \text{ such that } \sqcap_{j\in J}(\mu_j, M_j) = (\mu, M)$$

where $\mu(x) = \bigwedge_{j\in J} \mu_j(x), \forall\, x \in X$ and $M = \bigcap_{j\in J} M_j$.

Proposition 2.6 ([7]). *If $(\mu_j, M_j)_{j\in J} \in SF(X)$, then the family $\{(\mu_j, M_j)| j \in J\}$ has a join,that is least upper bound, in $(SF(X), \sqsubseteq)$, denoted by*

$$\sqcup_{j\in J}(\mu_j, M_j) \text{ such that } \sqcup_{j\in J}(\mu_j, M_j) = (\mu, M)$$

where $\mu(x) = \bigvee_{j\in J} \mu_j(x), \forall\, x \in X$ and $M = \bigcup_{j\in J} M_j$.

Definition 2.7 ([7]). Let X be a non-empty set and the soft fuzzy sets A and B be in the form,

$$A = \{(\mu, M)/\mu(x) \in I^X, \forall\, x \in X, M \subseteq X\}$$
$$B = \{(\lambda, N)/\lambda(x) \in I^X, \forall\, x \in X, N \subseteq X\}$$

Then,

(1) $A \sqsubseteq B \Leftrightarrow \mu(x) \leq \lambda(x), \forall\, x \in X, M \subseteq N$.
(2) $A = B \Leftrightarrow \mu(x) = \lambda(x), \forall\, x \in X, M = N$.
(3) $A' \Leftrightarrow 1 - \mu(x), \forall\, x \in X, X \mid M$.
(4) $A \sqcap B \Leftrightarrow \mu(x) \wedge \lambda(x), \forall\, x \in X, M \cap N$.
(5) $A \sqcup B \Leftrightarrow \mu(x) \vee \lambda(x), \forall\, x \in X, M \cup N$.

Definition 2.8 ([7]). $(0, \phi)$= $\{(\lambda, N)/\lambda = 0, N = \phi\}$
$(1, X)$= $\{(\lambda, N)/\lambda = 1, N = X\}$

Definition 2.9 ([7]). For $(\mu, M) \in SF(X)$ the soft fuzzy set $(\mu, M)^{'} = (1-\mu, X|M)$ is called the complement of (μ, M).

Definition 2.10 ([7]). A subset T⊆SF(X) is called an SF-topology on X if

(1) $(0, \phi)$ and $(1, X) \in T$.

(2) $(\mu_j, M_j) \in T, j = 1, 2, 3, ...n \Rightarrow \sqcap_{j=1}^{n}(\mu_j, M_j) \in T$.

(3) $(\mu_j, M_j), j \in J \Rightarrow \sqcup_{j\in J}(\mu_j, M_j) \in T$.

the elements of T are called soft fuzzy open, and those of $T^{'} = \{(\mu, M)/(\mu, M)^{'} \in T\}$ soft fuzzy closed.

If T is a SF-topology on X we call the pair (X,T) an SF-topological space(in short, SFTS).

3. Soft fuzzy quasi uniform topological space

Let D denote the family of all functions f: SF(X) → SF(X) with the following properties

(1) $f(0, \phi) = (0, \phi)$.

(2) $(\mu, M) \sqsubseteq f(\mu, M)$, for every $(\mu, M) \in$ SF(X).

(3) $f(\sqcup_i(\mu_i, M_i)) = \sqcup_i f(\mu_i, M_i)$, for each family $\{(\mu_i, M_i) : i \in J\} \subset SF(X)$.

Definition 3.1. A soft fuzzy quasi uniformity on X is a subcollection $\mathcal{U} \subset D$ satisfying the following axioms

(i) If $f \in \mathcal{U}$, $f \sqsubseteq g$ and $g \in D$, then $g \in \mathcal{U}$.

(ii) If $f_1, f_2 \in \mathcal{U}$, then there exists $g \in \mathcal{U}$ such that $g \sqsubseteq f_1 \sqcap f_2$.

(iii) For every $f \in \mathcal{U}$ there exists $g \in \mathcal{U}$ such that $g \circ g \sqsubseteq f$.

The pair $(X,\mathcal{U})$ is called soft fuzzy quasi uniform space.

Definition 3.2. Let $(X,\mathcal{U})$ be a soft quasi uniform space. The operator Int : SF(X) → SF(X) defined by

$Int(\mu, M) = \sqcup\{(\lambda, N) \in SF(X) : f(\lambda, N) \sqsubseteq (\mu, M) for some f \in \mathcal{U}\}$.

Definition 3.3. Let $(X,\mathcal{U})$ be a soft fuzzy quasi uniform space. Then $T_{\mathcal{U}} = \{(\mu, M) \in SF(X) : Int(\mu, M) = (\mu, M)\}$ be the soft fuzzy topology generated by $\mathcal{U}$. The pair $(X, T_{\mathcal{U}})$ is called soft fuzzy quasi uniform topological space. The members of $T_{\mathcal{U}}$ are called soft fuzzy $T_{\mathcal{U}}$ open set. The complement of a soft fuzzy $T_{\mathcal{U}}$ open set is soft fuzzy $T_{\mathcal{U}}$ closed.

Definition 3.4. Let $(X, T_{\mathcal{U}})$ be a soft fuzzy quasi uniform topological space. Then the interior of a soft fuzzy set (λ, M) in $(X, T_{\mathcal{U}})$ is defined as follows

$Int_{T_{\mathcal{U}}}(\lambda, M) = \sqcup\{(\mu, N) : (\mu, N) \sqsubseteq (\lambda, M)$ and (μ, N) is soft fuzzy $T_{\mathcal{U}}$ open in $(X, T_{\mathcal{U}})\}$

The closure of a soft fuzzy set (μ, M) in $(X, T_{\mathcal{U}})$ is defined as follows

$Cl_{T_{\mathcal{U}}}(\mu, M) = \sqcap\{(\lambda, N) : (\mu, M) \sqsubseteq (\lambda, N)$ and (λ, N) is soft fuzzy $T_{\mathcal{U}}$ closed in $(X, T_{\mathcal{U}})\}$.

Definition 3.5. $(X, T_{\mathcal{U}})$ denote the soft fuzzy quasi uniform topological space.

(i) A soft fuzzy set (λ, N) in $(X, T_{\mathcal{U}})$ is said to be soft fuzzy G_δ $T_{\mathcal{U}}$ open if $(\lambda, N) = \sqcap_{i=1}^{\infty}(\lambda_i, N_i)$ where each (λ_i, N_i) is soft fuzzy $T_{\mathcal{U}}$ open sets. The complement of soft fuzzy G_δ $T_{\mathcal{U}}$ open is soft fuzzy F_σ $T_{\mathcal{U}}$ closed set.

(ii) A soft fuzzy set (λ, M) in $(X, T_{\mathcal{U}})$ is said to be soft fuzzy pre $T_{\mathcal{U}}$ open set if $(\lambda, M) \sqsubseteq Int_{T_{\mathcal{U}}}(Cl_{T_{\mathcal{U}}}(\lambda, M))$. The complement of soft fuzzy pre $T_{\mathcal{U}}$ open is soft fuzzy pre $T_{\mathcal{U}}$ closed set.

(iii) A soft fuzzy set (λ, N) in $(X, T_{\mathcal{U}})$ is said to be soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set if $(\lambda, N) = (\mu, M) \sqcap (\gamma, L)$ where (μ, M) is soft fuzzy G_δ $T_{\mathcal{U}}$ open set and (γ, L) is soft fuzzy pre $T_{\mathcal{U}}$ open set. The complement of soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set is soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set.

(iv) For any fuzzy set (λ, N) in $(X, T_{\mathcal{U}})$ the soft fuzzy G_δ pre $T_{\mathcal{U}}$ int is defined as follows
$G_\delta preint_{T_{\mathcal{U}}}(\lambda, N) = \sqcup\{(\mu, M)/(\mu, M) \sqsubseteq (\lambda, N)$ and (μ, M) is soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set in $(X, T_{\mathcal{U}})\}$.

(v) For any fuzzy set (λ, N) in $(X, T_{\mathcal{U}})$ the soft fuzzy F_σ pre $T_{\mathcal{U}}$ cl is defined as follows
$F_\sigma precl_{T_{\mathcal{U}}}(\lambda, N) = \sqcap\{(\mu, M)/(\mu, M) \sqsupseteq (\lambda, N)$ and (μ, M) is soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set in $(X, T_{\mathcal{U}})\}$.

Proposition 3.6. *For any soft fuzzy set (λ, N) in $(X, T_{\mathcal{U}})$ the following hold*
(i) $G_\delta preint_{T_{\mathcal{U}}}((1, X) - (\lambda, N)) = (1, X) - F_\sigma precl_{T_{\mathcal{U}}}(\lambda, N)$.
(ii) $F_\sigma precl_{T_{\mathcal{U}}}((1, X) - (\lambda, N)) = (1, X) - G_\delta preint_{T_{\mathcal{U}}}(\lambda, N)$.

4. Somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous function

Definition 4.1. Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{V}})$ be any two soft fuzzy quasi uniform topological spaces. A function $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ is soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous if for each soft fuzzy $S_{\mathcal{V}}$ open set (λ, N) in $(Y, S_{\mathcal{V}})$, the inverse image $f^{-1}(\lambda, N)$ is a soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set in $(X, T_{\mathcal{U}})$.

Definition 4.2. Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{V}})$ be any two soft fuzzy quasi uniform topological spaces. A function $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous if (λ, N) is soft fuzzy $S_{\mathcal{V}}$ open set and $f^{-1}(\lambda, N) \neq (0, \phi)$ then there exists a soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set (μ, M) such that $(\mu, M) \neq (0, \phi)$ and $(\mu, M) \sqsubseteq f^{-1}(\lambda, N)$.

Definition 4.3. A soft fuzzy set (λ, N) in $(X, T_{\mathcal{U}})$ is called soft fuzzy $T_{\mathcal{U}}$ dense if there exists no soft fuzzy $T_{\mathcal{U}}$ closed set (μ, M) such that $(\lambda, N) \sqsubset (\mu, M) \sqsubset (1, X)$.

Definition 4.4. A soft fuzzy set (λ, N) in $(X, T_{\mathcal{U}})$ is called soft fuzzy G_δ pre $T_{\mathcal{U}}$ dense if there exists no soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set (μ, M) such that $(\lambda, N) \sqsubset (\mu, M) \sqsubset (1, X)$.

Proposition 4.5. *Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{V}})$ be any two soft fuzzy quasi uniform topological spaces. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ be a bijective function then the following are equivalent:*
(i) *f is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous function.*
(ii) *If (λ, N) is soft fuzzy $S_{\mathcal{V}}$ closed set such that $f^{-1}(\lambda, N) \neq (1, X)$, then there exists a proper soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set (μ, M) such that $(\mu, M) \sqsupset f^{-1}(\lambda, N)$.*
(iii) *If (λ, N) is a soft fuzzy G_δ pre $T_{\mathcal{U}}$ dense set in $(X, T_{\mathcal{U}})$ then $f(\lambda, N)$ is a soft fuzzy $S_{\mathcal{V}}$ dense set in $(Y, S_{\mathcal{V}})$.*

Proof. (i) $\Rightarrow$ (ii) Suppose that f is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous and (λ, N) is soft fuzzy $S_{\mathcal{V}}$ closed set such that $f^{-1}(\lambda, N) \neq (1, X)$. Therefore clearly $(1, Y) - (\lambda, N)$ is soft fuzzy $S_{\mathcal{V}}$ open set and $f^{-1}[(1, Y) - (\lambda, N)] = (1, X) - f^{-1}(\lambda, N) \neq (0, \phi)$. From the hypothesis, there exists a soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set (μ, M) such that $(\mu, M) \neq (0, \phi)$ and $(\mu, M) \sqsubseteq f^{-1}((1, Y) - (\lambda, N))$. Therefore $(1, X) - (\mu, M) \neq (1, X)$ is soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set. Now, $(1, X) - (\mu, M) \sqsupset (1, X) - f^{-1}((1, Y) - (\lambda, N)) = f^{-1}(\lambda, N)$.

(ii) $\Rightarrow$ (iii) Let (λ, N) be a soft fuzzy G_δ pre $T_{\mathcal{U}}$ dense set in $(X, T_{\mathcal{U}})$. Suppose that $f(\lambda, N)$ is not a soft fuzzy $S_{\mathcal{V}}$ dense set in $(Y, S_{\mathcal{V}})$. Then there exists soft fuzzy $S_{\mathcal{V}}$ closed set (μ, M) such that $f(\lambda, N) \sqsubset (\mu, M) \sqsubset (1, Y)$. Since $(\mu, M) \sqsubset (1, Y)$, therefore $f^{-1}(f(\lambda, N)) \sqsubset f^{-1}(1, Y) = (1, X)$. That is $f^{-1}(\mu, M) \neq (1, X)$. By (ii), there exists a proper soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set $(\delta, L) \neq (1, X)$ such that $(\delta, L) \sqsupset f^{-1}(\mu, M) \neq (1, X)$. But $(\delta, L) \sqsupset f^{-1}(\mu, M) \sqsupset f^{-1}(f(\lambda, N)) = (\lambda, N)$. That is there exists a proper soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set (δ, L) such that $(\lambda, N) \sqsubset (\delta, L) \neq (1, X)$. Which is a contradiction. Therefore $f(\lambda, N)$ soft fuzzy G_δ pre $S_{\mathcal{V}}$ dense set in $(Y, S_{\mathcal{V}})$.

(iii) $\Rightarrow$ (i) Suppose that (λ, N) is a soft fuzzy $S_{\mathcal{V}}$ open set and $f^{-1}(\lambda, N) \neq (0, \phi)$. Which implies $(\lambda, N) \neq (0, \phi)$. It must be shown that G_δ pre $int_{T_{\mathcal{U}}}(f^{-1}(\lambda, N)) \neq (0, \phi)$. On the contrary assume that G_δ pre $int_{T_{\mathcal{U}}}(f^{-1}(\lambda, N)) = (0, \phi)$. Then, $F_\sigma precl_{T_{\mathcal{U}}}[(1, X)$ - $f^{-1}(\lambda, N)] = (1, X)$ - $G_\delta preint_{T_{\mathcal{U}}}(f^{-1}(\lambda, N)) = (1, X) - (0, \phi)$ $= (1, X)$. Which implies $(1, X) - f^{-1}(\lambda, N)$ is a soft fuzzy G_δ pre $T_{\mathcal{U}}$ dense set in $(X, T_{\mathcal{U}})$. By hypothesis, $f[(1, X) - f^{-1}(\lambda, N)]$ is a soft fuzzy $S_{\mathcal{V}}$ dense set in $(Y, S_{\mathcal{V}})$. But $f((1, X) - f^{-1}(\lambda, N)) = f(f^{-1}((1, Y) - (\lambda, N))) = (1, Y) - (\lambda, N)$. Since $(1, Y) - (\lambda, N)$ is a soft fuzzy $S_{\mathcal{V}}$ closed set and $f((1, X) - f^{-1}(\lambda, N)) = (1, Y) - (\lambda, N)$. Therefore $Cl_{S_{\mathcal{V}}}[f((1, X) - f^{-1}(\lambda, N))] = (1, Y) - (\lambda, N)$. From hypothesis, $Cl_{S_{\mathcal{V}}}[f((1, X) - f^{-1}(\lambda, N))] = (1, Y)$. Therefore $(1, Y) = (1, Y) - (\lambda, N)$. Which implies $(\lambda, N) = (0, \phi)$. Which is contradiction. Hence $G_\delta preint_{T_{\mathcal{U}}}(f^{-1}(\lambda, N)) \neq (0, \phi)$. Thus f is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous. $\square$

Proposition 4.6. *Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{V}})$ be any two soft fuzzy quasi uniform topological spaces. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ be somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous function. Let $A \subseteq X$ such that $(1_A, A) \sqcap (\mu, M) \neq (0, \phi)$, for all soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set. Let $T_{\mathcal{U}} \mid A$ be the induced soft fuzzy quasi uniform topological space on A. Then $f \mid A : (A, T_{\mathcal{U}} \mid A) \to (Y, S_{\mathcal{V}})$ is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous.*

Proof. Let (λ, N) be a soft fuzzy $S_{\mathcal{V}}$ open set such that $f^{-1}(\lambda, N) \neq (0, \phi)$. Since f is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous, there exists a soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set $(\mu, M) \neq (0, \phi)$ and $(\mu, M) \sqsubseteq f^{-1}(\lambda, N)$. Now clearly $(\mu, M) \mid A$ is soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set $(\mu, M) \mid A \neq (0, \phi)$.

Also,
$$\begin{aligned}(f \mid A)^{-1}(\lambda, N) &= (\lambda \circ (f \mid A), (f \mid A)^{-1}(N))\\ &= (\lambda(f \mid A), (f \mid A)^{-1}(N))\\ &= (\lambda(f), f^{-1}(N))\\ &= f^{-1}(\lambda, N)\end{aligned}$$
Hence $f^{-1}(\lambda, N) \neq (0, \phi)$ and $f^{-1}(\lambda, N) \sqsupseteq (\mu, M) \sqsupseteq (\mu, M) \mid A$. $\square$

The following example shows that the above proposition is invalid if the hypothesis fails.

Example 4.7. Let $X = \{a, b, c\}$ and let D_1 denote the family of functions $f_1, f_2 : SF(X) \to SF(X)$ is defined as follows

$$f_1(\lambda, M) = \begin{cases} (0, \phi) & \text{if } (\lambda, M) = (0, \phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$f_2(\lambda, M) = \begin{cases} (0, \phi) & \text{if } (\lambda, M) = (0, \phi); \\ (\lambda_1, M_1) & (0, \phi) \neq (\lambda, M) \sqsubseteq (\lambda_1, M_1); \\ (\lambda_2, M_2) & (0, \phi) \neq (\lambda, M) \sqsubseteq (\lambda_2, M_2); \\ (\lambda_3, M_3) & (0, \phi) \neq (\lambda_1, M_1) \sqsubseteq (\lambda_3, M_3) or (0, \phi) \neq (\lambda_2, M_2) \sqsubseteq (\lambda_3, M_3); \\ (1, X) & \text{otherwise.} \end{cases}$$

where $(\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3)$ are defined as follows $\lambda_1(a) = 1, \lambda_1(b) = 0, \lambda_1(c) = 0$,$M_1 = \{c\}$; $\lambda_2(a) = 0, \lambda_2(b) = 0, \lambda_2(c) = 1$,$M_2 = \{c\}$, $\lambda_3(a) = 1, \lambda_3(b) = 0, \lambda_3(c) = 1$,$M_3 = \{c\}$ $\mathcal{U}$ is a subcollection of D_1. Thus $T_{\mathcal{U}} = \{(0, \phi), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{U}$. Thus $(X, T_{\mathcal{U}})$ is a soft fuzzy quasi uniform topological space.

Let $Y = \{p, q, r\}$ and let D_2 denote the family of functions $g_1, g_2 : SF(Y) \to SF(Y)$ is defined as follows

$$g_1(\lambda, M) = \begin{cases} (0, \phi) & \text{if } (\lambda, M) = (0, \phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$g_2(\lambda, M) = \begin{cases} (0, \phi) & \text{if } (\lambda, M) = (0, \phi); \\ (\gamma, L) & (0, \phi) \neq (\lambda, M) \sqsubseteq (\gamma, L); \\ (1, X), & \text{otherwise.} \end{cases}$$

where (γ, L) is defined as follows $\gamma(p) = 0, \gamma(q) = 1, \gamma(r) = 1$,$L = \{p, r\}$. $\mathcal{V}$ is a subcollection of D_2. Thus $S_{\mathcal{V}} = \{(0, \phi), (1, Y), (\gamma, L)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{V}$. Thus $(Y, S_{\mathcal{V}})$ is a soft fuzzy quasi uniform topological space. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ defined as follows $f(a) = p$,$f(b) = q$,$f(c) = r$. f is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous. Let $A = \{a, b\}$. Since $\{\chi_A, A\} \sqcap (\lambda_2, M_2) = (0, \phi)$, A does not satisfy the hypothesis of the above property. Consider $f \mid A : (A, T_{\mathcal{U}} \mid A) \to (Y, S_{\mathcal{V}})$ defined as follows $f \mid A(a) = f(a) = p$, $f \mid A(b) = f(b) = q$. $T_{\mathcal{U}} \mid A = \{(0, \phi), (1, A), (\lambda_1, M_1) \mid A, (\lambda_2, M_2) \mid A, (\lambda_3, M_3) \mid A\}$. Since $f^{-1}(\gamma, L)$ is not soft fuzzy G_δ pre $T_{\mathcal{U}}$ open in $(A, T_{\mathcal{U}} \mid A)$. Thus $f \mid A$ is not somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous.

The following example shows that an extension of somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous function need not be somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous.

Example 4.8. Let $X = \{a, b, c\}$ and let D_1 denote the family of functions $f_1, f_2 : SF(X) \to SF(X)$ is defined as follows

$$f_1(\lambda, M) = \begin{cases} (0, \phi) & \text{if } (\lambda, M) = (0, \phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$f_2(\lambda, M) = \begin{cases} (0, \phi) & \text{if } (\lambda, M) = (0, \phi); \\ (\lambda_1, M_1) & (0, \phi) \neq (\lambda, M) \sqsubseteq (\lambda_1, M_1); \\ (\lambda_2, M_2) & (0, \phi) \neq (\lambda, M) \sqsubseteq (\lambda_2, M_2); \\ (1, X) & \text{otherwise.} \end{cases}$$

where (λ_1, M_1), (λ_2, M_2) are defined as follows $\lambda_1(a) = 0, \lambda_1(b) = 1, \lambda_1(c) = 0, M_1 = \{a\}$; $\lambda_2(a) = 0, \lambda_2(b) = 0, \lambda_2(c) = 1, M_2 = \{c\}$, is a subcollection of D_1. Thus $T_{\mathcal{U}} = \{(0,\phi), (1, X), (\lambda_1, M_1), (\lambda_2, M_{21})\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{U}$. Thus $(X, T_{\mathcal{U}})$ is a soft fuzzy quasi uniform topological space.

Let $Y = \{p, q, r\}$ and let D_2 denote the family of functions $g_1, g_2 : SF(Y) \to SF(Y)$ is defined as follows

$$g_1(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$g_2(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (\gamma, L) & (0,\phi) \neq (\lambda, M) \sqsubseteq (\gamma, L); \\ (1, X) & \text{otherwise.} \end{cases}$$

where (γ, L) is defined as follows $\gamma(p) = 1, \gamma(q) = 0, \gamma(r) = 0, L = \{p, q\}$. $\mathcal{V}$ is a subcollection of D_2. Thus $S_{\mathcal{V}} = \{(0,\phi), (1, Y), (\gamma, L)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{V}$. Thus $(Y, S_{\mathcal{V}})$ is a soft fuzzy quasi uniform topological space.

Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ defined as follows $f(a) = p, f(b) = q, f(c) = r$. Since $f^{-1}(\gamma, L)$ is not soft fuzzy G_δ pre $T_{\mathcal{U}}$ open in $(X, T_{\mathcal{U}})$. Hence f is not somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous. Let $A = \{a, b\}$. Now $f \mid A : (A, T_{\mathcal{U}} \mid A) \to (Y, S_{\mathcal{V}})$ defined as follows $f \mid A(a) = f(a) = p$, $f \mid A(b) = f(b) = q$. But $f \mid A$ is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous.

5. Somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ open function

Definition 5.1. Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{V}})$ be any two soft fuzzy quasi uniform topological spaces. A function $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ is soft fuzzy G_δ pre $T_{\mathcal{U}}$ open function if (λ, N) is soft fuzzy $T_{\mathcal{U}}$ open set in $(X, T_{\mathcal{U}})$, $f(\lambda, N)$ is soft fuzzy G_δ pre $S_{\mathcal{V}}$ open set.

Definition 5.2. Let $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{V}})$ be any two soft fuzzy quasi uniform topological spaces. A function $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ open function iff (λ, N) is soft fuzzy $T_{\mathcal{U}}$ open set and $(\lambda, N) \neq (0,\phi)$ then there exists a soft fuzzy G_δ pre $S_{\mathcal{V}}$ open set (μ, M) such that $(\mu, M) \neq (0,\phi)$ and $(\mu, M) \sqsubseteq f(\lambda, N)$.

Clearly every soft fuzzy G_δ pre $T_{\mathcal{U}}$ open function is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ open function but the converse need not be true as shown in the following example.

Example 5.3. Let $X = \{a, b, c\}$ and let D_1 denote the family of functions $f_1, f_2 : SF(X) \to SF(X)$ is defined as follows

$$f_1(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$f_2(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (\lambda_1, M_1) & (0,\phi) \neq (\lambda, M) \sqsubseteq (\lambda_1, M_1); \\ (1, X) & \text{otherwise.} \end{cases}$$

where (λ_1, M_1) is defined as follows $\lambda_1(a) = 0.5, \lambda_1(b) = 0.4, \lambda_1(c) = 0.4, M_1 = \{a, b\}$; $\mathcal{U}$ is a subcollection of D_1. Thus $T_{\mathcal{U}} = \{(0,\phi), (1, X), (\lambda_1, M_1)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{U}$. Thus $(X, T_{\mathcal{U}})$ is a soft fuzzy quasi uniform topological space.

Let $Y = \{p, q, r\}$ and let D_2 denote the family of functions $g_1, g_2 : SF(Y) \to SF(Y)$ is defined as follows

$$g_1(\lambda, M) = \begin{cases} (0, \phi) & \text{if } (\lambda, M) = (0, \phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$g_2(\lambda, M) = \begin{cases} (0, \phi) & \text{if } (\lambda, M) = (0, \phi); \\ (\gamma_1, L_1) & (0, \phi) \neq (\lambda, M) \sqsubseteq (\gamma_1, L_1); \\ (\gamma_2, L_2) & (\gamma_1, L_1) \neq (\lambda, M) \sqsubseteq (\gamma_2, L_2); \\ (\gamma_3, L_3) & (\gamma_1, L_1) \neq (\lambda, M) \sqsubseteq (\gamma_3, L_3); \\ (1, X) & \text{otherwise.} \end{cases}$$

where (γ_1, L_1), (γ_2, L_2), (γ_3, L_3) are defined as follows $\gamma_1(p) = 0.4, \gamma_1(q) = 0.4, \gamma_1(r) = 0.4$, $L_1 = \{\phi\}$; $\gamma_2(p) = 0.4, \gamma_2(q) = 0.4, \gamma_2(r) = 0.4$, $L_2 = \{r\}$; $\gamma_3(p) = 0.4, \gamma_3(q) = 0.4, \gamma_3(r) = 0.4$, $L_3 = \{p, q\}$. $\mathcal{V}$ is a subcollection of D_2. Thus $S_{\mathcal{V}} = \{(0, \phi), (1, Y), (\gamma_1, L_1), (\gamma_2, L_2), (\gamma_3, L_3)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{V}$. Thus $(Y, S_{\mathcal{V}})$ is a soft fuzzy quasi uniform topological space. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ defined as follows $f(a) = p, f(b) = q, f(c) = r$. f is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ open function. Since $f(\lambda_1, M_1)$ is not soft fuzzy G_δ pre $T_{\mathcal{U}}$ open in $(Y, S_{\mathcal{V}})$. Thus f is not soft fuzzy G_δ pre $T_{\mathcal{U}}$ open function.

Proposition 5.4. *Suppose $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{V}})$ be any two soft fuzzy quasi uniform topological spaces. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ be a bijective function. Then the following conditions are equivalent.*

(i) *f is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ open function.*

(ii) *If (λ, N) is a soft fuzzy G_δ pre $S_{\mathcal{V}}$ dense set in $(Y, S_{\mathcal{V}})$, then $f^{-1}(\lambda, N)$ is a soft fuzzy $T_{\mathcal{U}}$ dense set in $(X, T_{\mathcal{U}})$.*

Proof. (i) $\Rightarrow$ (ii) Assume that f is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ open function. Suppose that (λ, N) is a soft fuzzy G_δ pre $S_{\mathcal{V}}$ dense set in $(Y, S_{\mathcal{V}})$. It must be shown that $f^{-1}(\lambda, N)$ is a soft fuzzy $T_{\mathcal{U}}$ dense set in $(X, T_{\mathcal{U}})$. Suppose if it is not. Then there exists a soft fuzzy $T_{\mathcal{U}}$ closed set (μ, M) such that $f^{-1}(\lambda, N) \sqsubset (\mu, M) \sqsubset (1, X)$. That is $(\lambda, N) = f(f^{-1}(\lambda, N)) \sqsubset f(\mu, M) \sqsubset f(1, X) = (1, Y)$. Since f is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ open function, there exists a soft fuzzy G_δ pre $S_{\mathcal{V}}$ open set (δ, L) such that $(\delta, L) \neq (0, \phi)$ and $(\delta, L) \sqsubseteq f((1, X) - (\mu, M))$. That is $(1, Y) - (\delta, L) \sqsupset f(\mu, M)$. Which implies $f(\mu, M) \sqsubset (1, Y) - (\delta, L) \sqsubset (1, Y)$. Therefore $f(\mu, M) \sqsubset F_\sigma precl_{S_{\mathcal{V}}}((1, Y) - (\delta, L)) \sqsubset (1, Y)$. That is $(\lambda, N) \sqsubset F_\sigma precl_{S_{\mathcal{V}}}((1, Y) - (\delta, L)) \sqsubset (1, Y)$. Which is a contradiction to our assumption that (λ, N) is soft fuzzy G_δ pre $S_{\mathcal{V}}$ dense set in $(Y, S_{\mathcal{V}})$. Hence $f^{-1}(\lambda, N)$ is a soft fuzzy G_δ pre $T_{\mathcal{U}}$ dense set in $(X, T_{\mathcal{U}})$.

(ii) $\Rightarrow$ (i) Suppose $(\lambda, N) \neq (0, \phi)$ is a soft fuzzy $T_{\mathcal{U}}$ open set. We have to show that $G_\delta preint_{S_{\mathcal{V}}}(f(\lambda, N)) \neq (0, \phi)$. Suppose that $G_\delta preint_{S_{\mathcal{V}}}(f(\lambda, N)) = (0, \phi)$. Now $F_\sigma precl_{S_{\mathcal{V}}}((1, Y) - f(\lambda, N)) = (1, Y) - G_\delta preint_{S_{\mathcal{V}}}(f(\lambda, N)) = (1, Y)$. Therefore by assumption (ii) $f^{-1}((1, Y) - f(\lambda, N))$ is soft fuzzy G_δ pre $T_{\mathcal{U}}$ dense in $(X, T_{\mathcal{U}})$. But $f^{-1}((1, Y) - f(\lambda, N)) = (1, X) - (\lambda, N)$, $F_\sigma precl_{T_{\mathcal{U}}}(f^{-1}((1, Y) - f(\lambda, N))) = (1, X) - (\lambda, N)$. Therefore $(1, X) = (1, X) - (\lambda, N)$. Which implies $(\lambda, N) = (0, \phi)$. Which is a contradiction. Therefore $G_\delta preint_{S_\mathcal{V}}(f(\lambda,N)) \neq (0,\phi)$. Thus f is somewhat soft fuzzy G_δ pre $T_\mathcal{U}$ open function. □

Proposition 5.5. *Suppose $(X,T_\mathcal{U})$ and $(Y,S_\mathcal{V})$ be any two soft fuzzy quasi uniform topological spaces. Let $f:(X,T_\mathcal{U}) \to (Y,S_\mathcal{V})$ be bijective function. Then the following conditions are equivalent.*

(i) *f is somewhat soft fuzzy G_δ pre $T_\mathcal{U}$ open function.*

(ii) *If (λ,N) is a soft fuzzy $T_\mathcal{U}$ closed set such that $f(\lambda,N) \neq (1,Y)$, then there exists soft fuzzy F_σ pre $S_\mathcal{V}$ closed set (μ,M) in $(Y,S_\mathcal{V})$ such that $(\mu,M) \neq (1,Y)$ and $(\mu,M) \sqsupset f(\lambda,N)$.*

Proof. (i) ⇒ (ii) Let (λ,N) be a soft fuzzy $T_\mathcal{U}$ closed set in $(X,T_\mathcal{U})$ such that $f(\lambda,N) \neq (1,Y)$. Then, $(1,X)-(\lambda,N)$ is a soft fuzzy $T_\mathcal{U}$ open set such that $f((1,X)-(\lambda,N)) = (1,Y)-f(\lambda,N) \neq (0,\phi)$. As f is somewhat soft fuzzy G_δ pre $T_\mathcal{U}$ open function, there exists a soft fuzzy G_δ pre $S_\mathcal{V}$ open set (μ,M) such that $(\mu,M) \neq (0,\phi)$ and $(\mu,M) \sqsubseteq f[(1,X)-(\lambda,N)]$. Since (μ,M) is soft fuzzy G_δ pre $S_\mathcal{V}$ open set such that $(\mu,M) \neq (0,\phi)$, $(1,Y)-(\mu,M)$ is soft fuzzy F_σ pre $S_\mathcal{V}$ closed set in $(Y,S_\mathcal{V})$ such that $(1,Y)-(\mu,M) \neq (1,Y)$ and $(1,Y)-(\mu,M) \sqsupset f(\lambda,N)$.

(ii) ⇒ (i) Let (λ,N) be a soft fuzzy $T_\mathcal{U}$ open set such that $(\lambda,N) \neq (0,\phi)$. Then $(1,X)-(\lambda,N)$ is a soft fuzzy $T_\mathcal{U}$ closed set and $(1,X)-(\lambda,N) \neq (1,X)$. Now $f((1,X)-(\lambda,N)) = (1,Y)-f(\lambda,N) \neq (1,Y)$. Hence by hypothesis, there exists soft fuzzy F_σ pre $S_\mathcal{V}$ closed set (μ,M) in $(Y,S_\mathcal{V})$ such that $(\mu,M) \neq (1,Y)$ and $(\mu,M) \sqsupset f((1,X)-(\lambda,N)) = (1,Y)-f(\lambda,N)$. Which implies $f(\lambda,N) \sqsupseteq (1,Y)-(\mu,M)$. Clearly $(1,Y)-(\mu,M)$ is soft fuzzy G_δ pre $S_\mathcal{V}$ open set in $(Y,S_\mathcal{V})$ such that $(1,Y)-(\mu,M) \sqsubseteq f(\lambda,N)$ and $(1,Y)-(\mu,M) \neq (0,\phi)$. This shows that f is somewhat soft fuzzy G_δ pre $T_\mathcal{U}$ open function. □

6. Somewhat and soft fuzzy almost G_δ pre $T_\mathcal{U}$ continuous function

Definition 6.1. Let $(X,T_\mathcal{U})$ be a soft fuzzy quasi uniform topological space. A soft fuzzy set (λ,N) is said to be soft fuzzy $T_\mathcal{U}$ regular open set in $(X,T_\mathcal{U})$ if $(\lambda,N) = Int_{T_\mathcal{U}}(Cl_{T_\mathcal{U}}(\lambda,N))$. The complement of a soft fuzzy $T_\mathcal{U}$ regular open set is soft fuzzy $T_\mathcal{U}$ regular closed set.

Definition 6.2. Let $(X,T_\mathcal{U})$ and $(Y,S_\mathcal{V})$ be any two soft fuzzy quasi uniform topological spaces. A function $f:(X,T_\mathcal{U}) \to (Y,S_\mathcal{V})$ is soft fuzzy almost G_δ pre $T_\mathcal{U}$ continuous function if the inverse image of each soft fuzzy $S_\mathcal{V}$ regular open set in $(Y,S_\mathcal{V})$ is a soft fuzzy G_δ pre $T_\mathcal{U}$ open set in $(X,T_\mathcal{U})$.

Proposition 6.3. *Suppose $(X,T_\mathcal{U})$ and $(Y,S_\mathcal{V})$ be any two soft fuzzy quasi uniform topological spaces. Let $f:(X,T_\mathcal{U}) \to (Y,S_\mathcal{V})$ be a function. Then the following conditions are equivalent.*

(i) *f is soft fuzzy almost G_δ pre $T_\mathcal{U}$ continuous function.*

(ii) *$f^{-1}(\lambda,N) \sqsubseteq G_\delta preint_{T_\mathcal{U}}(f^{-1}(Int_{S_\mathcal{V}}(Cl_{S_\mathcal{V}}(\lambda,N))))$ for each soft fuzzy $S_\mathcal{V}$ open set (λ,N) in $(Y,S_\mathcal{V})$.*

(iii) *$F_\sigma precl_{T_\mathcal{U}}(f^{-1}(Cl_{S_\mathcal{V}}(Int_{S_\mathcal{V}}(\mu,M)))) \sqsubseteq f^{-1}(\mu,M)$ for each soft fuzzy $S_\mathcal{V}$ closed set (μ,M) in $(Y,S_\mathcal{V})$.*

(iv) *$f^{-1}(\lambda,N)$ is a soft fuzzy F_σ pre $T_\mathcal{U}$ closed set , for each soft fuzzy $S_\mathcal{V}$ regular closed set (λ,N) in $(Y,S_\mathcal{V})$.*

Proof. (i) $\Rightarrow$ (ii) Let (λ, N) be a soft fuzzy $S_{\mathcal{V}}$ open set in $(Y, S_{\mathcal{V}})$. Now $(\lambda, N) \sqsubseteq Cl_{S_{\mathcal{V}}}(\lambda, N)$. Which implies $(\lambda, N) = Int_{S_{\mathcal{V}}}(\lambda, N) \sqsubseteq Int_{S_{\mathcal{V}}}(Cl_{S_{\mathcal{V}}}(\lambda, N))$. Therefore $f^{-1}(\lambda, N) \sqsubseteq f^{-1}(Int_{S_{\mathcal{V}}}(Cl_{S_{\mathcal{V}}}(\lambda, N)))$. Thus $Int_{S_{\mathcal{V}}}(Cl_{S_{\mathcal{V}}}(\lambda, N))$ is soft fuzzy $S_{\mathcal{V}}$ regular open set. Since f is soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous, $f^{-1}(Int_{S_{\mathcal{V}}}(Cl_{S_{\mathcal{V}}}(\lambda, N)))$ is soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set in $(X, T_{\mathcal{U}})$. Therefore $f^{-1}(Int_{S_{\mathcal{V}}}(Cl_{S_{\mathcal{V}}}(\lambda, N))) = G_\delta preint_{T_{\mathcal{U}}}(f^{-1}(Int_{S_{\mathcal{V}}}(Cl_{S_{\mathcal{V}}}(\lambda, N))))$. Also $f^{-1}(\lambda, N) \sqsubseteq G_\delta preint_{T_{\mathcal{U}}}(f^{-1}(Int_{S_{\mathcal{V}}}(Cl_{S_{\mathcal{V}}}(\lambda, N))))$.

(ii) $\Rightarrow$ (iii) Let (μ, M) be a soft fuzzy $S_{\mathcal{V}}$ closed set in $(Y, S_{\mathcal{V}})$. Therefore $(1, Y) - (\mu, M)$ is a soft fuzzy $S_{\mathcal{V}}$ open set in $(Y, S_{\mathcal{V}})$. Now by hypothesis, $f^{-1}[(1, Y) - (\mu, M)] \sqsubseteq G_\delta preint_{T_{\mathcal{U}}}\ \{f^{-1}[Int_{S_{\mathcal{V}}}(Cl_{S_{\mathcal{V}}}((1, Y) - (\mu, M)))]\}$. This implies that $(1, X) - f^{-1}(\mu, M) \sqsubseteq G_\delta preint_{T_{\mathcal{U}}}\{f^{-1}[(1, Y) - Cl_{S_{\mathcal{V}}}(Int_{S_{\mathcal{V}}}(\mu, M))]\}$. Then $(1, X) - f^{-1}(\mu, M) \sqsubseteq (1, X) - F_\sigma precl_{T_{\mathcal{U}}}\{f^{-1}(Cl_{S_{\mathcal{V}}}(Int_{S_{\mathcal{V}}}(\mu, M)))\}$. Therefore $f^{-1}(\mu, M) \sqsupseteq F_\sigma precl_{T_{\mathcal{U}}}\{f^{-1}(Cl_{S_{\mathcal{V}}}(Int_{S_{\mathcal{V}}}(\mu, M)))\}$.

(iii) $\Rightarrow$ (iv) Let (λ, N) be a soft fuzzy $S_{\mathcal{V}}$ regular closed set in $(Y, S_{\mathcal{V}})$. Then we have $F_\sigma precl_{T_{\mathcal{U}}}(f^{-1}(\lambda, N)) = F_\sigma precl_{T_{\mathcal{U}}}(f^{-1}(Cl_{S_{\mathcal{V}}}(Int_{S_{\mathcal{V}}}(\lambda, N))))$. Since (λ, N) is a soft fuzzy $S_{\mathcal{V}}$ regular closed set, (λ, N) is a soft fuzzy $S_{\mathcal{V}}$ closed set. Therefore by hypothesis, $F_\sigma precl_{T_{\mathcal{U}}}(f^{-1}(Cl_{S_{\mathcal{V}}}(Int_{S_{\mathcal{V}}}(\lambda, N)))) \sqsubseteq f^{-1}(\lambda, N)$. Therefore $F_\sigma precl_{T_{\mathcal{U}}}(f^{-1}(\lambda, N)) \sqsubseteq f^{-1}(\lambda, N)$. But clearly $F_\sigma precl_{T_{\mathcal{U}}}(f^{-1}(\lambda, N)) \sqsupseteq f^{-1}(\lambda, N)$. Thus $F_\sigma precl_{T_{\mathcal{U}}}(f^{-1}(\lambda, N)) = f^{-1}(\lambda, N)$. Which implies $f^{-1}(\lambda, N)$ is a soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set.

(iv) $\Rightarrow$ (i) Let (λ, N) be a soft fuzzy $S_{\mathcal{V}}$ regular open set in $(Y, S_{\mathcal{V}})$. Then $(1, Y) - (\lambda, N)$ is a soft fuzzy $S_{\mathcal{V}}$ regular closed set in $(Y, S_{\mathcal{V}})$. Then by hypothesis, $f^{-1}((1, Y) - (\lambda, N))$ is a soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set. Therefore $(1, X) - f^{-1}(\lambda, N)$ is a soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set. Which implies $f^{-1}(\lambda, N)$ is a soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set. Hence f is soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous function. □

Definition 6.4. A soft fuzzy set (λ, N) in $(X, T_{\mathcal{U}})$ is called soft fuzzy $T_{\mathcal{U}}$ regular dense if there exists no soft fuzzy $T_{\mathcal{U}}$ regular closed set (μ, M) such that $(\lambda, N) \sqsubset (\mu, M) \sqsubset (1, X)$.

Definition 6.5. Let $(X, T_{\mathcal{U}})$ be a soft fuzzy quasi uniform topological space. Then the regular interior of a soft fuzzy set (λ, M) in $(X, T_{\mathcal{U}})$ is defined as follows

$R - Int_{T_{\mathcal{U}}}(\lambda, M) = \sqcup\{(\mu, N) : (\mu, N) \sqsubseteq (\lambda, M)$ and (μ, N) is soft fuzzy $T_{\mathcal{U}}$ regular open in $(X, T_{\mathcal{U}})\}$

The regular closure of a soft fuzzy set (μ, M) in $(X, T_{\mathcal{U}})$ is defined as follows

$R - Cl_{T_{\mathcal{U}}}(\mu, M) = \sqcap\{(\lambda, N) : (\mu, M) \sqsubseteq (\lambda, N)$ and (λ, N) is soft fuzzy $T_{\mathcal{U}}$ regular closed in $(X, T_{\mathcal{U}})\}$.

Proposition 6.6. *Suppose $(X, T_{\mathcal{U}})$ and $(Y, S_{\mathcal{V}})$ be any two soft fuzzy quasi uniform topological spaces. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ be a bijective function. Then the following conditions are equivalent.*

(i) *f is somewhat soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous function.*

(ii) *If (λ, N) is a soft fuzzy $S_{\mathcal{V}}$ regular closed set such that $f^{-1}(\lambda, N) \neq (1, X)$, then there exists a proper soft fuzzy F_σ pre $T_{\mathcal{U}}$ closed set (μ, M) such that $(\mu, M) \sqsupseteq f^{-1}(\lambda, N)$.*

(iii) *If (λ, N) is a soft fuzzy G_δ pre $T_{\mathcal{U}}$ dense set in $(X, T_{\mathcal{U}})$, then $f(\lambda, N)$ is a soft fuzzy $T_{\mathcal{U}}$ regular dense set in $(Y, S_{\mathcal{V}})$.*

Proof. (i) $\Rightarrow$ (ii) Suppose f is somewhat soft fuzzy almost G_δ pre $T_\mathcal{U}$ continuous and (λ, N) is soft fuzzy $S_\mathcal{V}$ regular closed set in $(Y, S_\mathcal{V})$ such that $f^{-1}(\lambda, N) \neq (1, X)$. Therefore clearly $(1, Y) - (\lambda, N)$ is soft fuzzy $S_\mathcal{V}$ regular open set and $f^{-1}((1, Y) - (\lambda, N)) = f^{-1}(1, Y) - f^{-1}(\lambda, N) \neq (0, \phi)$. By (i) there exists a soft fuzzy G_δ pre $T_\mathcal{U}$ open set (μ, M) in $(X, T_\mathcal{U})$ such that $(\mu, M) \neq (0, \phi)$ and $(\mu, M) \sqsubseteq f^{-1}((1, Y) - (\lambda, N)$. That is $(1, X) - (\mu, M) \neq (1, X)$ is a soft fuzzy F_σ pre $T_\mathcal{U}$ closed set in $(X, T_\mathcal{U})$ and $(1, X) - (\mu, M) \sqsupseteq (1, X) - f^{-1}((1, Y) - (\lambda, N))$. Which implies $(1, X) - (\mu, M) \sqsupseteq f^{-1}(\lambda, N)$.

(ii) $\Rightarrow$ (iii) Let (λ, N) be a soft fuzzy G_δ pre $T_\mathcal{U}$ dense set in $(X, T_\mathcal{U})$ and $f(\lambda, N)$ not soft fuzzy $S_\mathcal{V}$ regular dense set in $(Y, S_\mathcal{V})$. Then there exists a soft fuzzy $S_\mathcal{V}$ regular closed set (μ, M) such that $f(\lambda, N) \sqsubseteq (\mu, M) \sqsubseteq (1, Y)$. Since $(\mu, M) \sqsubseteq (1, Y)$, $f^{-1}(\mu, M) \neq (1, X)$ and by (ii) there exists a soft fuzzy F_σ pre $T_\mathcal{U}$ closed set (δ, L) such that $(\delta, L) \sqsupseteq f^{-1}(\mu, M)$. But $f^{-1}(f(\lambda, N)) \sqsubseteq f^{-1}(\mu, M) \sqsubseteq (\delta, L)$. That is there exists a soft fuzzy F_σ pre $T_\mathcal{U}$ closed set such that $(1, X) \neq (\delta, L) \sqsupseteq (\lambda, N)$, which is a contradiction. Hence $f(\lambda, N)$ is a soft fuzzy $S_\mathcal{V}$ regular dense set in $(Y, S_\mathcal{V})$.

(iii) $\Rightarrow$ (i) Suppose (λ, N) is soft fuzzy $S_\mathcal{V}$ regular open set and $f^{-1}(\lambda, N) \neq (0, \phi)$ and therefore $(\lambda, N) \neq (0, \phi)$. We have to show that $G_\delta preint_{T_\mathcal{U}}(f^{-1}(\lambda, N)) \neq (0, \phi)$. Suppose $G_\delta preint_{T_\mathcal{U}}(f^{-1}(\lambda, N)) = (0, \phi)$. Thus $F_\sigma precl_{T_\mathcal{V}}((1, X) - f^{-1}(\lambda, N))$ $= (1, X)$. This shows that $(1, X) - f^{-1}(\lambda, N)$is a soft fuzzy G_δ pre $T_\mathcal{U}$ dense set in $(X, T_\mathcal{U})$. By (iii) $f((1, X) - f^{-1}(\lambda, N))$ is a soft fuzzy $S_\mathcal{V}$ regular dense set in $(Y, S_\mathcal{V})$. But $f((1, X) - f^{-1}(\lambda,)) = f(f^{-1}((1, Y) - (\lambda, N) = (1, Y) - (\lambda, N)$. Since $(1, Y) - (\lambda, N)$ is soft fuzzy $S_\mathcal{V}$ regular closed set and $f((1, X) - f^{-1}(\lambda, N)) = (1, Y) - (\lambda, N)$. From hypothesis $R - cl_{S_\mathcal{V}}(f((1, X) - f^{-1}(\lambda, N))) = (1, Y)$. Therefore $(1, Y) = (1, Y) - (\lambda, N)$. Which implies $(\lambda, N) = (0, \phi)$. Which is a contradiction. Hence $G_\delta preint_{T_\mathcal{U}}(f^{-1}(\lambda, N)) \neq (0, \phi)$. ☐

7. Interrelations between above discussed continuous functions

From the above defined continuous functions the following interrelation obtained.

$$\begin{array}{ccc}
\text{Soft fuzzy} & \rightarrow & \text{Somewhat soft fuzzy} \\
G_\delta \text{ pre } T_\mathcal{U} \text{ continuous function} & \not\leftarrow & G_\delta \text{ pre } T_\mathcal{U} \text{ continuous function} \\
\downarrow \not\uparrow & & \downarrow \not\uparrow \\
\text{Soft fuzzy almost} & \rightarrow & \text{Somewhat soft fuzzy almost} \\
G_\delta \text{ pre } T_\mathcal{U} \text{ continuous function} & \not\leftarrow & G_\delta \text{ pre } T_\mathcal{U} \text{ continuous function}
\end{array}$$

The converse of the implication need not be true shown in the following the examples.

Example 7.1. Let $X = \{a, b, c\}$ and let D_1 denote the family of functions $f_1, f_2, f_3, f_4 : SF(X) \rightarrow SF(X)$ is defined as follows

$$f_1(\lambda, M) = \begin{cases} (0, \phi) & \text{if } (\lambda, M) = (0, \phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$f_2(\lambda, M) = \begin{cases} (0, \phi) & \text{if } (\lambda, M) = (0, \phi); \\ (\lambda_1, M_1) & (0, \phi) \neq (\lambda, M) \sqsubseteq (\lambda_1, M_1); \\ \text{(1,X),} & \text{otherwise.} \end{cases}$$

$$f_3(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (\lambda_2, M_2) & (\lambda_1, M_1) \neq (\lambda, M) \sqsubseteq (\lambda_2, M_2); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$f_4(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (\lambda_3, M_3) & (\lambda_1, M_1) \neq (\lambda, M) \sqsubseteq (\lambda_3, M_3); \\ (1, X) & \text{otherwise.} \end{cases}$$

where (λ_1, M_1), (λ_2, M_2), (λ_3, M_3) are defined as follows $\lambda_1(a) = 0.4, \lambda_1(b) = 0.4, \lambda_1(c) = 0.4$, $M_1 = \{\phi\}$; $\lambda_2(a) = 0.4, \lambda_2(b) = 0.4, \lambda_2(c) = 0.4$, $M_2 = \{c\}$; $\lambda_3(a) = 0.4, \lambda_3(b) = 0.4, \lambda_3(c) = 0.4$,$M_3 = \{a, b\}$; $\mathcal{U}$ is a subcollection of D_1. Thus $T_{\mathcal{U}} = \{(0,\phi), (1,X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{U}$. Thus $(X, T_{\mathcal{U}})$ is a soft fuzzy quasi uniform topological space.

Let $Y = \{p, q, r\}$ and let D_2 denote the family of functions $g_1, g_2 : SF(Y) \to SF(Y)$ is defined as follows

$$g_1(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$g_2(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (\gamma, L) & (0,\phi) \neq (\lambda, M) \sqsubseteq (\gamma, L); \\ (1, X) & \text{otherwise.} \end{cases}$$

where (γ, L) is defined as follows $\gamma(p) = 0.5, \gamma(q) = 0.4, \gamma(r) = 0.4$, $L = \{p, q\}$; $\mathcal{V}$ is a subcollection of D_2. Thus $S_{\mathcal{V}} = \{(0,\phi), (1,Y), (\gamma, L)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{V}$. Thus $(Y, S_{\mathcal{V}})$ is a soft fuzzy quasi uniform topological space. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ defined as follows $f(a) = p, f(b) = q, f(c) = r$. f is somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous function. Since $f^{-1}(\gamma, L)$ is not soft fuzzy G_δ pre $T_{\mathcal{U}}$ open in $(X, T_{\mathcal{U}})$. Thus f is not soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous function.

Example 7.2. Let $X = \{a, b, c\}$ and let D_1 denote the family of functions $f_1, f_2 : SF(X) \to SF(X)$ is defined as follows

$$f_1(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$f_2(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (\lambda_1, M_1) & (0,\phi) \neq (\lambda, M) \sqsubseteq (\lambda_1, M_1); \\ (1, X) & \text{otherwise.} \end{cases}$$

where (λ_1, M_1) is defined as follows $\lambda_1(a) = 0.6, \lambda_1(b) = 0.7, \lambda_1(c) = 0.8$, $M_1 = \{a, b\}$; $\mathcal{U}$ is a subcollection of D_1. Thus $T_{\mathcal{U}} = \{(0,\phi), (1,X), (\lambda_1, M_1)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{U}$. Thus $(X, T_{\mathcal{U}})$ is a soft fuzzy quasi uniform topological space.

Let $Y = \{p, q, r\}$ and let D_2 denote the family of functions $g_1, g_2 : SF(Y) \to SF(Y)$ is defined as follows

$$g_1(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$g_2(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (\gamma_1, L_1) & (0,\phi) \neq (\lambda, M) \sqsubseteq (\gamma_1, L_1); \\ (\gamma_2, L_2) & (\gamma_1, L_1) \neq (\lambda, M) \sqsubseteq (\gamma_2, L_2); \\ (\gamma_3, L_3) & (\gamma_1, L_1) \neq (\lambda, M) \sqsubseteq (\gamma_3, L_3); \\ (1, X), & \text{otherwise.} \end{cases}$$

where (γ_1, L_1), (γ_2, L_2), (γ_3, L_3) are defined as follows $\gamma_1(p) = 0.4, \gamma_1(q) = 0.3, \gamma_1(r) = 0.2$, $L_1 = \{\phi\}$; $\gamma_2(p) = 0.4, \gamma_2(q) = 0.3, \gamma_2(r) = 0.2$, $L_2 = \{r\}$; $\gamma_3(p) = 0.5, \gamma_3(q) = 0.4, \gamma_3(r) = 0.4$, $L_3 = \{p, q\}$. $\mathcal{V}$ is a subcollection of D_2. Thus $S_{\mathcal{V}} = \{(0,\phi), (1, Y), (\gamma_1, L_1), (\gamma_2, L_2), (\gamma_3, L_3)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{V}$. Thus $(Y, S_{\mathcal{V}})$ is a soft fuzzy quasi uniform topological space. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ defined as follows $f(a) = p, f(b) = q, f(c) = r$. f is soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous function. Since $f^{-1}(\gamma_1, L_1)$ is not soft fuzzy G_δ pre $T_{\mathcal{U}}$ open in $(X, T_{\mathcal{U}})$. Thus f is not soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous function.

Example 7.3. In the same example above defined f is somewhat soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous function. Since $f^{-1}(\gamma_1, L_1)$ is not soft fuzzy G_δ pre $T_{\mathcal{U}}$ open in $(X, T_{\mathcal{U}})$ and also there exists no soft fuzzy G_δ pre $T_{\mathcal{U}}$ open set $(\mu, M) \sqsubseteq f^{-1}(\gamma_1, L_1)$. Thus f is not somewhat soft fuzzy G_δ pre $T_{\mathcal{U}}$ continuous function.

Example 7.4. Let $X = \{a, b, c\}$ and let D_1 denote the family of functions $f_1, f_2 : SF(X) \to SF(X)$ is defined as follows

$$f_1(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$f_2(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (\lambda_1, M_1) & (0,\phi) \neq (\lambda, M) \sqsubseteq (\lambda_1, M_1); \\ (1, X) & \text{otherwise.} \end{cases}$$

where (λ_1, M_1) is defined as follows $\lambda_1(a) = 0.4, \lambda_1(b) = 0.3, \lambda_1(c) = 0.2$, $M_1 = \{\phi\}$; $\mathcal{U}$ is a subcollection of D_1. Thus $T_{\mathcal{U}} = \{(0,\phi), (1, X), (\lambda_1, M_1)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{U}$. Thus $(X, T_{\mathcal{U}})$ is a soft fuzzy quasi uniform topological space.

Let $Y = \{p, q, r\}$ and let D_2 denote the family of functions $g_1, g_2 : SF(Y) \to SF(Y)$ is defined as follows

$$g_1(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (1, X) & \text{otherwise.} \end{cases}$$

$$g_2(\lambda, M) = \begin{cases} (0,\phi) & \text{if } (\lambda, M) = (0,\phi); \\ (\gamma_1, L_1) & (0,\phi) \neq (\lambda, M) \sqsubseteq (\gamma_1, L_1); \\ (\gamma_2, L_2) & (\gamma_1, L_1) \neq (\lambda, M) \sqsubseteq (\gamma_2, L_2); \\ (\gamma_3, L_3) & (\gamma_1, L_1) \neq (\lambda, M) \sqsubseteq (\gamma_3, L_3); \\ (\gamma_4, L_4) & (\gamma_2, L_2) \neq (\lambda, M) \sqsubseteq (\gamma_4, L_4); \\ (1, X), & \text{otherwise.} \end{cases}$$

where (γ_1, L_1), (γ_2, L_2), (γ_3, L_3), (γ_4, L_4) are defined as follows $\gamma_1(p) = 0.4, \gamma_1(q) = 0.3, \gamma_1(r) = 0.2$, $L_1 = \{\phi\}$; $\gamma_2(p) = 0.4, \gamma_2(q) = 0.3, \gamma_2(r) = 0.2$, $L_2 = \{q\}$; $\gamma_3(p) = 0.5, \gamma_3(q) = 0.6, \gamma_3(r) = 0.4$, $L_3 = \{r\}$; $\gamma_4(p) = 0.5, \gamma_4(q) = 0.6, \gamma_4(r) = 0.4$, $L_4 = \{q, r\}$. $\mathcal{V}$ is a subcollection of D_2. Thus $S_{\mathcal{V}} = \{(0,\phi), (1, Y), (\gamma_1, L_1), (\gamma_2, L_2), (\gamma_3, L_3), (\gamma_4, L_4)\}$ is a soft fuzzy quasi uniform topology generated by $\mathcal{V}$. Thus $(Y, S_{\mathcal{V}})$ is a soft fuzzy quasi uniform topological space. Let $f : (X, T_{\mathcal{U}}) \to (Y, S_{\mathcal{V}})$ defined as follows $f(a) = p, f(b) = q, f(c) = r$. f is somewhat soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous function but f is not soft fuzzy almost G_δ pre $T_{\mathcal{U}}$ continuous function.

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