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Soft filters and their convergence properties

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Abstract. In the present paper, we introduce concepts of soft filter and soft ideal by using soft sets on an universal set, and present their related properties. Also, we investigate the convergence theory of the soft filter in a soft topological space.

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1. Introduction

The real world is inherently uncertain, imprecise and vague. To solve complex problems in economics, engineering, environment, sociology, medical science, business management, etc. we cannot successfully use classical methods because of various uncertainties typical for those problems. In recent years, a number of theories have been proposed for dealing which such systems in an effective way. Some of these are theory of probability, theory of fuzzy sets [24], theory of intuitionistic fuzzy sets [3], theory of vague sets [10], theory of interval mathematics [11], theory of rough set [21], etc. and these many be utilized as mathematical tools for dealing with diverse types of uncertainties and imprecision embedded in a system. But all these theories have their inherent difficulties. To overcome these kinds of difficulties, Molodtsov [20] proposed a completely new approach, which is called soft set theory, for modelling vagueness and uncertainty.

Soft set theory is growing very rapidly nowadays. The basic properties of the theory may be found in [17]. Ali et al. [2] presented some new algebraic operations on soft sets. Aktaş et al. [1] introduced the soft group and also compared soft sets to fuzzy set and rough set. Feng et al. [7] investigated the concept of soft semirings.

Many efforts have been devoted to further generalizations and extensions of Molodtsov's soft sets. Maji et al. [15] presented the concept of fuzzy soft set by combining the fuzzy set and soft set. Ahmad and Kharal [13] revised and improved some results in fuzzy soft set theory. Xu et al. [23] introduced vague soft sets which is a combination of soft sets and vague sets. Feng et al. [9] defined soft rough approximations and soft rough sets. Some applications of soft sets may be found in [6, 8, 14, 16, 18, 25].

Researches on soft set theory have been progressing in several directions. Shabir and Naz [22] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Çağman et al. [5] defined the soft topology on a soft set, and showed its related properties. However, the notion of soft topology in [5] is more general than in [22]. Then some authors studied some of basic concepts and properties of soft topological spaces, see [4, 12, 19, 26]. In particular, Zorlutuna et al. [26] showed that a fuzzy topological space is a special case of the soft topological space. Aygünoğlu and Aygün [4] introduced the soft continuity of soft mapping, soft product topology and studied soft compactness and generalized Tychonoff theorem to the soft topological space.

In the present paper, firstly we give, as a preliminaries, some basis facts in soft set theory and the properties of image and preimage of soft sets under soft mapping as well as soft topological spaces and its properties. Secondly, we introduce concepts of soft filter and soft ideal by using soft sets on an universal set, and give several interesting properties. Finally, we investigate the convergence theory of soft filter in a soft topological space.

2. Preliminaries

In this section, we give some preliminaries about soft set. We make some small modifications to some of them in order to make theoretical study in detail.

Throughout this paper, X refers to an initial universal set, E is a set of all possible parameters, $P(X)$ is the power set of X, and $A \subseteq E$. Moreover, $S(X, E)$ denotes the family of all soft sets over X.

Definition 2.1 ([6, 18])**.** A soft set F_A on the universe X is defined by the set of ordered pairs $F_A = \{(e, F_A(e)) : e \in E, F_A(e) \in P(X)\}$, where $F_A : E \to P(X)$ is a function such that $F_A(e) = \varnothing$ if $e \notin A$.

In other words, a soft set over X is a parametrized family of subsets of the universe X. For $e \in A$, $F_A(e)$ may be considered as the set of e-approximate elements of the soft set F_A. Clearly, a soft set is not a set.

Definition 2.2 ([6])**.** The soft set $F_\varnothing \in S(X, E)$ is called null soft set, denoted by Φ if $F_\varnothing(e) = \varnothing$ for every $e \in E$.

Definition 2.3 ([6])**.** Let $F_A \in S(X, E)$. If $F_A(e) = X$, $\forall e \in A$, then F_A is called $A-$absolute soft set, denoted by $\widetilde{A}$. If $A = E$, then the $A-$absolute soft set is called absolute soft set and denoted by $\widetilde{X}$.

Definition 2.4 ([6])**.** Let $F_A, G_B \in S(X, E)$. F_A is a soft subset of G_B, denoted $F_A \breve{\subseteq} G_B$ if $F_A(e) \subseteq G_B(e)$, for each $e \in E$.

Definition 2.5 ([6])**.** Let $F_A, G_B \in S(X, E)$. F_A and G_B are soft equal, denote by $F_A = G_B$ if $F_A \breve{\subseteq} G_B$ and $G_B \breve{\subseteq} F_A$.

Definition 2.6 ([6]). Let $F_A, G_B \in S(X,E)$. Union of F_A and G_B is a soft set H_C defined by $H_C(e) = F_A(e) \cup G_B(e)$, $\forall e \in E$, where $C = A \cup B$. That is, $H_C = F_A \breve{\cup} G_B$.

Definition 2.7 ([6]). Let $F_A, G_B \in S(X,E)$. Intersection of F_A and G_B is a soft set H_C defined by $H_C(e) = F_A(e) \cap G_B(e)$, $\forall e \in E$, where $C = A \cap B$. That is, $H_C = F_A \breve{\cap} G_B$.

Definition 2.8 ([2]). Let $F_A \in S(X,E)$. Complement of F_A, denoted by F_A^c and is defined by $F_A^c(e) = X - F_A(e)$ for all $e \in A$.

Definition 2.9 ([12]). Difference H_C of two soft sets F_A and G_B over X such that, denoted by $H_C = F_A \breve{-} G_B$, is defined as $H_C(e) = F_A(e) - G_B(e)$ for all $e \in E$.

Definition 2.10 ([22]). Let F_A be a soft set over X and $x \in X$. Then $x \breve{\in} F_A$ read as x belongs to the soft set F_A whenever $x \in F_A(e)$ for all $e \in E$.

Note that for any $x \in X$, $x \breve{\notin} F_A$, if $x \notin F_A(e)$ for some $e \in E$.

Proposition 2.11 ([6]). *Let $F_A, G_B, H_C \in S(X,E)$. Then*
(1) $F_A \breve{\subseteq} \widetilde{X}$,
(2) $F_A \breve{\subseteq} F_A$,
(3) $F_A \breve{\subseteq} G_B$ *and* $G_B \breve{\subseteq} H_C \Rightarrow F_A \breve{\subseteq} H_C$.

Definition 2.12 ([14]). Let $S(X,E)$ and $S(Y,K)$ be the families of all soft sets over X and Y, respectively. The mapping φ_ψ is called a soft mapping from X to Y, denoted by $\varphi_\psi : S(X,E) \longrightarrow S(Y,K)$ where $\varphi : X \longrightarrow Y$ and $\psi : E \longrightarrow K$ are two mappings.

(1) Let $F_A \in S(X,E)$, then the image of F_A under the soft mapping φ_ψ is the soft set over Y denoted by $\varphi_\psi(F_A)$ and defined by
$$\varphi_\psi(F_A)(k) = \begin{cases} \bigcup\limits_{e \in \psi^{-1}(k) \cap A} \varphi(F_A(e)), & \text{if } \psi^{-1}(k) \cap A \neq \varnothing; \\ \varnothing, & \text{otherwise.} \end{cases}$$
(2) Let $G_B \in S(Y,K)$, then the pre-image of G_B under the soft mapping φ_ψ is the soft set over X denoted by $\varphi_\psi^{-1}(G_B)$, where
$$\varphi_\psi^{-1}(G_B)(e) = \begin{cases} \varphi^{-1}(G_B(\psi(e))), & \text{if } \psi(e) \in B; \\ \varnothing, & \text{otherwise.} \end{cases}$$

The soft mapping φ_ψ is called injective, if φ and ψ are injective. The soft mapping φ_ψ is called surjective, if φ and ψ are surjective.

Proposition 2.13 ([14]). *Let φ_ψ be a soft mapping from $S(X,E)$ to $S(Y,K)$, where $\varphi : X \longrightarrow Y$ and $\psi : E \longrightarrow K$ are two mappings. Then for soft sets $(F_A)_1$, $(F_A)_2$ over X and $(G_B)_1$, $(G_B)_2$ over Y we have:*
(1) $\varphi_\psi^{-1}(\Phi_Y) = \Phi_X$ *and* $\varphi_\psi(\Phi_X) = \Phi_Y$,
(2) $\varphi_\psi^{-1}\left(\widetilde{Y}\right) = \widetilde{X}$ *and* $\varphi_\psi\left(\widetilde{X}\right) \breve{\subseteq} \widetilde{Y}$,
(3) $\varphi_\psi^{-1}\left((G_B)_1 \breve{\cup} (G_B)_2\right) = \varphi_\psi^{-1}((G_B)_1) \breve{\cup} \varphi_\psi^{-1}((G_B)_2)$ *and* $\varphi_\psi\left((F_A)_1 \breve{\cup} (F_A)_2\right) = \varphi_\psi((F_A)_1) \breve{\cup} \varphi_\psi((F_A)_2)$,
(4) $\varphi_\psi^{-1}\left((G_B)_1 \breve{\cap} (G_B)_2\right) = \varphi_\psi^{-1}((G_B)_1) \breve{\cap} \varphi_\psi^{-1}((G_B)_2)$ *and* $\varphi_\psi\left((F_A)_1 \breve{\cap} (F_A)_2\right) \breve{\subseteq} \varphi_\psi((F_A)_1) \breve{\cap} \varphi_\psi((F_A)_2)$,

(5) *if* $(F_A)_1 \breve{\subseteq} (F_A)_2$, *then* $\varphi_\psi((F_A)_1)\ \breve{\subseteq} \varphi_\psi((F_A)_2)$,
(6) *if* $(G_B)_1 \breve{\subseteq} (G_B)_2$, *then* $\varphi_\psi^{-1}((G_B)_1)\ \breve{\subseteq} \varphi_\psi^{-1}((G_B)_2)$.

2.1. **Soft topology.** In this section, we give some basic results of soft topological spaces which we need next section.

Definition 2.14 ([22])**.** A soft topology $\mathcal{T}$ is a family of soft sets over X satisfying the following properties.

(1) $\Phi, \breve{\widetilde{X}} \in \mathcal{T}$
(2) If $F_A, G_B \in \mathcal{T}$, then $F_A \breve{\cap} G_B \in \mathcal{T}$
(3) If $(F_A)_i \in \mathcal{T}$, $\forall i \in I$, then $\bigcup\limits_{i=1} (F_A)_i \in \mathcal{T}$.

The pair $(X, \mathcal{T})$ is called a soft topological space.

Definition 2.15 ([22])**.** Let $(X, \mathcal{T})$ be a soft topological space and E be a set of all parameters. Then the collection $\mathcal{T}_e = \{F_A(e) : F_A \in \mathcal{T}\}$ for each $e \in E$, defines a topology on X. It is called $e-$parameter topology on X.

Above definition shows that corresponding to each parameter $e \in E$, we have a topology $\mathcal{T}_e$ on X. Thus a soft topology on X gives a parameterized family of topologies on X.

Definition 2.16 ([22])**.** Let $(X, \mathcal{T})$ be a soft topological space. Then every element of $\mathcal{T}$ is called a soft open set. Clearly, Φ and $\widetilde{X}$ are soft open sets.

Definition 2.17 ([22])**.** Let $(X, \mathcal{T})$ be a soft topological space and $F_A \in S(X, E)$. Then F_A is said to be soft closed if the soft set F_A^c is soft open.

Definition 2.18 ([22])**.** Let $(X, \mathcal{T})$ be a soft topological space, F_A be a soft set over X and $x \in X$. Then F_A is called a soft neighborhood of x if there exists a soft open set G_B such that $x \breve{\in} G_B \breve{\subseteq} F_A$.

The soft neighborhood system of a point x, denoted by $\breve{N}(x)$, is the family of all its soft neighborhoods. That is, $\breve{N}(x) = \left\{F_A : \exists G_B \in \mathcal{T},\ x \breve{\in} G_B \breve{\subseteq} F_A\right\}$.

We give again concept of soft limit point of Çağman et al. [5] by making some small modifications with respect to definition of topological space in Shabir and Naz [22] as follows:

Definition 2.19. Let $(X, \mathcal{T})$ be a soft topological space, F_A be a soft set over X and $x \in X$. If every soft open set containing x contains a point of F_A different from x, then x is called a soft limit point (or soft accumulation point) of F_A. The set of all soft accumulation points of F_A is denoted by $F_A^{\prime}$.

In other words, if $(X, \mathcal{T})$ is a soft topological space, F_A is a soft set over X and $x \in X$, then $x \breve{\in} F_A^{\prime} \Leftrightarrow G_B \breve{\cap} \left(F_A \breve{-} \{x\}\right) \neq \Phi$ for all $G_B \in \mathcal{T}$ such that $x \breve{\in} G_B$.

Definition 2.20 ([22])**.** Let $(X, \mathcal{T})$ be a soft topological space and F_A be a soft set over X. Then the soft closure of F_A, denoted by $\overline{F_A}$ is the intersection of all soft closed super sets of F_A. Clearly, F_A is the smallest soft closed set over X which contains F_A. Moreover, (i) $F_A \subseteq \overline{F_A}$, (ii) F_A is a soft closed set if and only if $F_A = \overline{F_A}$, for $F_A \in S(X, E)$.

Theorem 2.21 ([5]). *Let $(X, \mathcal{T})$ be a soft topological space and F_A be a soft set over X. Then*

$$F_A \breve{\cup} F'_A = \overline{F_A}$$

Definition 2.22 ([22]). Let X be an initial universal set, E a set of parameters and Y be a non-empty classical subset of X. Then the sub-soft set of F_A over Y denoted by ${}^Y F_A$, is defined as follows

$${}^Y F_A(e) = Y \cap F_A(e), \text{ for all } e \in E$$

In other words, ${}^Y F_A = \widetilde{Y} \breve{\cap} F_A$.

Definition 2.23 ([22]). Let $(X, \mathcal{T})$ be a soft topological space and Y be a non-empty subset of X. Then

$$\mathcal{T}_Y = \{{}^Y F_A : F_A \in \mathcal{T}\}$$

is said to be the soft relative topology on Y and $(Y, \mathcal{T}_Y)$ is called a soft subspace of $(X, \mathcal{T})$.

Definition 2.24 ([4]). Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two soft topological spaces. A soft mapping $\varphi_\psi : (X, \mathcal{T}_1) \longrightarrow (Y, \mathcal{T}_2)$ is called soft continuous if $\varphi_\psi^{-1}(G_B) \in \mathcal{T}_1$, $\forall G_B \in \mathcal{T}_2$.

Note that the above definition can be given by means of soft closed sets instead of soft open sets.

Definition 2.25 ([22]). Let $(X, \mathcal{T})$ be a soft topological space and $x, y \in X$ such that $x \neq y$. $(X, \mathcal{T})$ is called soft Hausdorff space or soft T_2-space if there exist soft open sets G_B and F_A such that $x \breve{\in} F_A$, $y \breve{\in} G_B$ and $F_A \breve{\cap} G_B = \Phi$.

Definition 2.26 ([22]). Let $(X, \mathcal{T})$ be a soft topological space, G_B be a soft closed set in X and $x \in X$ such that $x \breve{\notin} G_B$. If there exist soft open sets F_A and H_C such that $x \breve{\in} F_A, G_B \breve{\subseteq} H_C$ and $F_A \breve{\cap} H_C = \Phi$, then $(X, \mathcal{T})$ is called a soft regular space.

Definition 2.27 ([4]). Let $(X, \mathcal{T})$ be a soft topological space.

(1) A family $\mathcal{C} = \{(F_A)_i : i \in I\}$ of open soft sets in $(X, \mathcal{T})$ is called a soft open cover of X, if it satisfies $\bigcup_{i \in I} (F_A)_i = \widetilde{X}$. A finite subfamily of a soft open cover $\mathcal{C}$ of X is called a finite subcover of $\mathcal{C}$, if it is also a soft open cover of X.
(2) X is called soft compact if every soft open cover of X has a finite subcover.

Definition 2.28 ([26]). A family $\mathcal{C} = \{(F_A)_i : i \in I\}$ of soft sets on X has the finite intersection property if the intersection of the members of each finite subfamily of $\mathcal{C}$ is not null soft set.

Theorem 2.29 ([26]). *A soft topological space is compact if and only if each family of soft closed sets with the finite intersection property has a non null intersection.*

3. Soft filters and soft ideals

Filters and ideals play an important role in several mathematical disciplines such as algebra, topology, logic, measure theory. In this chapter we introduce the concepts of soft filter and soft ideal on a nonempty universal set X, and give several related properties. Also, we investigate their relation with concepts of an ideal and filter defined on X.

Definition 3.1. A soft filter $\mathcal{F}$ over X is a collection of soft sets over X which satisfies the following properties:

(1) $\widetilde{X} \in \mathcal{F}$ and $\Phi \notin \mathcal{F}$
(2) If $F_A \in \mathcal{F}$ and $G_B \in \mathcal{F}$, then $F_A \breve{\cap} G_B \in \mathcal{F}$
(3) If $F_A \in \mathcal{F}$ and $F_A \breve{\subseteq} G_B \breve{\subseteq} \widetilde{X}$ implies that $G_B \in \mathcal{F}$.

Example 3.2. $\mathcal{F} = \left\{ \widetilde{X} \right\}$ is a soft filter on X. It is called trivial soft filter.

Example 3.3. Let $F_A \in S(X, E)$. Then

$$\mathcal{F} = \left\{ G_B \breve{\subseteq} \widetilde{X} : F_A \breve{\subseteq} G_B \right\}$$

is a soft filter on X. It is called principal soft filter generated by F_A. Moreover, it is clearly the least soft filter on X containing F_A.

Example 3.4. Let X be an infinite set. $\mathcal{F} = \left\{ G_B \breve{\subseteq} \widetilde{X} : \widetilde{X} \breve{-} G_B \text{ is finite} \right\}$ is a soft filter on X. It is called cofinite soft filter.

Let X be an universal set and $\mathcal{F}$ be any collection of soft sets on X. Then the collection $\mathcal{F}_e$ on X, defined as follows

$$\mathcal{F}_e = \{ F_A(e) : F_A \in \mathcal{F} \}$$

for each $e \in E$, is a parameterized family of subsets of X derived from $\mathcal{F}$.

Remark 3.5. Let X be an universal set and $\mathcal{F}$ be any collection of soft sets on X. Then $\mathcal{F}$ need not to be a soft filter on X, even if $\mathcal{F}_e$ defines a filter in X for each $e \in E$.

Example 3.6. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\mathcal{F} = \{\widetilde{X}, (F_A)_1, (F_A)_2, (F_A)_3, (F_A)_4\}$ where $(F_A)_i$ is a soft set over X for $i \in I = \{1, 2, 3, 4\}$, defined as follows

$$\begin{array}{lcllcl}
(F_A)_1(e_1) & = & \{x_2\} & (F_A)_1(e_2) & = & \{x_1\} \\
(F_A)_2(e_1) & = & \{x_2, x_3\} & (F_A)_2(e_2) & = & \{x_1, x_2\} \\
(F_A)_3(e_1) & = & \{x_1, x_2\} & (F_A)_3(e_2) & = & \{x_1, x_2\} \\
(F_A)_4(e_1) & = & \{x_2\} & (F_A)_4(e_2) & = & \{x_1, x_3\}
\end{array}$$

Then $\mathcal{F}_{e_1} = \{X, \{x_2\}, \{x_2, x_3\}, \{x_1, x_2\}\}$ and $\mathcal{F}_{e_2} = \{X, \{x_1\}, \{x_1, x_3\}, \{x_1, x_2\}\}$ are filters on X. However, the $\mathcal{F}$ is not a soft filter on X because $(F_A)_2 \breve{\cap} (F_A)_3 = G_B$, where $G_B(e_1) = \{x_2\}$, and $G_B(e_2) = \{x_1, x_2\}$ and $G_B \notin \mathcal{F}$.

Remark 3.7. A soft filter $\mathcal{F}$ on X does not guarantee that $\mathcal{F}_e$ is a filter on X for each $e \in E$, as shown below.

Example 3.8. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\mathcal{F} = \left\{\widetilde{X}, (F_A)_1, (F_A)_2, (F_A)_3\right\}$ where $(F_A)_i$ is a soft set over X for $i \in I = \{1, 2, 3\}$, defined as follows

$$\begin{array}{lllllll} (F_A)_1(e_1) & = & \varnothing & & (F_A)_1(e_2) & = & X \\ (F_A)_2(e_1) & = & \{x_1\} & & (F_A)_2(e_2) & = & X \\ (F_A)_3(e_1) & = & \{x_2\} & & (F_A)_3(e_2) & = & X \end{array}$$

Then $\mathcal{F}$ is a soft filter on X. But $\mathcal{F}_{e_1}$ is not a filter on X because $\varnothing \in \mathcal{F}_{e_1}$ for $e_1 \in E$, even if $\mathcal{F}_{e_2}$ defines a filter in X, for $e_2 \in E$.

Theorem 3.9. *Let $S(X, E)$ and $S(Y, K)$ be the families of all soft sets on X and Y, respectively and φ_ψ be a soft mapping from $S(X, E)$ to $S(Y, K)$. Suppose that $\mathcal{F}$ is a soft filter on X. Then*

$$\varphi_\psi(\mathcal{F}) = \left\{G_B : \varphi_\psi^{-1}(G_B) \in \mathcal{F}\right\}$$

is a soft filter on Y.

Proof. (i) Let $\mathcal{F}$ be a soft filter on X. Since $\varphi_\psi^{-1}(\Phi) = \Phi \notin \mathcal{F}$, we have $\Phi \notin \varphi_\psi(\mathcal{F})$. Moreover, $\varphi_\psi^{-1}\left(\widetilde{Y}\right) = \widetilde{X} \in \mathcal{F}$ and hence $\widetilde{Y} \in \varphi_\psi(\mathcal{F})$.

(ii) Let $G_B, H_C \in \varphi_\psi(\mathcal{F})$. Then we have $\varphi_\psi^{-1}(G_B) \in \mathcal{F}$ and $\varphi_\psi^{-1}(H_C) \in \mathcal{F}$. Since $\mathcal{F}$ is a soft filter, $\varphi_\psi^{-1}(G_B) \breve{\cap} \varphi_\psi^{-1}(H_C) = \varphi_\psi^{-1}\left(G_B \breve{\cap} H_C\right) \in \mathcal{F}$ and so $G_B \breve{\cap} H_C \in \varphi_\psi(\mathcal{F})$.

(iii) Let $G_B \in \varphi_\psi(\mathcal{F})$ and $H_C \in S(Y, K)$ such that $G_B \breve{\subseteq} H_C$. By Proposition 2.13, $\varphi_\psi^{-1}(G_B) \breve{\subseteq} \varphi_\psi^{-1}(H_C)$. Since $\mathcal{F}$ is a soft filter and $\varphi_\psi^{-1}(G_B) \in \mathcal{F}$, this implies that $\varphi_\psi^{-1}(H_C) \in \mathcal{F}$ and so $H_C \in \varphi_\psi(\mathcal{F})$. □

Definition 3.10. A soft ideal $\mathcal{I}$ over X is a collection of soft sets over X which satisfies the following properties:

(1) $\widetilde{X} \notin \mathcal{I}$ and $\Phi \breve{\in} \mathcal{I}$
(2) If $F_A \in \mathcal{I}$ and $G_B \in \mathcal{I}$, then $F_A \breve{\cup} G_B \in \mathcal{I}$
(3) If $F_A \in \mathcal{I}$ and $G_B \breve{\subseteq} F_A \breve{\subseteq} \widetilde{X}$ implies that $G_B \in \mathcal{I}$.

If $\mathcal{F}$ is a soft filter on X, then $\mathcal{I} = \{\widetilde{X} \breve{-} G_B : G_B \in \mathcal{F}\}$ is a soft ideal on X, and conversely, if $\mathcal{I}$ is a soft ideal on X, then $\mathcal{F} = \{\widetilde{X} \breve{-} G_B : G_B \in \mathcal{I}\}$ is a soft filter on X. In the case, we say that $\mathcal{F}$ and $\mathcal{I}$ are dual to each other.

Example 3.11. $\mathcal{I} = \{\Phi\}$ is a soft ideal on X. It is called minimal soft ideal.

Remark 3.12. Let X be an universal set and $\mathcal{I}$ be a collection of soft sets on X. Then $\mathcal{I}$ need not to be a soft ideal on X, even if $\mathcal{I}_e$ defines an ideal in X, for each $e \in E$.

Example 3.13. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\mathcal{I} = \{\Phi, (F_A)_1, (F_A)_2, (F_A)_3, (F_A)_4\}$ where $(F_A)_i$ is a soft set over X for $i \in I = \{1, 2, 3, 4\}$, defined as follows

$$\begin{array}{lllllll} (F_A)_1(e_1) & = & \{x_2\} & & (F_A)_1(e_2) & = & \{x_1\} \\ (F_A)_2(e_1) & = & \{x_1, x_2\} & & (F_A)_2(e_2) & = & \{x_3\} \\ (F_A)_3(e_1) & = & \{x_2\} & & (F_A)_3(e_2) & = & \{x_1, x_3\} \\ (F_A)_4(e_1) & = & \{x_1\} & & (F_A)_4(e_2) & = & \{x_3\} \end{array}$$

Then $\mathcal{I}_{e_1} = \{\varnothing, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$ and $\mathcal{I}_{e_2} = \{\varnothing, \{x_1\}, \{x_3\}, \{x_1, x_3\}\}$ are ideals on X. However, the $\mathcal{I}$ is not a soft ideal on X because $(F_A)_1 \breve{\cup} (F_A)_2 = G_B$, where $G_B(e_1) = \{x_1, x_2\}$, and $G_B(e_2) = \{x_1, x_3\}$ and $G_B \notin \mathcal{I}$.

Remark 3.14. A soft ideal $\mathcal{I}$ on X does not guarantee that $\mathcal{I}_e$ is an ideal on X for each $e \in E$, as shown below.

Example 3.15. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\mathcal{I} = \{\Phi, (F_A)_1, (F_A)_2, (F_A)_3\}$ where $(F_A)_i$ is a soft set over X for $i \in I = \{1, 2, 3\}$, defined as follows

$$\begin{array}{lcllcl} (F_A)_1(e_1) & = & X & (F_A)_1(e_2) & = & \varnothing \\ (F_A)_2(e_1) & = & \{x_1\} & (F_A)_2(e_2) & = & \varnothing \\ (F_A)_3(e_1) & = & \{x_2\} & (F_A)_3(e_2) & = & \varnothing \end{array}$$

Then $\mathcal{I}$ is a soft ideal on X. However, the $\mathcal{I}_{e_1}$ is not an ideal on X because $X \in \mathcal{I}_{e_1}$ for $e_1 \in E$, even if $\mathcal{I}_{e_2}$ defines an ideal in X, for $e_2 \in E$.

Definition 3.16. Given two soft filters $\mathcal{F}$, $\mathcal{F}^*$ on the same set X, $\mathcal{F}^*$ is said to be finer than $\mathcal{F}$, or $\mathcal{F}$ is coarser than $\mathcal{F}^*$, if $\mathcal{F} \subseteq \mathcal{F}^*$. If also $\mathcal{F} \neq \mathcal{F}^*$, then $\mathcal{F}^*$ is said to be strictly finer than $\mathcal{F}$, or $\mathcal{F}$ is said to be strictly coarser than $\mathcal{F}^*$.

Two soft filter are said to be comparable if one is finer than the other. The set of all soft filters on X is may ordered by the relation "$\mathcal{F}$ is coarser than $\mathcal{F}^*$".

Recall that any subset A of some partially ordered set $(X, \leq)$ is called a chain if it is total ordered which is $x \leq y$ or $y \leq x$, for each $x, y \in A$. Moreover by Zorn's Lemma, if X is any nonempty partially ordered set in which every chain has an upper bound, then X has a maximal element.

Proposition 3.17. *Let $\mathcal{F}_0$ be a soft filter on X. Let P be the set of all soft filters $\mathcal{F}$ on X such that $\mathcal{F}_0 \subseteq \mathcal{F}$. Then $(P, \subseteq)$ is a partially ordered set.*

Proof. By Proposition 2.11, it is clear. □

Lemma 3.18. *Let X be an universal set. For $i \in I$, $P = \{\mathcal{F}_i : \mathcal{F}_i$ is a soft filter on $X\}$. Then,*

1. *If P is a nonempty family of soft filters on X, then $\bigcap_{\mathcal{F}_i \in P} \mathcal{F}_i$ is a soft filter on X.*
2. *If P is a $\subseteq -$chain of soft filters on X, then $\bigcup_{\mathcal{F}_i \in P} \mathcal{F}_i$ is a soft filter on X.*
3. *If $\mathcal{C}$ is a family of soft sets with the finite intersection property, then there is a soft filter $\mathcal{F}$ on X such that $\mathcal{C} \subseteq \mathcal{F}$.*

Proof. (1) and (2) are easy to verify. (3) If $\mathcal{C}$ is a collection of elements of $S(X, E)$ with the finite intersection property such that for all $m \in \mathbb{N}$ and $(F_A)_1, ..., (F_A)_m \in \mathcal{C}$, we have $\bigcap_{i=1}^{m} (F_A)_i \neq \Phi$. Then $\mathcal{F} = \{G_B \in S(X, E) : \exists (F_A)_1, ..., (F_A)_m \in \mathcal{C}$ and $\bigcap_{i=1}^{m} (F_A)_i \breve{\subseteq} G_B\}$. Thus $\mathcal{F}$ is a soft filter containing $\mathcal{C}$. We say that the soft filter $\mathcal{F}$ is generated by $\mathcal{C}$. □

Corollary 3.19. *Let $\mathcal{C}$ be a collection of subsets of $S(X,E)$. Then $\mathcal{C}$ generates a soft filter if it has the finite intersection property.*

Proposition 3.20. *Every soft filter has the finite intersection property.*

Proof. The proof is trivial. □

Definition 3.21. Let $\mathcal{F}$ be a soft filter on X. Then a subfamily $\mathcal{C}$ of $\mathcal{F}$ is called a soft filter base for $\mathcal{F}$ if for any $F_A \in \mathcal{F}$ there exists $G_B \in \mathcal{C}$ such that $G_B \breve{\subseteq} F_A$.

If $\mathcal{C}$ is a soft filter base for $\mathcal{F}$, then it is clear that

$$\mathcal{F} = \left\{F_A \in S(X,E) : \exists G_B \in \mathcal{C} \text{ and } G_B \breve{\subseteq} F_A\right\}$$

Definition 3.22. Let $\mathcal{F}$ be a soft filter on X. Then a subfamily $\mathcal{C}$ of $\mathcal{F}$ is called a soft filter subbase for $\mathcal{F}$ if the family of all finite intersections of elements of $\mathcal{C}$ is a soft filter base for $\mathcal{F}$. We also say that $\mathcal{C}$ generates $\mathcal{F}$.

Definition 3.23. Let $\mathcal{F}$ be a soft filter on X.

(1) $\mathcal{F}$ is a soft ultrafilter if either $F_A \in \mathcal{F}$ or $F_A^c \in \mathcal{F}$, for every $F_A \in S(X,E)$.
(2) $\mathcal{F}$ is a prime soft filter if either $F_A \in \mathcal{F}$ or $G_B \in \mathcal{F}$, for every $F_A, G_B \in S(X,E)$ such that $F_A \breve{\cup} G_B \in \mathcal{F}$.
(3) $\mathcal{F}$ is a maximal soft filter if every soft filter containing $\mathcal{F}$ coincides with $\mathcal{F}$.

Example 3.24. If $x \in X$ and $\mathcal{F} = \{F_A \in S(X,E) : x \breve{\in} F_A\}$, then $\mathcal{F}$ is a soft ultrafilter.

Theorem 3.25. *Let $\mathcal{F}$ be a soft filter on X. Then the properties of being soft ultrafilter, prime soft filter and maximal soft filter are equivalent.*

Proof. Suppose that $\mathcal{F}$ is a soft ultrafilter on X and $F_A, G_B \notin \mathcal{F}$ for $F_A, G_B \in S(X,E)$. Then $F_A^c, G_B^c \in \mathcal{F}$ and so their intersection $\widetilde{X} \breve{-} \left(F_A \breve{\cup} G_B\right)$ is in $\mathcal{F}$. However since $\mathcal{F}$ is a soft ultrafilter, $\left(F_A \breve{\cup} G_B\right)$ is not in $\mathcal{F}$. Then $\mathcal{F}$ is a prime soft filter.

Suppose that $\mathcal{F}$ is a prime soft filter on X and $G_B \notin \mathcal{F}$ for some $G_B \in S(X,E)$. To prove maximalty of $\mathcal{F}$, we must show that $\mathcal{F} \breve{\cup} \{G_B\}$ is not a soft filter and so it does not have the finite intersection property. Since $\mathcal{F}$ is a prime soft filter and $\widetilde{X} = G_B \breve{\cup} \left\{\widetilde{X} \breve{-} G_B\right\}$, we have $\widetilde{X} \breve{-} G_B \in \mathcal{F}$ and $G_B \breve{\cap} \left\{\widetilde{X} \breve{-} G_B\right\} = \Phi$. This is show that $\mathcal{F} \breve{\cup} \{G_B\}$ does not have the finite intersection, so $\mathcal{F}$ is a maximal soft filter.

Now, we show that a maximal soft filter on X is a soft ultrafilter. Suppose that $\mathcal{F}$ is not a soft ultrafilter on X. We will show that $\mathcal{F}$ is not maximal. Let $F_A \in S(X,E)$ be such that neither F_A and F_A^c is in $\mathcal{F}$. Take the collection $\mathcal{C} = \mathcal{F} \breve{\cup} \{F_A\}$; we claim that $\mathcal{C}$ has the finite intersection property. If $G_B \in \mathcal{F}$, $F_A \breve{\cap} G_B \neq \Phi$, for otherwise we would have $G_B \breve{\subseteq} \widetilde{X} \breve{-} F_A$ and $\widetilde{X} \breve{-} F_A \in \mathcal{F}$. Thus, if $(G_B)_1, ..., (G_B)_m \in \mathcal{F}$, then $(G_B)_1 \breve{\cap} ... \breve{\cap} (G_B)_m \in \mathcal{F}$ and so $F_A \breve{\cap} (G_B)_1 \breve{\cap} ... \breve{\cap} (G_B)_m \neq \Phi$. Hence $\mathcal{C}$ has the finite intersection property, and by Lemma 3.18 (3), there is a soft filter $\mathcal{F}'$ such that $\mathcal{C} \subseteq \mathcal{F}'$. Since $F_A \in \mathcal{F}' - \mathcal{F}$, $\mathcal{F}$ is not maximal. □

Proposition 3.26. *Let $S(X,E)$ and $S(Y,K)$ be the families of all soft sets on X and Y, respectively and φ_ψ be a soft mapping from $S(X,E)$ to $S(Y,K)$. If $\mathcal{F}$ is a soft ultrafilter on X, then $\varphi_\psi(\mathcal{F})$ is a soft ultrafilter on Y.*

Proof. Let $\mathcal{F}$ be a soft ultrafilter on X. By Theorem 3.9, $\varphi_\psi(\mathcal{F})$ is a soft filter on Y. Suppose that $\mathcal{G}$ is a soft filter on Y such that $\varphi_\psi(\mathcal{F})\breve{\subseteq}\mathcal{G}$. We have to show $\varphi_\psi(\mathcal{F}) = \mathcal{G}$. Let $F_A \in \mathcal{G}$ such that $F_A \notin \varphi_\psi(\mathcal{F})$. Then $\varphi_\psi^{-1}(F_A) \notin \mathcal{F}$. By Definition 3.23, $G_B = \left[\varphi_\psi^{-1}(F_A)\right]^c \in \mathcal{F}$. Since $G_B \breve{\subseteq} \varphi_\psi^{-1}(\varphi_\psi(G_B))$, we have $\varphi_\psi^{-1}(\varphi_\psi(G_B)) \in \mathcal{F}$ and so $\varphi_\psi(G_B) \in \varphi_\psi(\mathcal{F})$. By assumption, $\varphi_\psi(\mathcal{F})\breve{\subseteq}\mathcal{G}$ implies that $\varphi_\psi(G_B) \in \mathcal{G}$. Moreover, $\varphi_\psi(G_B)\breve{\cap}F_A = \varphi_\psi\left(\left[\varphi_\psi^{-1}(F_A)\right]^c\right)\breve{\cap}F_A = \Phi \notin \mathcal{G}$. Thus, $\mathcal{G}$ is not a soft filter and this is a contradiction to our assumptions. Then $F_A \in \varphi_\psi(\mathcal{F})$ and so $\varphi_\psi(\mathcal{F}) = \mathcal{G}$. Hence $\varphi_\psi(\mathcal{F})$ is a soft ultrafilter on Y. □

Theorem 3.27. *Every soft filter can be extended to a soft ultrafilter.*

Proof. Suppose that $\mathcal{F}_0$ is a soft filter on X . $(P, \breve{\subseteq})$, where P is the set of all soft filters $\mathcal{F}$ on X such that $\mathcal{F}_0\breve{\subseteq}\mathcal{F}$ is the partially ordered set. If $\mathcal{C}$ is a $\breve{\subseteq}-$chain in P then by Lemma 3.18 (2), $\bigcup C$ is a soft filter and so it is an upper bound of $\mathcal{C}$ in P. By Zorn's Lemma, there exists a maximal element $\mathcal{F}$ in P. By Theorem 3.25, this $\mathcal{F}$ is a soft ultrafilter. □

3.1. **Convergence of Soft Filters.** In this section, we investigate the convergence theory of the soft filter in a soft topological space.

Proposition 3.28. *Let $(X, \mathcal{T})$ be a soft topological space and $\breve{N}(x)$ be the family of all soft neighborhoods of $x \in X$, then $\breve{N}(x)$ is a soft filter on X. $\breve{N}(x)$ is called soft neighbourhood filter of x.*

Proof. It is obvious. □

Definition 3.29. Let $(X, \mathcal{T})$ be a soft topological space, and $\mathcal{F}$ be a soft filter on X and $x \in X$. We say that $\mathcal{F}$ converges to x, or that x is a soft limit of $\mathcal{F}$ if $\breve{N}(x)\breve{\subseteq}\mathcal{F}$. If $\mathcal{F}$ converges to x, we denote by $\mathcal{F}\breve{\longrightarrow}x$.

Definition 3.29 show that $\mathcal{F}$ converges to x if every soft neighbourhood of x is a member of $\mathcal{F}$.

Theorem 3.30. *Let $(X, \mathcal{T})$ be a soft topological space. Then the following statements are equivalent.*

(1) *X is soft Hausdorff, that is any two distinct points of X have disjoint soft neighbourhoods.*
(2) *Every soft filter on X has at most one soft limit.*

Proof. Let $(X, \mathcal{T})$ is soft Hausdorff and let $x_1, x_2 \in X$ $(x_1 \neq x_2)$. Then there exist soft open sets G_B and F_A such that $x_1\breve{\in}F_A$, $x_2\breve{\in}G_B$ and $F_A\breve{\cap}G_B = \Phi$. Thus no soft filter contains both F_A and G_B, and so no soft filter can converge to both x_1 and x_2. Then all soft filters have at most one soft limit.

Conversely, suppose that $\mathcal{F}$ is a soft filter on X, and every soft neighbourhood F_A of x_1 meets every soft neighbourhood G_B of x_2 such that $x_1 \neq x_2$. Then $F_A\breve{\cap}G_B$ form a soft filter base for the $\mathcal{F}$ which has both x_1 and x_2 as soft limits, which is contrary to hypothesis. Hence, X is soft Hausdorff if every soft filter has at most one soft limit. □

Theorem 3.31. *Let $S(X,E)$ and $S(Y,K)$ be the families of all soft sets on X and Y, respectively. Suppose that $\varphi_\psi : (X,\mathcal{T}) \longrightarrow (Y,\mathcal{T}^*)$ is a soft mapping, where $\varphi : X \longrightarrow Y$ and $\psi : E \longrightarrow K$ are two mappings. If φ_ψ is a soft continuous function and $\mathcal{F}$ is a soft filter on X such that $\mathcal{F}\breve{\longrightarrow}x$, then we have $\varphi_\psi(\mathcal{F})\breve{\longrightarrow}\varphi(x)$.*

Proof. Let $G_B \in \breve{N}(\varphi(x))$ such that $\varphi(x)\breve{\in}G_B$. Then there exist a $F_A \in \mathcal{T}^*$ such that $\varphi(x)\breve{\in}F_A\breve{\subseteq}G_B$. Thus, we have $x \in \varphi^{-1}(F_A(\psi(e)))$ for each $\psi(e) \in K$. Since φ_ψ is a soft continuous function, then $\varphi_\psi^{-1}(F_A)$ is soft open in X such that $x\breve{\in}\varphi_\psi^{-1}(F_A)$ and hence $\varphi_\psi^{-1}(F_A) \in \mathcal{F}$. Since $F_A\breve{\subseteq}G_B$ and so $\varphi_\psi^{-1}(F_A)\breve{\subseteq}\varphi_\psi^{-1}(G_B)$, then we have $\varphi_\psi^{-1}(G_B) \in \mathcal{F}$. This implies that $G_B \in \varphi_\psi(\mathcal{F})$ and so $\varphi(x)\breve{\in}G_B \in \varphi_\psi(\mathcal{F})$. It follows that $\varphi_\psi(\mathcal{F})\breve{\longrightarrow}\varphi(x)$. □

Theorem 3.32. *Let $(X,\mathcal{T})$ be a soft topological space. Then it is soft compact if and only if every soft ultrafilter on X converges to at least one point.*

Proof. Let $\mathcal{F}$ be a soft ultrafilter on X. By Theorem 2.29, since X is a soft compact, the collection of closures of elements of $\mathcal{F}$ satisfies the finite intersection property, so by assumption there exists some x which is in the closure of every element of $\mathcal{F}$. For any $F_A \in N(x)$, the closure of F_A^c does not contain x, hence $\mathcal{F}$ cannot contain F_A^c and must contain F_A. It follows that $\mathcal{F}\breve{\longrightarrow}x$.

Conversely, suppose that any collection of soft closed sets $(F_A)_i$ with the finite intersection property is contained in a soft ultrafilter $\mathcal{F}$. By assumption, this soft ultrafilter converges to some point x. Since $\mathcal{F}$ contains each $(F_A)_i$, it cannot contain their complements $(F_A)_i^c$, so $\mathcal{F}$ cannot converge to any element of $\bigcup(F_A)_i^c$. It follows that $x\breve{\in}\bigcap(F_A)_i \neq \Phi$. By Theorem 2.29, $(X,\mathcal{T})$ is a compact soft topological space. □

Let $(X,\mathcal{T})$ be a soft topological space and Y be a classical subset of X. Suppose that $\mathcal{F}$ is a soft filter on Y. Then $\mathcal{F}$ forms a soft filter base in X, and so generates a new soft filter $\mathcal{F}^*$.

Theorem 3.33. *Let $(X,\mathcal{T})$ be a soft topological space, Y a soft subspace of X and $\mathcal{F}$ be a soft filter on Y. Then $\mathcal{F}$ converges to x in Y, for any $x \in Y$ if and only if $\mathcal{F}^*$, which is generated by the soft filter $\mathcal{F}$ converges to x in X.*

Proof. Since $\mathcal{F}$ forms a soft filter base for $\mathcal{F}^*$, we have $\mathcal{F}^* = \{G_B : \exists F_A \in \mathcal{F}, F_A\breve{\subseteq} G_B\}$. Let $\mathcal{F}$ be a soft filter converging to x in Y and G_B be a soft neighbourhood of x in X. Then there exists an open soft set F_A such that $x\breve{\in}F_A\breve{\subseteq}G_B$, Since $F_A\breve{\cap}\widetilde{Y}$ is soft open over subspace Y, it is a soft neighbourhood of x in Y and so $F_A\breve{\cap}\widetilde{Y} \in \mathcal{F}$. Since $F_A\breve{\cap}\widetilde{Y}\breve{\subseteq}F_A\breve{\subseteq}G_B$, we have $G_B \in \mathcal{F}^*$. Hence $\mathcal{F}^*$ converges to x in X.

Conversely, Let $\mathcal{F}^*$ be a soft filter converging to x in X and G_B be a soft neighbourhood of x in X. Then $G_B \in \mathcal{F}^*$. By Definition 3.21, there exist $F_A \in \mathcal{F}$ such that $F_A\breve{\subseteq}G_B$. Since $F_A\breve{\subseteq}\widetilde{Y}$, we have $F_A\breve{\subseteq}G_B\breve{\cap}\widetilde{Y}$. Since $\mathcal{F}$ is a soft filter on Y, $G_B\breve{\cap}\widetilde{Y} \in \mathcal{F}$. Thus, every soft neighbourhood of x in Y is of the form $G_B\breve{\cap}\widetilde{Y}$. Then $\mathcal{F}$ converges to x in Y. □

Lemma 3.34. *Let $(X,\mathcal{T})$ be a soft topological space and G_B be a soft set on X. Then the following statements are equivalent for any $x\breve{\in}X$.*

(1) $x \in \overline{G_B}$
(2) $F_A \breve{\cap} G_B \neq \Phi$, *for each* $F_A \in \mathcal{T}$ *such that* $x \breve{\in} F_A$.

Proof. (i) $\Rightarrow$ (ii) Let $x \breve{\in} \overline{G_B}$ and $F_A \in \mathcal{T}$ such that $x \breve{\in} F_A$. Then either $x \breve{\in} G_B$ or $x \breve{\notin} G_B$. If $x \breve{\in} G_B$, then $F_A \breve{\cap} G_B \neq \Phi$ since $x \breve{\in} F_A \breve{\cap} G_B$. If $x \breve{\notin} G_B$, then $x \breve{\in} G'_B$. Thus we have $G_B \breve{\cap} \{F_A \breve{-} \{x\}\} \neq \Phi$ for each $F_A \in \mathcal{T}$ such that $x \breve{\in} F_A$. This implies that $F_A \breve{\cap} G_B \neq \Phi$ because of $x \breve{\notin} G_B$.

(ii) $\Rightarrow$ (i) Let $F_A \breve{\in} \mathcal{T}$ and $x \breve{\in} F_A$. Since $x \in X$, either $x \breve{\in} G_B$ or $x \breve{\notin} G_B$. If $x \breve{\in} G_B$, then $x \breve{\in} \overline{G_B}$. Suppose that $x \breve{\notin} G_B$. Since $F_A \breve{\cap} G_B \neq \Phi$, we have $G_B \cap \{F_A \breve{-} \{x\}\} \neq \Phi$. Hence $x \breve{\in} G'_B$ and so $x \breve{\in} \overline{G_B}$. □

Proposition 3.35. *Let* $(X, \mathcal{T})$ *be a soft topological space. Then the following statements are equivalent.*

(1) $(X, \mathcal{T})$ *is a soft Hausdorff space.*
(2) *There exist a soft open set* F_A *such that* $x \breve{\in} F_A$ *and* $y \breve{\notin} \overline{F_A}$ *for all* $x, y \in X$ $(x \neq y)$.

Proof. (i) $\Rightarrow$ (ii) Let $x, y \in X$ such that $x \neq y$. Since $(X, \mathcal{T})$ is a soft Hausdorff space, there exist soft open sets G_B and F_A such that $x \breve{\in} F_A$, $y \breve{\in} G_B$ and $F_A \breve{\cap} G_B = \Phi$. By Lemma 3.34, we have $y \breve{\notin} \overline{F_A}$.

(ii) $\Rightarrow$ (i) Let F_A be a soft open set and $x \breve{\in} F_A$ and $y \breve{\notin} \overline{F_A}$ for all $x, y \in X$, $(x \neq y)$. Take $G_B = \widetilde{X} \breve{-} \overline{F_A}$. Since $y \breve{\notin} \overline{F_A}$, we have $y \breve{\in} G_B$ and so $F_A \breve{\cap} G_B = \Phi$. Hence, $(X, \mathcal{T})$ is a soft Hausdorff space. □

Definition 3.36. Let $(X, \mathcal{T})$ be a soft topological space and $\mathcal{F}$ be a soft ultrafilter on X. $\mathcal{F}$ is said to be a soft compact if it contains some F_A such that $\overline{F_A}$ is soft compact.

Theorem 3.37. *Let* $(X, \mathcal{T})$ *be a soft Hausdorff space and* $\mathcal{F}$ *be a compact soft ultrafilter on* X. *Then*

$$\bigcap \{\overline{F_A} : F_A \in \mathcal{F}\} = G_B$$

is a singletion soft set.

Proof. Let $(F_A)_0 \in \mathcal{F}$ such that $\overline{(F_A)_0}$ is soft compact. We show that G_B contains at least one point that is, exist at least one $x \in X$ such that $x \in G_B(e)$ for all $e \in E$. Suppose that $G_B(e) = \varnothing$ for each $e \in E$. Then $\bigcup \left\{\widetilde{X} \breve{-} \overline{F_A} : F_A \in \mathcal{F}\right\} = \widetilde{X}$, so $\{F_A^c : F_A \in \mathcal{F}\}$ covers X and hence $\overline{(F_A)_0}$. Since $\overline{(F_A)_0}$ is soft compact, there exist a finite subcover $(F_A)_1$, $(F_A)_2$, ..., $(F_A)_m \in \mathcal{F}$ such that $(F_A)_0 \breve{\subseteq} \overline{(F_A)_0} \breve{\subseteq} (F_A)_1^c \breve{\cup} (F_A)_2^c \breve{\cup} ... \breve{\cup} (F_A)_m^c$, hence $(F_A)_0 \breve{\cap} (F_A)_1 \breve{\cap} (F_A)_2 \breve{\cap} ... \breve{\cap} (F_A)_m = \Phi \in \mathcal{F}$: i.e. for each $e \in E$, $\{(F_A)_0 \cap (F_A)_1 \cap (F_A)_2 \cap ... \cap (F_A)_m\}(e) = \varnothing$. This contradicts with the fact that $\mathcal{F}$ is a soft ultrafilter. Then G_B contains at least one point.

Now we show that G_B contains at most one point. Let $x_1, x_2 \in X$ such that $x_1 \neq x_2$. Suppose that $x_1, x_2 \breve{\in} G_B$: i.e. $x_1, x_2 \in G_B(e)$ for each $e \in E$. Since X is soft Hausdorff, there exist $(F_A)_1, (F_A)_2 \in \mathcal{T}$ such that $x_1 \breve{\in} (F_A)_1$, $x_2 \breve{\in} (F_A)_2$ and $(F_A)_1 \breve{\cap} (F_A)_2 = \Phi$. Then $x_1 \breve{\notin} \overline{(F_A)_1^c}$ and by Proposition 3.35, $x_2 \breve{\notin} \overline{(F_A)_1}$. Since $\mathcal{F}$ is soft ultrafilter, either $(F_A)_1$ or $(F_A)_1^c$ is in $\mathcal{F}$ and so either x_1 or x_2 is not in G_B. Hence G_B contains at least one point. □

Lemma 3.38. *Let $(X, \mathcal{T})$ be a soft topological space and $\mathcal{F}$ be a soft ultrafilter converging x with $x \in X$. If $x \breve{\in} F_A \in \mathcal{T}$, then $F_A \in \mathcal{F}$. Moreover, if X is soft regular, then there exists some $G_B \in \mathcal{F}$ such that $x \breve{\in} G_B$ and $\overline{G_B} \breve{\subseteq} F_A$.*

Proof. Let $\mathcal{F}$ be a soft ultrafilter converging $x \in X$ and $x \breve{\in} F_A \in \mathcal{T}$. Since every soft open set containing x is a soft neighborhood of x, we have $F_A \in \mathcal{F}$. For the second part of the Lemma, since F_A is soft open in X, F_A^c is a soft closed set. There exist $G_B, H_C \in \mathcal{T}$ such that $x \breve{\in} G_B$, $F_A^c \breve{\subseteq} H_C$ and $G_B \breve{\cap} H_C = \Phi$, since X is soft regular. Thus $\overline{G_B} \breve{\subseteq} F_A$ and by the first claim of Lemma, $G_B \in \mathcal{F}$. □

Definition 3.39. Let $\mathcal{F}$ and $\mathcal{F}^*$ be a compact soft ultrafilters on X. If $\overline{F_A} \cap \overline{G_B} \neq \Phi$ for all $F_A \in \mathcal{F}$, $G_B \in \mathcal{F}^*$, then we say $\mathcal{F}$ and $\mathcal{F}^*$ are equivalent.

Theorem 3.40. *Let $(X, \mathcal{T})$ be a Hausdorff and regular soft topological space. If $\mathcal{F}$ and $\mathcal{F}^*$ are compact soft ultrafilters such that converge to x and y, respectively then $x = y$ if and only if $\mathcal{F}$ and $\mathcal{F}^*$ are equivalent.*

Proof. The only if direction is clear. Let $x \neq y$ for the if direction. Since X is soft Hausdorff, there exist $F_A, G_B \in \mathcal{T}$ such that $x \breve{\in} F_A$, $y \breve{\in} G_B$ and $F_A \breve{\cap} G_B = \Phi$. Thus, $G_B \in \mathcal{F}^*$ and by Lemma 3.38, there exist $H_C \in \mathcal{F}$ such that $\overline{H_C} \breve{\subseteq} F_A$. Hence, $\overline{H_C} \cap \overline{G_B} = \Phi$ contradicting the equivalence of $\mathcal{F}$ and $\mathcal{F}^*$. Then $x = y$. □

Proposition 3.41. *Let $(X, \mathcal{T})$ be a soft compact topological space and F_A be a soft closed set on X. Then F_A is a soft compact set on X.*

Proof. Let $\mathcal{C} = \{(G_B)_i : i \in I\}$ be a soft open cover of F_A. Then $F_A \breve{\subseteq} \cup_{i\in I} (G_B)_i$. Hence $\widetilde{X} = \cup_{i\in I} (G_B)_i \cup F_A^c$, that is, $\mathcal{C}^* = \{(G_B)_i : i \in I\} \breve{\cup} F_A^c$ is a soft cover of X. But F_A^c is soft open since F_A is soft closed, so $\mathcal{C}^*$ is a soft open cover of X. By hypothesis, X is soft compact, there exist a finite cover of X such that

$$\widetilde{X} = (G_B)_1 \breve{\cup} \cdots \breve{\cup} (G_B)_m \breve{\cup} F_A^c$$

But F_A and F_A^c are disjoint; hence

$$F_A \breve{\subseteq} (G_B)_1 \breve{\cup} \cdots \breve{\cup} (G_B)_m$$

Then any soft open cover $\mathcal{C}$ of F_A contains a finite subcover, i.e F_A is soft compact. □

Proposition 3.42. *Let φ_ψ be a soft continuous from $(X, \mathcal{T})$ to $(Y, \mathcal{T}^*)$ and G_B be a soft set on Y. Then*

$$\overline{\varphi_\psi^{-1}(G_B)} \breve{\subseteq} \varphi_\psi^{-1}(\overline{G_B})$$

Proof. Let φ_ψ be a soft continuous function and G_B a soft set on Y. Since $G_B \subseteq \overline{G_B}$, we have $\varphi_\psi^{-1}(G_B) \subseteq \varphi_\psi^{-1}\left(\overline{G_B}\right)$. Since φ_ψ is a soft continuous, $\varphi_\psi^{-1}\left(\overline{G_B}\right)$ is a soft closed set on X. Then $\overline{\varphi_\psi^{-1}(\overline{G_B})} = \varphi_\psi^{-1}\left(\overline{G_B}\right)$ and so $\overline{\varphi_\psi^{-1}(G_B)} \subseteq \overline{\varphi_\psi^{-1}\left(\overline{G_B}\right)} = \varphi_\psi^{-1}\left(\overline{G_B}\right)$. Hence, $\overline{\varphi_\psi^{-1}(G_B)} \subseteq \varphi_\psi^{-1}(\overline{G_B})$. □

Theorem 3.43. *Let φ_ψ be a soft continuous function from $(X, \mathcal{T})$ to $(Y, \mathcal{T}^*)$ and $\mathcal{F}$, $\mathcal{F}^*$ be equivalent compact soft ultrafilters on X. If $(Y, \mathcal{T}^*)$ is a compact soft topological space, then $\varphi_\psi(\mathcal{F})$ and $\varphi_\psi(\mathcal{F}^*)$ are equivalent compact soft ultrafilters on Y.*

Proof. Let $\mathcal{F}$ and $\mathcal{F}^*$ be equivalent compact soft ultrafilters on X, and $\varphi_\psi : (X, \mathcal{T}) \longrightarrow (Y, \mathcal{T}^*)$ be a soft continuous. By Theorem 3.26, $\varphi_\psi(\mathcal{F})$ and $\varphi_\psi(\mathcal{F}^*)$ are soft ultrafilters on Y. Since $\mathcal{F}$ is compact soft ultrafilter on X, there exist a $H_C \in \mathcal{F}$ such that $\overline{H_C}$ is compact soft set on X. By Proposition 3.41 , $\overline{\varphi_\psi(H_C)}$ is a compact soft set in Y such that $\varphi_\psi(H_C) \in \varphi_\psi(\mathcal{F})$, since Y is soft compact. Then $\varphi_\psi(\mathcal{F})$ (similarly, $\varphi_\psi(\mathcal{F}^*)$) is a compact soft ultrafilter on Y. Now, let $F_A \in \varphi_\psi(\mathcal{F})$, $G_B \in \varphi_\psi(\mathcal{F}^*)$. Then $\varphi_\psi^{-1}(F_A) \in \mathcal{F}$ and $\varphi_\psi^{-1}(G_B) \in \mathcal{F}^*$. Since $\mathcal{F}$ and $\mathcal{F}^*$ are equivalent compact soft ultrafilters on X, we have $\overline{\varphi_\psi^{-1}(F_A)} \tilde{\cap} \overline{\varphi_\psi^{-1}(G_B)} \neq \Phi$. Since φ_ψ is a soft continuous mapping, by Proposition 3.42

$$\overline{\varphi_\psi^{-1}(F_A)} \tilde{\cap} \overline{\varphi_\psi^{-1}(G_B)} \tilde{\subseteq} \varphi_\psi^{-1}\left(\overline{F_A}\right) \tilde{\cap} \varphi_\psi^{-1}\left(\overline{G_B}\right) \neq \Phi$$

and so $\varphi_\psi^{-1}\left(\overline{F_A} \tilde{\cap} \overline{G_B}\right) \neq \Phi$. Thus, $\overline{F_A} \tilde{\cap} \overline{G_B} \neq \Phi$. Then $\varphi_\psi(\mathcal{F})$ and $\varphi_\psi(\mathcal{F}^*)$ are equivalent compact soft ultrafilters on Y. □

4. Conclusions

The soft set theory of Molodtsov [20] offers a general mathematical tool for dealing with uncertain, fuzzy, or vague objects. Molodtsov in [20] has give several possible applications of soft set theory. In this paper, we define the notions of soft filter and soft ideal by using soft sets on an universal set. Also, we investigate their relationships with concepts of filter and ideal corresponding to each parameter defined on the same universal set and support by examples and counterexamples. We hope that the findings in this paper will help researcher enhance and promote the further study on soft set theory to carry out a general framework for their applications.

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